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Research Article

Dynamic Analysis and Chaos of the 4D Fractional-Order Power System

Fengyun Sun and Qin Li

School of Mathematics and Computational Science, Yantai University, Yantai 264005, China

Correspondence should be addressed to Fengyun Sun; mathsfy@163.com

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Now dealing with power system became the most important arts. In this paper we report the dynamic analysis of a fractional-order power system with parameter Q_1. To the best know of our knowledge, that was the first report about bifurcation analysis of the fractional order power system. So first we discuss the dynamic analysis with different fractional order and different parameters. Furthermore we will establish its numerical simulations which are provided to demonstrate the feasibility and efficacy of our analysis.

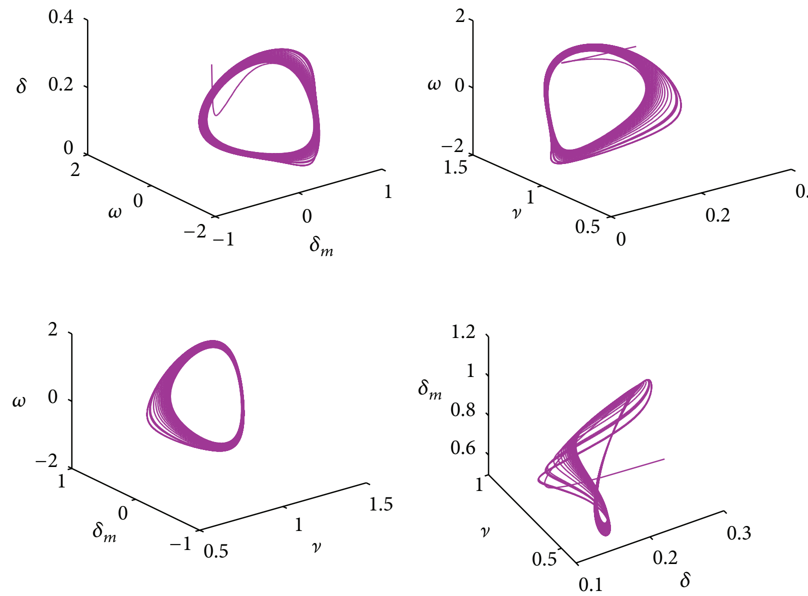
1. Introduction

For a long time, many researchers have been growing interest in investigating the potential use of dynamics in applications, taking power system as an example. Under normal and emergency operating conditions, power systems always can show rich nonlinear dynamic phenomena. From a mathematical point of view, differential equations can be used to describe the dynamic behavior of the power system. The power system is usually run under equilibrium conditions, but in engineering practice, the stability of the equilibrium point form sometimes changes with the range of small perturbations. After 1980s, with the development of the power system, even if a small perturbation can also cause the system to lose stability, due to the changing of some control parameters. Researchers focus on the study of the critical state of the system.

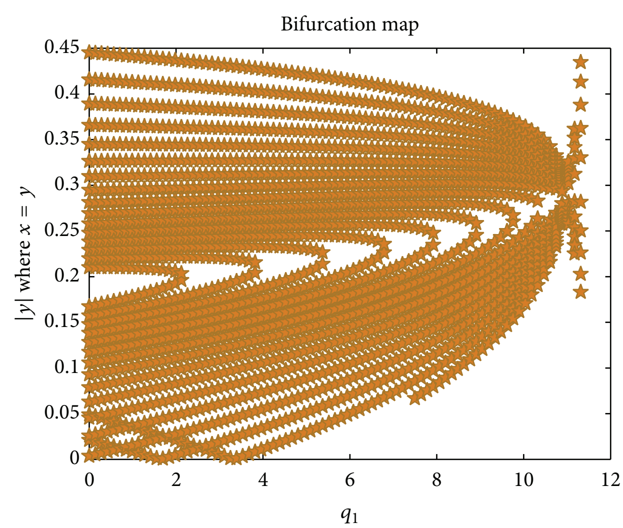
Along with the 300-year-old history of the fractional calculus, its applications to electrical engineering have received more attention. Typically, chaotic systems are still chaotic even when their differential equations become fractional-order differential systems [1–9]. We all know there are many results on fractional-order system, but the bifurcation of fractional-order nonlinear systems has not been well studied. Obviously there are two kinds of methods which have been used in the previous paper to solve fractional-order differential systems: the frequency-domain methods [10] and the time-domain methods [11]. Some researcher found it is sometimes invalid to research some chaotic systems who used the frequently method to investigate chaos [12, 13]. However, we give up the first method and choice the time domain method to make the numerical simulations in this paper. The main idea was introduced by Diethelm et al. [14] and has been used by the following paper [15–17].

In this paper, we report the first investigation of a fractional-order power system using the time domain method. Recently, a simplified 4D power system was reported by Chiang et al. [18]. Its chaotic and bifurcation analysis have been investigated and got well results [19, 20]. But to the best know of our knowledge, that was the first report about bifurcation analysis of the fractional-order system. According to the different values we will show the bifurcation diagrams of the system which chaos exists.

The paper is organized as follows. In Section 2, the numerical algorithm for the fractional-order power system is presented. In Section 3, the chaotic behavior and bifurcations of the system are studied. Finally, we summarize the results.



(a)



(b)

Figure 1: (a) The 3D trajectory in the integer order power system. (b) The related bifurcation diagram of the parameter Q_1.

2. Numerical Algorithm for Fractional-Order Power System

2.1. System Description [21]. In this paper we consider the following power system in [21] (see Figure 1). Here we use Q_1 as the varying parameter to derive the corresponding attractor.

Now we consider the fraction order power system by

$$\begin{aligned}
\frac{d^{q_1}\delta_m}{dt^q} &= \omega;\\
\frac{d^{q_2}\omega}{dt^q} &= \frac{50}{3}\nu\sin(\delta-\delta_m+0.0873)-\frac{1}{6}\omega+\frac{47}{25};\\
\frac{d^{q_3}\delta}{dt^q} &= 496.8\nu^2-\frac{50}{3}\nu\cos(\delta-\delta_m-0.0873)\\
&\quad-\frac{2000}{3}\nu\cos(\delta-0.2094)\\
&\quad-\frac{280}{3}\nu+\frac{100}{3}Q_1+\frac{130}{3};\\
\frac{d^{q_4}\nu}{dt^q} &= -78.7\nu^2+26.2\nu\cos(\delta-\delta_m-0.0124)\\
&\quad+\frac{524}{5}\nu\cos(\delta-0.1346)+\frac{29}{2}\nu-5.3Q_1-\frac{703}{100};
\end{aligned} \tag{1}$$

here $\delta_m, \omega, \delta,$ and ν are variables and $d^{q_i}/dt^{q_i} = D_*^{q_i}$ $(i = 1,2,3,4)$, its order is denoted by $q = (q_1, q_2, q_3, q_4)$. When $q_i = 1$, $i = 1,2,3,4$, system (1) becomes the integer order power system in [21].

2.2. Fractional Derivative [22]. There are several definitions of a fractional-order differential system. In the following, we introduce the most common one of them [22]:

$$D_*^{\alpha} x(t) = J^{m-\alpha} x(t)^{(m)}, \quad \text{with } \alpha > 0 \tag{2}$$

with $m = [\alpha]$; that is, m is the first integer which is not less than α, x^m is the m-order derivative in the usual sense, and J^{β} $(\beta > 0)$ is the β-order Reimann-Liouville integral operator with expression

$$J^{\beta} y(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} y(\tau)\, d\tau. \tag{3}$$

Here τ stands for Gamma function, and the operator D_*^{α} is generally called "α-order Caputo differential operator" [22].

Next, we consider the fractional-order differential equation with related initial conditions

$$\begin{aligned} &D_*^{\alpha} x(t) = f(t, x(t)), \quad 0 < t < T, \\ &x^{(k)}(0) = x_0^{(k)}, \quad k = 0,1,2,\ldots n-1. \end{aligned} \tag{4}$$

Combined with the methods [11, 14] and the above definition of fractional-order differential equation. Set $\tau = T/N$, the fractional-order power system can be rewritten

$$\begin{aligned}
(\delta_m)_{n+1} &= (\delta_m)_0 + \frac{\tau^{q_1}}{\Gamma(q_1+2)} \times \left\{ \omega_{n+1}^p + \sum_{j=0}^{n} (a_1)_{j,n+1} \omega_j \right\}; \\
\omega_{n+1} &= \omega_0 + \frac{\tau^{q_2}}{\Gamma(q_2+2)} \\
&\quad \times \Bigg\{ \frac{50}{3} \nu_{n+1}^p \sin\left(\delta_{n+1}^p - (\delta_m)_{n+1}^p + 0.0873\right) - \frac{1}{6}\omega_{n+1}^p + \frac{47}{25} \\
&\quad + \sum_{j=0}^{n} (a_2)_{j,n+1} \left[\frac{50}{3} \nu_j \sin\left(\delta_j - (\delta_m)_j + 0.0873\right) - \frac{1}{6}\omega_j + \frac{47}{25} \right] \Bigg\}; \\
\delta_{n+1} &= \delta_0 + \frac{\tau^{q_3}}{\Gamma(q_3+2)} \\
&\quad \times \Bigg\{ 496.8 \left(\nu_{n+1}^p\right)^2 - \frac{50}{3} \nu_{n+1}^p \times \cos\left(\delta_{n+1}^p - (\delta_m)_{n+1}^p - 0.0873\right) \\
&\quad - \frac{2000}{3} \nu_{n+1}^p \cos\left(\delta_{n+1}^p - 0.2094\right) - \frac{280}{3} \nu_{n+1}^p + \frac{100}{3} q_1 + \frac{130}{3} \\
&\quad + \sum_{j=0}^{n} (a_3)_{j,n+1} \bigg[496.8 \left(\nu_j\right)^2 - \frac{50}{3} \nu_j \times \cos\left(\delta_j - (\delta_m)_j - 0.0873\right) \\
&\quad - \frac{2000}{3} \nu_j \times \cos\left(\delta_j - 0.2094\right) - \frac{280}{3} \nu_j + \frac{100}{3} q_1 + \frac{130}{3} \bigg] \Bigg\}; \\
\nu_{n+1} &= \nu_0 + \frac{\tau^{q_4}}{\Gamma(q_4+2)} \\
&\quad \times \Bigg\{ -78.7 \left(\nu_{n+1}^p\right)^2 - 26.2 \nu_{n+1}^p \times \cos\left(\delta_{n+1}^p - (\delta_m)_{n+1}^p - 0.0124\right) \\
&\quad - \frac{524}{5} \nu_{n+1}^p \cos\left(\delta_{n+1}^p - 0.1346\right) + \frac{29}{2} \nu_{n+1}^p - 5.3 q_1 - \frac{703}{100} \\
&\quad + \sum_{j=0}^{n} (a_4)_{j,n+1} \bigg[-78.7 \left(\nu_j\right)^2 - 26.2 \nu_j \times \cos\left(\delta_j - (\delta_m)_j - 0.0124\right) \\
&\quad - \frac{524}{5} \nu_j \cos\left(\delta_j - 0.1346\right) + \frac{29}{2} \nu_j - 5.3 q_1 - \frac{703}{100} \bigg] \Bigg\};
\end{aligned} \tag{5}$$

in which

$$(\delta_m)_{n+1}^p = (\delta_m)_0 + \frac{1}{\Gamma(q_1)} \sum_{j=0}^{n} (b_1)_{j,n+1} \omega_j;$$

$$\omega_{n+1}^p = \omega_0 + \frac{1}{\Gamma(q_2)}$$

$$\times \sum_{j=0}^{n} (b_2)_{j,n+1} \left[\frac{50}{3} \nu_j \times \sin\left(\delta_j - (\delta_m)_j + 0.0873\right) - \frac{1}{6}\omega_j + \frac{47}{25} \right];$$

$$\delta_{n+1}^{p} = \delta_0 + \frac{1}{\Gamma(q_3)} \times \sum_{j=0}^{n} (b_3)_{j,n+1} \left[496.8\left(\nu_j\right)^2 - \frac{50}{3}\nu_j \cos\left(\delta_j - (\delta_m)_j - 0.0873\right) - \frac{2000}{3}\nu_j \cos\left(\delta_j - 0.2094\right) - \frac{280}{3}\nu_j + \frac{100}{3} q_1 + \frac{130}{3} \right];$$

$$\nu_{n+1}^{p} = \nu_0 + \frac{1}{\Gamma(q_4)} \times \sum_{j=0}^{n} (b_4)_{j,n+1} \left[-78.7\left(\nu_j\right)^2 - 26.2\nu_j \times \cos\left(\delta_j - (\delta_m)_j - 0.0124\right) - \frac{524}{5}\nu_j \cos\left(\delta_j - 0.1346\right) + \frac{29}{2}\nu_j - 5.3 q_1 - \frac{703}{100} \right]; \tag{6}$$

$$(a_1)_{j,n+1} = \begin{cases} n^{q_1} - (n - q_1)(n+1)^{q_1}, & j = 0, \\ (n-j+2)^{q_1+1} + (n-j)^{q_1+1} - 2(n-j+1)^{q_1+1}, & 0 \le j \le n, \end{cases}$$

$$(a_2)_{j,n+1} = \begin{cases} n^{q_2} - (n - q_2)(n+1)^{q_2}, & j = 0, \\ (n-j+2)^{q_2+1} + (n-j)^{q_2+1} - 2(n-j+1)^{q_2+1}, & 0 \le j \le n, \end{cases}$$

$$(a_3)_{j,n+1} = \begin{cases} n^{q_3} - (n - q_3)(n+1)^{q_3}, & j = 0, \\ (n-j+2)^{q_3+1} + (n-j)^{q_3+1} - 2(n-j+1)^{q_3+1}, & 0 \le j \le n, \end{cases}$$

$$(a_4)_{j,n+1} = \begin{cases} n^{q_4} - (n - q_4)(n+1)^{q_4}, & j = 0, \\ (n-j+2)^{q_4+1} + (n-j)^{q_4+1} - 2(n-j+1)^{q_4+1}, & 0 \le j \le n, \end{cases} \tag{7}$$

$$\begin{aligned} (b_1)_{j,n+1} &= \frac{\tau^{q_1}}{q_1}\left((n-j+1)^{q_1} - (n-j)^{q_1}\right), \quad 0 \le j \le n, \\ (b_2)_{j,n+1} &= \frac{\tau^{q_2}}{q_2}\left((n-j+1)^{q_2} - (n-j)^{q_2}\right), \quad 0 \le j \le n, \\ (b_3)_{j,n+1} &= \frac{\tau^{q_3}}{q_3}\left((n-j+1)^{q_3} - (n-j)^{q_3}\right), \quad 0 \le j \le n, \\ (b_4)_{j,n+1} &= \frac{\tau^{q_4}}{q_4}\left((n-j+1)^{q_4} - (n-j)^{q_4}\right), \quad 0 \le j \le n. \end{aligned} \tag{8}$$

3. Bifurcations Analysis of the Fractional-Order Power System

3.1. Integer-Order Power System. In the following discussion, we mainly use the simulation method to analyze the fractional-order power system by calculating the largest Lyapunov exponent using the Wolf algorithm [6]. Because the nonperiodicity solutions and sensitive dependence on initial conditions are important reason of chaos, here, we let the initial condition be defined as $((\delta_m)_0, \omega_0, \delta_0, \nu_0) = (0.348; 0.1; 0.138; 0.925)$ [21] and vary $q_i = q$ $(i = 1, 2, 3, 4)$. According to our numerical simulations, we have visually founded the difference between the integer order and the fractional order. Here we let $q_i = 1$ $(i = 1, 2, 3, 4)$, that is, integer order, the 3D trajectory in the power system is shown in Figure 1. The related bifurcation diagram of the parameter Q_1 is shown in Figure 1, and the largest Lyapunov exponent diagram is shown in Figure 2.

3.2. Dynamic Analysis of the Fractional-Order System. Next we correspond to discuss the fractional-order power system. The fractional orders q_i $(i = 1, 2, 3, 4)$ are equal and fixed with at some value which is from 0 to 1. On the basis of Section 2, first we find that the range of parameter is changing along with the changing of fractional order. When we let $q_i = 0.99$ $(i = 1, 2, 3, 4)$ be an example while the parameter Q_1 is changed from 0 to 11.4 [21]. For a step size in Q_1 is 0.01, we clearly got the range of $Q_1 \in (6, 11.4)$ in which the solutions of the fractional-order power system are bounded. We take $Q_1 = 11.37$ as an example, the 2D trajectory is shown in Figure 3 which clearly finds the difference between two orders 0.99 and 1. The difference is that the former generates chaotic phenomena and the later happens that after a period of offloading shock of system voltage, ultimately stabilize at a certain value, corresponding to the equilibrium point is asymptotically stable focus. Further, we let $q_i = 0.9$ $(i = 1, 2, 3, 4)$; we clearly got the range of $Q_1 \in (1, 5.9)$. The 2D trajectory is shown in Figure 4 in which $Q_1 = 1$ and $Q_1 = 5.9$.

3.3. Dynamic Analysis with Different Order

(1) Fix $q_2 = q_3 = q_4 = 1$, $Q_1 = 11$, and let q_1 vary. We calculate numerically the fractional-power system for $q_1 \in (0.9, 0.99)$ with a varied step size equal to 0.01. The related Largest Lyapunov exponent is shown in Figure 5.

(2) Fix $q_1 = q_3 = q_4 = 1$, $Q_1 = 11$, and let q_2 vary. We calculate numerically the fractional-power system for $q_2 \in (0.9, 0.99)$ with a varied step size equal to 0.01. The related Largest Lyapunov exponent is shown in Figure 5.

(3) Fix $q_1 = q_2 = q_4 = 1$, $Q_1 = 11$, and let q_3 vary. We calculate numerically the fractional-power system for $q_3 \in (0.9, 0.99)$ with a varied step size equal to 0.01. The related Largest Lyapunov exponent is shown in Figure 6.

(4) Fix $q_1 = q_2 = q_3 = 1$, $Q_1 = 11$, and let q_4 vary. We calculate numerically the fractional-power system for

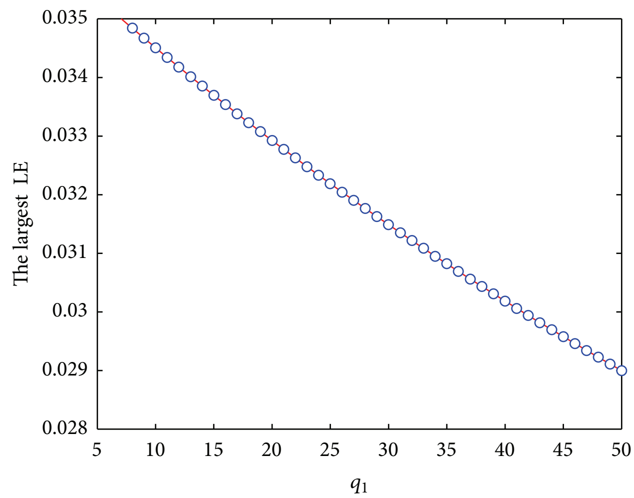


FIGURE 2: The largest Lyapunov exponent diagram.

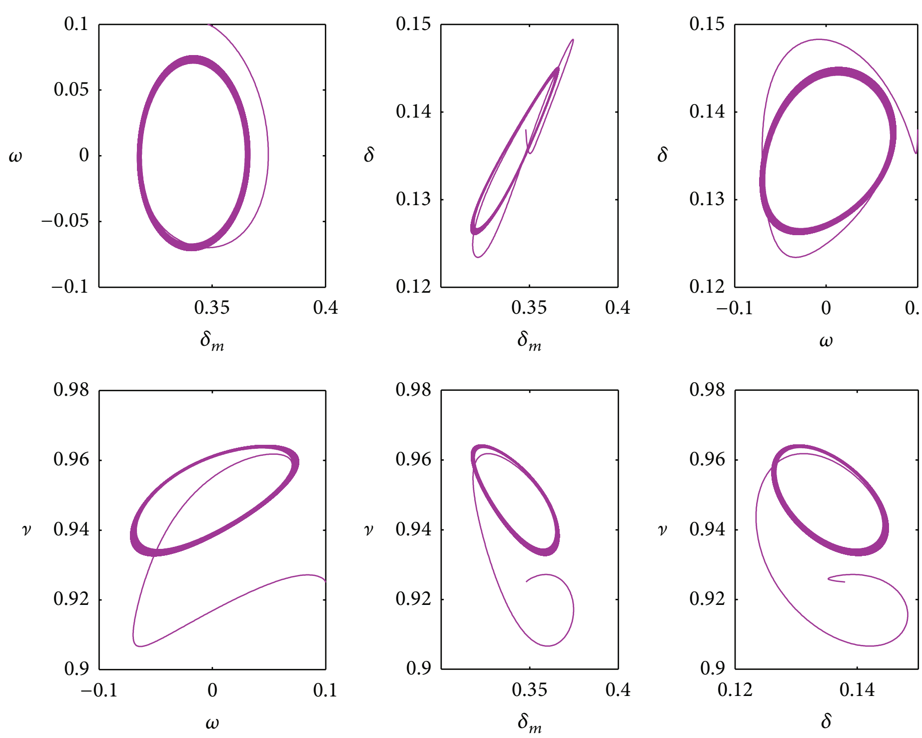


FIGURE 3: The 2D trajectory in the fractional-order power system with $q_i = 0.99$, $i = 1, 2, 3, 4$.

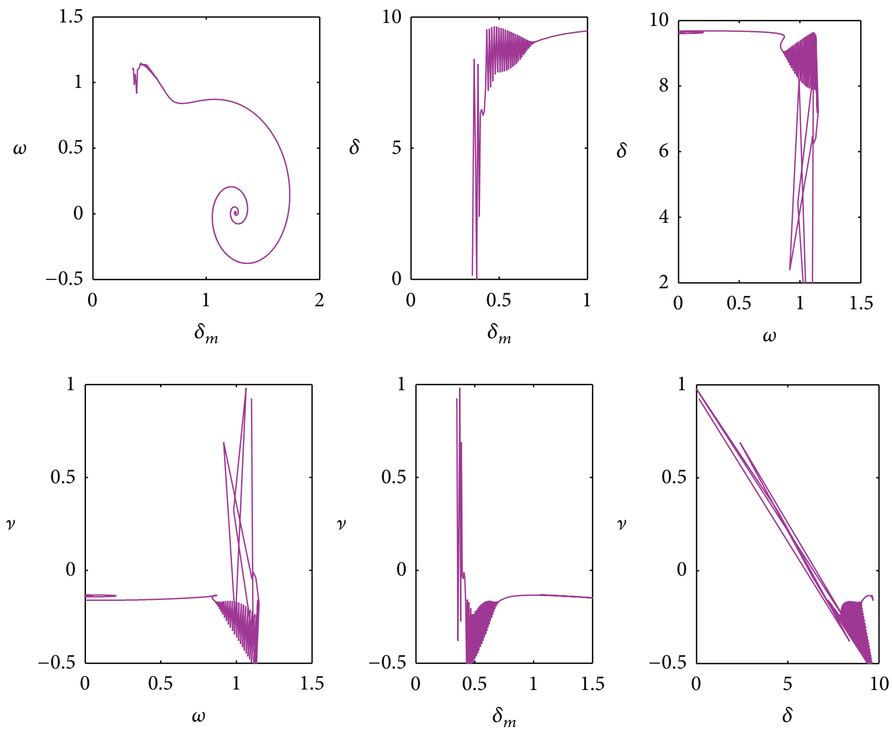
$q_4 \in (0.9, 0.99)$ with a varied step size equal to 0.01. The related Largest Lyapunov exponent is shown in Figure 6.

4. Conclusions

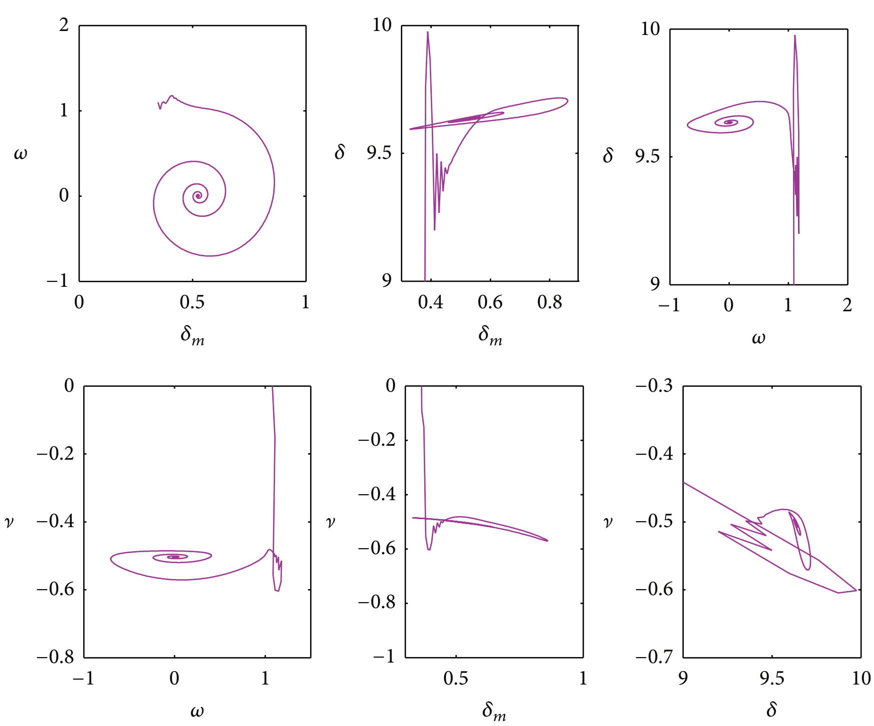
Through the above discussion, we successfully have numerically studied the existence of the bifurcations type of the fractional order power system with the difference orders. According to the different initial conditions and fractional order, the largest Lyapunov exponent values and the bifurcation maps are given to verify the rationality of the analysis. Of course, the next work on this topic should include the depth study in chaos control and synchronization.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

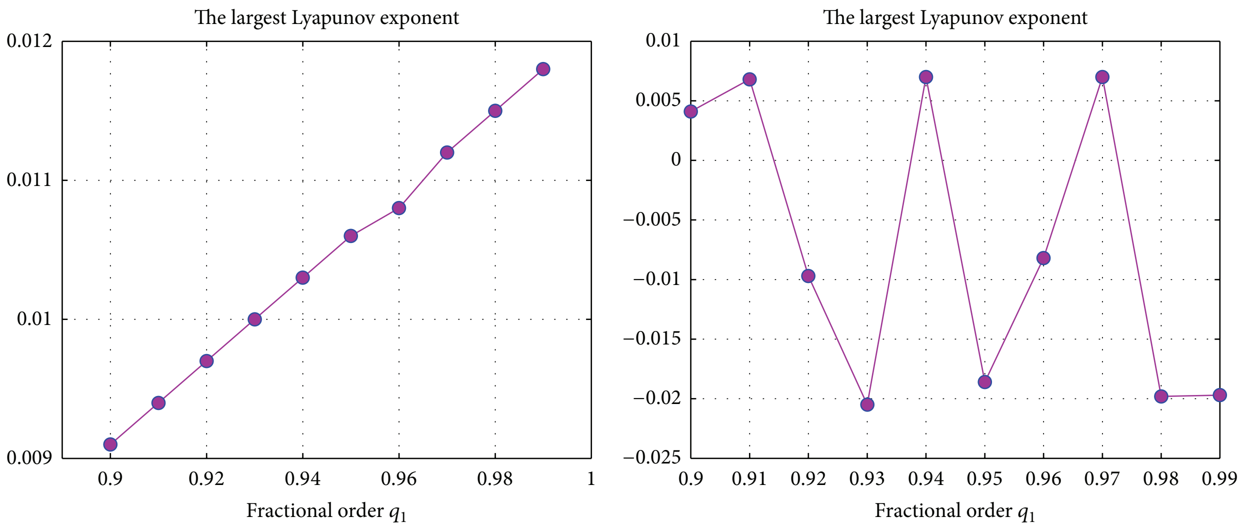


(a)



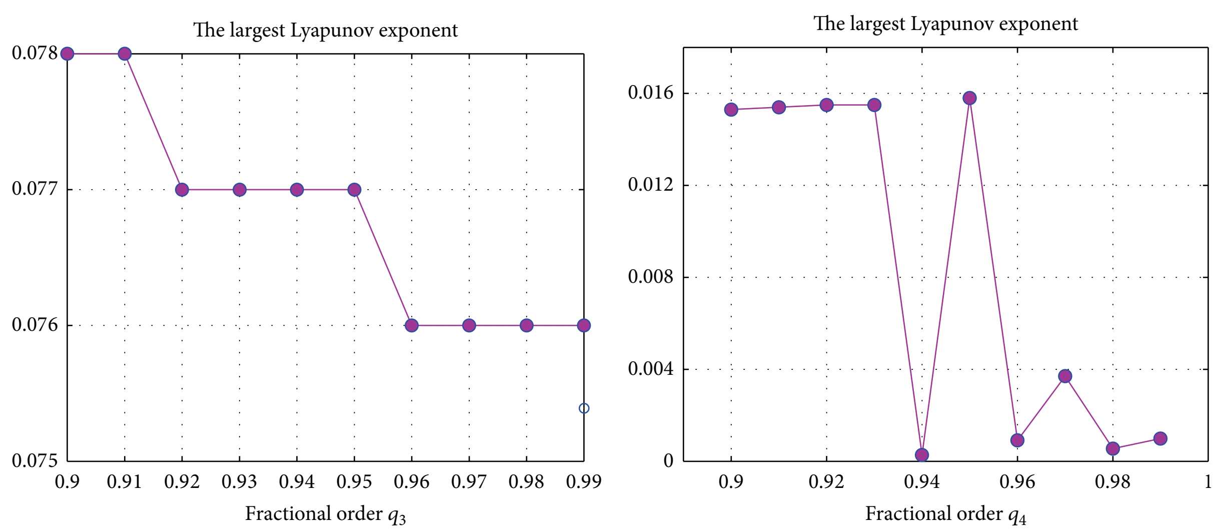
(b)

FIGURE 4: The 2D trajectory in the fractional-order power system with $q_i = 0.9$, $i = 1, 2, 3, 4$. (a) $Q_1 = 1$; (b) $Q_1 = 5.9$.



(a) (b)

Figure 5: The Largest Lyapunov Exponent of the fractional-order power system; (a) $q_2 = q_3 = q_4 = 1$, $q_1 = 0.99$, and (b) $q_1 = q_3 = q_4 = 1$, $q_2 = 0.99$.



(a) (b)

Figure 6: The Largest Lyapunov Exponent of the fractional-order power system; (a) $q_2 = q_1 = q_4 = 1$, $q_3 = 0.99$, and (b) $q_2 = q_3 = q_1 = 1$, $q_4 = 0.99$.

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