Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 495680, 9 pages
http://dx.doi.org/10.1155/2014/495680

Research Article

Super-Twisting-Algorithm-Based Terminal Sliding Mode Control for a Bioreactor System

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Received 27 February 2014; Accepted 24 April 2014; Published 25 May 2014

Academic Editor: Hamid Reza Karimi

This study proposes a class of super-twisting-algorithm-based (STA-based) terminal sliding mode control (TSMC) for a bioreactor system with second-order type dynamics. TSMC not only can retain the advantages of conventional sliding mode control (CSMC), including easy implementation, robustness to disturbances, and fast response, but also can make the system states converge to the equivalent point in a finite amount of time after the system states intersect the sliding surface. The chattering phenomena in TSMC will originally exist on the sliding surface after the system states achieve the sliding surface and before the system states reach the equivalent point. However, by using the super twisting algorithm (STA), the chattering phenomena can be obviously reduced. The proposed method is also compared with two other methods: (1) CSMC without STA and (2) TSMC without STA. Finally, the control schemes are applied to the control of a bioreactor system to illustrate the effectiveness and applicability. Simulation results show that it can achieve better performance by using the proposed method.

1. Introduction

It is known that nonlinearities exist almost everywhere in the real-world control systems. If the dynamical behavior is dominated by nonlinear phenomenon, it is in general inadequate to deal with this class of control systems simply by linear control schemes. Therefore, many studies on nonlinear systems and control have been discussed by many researchers [1–26]. The conventional sliding mode control (CSMC) scheme is known to be an effective robust nonlinear control approach for systems with uncertainties and/or disturbances. It has many advantages such as fast response, small sensitivity to system uncertainties and/or disturbances from the environment, and being easily designed. Based on these reasons, the CSMC approach has been popularly applied to a variety of control issues [1–3, 27–39]. The design of CSMC is known to consist of two main steps. The first step is to select an appropriate sliding manifold (or sliding surface). The selected surface should be an invariant manifold and therefore the desired control performance will be achieved if the system state remains on the sliding surface. The second step is the organization of a suitable control law, which forces the system state to reach the sliding surface in a finite amount of time and to make the sliding surface an invariant manifold.

The CSMC design usually adopts linear sliding manifold which can only guarantee system states to converge asymptotically. In other words, CSMC with linear sliding manifold cannot provide system states with finite-time convergence property. The concept of terminal sliding mode control (TSMC) was first proposed by Zak [40] and then has been studied by many other researchers [40–54]. It was mainly developed to achieve finite-time convergence of system dynamics on the nonlinear sliding surface. By suitably designing the parameter matrices of the TSMC, the system states can reach the control objective point on the sliding surface in finite time and then the closed loop system will be infinitely stable. Therefore, TSMC not only can preserve the advantages of CSMC, but also can make system states converge to the desired point in a finite amount of time.

Although the TSMC design can be used to solve the only "asymptotic tracking" feature in conventional sliding mode control design (CSMC), the TSMC inevitably experiences the "chattering effect" which is an undesired phenomenon in both CSMC and TSMC because the phenomenon may lead to exciting neglected resonant modes and low control accuracy. In 1993, the so-called super twisting algorithm (STA) was proposed by Professor Levant [55]. Moreno and Osorio discussed the strict Lyapunov functions for the super twisting algorithm [56]. It is a second-order sliding mode algorithm for solving chattering phenomenon. In this study, we will employ the TSMC incorporated with the STA to design the control law. This hybrid method not only can achieve tracking performance in a finite amount of time but also can reduce the chattering phenomenon in original TSMC design. Finally, the control schemes will be applied to the control of a bioreactor system to illustrate the effectiveness and applicability.

Here, the concept of a bioreactor is simply reviewed from [57–60]. A bioreactor means a device system where the biochemical reactions can proceed in vitro. It can be relatively simple when they own few variables. However, it is still very difficult to control due to strong nonlinearities which are difficult to accurately model. In its simplest form, a bioreactor is simply a tank containing water and cells (e.g., yeast or bacteria) which consume nutrients (substrate) and produce products (both desired and undesired). It can also be quite complex since cells are self-regulatory mechanisms which can adjust their growth rates and production of different products radically depending on temperature and concentrations of waste product (e.g., alcohol). Systems with heating or cooling, multiple reactors, or unsteady operation greatly complicate analysis. For a benchmark, however, a relatively simple system will be better to deal with the control issue. For more details, the reader can refer to [57–60].

The rest of this paper is organized as follows. Section 2 states the problem formulation and the main goal of this paper. It is followed by the description of the controllers design, including CSMC, TSMC, and STA-based TSMC. In Section 4, the obtained analytical results are applied to a bioreactor system. Finally, Section 5 gives the conclusions.

2. Problem Formulation

In this study, we mainly focus on the control issue of a class of bioractor systems with second-order dynamics. One of the simplest versions of the bioreactor problems is a continuous flow stirred tank reactor (CFSTR) in which cell growth depends only on the nutrient being fed to the system. The target value to be controlled is the cell mass. The normalized dynamic model of the bioreactor system can be written as [57–60]

$$\begin{aligned} \dot{c}_1 &= -c_1(t)\,u(t) + c_1(t)\left(1-c_2(t)\right)e^{c_2(t)/\gamma}, \\ \dot{c}_2 &= -c_2(t)\,u(t) + c_1(t)\left(1-c_2(t)\right)e^{c_2(t)/\gamma}\frac{1+\beta}{1+\beta-c_2(t)} \end{aligned} \tag{1}$$

in which $c_1(t)$ denotes the normalized cell concentration, $c_2(t)$ represents the normalized amount of nutrient per unit volume, β and γ denote the growth rate and nutrient inhibition parameters, respectively, and $u(t)$ is the normalized flow rate (control input).

This system is difficult to control for several reasons. (1) The uncontrolled equations are highly nonlinear and exhibit limit cycles. (2) Optimal behavior occurs in or near an unstable region. (3) The problem exhibits multiplicity: two different values of the control parameters, that is, flow rate, can lead to the same desired set point in cell mass yield.

From [57–60], we know that the normalized $c_1(t)$, $c_2(t)$, and $u(t)$ satisfy the conditions given as follows:

$$0 < c_1(t) < 1, \qquad 0 < c_2(t) < 1, \qquad 0 < u(t) < 2. \tag{2}$$

The main control objective is to organize a sliding mode control law for $u(t)$ so that the normalized $c_1(t)$ and $c_2(t)$ of bioreactor system converge to the desired values $c_{1d}(t)$ and $c_{2d}(t)$.

Before the control law design, we introduce the coordinate transformation $(x_1, x_2) =: T(c_1, c_2)$ (change variables) [60] as follows, such that the bioreactor system (1) can be represented in a regular form:

$$x_1(t) = \ln\left(c_1(t)\right) - \ln\left(c_2(t)\right) = \ln\left(\frac{c_1(t)}{c_2(t)}\right), \tag{3}$$

$$x_2(t) = \left(1-c_2(t)\right)\left(1-\frac{c_1(t)}{c_2(t)}p\right)e^{c_2(t)/\gamma}, \tag{4}$$

where

$$p = \frac{1+\beta}{1+\beta-c_2}. \tag{5}$$

Under the transformation (3)–(5), the bioreactor system (1) can be rewritten in the regular form of a nonlinear system [3]:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f(x_1, x_2) + g(x_1, x_2)\,u(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} f(x_1, x_2) &= c_1(1-c_2)\cdot p\cdot e^{c_2(t)/\gamma} \\ &\quad\cdot\left[-1+\frac{c_1}{c_2}\cdot p-\frac{1-c_2}{c_2}+\frac{c_1(1-c_2)}{c_2^2}\right. \\ &\quad\cdot p-\frac{c_1(1-c_2)}{c_2(1+\beta-c_2)}\cdot p \\ &\quad\left.\left.+\left(1-\frac{c_1}{c_2}\cdot p\right)\cdot\frac{1}{\gamma}\cdot(1-c_2)\right]\right|_{(c_1,c_2)=T^{-1}(x_1,x_2)} \\ &=: f_c(c_1,c_2)\big|_{(c_1,c_2)=T^{-1}(x_1,x_2)}, \end{aligned} \tag{7}$$

$$\begin{aligned} g(x_1, x_2) &= c_2 e^{c_2(t)/\gamma} \\ &\quad\cdot\left[1-\frac{c_1}{c_2}p-\frac{1}{\gamma}(1-c_2)\left(1-\frac{c_1}{c_2}p\right)\right. \\ &\quad\left.\left.+\frac{(1-c_2)(1+\beta)c_1c_2}{c_2^2(1+\beta-c_2)^2}\right]\right|_{(c_1,c_2)=T^{-1}(x_1,x_2)} \\ &= g_c(c_1,c_2)\big|_{(c_1,c_2)=T^{-1}(x_1,x_2)}. \end{aligned} \tag{8}$$

Clearly, if condition (2) is satisfied, then g given in (8) is bounded away from zero [60]. That is, $1/g$ exists.

3. Controllers Design

In this section, we design three types of controllers for the bioreactor systems and consider the finite-time control scheme to improve the performance.

3.1. Design of Conventional Sliding Mode Controller. According to the control goal stated in Section 2, let c_{1d} and c_{2d} be the desired constant values of c_1 and c_2. Define the tracking error as follows:

$$e_1 = x_1 - x_{1d}, \tag{9}$$

where $x_{1d} := \ln(c_{1d}/c_{2d})$. Note that c_{1d} and c_{2d}, in general, are chosen to satisfy [60]

$$c_{2d}\left(1+\beta-c_{2d}\right) = c_{1d}\left(1+\beta\right). \tag{10}$$

Hence, the tracking performance (i.e., $c_1 \to c_{1d}$ and $c_2 \to c_{2d}$) is achieved when e_1 approaches zero as time t tends to infinity. This fact was proven in paper [60]. Here, the result is recalled in the following lemma.

Lemma 1. *If e_1, given in* (9)*, together with $\dot{e}_1$ converges to zero as time t tends to infinity, then tracking performance is achieved.*

Proof. From (3) and (9), it is clear that $c_1/c_2 \to c_{1d}/c_{2d}$ when e_1 converges to zero. Moreover, due to $\dot{e}_1 = x_2$, it also follows from (2) $(0 < c_2 < 1)$ and (4) that $((c_1/c_2)((1+\beta)/(1+\beta-c_2))) \to 1$ when $\dot{e}_1$ converges to zero. Hence, when e_1 and $\dot{e}_1$ converge to zero, we then have $((c_{1d}/c_{2d})((1+\beta)/(1+\beta-c_2))) \to 1$. Finally, from (10) we can claim that $c_1 \to c_{1d}$ and $c_2 \to c_{12d}$ when e_1 and $\dot{e}_1$ converge to zero. □

In the following, a conventional sliding mode control (CSMC) law will be designed to perform tracking control objective. According to the standard design procedure of sliding mode control, we introduce the sliding manifold $s \in \mathbb{R} = 0$, where

$$s = \dot{e}_1 + me_1, \quad m > 0. \tag{11}$$

To achieve the so-called reachability condition [27] of sliding mode control, we design the sliding mode control law to be

$$u = \frac{1}{g\left(c_1,c_2\right)}\left[-f_c\left(c_1,c_2\right) - m\left(1-c_2\right)\left(1-\frac{c_1}{c_2}p\right)e^{c_2(t)/\gamma} - \eta\cdot\operatorname{sign}\left(s\right)\right], \quad \eta > 0. \tag{12}$$

The main results of the design are summarized in the following theorem.

Theorem 2. *Consider the normalized dynamic model of the bioreactor system* (1)*, which is equal to system* (6)*; the sliding mode control law* (12) *can achieve the tracking performance for the system* (1)*; that is, $c_1 \to c_{1d}$ and $c_2 \to c_{2d}$ as time t tends to infinity.*

Proof. From (6)–(9) and (11), the time derivative of sliding variable s is

$$\begin{aligned}\dot{s} &= f\left(x_1,x_2\right) + g\left(x_1,x_2\right)u \\ &\quad + m\left(1-c_2\left(t\right)\right)\left(1-\frac{c_1\left(t\right)}{c_2\left(t\right)}p\right)e^{c_2(t)/\gamma} \\ &= f\left(T\left(c_1,c_2\right)\right) + g\left(T\left(c_1,c_2\right)\right)u \\ &\quad + m\left(1-c_2\left(t\right)\right)\left(1-\frac{c_1\left(t\right)}{c_2\left(t\right)}p\right)e^{c_2(t)/\gamma} \\ &= f_c\left(c_1,c_2\right) + g_c\left(c_1,c_2\right)u \\ &\quad + m\left(1-c_2\left(t\right)\right)\left(1-\frac{c_1\left(t\right)}{c_2\left(t\right)}p\right)e^{c_2(t)/\gamma}.\end{aligned} \tag{13}$$

Consider a Lyapunov function candidate $V = (1/2)s^2$ for s and substitute u given in (12) into (13). Then the time derivative of V along the trajectory of system (6) is

$$\dot{V} = s\dot{s} = -\eta s\cdot\operatorname{sign}\left(s\right) = -\eta\cdot\left|s\right|. \tag{14}$$

That is, the so-called reachability condition [27] is satisfied. Thus, the sliding mode $s = 0$ can be achieved in a finite amount of time. When the system (6) is under sliding mode, it follows from (11) that $\dot{e}_1 = -me_1$, which implies that $e_1(t)$ converges to zero as time t tends to infinity, and then $\dot{e}_1(t)$ also approaches zero as time t tends to infinity. Finally, according to the result given in Lemma 1, the proof is completed. □

3.2. Design of Terminal Sliding Mode Controller. In the former subsection, we have completed the design of conventional sliding mode control (CSMC) for the normalized dynamic model of the bioreactor system so that the tracking performance is performed. However, the tracking mission is actually achieved in the sense of "asymptotic" rather than in the sense of "finite-time achievement." To solve this "asymptotic tracking" problem in the traditional sliding mode design, in this subsection, we use the so-called terminal sliding mode control (TSMC) technique to organize the control law, so that the tracking performance can be achieved in a finite amount of time rather than in the sense of asymptotic tracking.

Consider the same normalized dynamic model (1) as given in the former subsection; for simplicity, we start our design from considering the system (6). According to the standard design procedure of terminal sliding mode, we introduce the sliding manifold

$$s = k\left(e_1\right)^{q_1/q_2} + \dot{e}_1, \quad k > 0, \tag{15}$$

where $s \in \mathbb{R}$, and q, p are positive odd integers satisfying the condition $q_1 < q_2$ [41, 47]. To achieve the so-called reachability condition [27] of (terminal) sliding mode control, we design the sliding mode control law to be

$$u = \frac{1}{g(c_1, c_2)}\left[-f_c(c_1, c_2) - k \cdot (e_1)^{(q_1-q_2)/q_2}(1-c_2) \times \left(1 - \frac{c_1}{c_2}p\right)e^{c_2(t)/\gamma} - \nu \cdot \operatorname{sign}(s)\right], \quad \nu > 0. \tag{16}$$

The main results of the design are summarized in the following theorem.

Theorem 3. *Consider the normalized dynamic model of the bioreactor system* (1)*, which is equal to system* (6)*. The terminal sliding mode control law* (16) *can achieve the tracking performance in a finite amount of time for the system* (1)*; that is,* $c_1 \to c_{1d}$ *and* $c_2 \to c_{2d}$ *as time* $t \le t_r$*, where* $t_r = t_{reach} + (p/k(p-q)) \cdot |e_1(t_{reach})|^{(p-q)/p}$ *and* $t_{reach} > 0$ *is the time of achieving sliding mode.*

Proof. From (6)–(9) and (15), the time derivative of sliding variable s is

$$\dot{s} = f_c(c_1, c_2) + g_c(c_1, c_2)u + k(e_1)^{(q-p)/p} \times (1 - c_2(t))\left(1 - \frac{c_1(t)}{c_2(t)}p\right)e^{c_2(t)/\gamma}. \tag{17}$$

Consider a Lyapunov function candidate $V = (1/2)s^2$ for s and substituting u given in (16) into (17), it follows that the time derivative of V along the trajectory of system (6) is

$$\dot{V} = s\dot{s} = -\eta s \cdot \operatorname{sign}(s) = -\eta \cdot |s|. \tag{18}$$

That is, the so-called reachability condition [27] is satisfied. Thus, the sliding mode $s = 0$ can be achieved in a finite amount of time. When the system (6) is under sliding mode (i.e., after $t \ge t_{\text{reach}}$), it follows from (15) that $\dot{e}_1 = -k(e_1)^{q/p}$, which implies that $e_1(t)$ converges in finite time $t \le t_{\text{reach}} + (p/k(p-q)) \cdot |e_1(t_{\text{reach}})|^{(p-q)/p}$ [41–47], and then $\dot{e}_1(t)$ also approaches zero in finite time $t \le t_{\text{reach}} + (p/k(p-q)) \cdot |e_1(t_{\text{reach}})|^{(p-q)/p}$. Finally, according to the result given in Lemma 1, the proof is completed. □

3.3. Design of Super-Twisting-Algorithm-Based Terminal Sliding Mode Controller. Although the terminal sliding control design can be used to solve the only “asymptotic tracking” feature in traditional sliding mode control design, the terminal sliding control design inevitably which has undergone the “chattering effect” which is an undesired phenomenon in sliding mode design because might lead to exciting neglected resonant modes and low control accuracy. In 1993, the so-called super twisting algorithm was proposed by Professor Levant [55]. This is a popular second-order sliding mode algorithm for solving chattering phenomenon in sliding mode design. In this subsection, we will employ the terminal sliding mode technique with super twisting algorithm to design the control law for a bioreactor system. The tracking performance can be achieved in a finite amount of time. Moreover, the chattering phenomena in the original terminal sliding mode design can also be reduced.

Consider the same normalized dynamic model (1) as given in Section 2. For simplicity, we start our design from considering the system (6). As the design of terminal sliding mode, we introduce the sliding manifold

$$s = k(e_1)^{q_1/q_2} + \dot{e}_1, \quad k > 0, \tag{19}$$

where $s \in \mathbb{R}$ and q, p are positive odd integers satisfying the condition $q_1 < q_2$ [41–47]. To combine the super twisting algorithm, the control law is organized as follows:

$$u = \frac{1}{g(c_1, c_2)}\left[-f_c(c_1, c_2) - k \cdot (e_1)^{(q_1-q_2)/q_2} \times (1-c_2)\left(1 - \frac{c_1}{c_2}p\right)e^{c_2(t)/\gamma} - u_{\text{sta}}\right], \tag{20}$$

$$u_{\text{sta}} = -k_1 \cdot |s|^{1/2} \cdot \operatorname{sign}(s) - \int_0^t k_2 \cdot \operatorname{sign}(s(\tau))\, d\tau. \tag{21}$$

Note that the control law (20)-(21) is continuous which is different from the original terminal sliding mode control (or traditional sliding mode control). Before we start to investigate the effectiveness of the proposed control law, it can be seen that the time derivative of sliding variable is

$$\dot{s} = f_c(c_1, c_2) + g_c(c_1, c_2)u + k(e_1)^{(q-p)/p} \times (1 - c_2(t))\left(1 - \frac{c_1(t)}{c_2(t)}p\right)e^{c_2(t)/\gamma}. \tag{22}$$

Substituting control law (20)-(21) into (22), we then have

$$\dot{s} = -k_1 \cdot |s|^{1/2} \cdot \operatorname{sign}(s) - \int_0^t k_2 \cdot \operatorname{sign}(s(\tau))\, d\tau. \tag{23}$$

Let $\xi = -\int_0^t k_2 \cdot \operatorname{sign}(s(\tau))d\tau$, and then (23) can be rewritten as

$$\dot{s} = -k_1 \cdot |s|^{1/2} \cdot \operatorname{sign}(s) + \xi, \tag{24}$$

$$\dot{\xi} = -k_2 \cdot \operatorname{sign}(s). \tag{25}$$

Clearly, if the variables s and ξ in (24)-(25) converge to zero at time t_s and keep zero after $t \ge t_s$, then it follows from (24) that $s(t) = 0$ and $\dot{s}(t) = 0$ for all $t \ge t_s$. That is, sliding mode is achieved after $t \ge t_s$. Note that from (24) we can find that $\dot{s}$ is continuous and the chattering is indeed reduced. Now we have to show the convergence (to zero) of variables s and ξ in (24)-(25). Fortunately, the convergence of variables s and ξ was proven in [56]. The result stated in [56] is recalled in the following lemma.

Lemma 4. *Consider* (24)-(25) *and constant gains* k_1, k_2. *Then the following statements are equivalent.*

(i) *The origin $x = 0$ of* (24)-(25) *is finite-time stable.*

(ii) *The constant gains are positive; that is,* $k_1 > 0$, $k_2 > 0$.

(iii) *The matrix*

$$A = \begin{bmatrix} -\frac{1}{2} \cdot k_1 & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix} \tag{26}$$

is Hurwitz; that is, all its eigenvalues have negative real parts.

(iv) *For every symmetric and positive definite* $Q = Q^T > 0$, *the algebra Lyapunov equation (ALE)*

$$A^T P + PA^T = -Q \tag{27}$$

has a unique symmetric and positive definite solution $P = P^T > 0$, *where A is the matrix defined in* (26). *In this case, the function* $V(x) = \boldsymbol{\zeta}^T P \boldsymbol{\zeta}$, *where* $\boldsymbol{\zeta} = (s, \xi)^T$ *is a global strict Lyapunov function for system* (24)-(25). *Its time derivative is* $\dot{V}(x)$ *along the trajectories of system* (24)-(25) *where σ is a constant depending on the gains* k_1, k_2 *and matrix* Q.

Proof. Refer to [56]. □

So far we know that the sliding mode can be achieved (in a finite amount of time) if k_1 and k_2 of control law (20)-(21) are chosen to be positive. The remainder is to investigate the system response in the sliding mode. Since the sliding surface (19) is designed to be the same as that of the original terminal sliding mode design, as given in (15), thus when the sliding mode is achieved, the performance of system (6) is the same as that of system (6) under terminal sliding mode control (16). The main results of this section are summarized in the following theorem.

Theorem 5. *Consider the normalized dynamic model of the bioreactor system* (1), *which is equal to system* (6). *The super-twisting-algorithm-based terminal sliding mode control law* (20)-(21) *can achieve the tracking performance in a finite amount of time for the system* (1); *that is,* $c_1 \rightarrow c_{1d}$ *and* $c_2 \rightarrow c_{2d}$ *as time* $t \le t_r$, *where* $t_r = t_{reach} + (p/k(p-q)) \cdot |e_1(t_{reach})|^{(p-q)/p}$ *and* $t_{reach} > 0$ *is the time of achieving sliding mode.*

Proof. Refer to the former discussion, and the details are omitted here. □

4. Application to a Bioreactor System

To demonstrate the effectiveness of the proposed scheme in the study, in this section, three control laws: conventional sliding mode (labeled SMC), terminal sliding mode (labeled TSMC), and super-twisting-algorithm-based terminal sliding mode (labeled STA+TSMC) control laws, as given by (12), (16), and (20)-(21), respectively, are employed for a bioreactor system (1) to fulfill the tracking task. The relevant parameters ($\beta = 0.02$ and $\gamma = 0.48$) of a bioreactor system can be referred to in [60]. The desired trajectories of normalized cell concentration and normalized cell concentration are chosen to be constant $c_{1d} = 0.1207$ and $c_{2d} = 0.8801$ [60]. The initial condition is chosen to be $(c_1(0), c_2(0)) = (0.12, 0.75)$ which is consistent with the requirement of lying within ±10% of the desired point (c_{1d}, c_{2d}). Besides, the control parameters of three schemes are selected below. For SMC, the parameters of sliding surface and control law are set to be $m = 1$ and $\eta = 1$, respectively; for TSMC, $k = 1$, $q_1 = 1$, and $q_2 = 1.5$ are selected for parameters of corresponding sliding surface, while the control gain is also chosen to be $\nu = 1$; finally, the control parameters of STA+TSMC are chosen as $k_1 = k_2 = 1$, while the parameters of sliding surface of STA+TSMC are the same as those of TSMC.

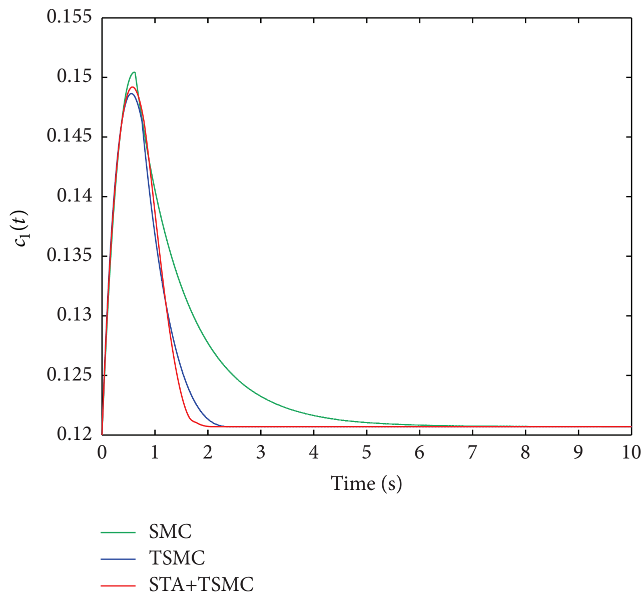

Figure 1: Time history of the $c_1(t)$ for all three schemes.

From Figures 1 and 2, we can find that the tracking task is, as expected, achieved for all three schemes. In addition, from Figure 3, it can be found that the sliding mode behavior can also be fulfilled for these three schemes; however, the chattering phenomena obviously appear in the timing responses of control input and corresponding sliding variables of both CSMC and TSMC, which can be observed in Figures 3, 4, and 6. In contrast to SMC and TSMC, the timing responses of control input and sliding variable of STA+SMC will be more smooth. This agrees with the main theoretical conclusion of this study. Figure 6 shows the control input for three different control schemes, respectively, and the comparison among three different control schemes. Moreover, from Figures 1 and 2, it can be observed that the convergent speed (to the desired point (c_{1d}, c_{2d})) of STA+TSMC is more faster than that of both SMC and TSMC, while the required maximum control magnitude of STA+TSMC is also smaller than that required by SMC and TSMC. A reasonable explanation is that since STA+TSMC does not undergo high-frequency chattering on sliding variable, it only requires relatively smaller control force than that required by SMC and TSMC to keep the system in sliding mode. Moreover, relatively

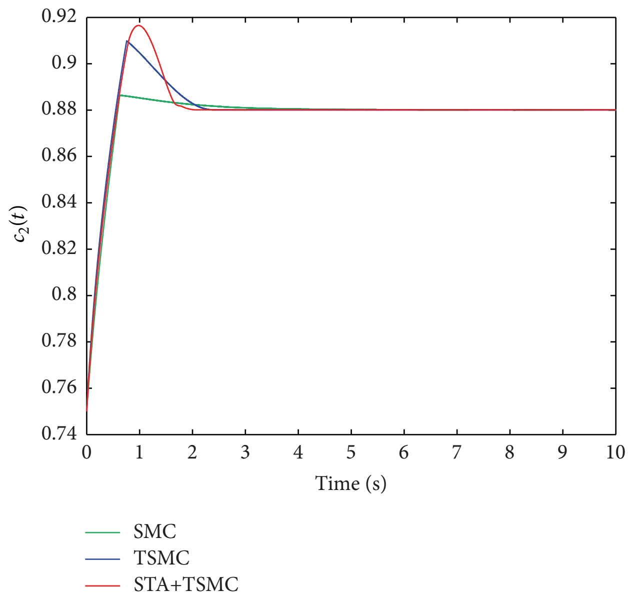

Figure 2: Time history of the $c_2(t)$ for all three schemes.

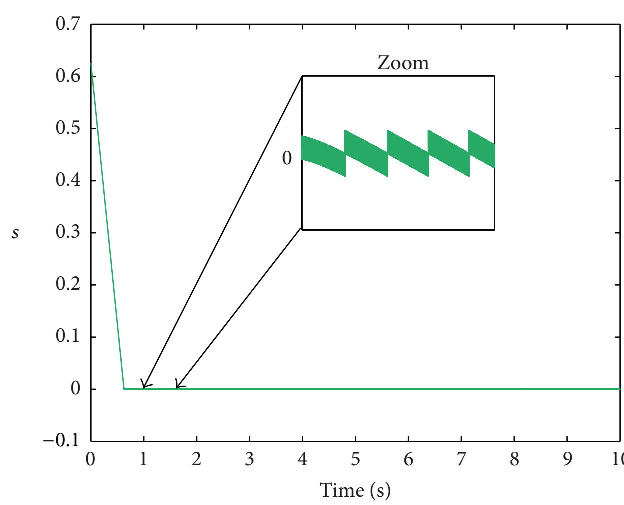

Figure 3: Time history of the sliding variable $s(t)$ for CSMC.

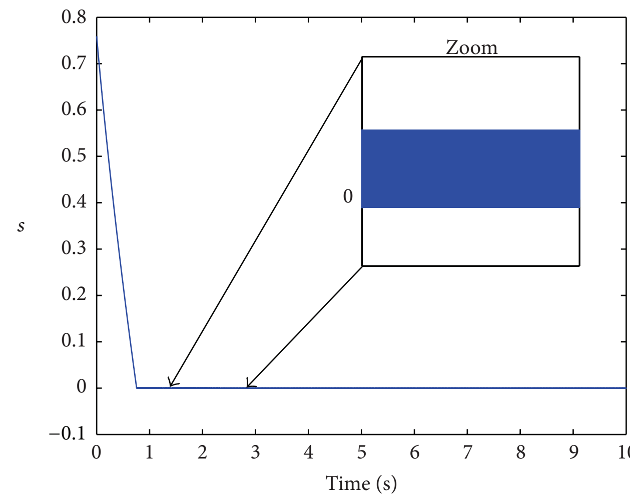

Figure 4: Time history of the sliding variable $s(t)$ for TSMC.

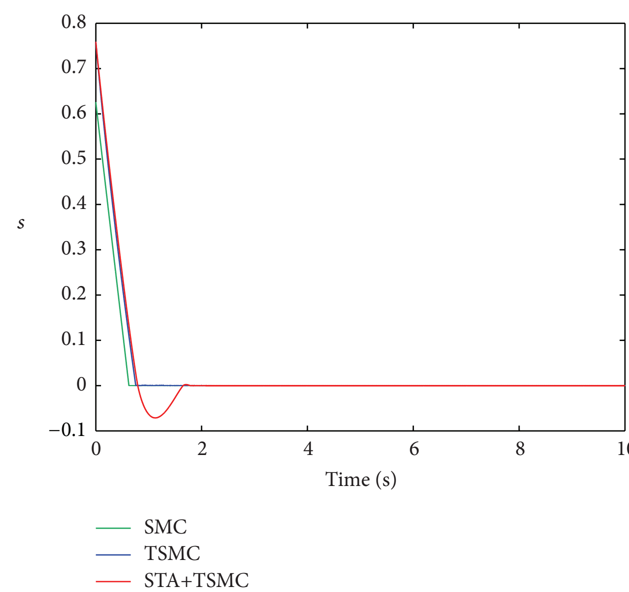

Figure 5: Time history of the sliding variable $s(t)$ for all three schemes.

lower accuracy of system states of SMC and TSMC lying on corresponding sliding surface compared with STA+TSMC might lead to a relatively slower converge speed. It is worth noting from Figure 5 that the sliding variable of STA+TSMC is not directly reaching zero and raises the oscillation before it converges to zero. This comes from the nature of the design of super twisting algorithm that it only makes s and $\dot{s}$ converge to zero eventually (with finite convergent time) rather than require the distance $|s(t)|$ to be a decreasing function. Finally, from this simulation, it is concluded that the STA+TSMC design not only can effectively reduce the harmful chattering of SMC and TSMC designs but also can retain the benefits of finite-time tracking achievement provided by TSMC design.

Remark 6. The computation complexity, concerning time and/or space, is an important index for an algorithm [23]. The lower the complexity is, the more valuable the algorithm will be. Due to the low computation complexity of the super twisting algorithm, it can thus be successfully applied to TSMC.

5. Conclusions

In this study, three types of nonlinear control schemes are applied to the tracking design in a bioreactor system with second-order type dynamics. The super-twisting-algorithm-based (STA-based) terminal sliding mode control (TSMC) is proposed. It not only can retain the advantages of conventional sliding mode control (CSMC), including easy implementation, robustness to disturbances, and fast response, but also can make the system states converge to the equivalent

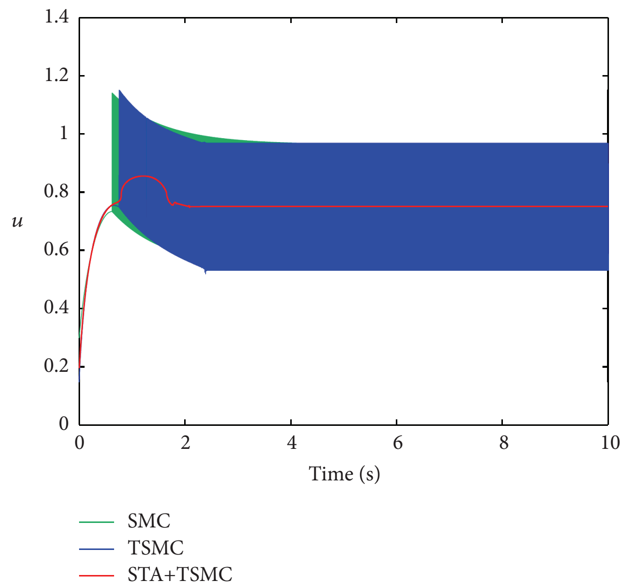

FIGURE 6: Time history of the control input $u(t)$ for all three schemes.

point in a finite amount of time after the system states intersect the sliding surface. The chattering phenomena will originally exist on the sliding surface after the system states achieve the sliding surface and before the system states reach the equivalent point. By using the super twisting algorithm (STA), the harmful chattering phenomena can be indeed reduced. The proposed method is also compared with two other methods: (1) CSMC without STA and (2) TSMC without STA. These control schemes are also applied to the control of a bioreactor system. Simulation results show that it can achieve better performance by using the proposed method.

In the future, the work might be further extended considering the other algorithms and the other possible applications with some conditions, such as networked-based environment, time delays, quantization, and data-driven schemes [4–7, 10, 24].

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The author would like to thank the Editor Professor Hamid Reza Karimi. The author would also like to express appreciation to anonymous reviewers for their helpful comments and valuable suggestions for the revision. This research was partially supported by the National Science Council, Taiwan, under the Grants NSC 101-2221-E-011-056 and NSC 102-2221-E-011-121, the Institute for Information Industry (III), Taiwan, and the National Taiwan University of Science and Technology (NTUST), Taiwan, under the Grants MMH-NTUST-103-06 and TMU-NTUST-103-12.

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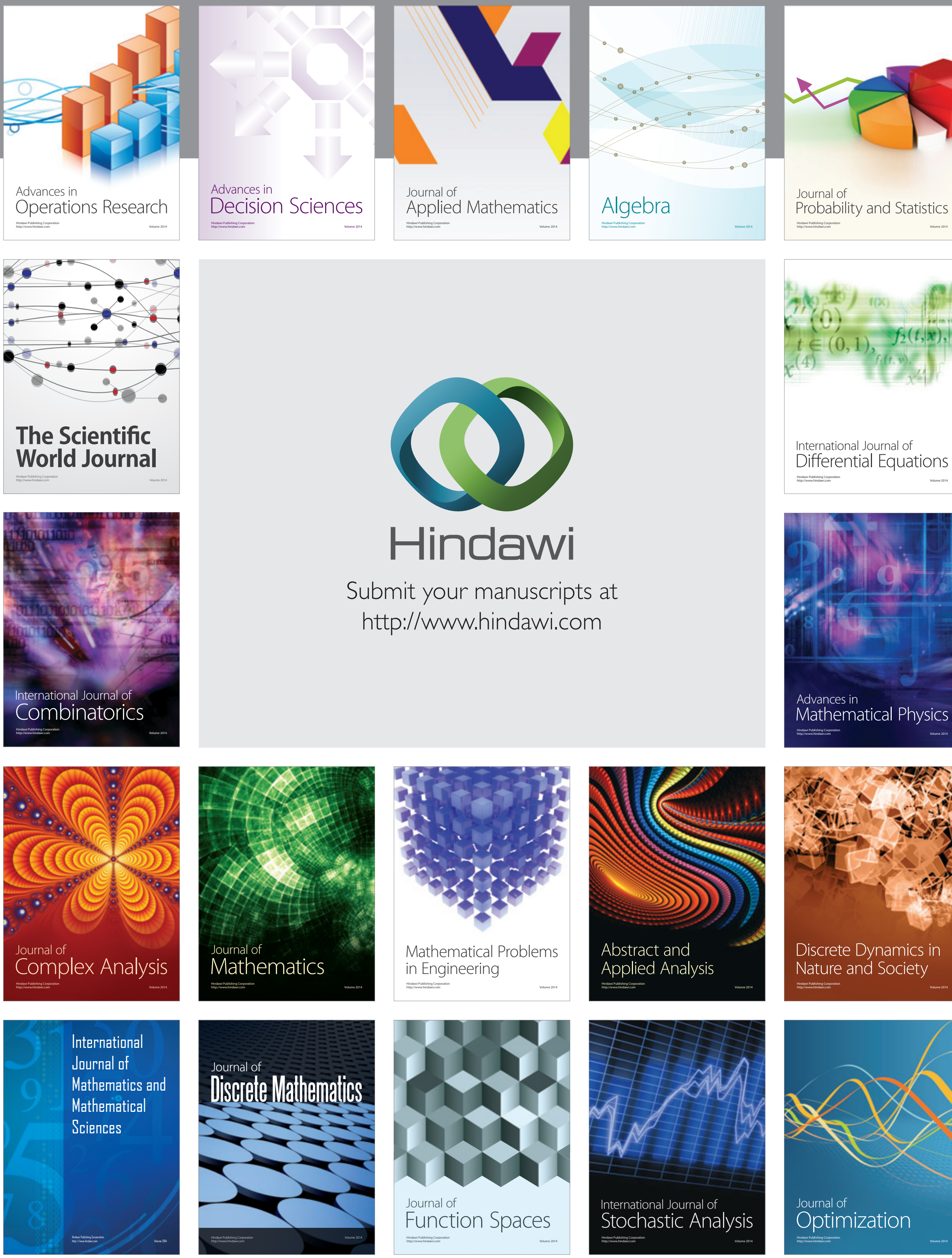
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