Research Article

Contractile Force of Human Extraocular Muscle: A Theoretical Analysis

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Aim. The length-contractile force relationships of six human extraocular muscles (EOMs) in primary innervations should be determined during eye movement modeling and surgery of clinical EOMs. This study aims to investigate these relationships. *Method*. The proposal is based on the assumption that six EOMs have similar constitutive relationships, with the eye suspended in the primary position. The constitutive relationships of EOMs are obtained by optimizing from previous experimental data and the theory of mechanical equilibrium using traditional model. Further, simulate the existing experiment of resistance force, and then compare the simulated results with the existing experimental results. Finally, the mechanical constitutive relationships of EOMs are obtained. *Results*. The results show that the simulated resistance forces from the other four EOMs except for the horizontal recti well agree with previous experimental results. *Conclusion*. The mechanical constitutive relationships of six EOMs in primary innervations are obtained, and the rationality of the constitutive relationships is verified. Whereafter, the active stress-strain relationships of the six EOMs in the primary innervations are obtained. The research results can improve the eye movement model to predict the surgical amounts of EOMs before EOM surgery more precisely.

1. Introduction

Eye movement is controlled by the six extraocular muscles (EOMs), namely, lateral rectus (LR), medial rectus (MR), superior rectus (SR), inferior rectus (IR), superior oblique (SO), and inferior oblique (IO). This biological regulation mainly depends on the effect of the active forces of the EOMs, which are governed by innervations. Such information helps in understanding the pathology of some eye diseases, for example, strabismus [1]. However, how innervations govern the activation of EOMs is unknown [2–4]. During eye movement modeling and ophthalmic surgery, the primary position (looking straight ahead) [5] as the reference position is important. According to Hering's law, when one of the two eyes fixes on the primary position actively, the innervations of the EOMs of the other eye, which is rotated to any position passively by an external force, are the same as the innervations of the corresponding EOMs of the active eye. In this case, the innervations of EOMs of the active eye and the passive eye are both called primary innervations. The length-contractile

force relationships of the six human EOMs in primary innervations are essential for eye movement modeling and EOM surgery. However, the active forces of the EOMs are hardly measured by the experimental installation because of the difficulty in anatomy [6, 7]. A mathematical model is often used to determine the mechanical properties of biological tissues and biomaterials [8–11]. The present work focuses on calculating the length-contractile force relationships of the EOMs in the primary innervations based on the mechanical equilibrium and the mathematical optimization.

Biomechanical tests of the horizontal EOMs (i.e., LR and MR muscles) have been performed by previous investigators [12–16]. Collins et al. [12, 14] determined the length-tension relationships of LR and MR muscles during strabismus surgery, respectively. The experimental surgeries include the horizontal recti of the experimental eye detached from the eyeball and connected by a transducer to measure its force with the normal eye fixed on the given target. Collins et al. [13] also determined the length-tension relationships of the horizontal recti of the normal human subject using a noninvasive
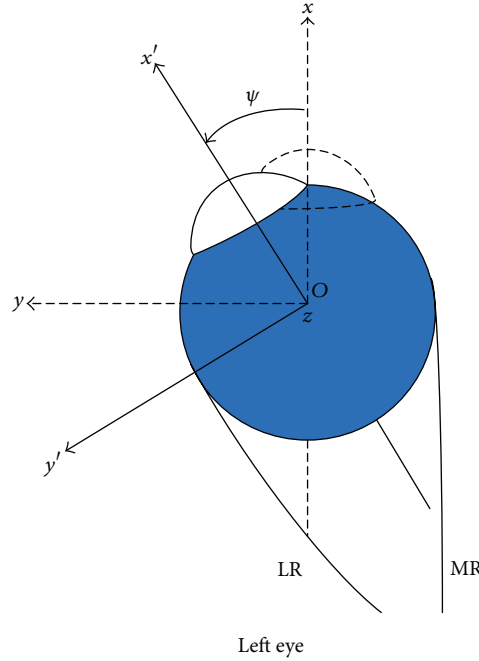
FIGURE 1: Schematic of horizontal movement of the left eye, in which only the lateral rectus (LR) and the medial rectus (MR) are plotted. $x(x, y, z)$ is the reference configuration, and $x'(x', y', z)$ is the current configuration after the eye rotating by ψ.

method. Recently, Lennerstrand et al. [15] determined the isometric forces of the human horizontal recti attached onto or detached from the eyeball. They reported that the values of these two forces have no consistent statistical differences. The same group investigated the isometric forces of the EOMs of the saccadic eye and found almost identical forces developed by tendons and pulleys [16].

In recent years, numerous eye movement models have been presented. A mechanical model proposed by Pascolo and Carniel [5] uses a modified Hill muscle model to simulate EOMs. The famous software of eye movement is Orbit™ 1.8 Gaze Mechanics Simulation, in which the EOMs model is established mainly by referring to the work of Robinson [17] and the work of Miller and Robinson [18, 19]. In the EOMs model of Orbit 1.8, the length-active force relationship and the length-passive force relationship of the LR are fitted by experimental data, and the corresponding relationships of the other EOMs are obtained by multiplying the relative relationships of LR by scale factors [20]. Wei et al. [2] further develop a novel eye movement model, in which the length-tension relationships of the SR, IR, SO, and IO are obtained based on the length-tension relationships of LR (based on Orbit 1.8) and scaled by their cross sections. In existing eye movement model, only the length-tension relationships of horizontal recti are obtained from experiment, while the length-tension relationships of the other EOMs are still at the exploration stage because few experiment data exist. But modeling in the eye movement requires knowledge of the mechanical properties of all six EOMs.

To improve the set of the mechanical constitutive relationships of the EOM model in eye movement software (e.g., Orbit 1.8) and provide a theoretical direction to

the operational decisions of strabismus surgery, the present study focuses on the length-contractile force relationships of the EOMs in the primary innervations, specifically the relationships of the other four EOMs, except for horizontal recti.

2. Methods

The contractile forces are calculated based on an optimization proposed herein and on the theory of mechanical equilibrium using the traditional model of eye movement. The geometry model and the coordinate system are shown in Figure 1, in which the geometry parameters (i.e., origins, insertions, and cross sections) of EOMs are the same as those in the work of Gao et al. [21].

2.1. Translation Relationships of the Key Points of EOMs. During the horizontal eye movement, the origins and the insertions of the EOMs can be translated from their reference configurations to current configurations by (1). Additionally, coordinates of origins are the same between two different configurations in the coordinate system

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x' \\ y' \\ z \end{pmatrix}. \qquad (1)$$

The tangent points of the EOMs can be calculated using

$$x_{C_i}^2 + y_{C_i}^2 + z_{C_i}^2 = R^2,$$

$$x_{C_i} \cdot x_{D_i} + y_{C_i} \cdot y_{D_i} + z_{C_i} \cdot z_{D_i} = R^2,$$

$$\left(y_{D_i}z_{B_i} - z_{D_i}y_{B_i}\right) \cdot x_{C_i} + \left(z_{D_i}x_{C_i} - x_{D_i}z_{C_i}\right) \cdot y_{C_i}$$
$$+ \left(x_{D_i}y_{C_i} - y_{D_i}x_{C_i}\right) \cdot z_{C_i} = 0, \tag{2}$$

where the subscripts B_i, C_i, and D_i represent insertion, tangent point, and origin of the ith EOM, respectively. The in-order Arabic numerals 1–6 are introduced to represent the LR, MR, SR, IR, SO, and IO muscle, respectively.

2.2. Total Force.

By fitting the experimental data of the work of Collins [12], the total force of the LR muscle (F_1) in the primary innervation can be described as

$$F_1 = 5.20\Delta L_1 + 10.66e^{\Delta L_1/(-6.32)} + 8.49, \tag{3}$$

where ΔL_1 denotes the elongation of the LR muscle relative to its primary length L_{11}, which is 50.21 mm.

The corresponding stress-strain relationship of the LR muscle is

$$\sigma_1 = 15.61\varepsilon_1 + 0.64e^{-7.94\varepsilon_1} + 0.51, \tag{4}$$

where $\sigma_1 = F_1/A_1$ is the total stress of the LR muscle, $A_1 = 16.73 \text{ mm}^2$ is the cross section of the LR muscle [5], and $\varepsilon_1 = \Delta L_1/L_{11}$ is the strain along the length direction of the LR.

By assuming that the other EOMs have similar stress-strain relationships to that of the LR muscle in the primary innervation, the stress of the ith EOM ($i = 2, 3, 4, 5, 6$) can be described as

$$\sigma_i = \sigma_{i0} + d\sigma_i, \tag{5}$$

where $\sigma_{i0} = 15.61\varepsilon_i + 0.64\exp(-7.94\varepsilon_i) + 0.51$ and $d\sigma_i$ is the supplementary variation.

Therefore, the total force of the ith EOM in the primary innervation is

$$F_i = \sigma_i \cdot A_i, \tag{6}$$

where i denotes the ith EOM.

In the primary position, the eye is kept at equilibrium with the contributions from the six EOMs. Equations (7a)–(7d) are the corresponding valid torque equilibrium equations:

$$\sum_{i=1}^{6} m_{x_i} = 0,$$
$$\sum_{i=1}^{6} m_{y_i} = 0, \tag{7a}$$
$$\sum_{i=1}^{6} m_{z_i} = 0,$$

where

$$m_{x_i} = y_{c_i}Z_i - z_{c_i}Y_i,$$
$$m_{y_i} = z_{c_i}X_i - x_{c_i}Z_i, \tag{7b}$$
$$m_{z_i} = x_{c_i}Y_i - y_{c_i}X_i,$$
$$(X_i, Y_i, Z_i)$$
$$= \left(F_i \cdot \frac{x_{D_i} - x_{C_i}}{\sqrt{*}}, F_i \cdot \frac{y_{D_i} - y_{C_i}}{\sqrt{*}}, F_i \cdot \frac{z_{D_i} - z_{C_i}}{\sqrt{*}}\right), \tag{7c}$$
$$* = \left(x_{D_i} - x_{C_i}\right)^2 + \left(y_{D_i} - y_{C_i}\right)^2 + \left(z_{D_i} - z_{C_i}\right)^2, \tag{7d}$$

where D_i and C_i are the origin and tangent point of the ith EOM, respectively.

Optimizing function (8) was used to make the value of the stress deviator $d\sigma_i$ as small as possible; that is, let the objective function Φ obtained be its minimum value:

$$\Phi = \sum_{i=2}^{6} \left(\frac{d\sigma_i}{\sigma_{i0}}\right)^2. \tag{8}$$

In the primary position, the values of $d\sigma_2$–$d\sigma_6$ in (5) could be obtained using the quadprog function [22] by solving a constrained minimal value of objective function Φ (with conditions that the total force of each EOM should subject to (7a)–(7d)). And the constitutive relationships (total stress-strain relationships) of the six EOMs in the primary innervations can be rewritten as

$$\sigma_1 = 15.61\varepsilon_1 + 0.64e^{-7.94\varepsilon_1} + 0.51, \tag{9a}$$
$$\sigma_2 = 15.61\varepsilon_2 + 0.64e^{-7.94\varepsilon_2} + 0.06, \tag{9b}$$
$$\sigma_3 = 15.61\varepsilon_3 + 0.64e^{-7.94\varepsilon_3} + 0.47, \tag{9c}$$
$$\sigma_4 = 15.61\varepsilon_4 + 0.64e^{-7.94\varepsilon_4} + 0.31, \tag{9d}$$
$$\sigma_5 = 15.61\varepsilon_5 + 0.64e^{-7.94\varepsilon_5} + 0.54, \tag{9e}$$
$$\sigma_6 = 15.61\varepsilon_6 + 0.64e^{-7.94\varepsilon_6} + 0.52. \tag{9f}$$

2.3. Validation of Total Force Formula.

Because of the difficulty of conducting experiments on human in vivo, only the resistance data of the experiment worked out by Collins [12] can be compared, and the corresponding method is shown in Figure 2. In Figure 2, the left eye, whose horizontal recti are detached, is passively rotated temporalward with an artificial external force F, whereas the right eye fixes on the primary position actively. In this paper, the above eye movement model (Figure 1) and the obtained total force formula are used to simulate the experiment of Collins [12]. The forces of the other four EOMs (i.e., SR, IR, SO, and IO) resisting the left eye movement (Figure 2) can be obtained using (6) and (9a)–(9f). Finally, compare the simulated results to the experimental results.
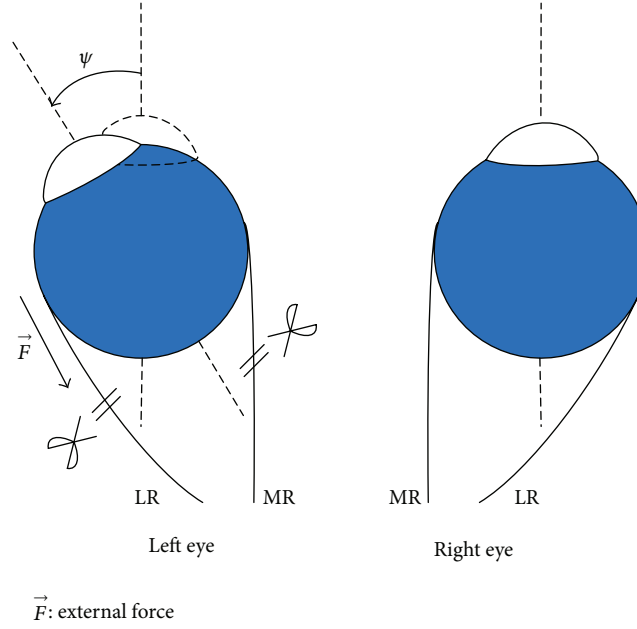
$\vec{F}$: external force

FIGURE 2: Schematic of double eyes only with the lateral rectus (LR) and the medial rectus (MR): not all the six EOMs are plotted.

2.4. Passive Force.

By fitting the experimental data of previous works [6, 12, 23], the passive force F_{p1} of the LR muscle can be described as

$$F_{p1} = -0.29\Delta L_1 + 4.79e^{\Delta L_1/4.57} - 3.01. \qquad (10)$$

The corresponding passive stress σ_{p1} of the LR muscle is

$$\sigma_{p1} = -0.87\varepsilon_1 + 0.29e^{10.99\varepsilon_1} - 0.18. \qquad (11)$$

Theoretically, passive stress is only relative to the material, and the passive stresses σ_{pi} of all six EOMs can be described as

$$\sigma_{pi} = -0.87\varepsilon_i + 0.29e^{10.99\varepsilon_i} - 0.18. \qquad (12)$$

The passive force of the ith EOM is

$$F_{pi} = \sigma_{pi} \cdot A_i. \qquad (13)$$

2.5. Active Force.

The active stresses σ_{ai} of the six EOMs in primary innervations can be obtained by the respective total stress σ_i (see (9a)–(9f)) minus the passive stress σ_{pi} (see (12)); that is,

$$\sigma_{a1} = 16.48\varepsilon_1 + 0.64e^{-7.94\varepsilon_1} - 0.29e^{10.99\varepsilon_1} + 0.69, \qquad (14a)$$

$$\sigma_{a2} = 16.48\varepsilon_2 + 0.64e^{-7.94\varepsilon_2} - 0.29e^{10.99\varepsilon_2} + 0.24, \qquad (14b)$$

$$\sigma_{a3} = 16.48\varepsilon_3 + 0.64e^{-7.94\varepsilon_3} - 0.29e^{10.99\varepsilon_3} + 0.65, \qquad (14c)$$

$$\sigma_{a4} = 16.48\varepsilon_4 + 0.64e^{-7.94\varepsilon_4} - 0.29e^{10.99\varepsilon_4} + 0.49, \qquad (14d)$$

$$\sigma_{a5} = 16.48\varepsilon_5 + 0.64e^{-7.94\varepsilon_5} - 0.29e^{10.99\varepsilon_5} + 0.72, \qquad (14e)$$

$$\sigma_{a6} = 16.48\varepsilon_6 + 0.64e^{-7.94\varepsilon_6} - 0.29e^{10.99\varepsilon_6} + 0.70. \qquad (14f)$$

The active force of the ith EOM in the primary innervation is

$$F_{ai} = \sigma_{ai} \cdot A_i. \qquad (15)$$

3. Results

3.1. Total Force.

The calculation results shown in Figure 3 reveal that with the eye movement the total force of the MR muscle linearly increases and reaches its maximum value of 617.25 mN. Meanwhile, the total force of the LR muscle decreases from 188.74 mN to 45.22 mN with the eye rotating temporalward by 35° and then slightly increases to 55.23 mN, with the eye rotating temporalward by 40°. The total forces of the SR and IR muscles are seemingly stable; that is, their respective minimum values are 123.90 and 147.54 mN in the primary position, and their respective maximum values are 147.93 and 184.72 mN in the temporal 25°. The total forces of the SO and IO muscles are slightly reduced from the primary position to the temporal 40°. Their maximum values are 223.86 and 224.94 mN in the primary position, whereas their minimum values are 154.61 and 127.53 mN in the temporal 40°, respectively.

3.2. Validation Results of Total Force Formula.

The comparison results shown in Figure 4 reveal that the calculation results well agree with the experimental data of Collins [12] when the eye rotates temporalward under 25°, which is the normal range of eye abduction [24]. Meanwhile, the errors between the calculation results and experimental data increase with eye rotation from 25° to 40°.

3.3. The Value of the Objective Function.

The values of the objective function Φ calculated by corresponding formulation (8) are summarized in Table 1. These values are small
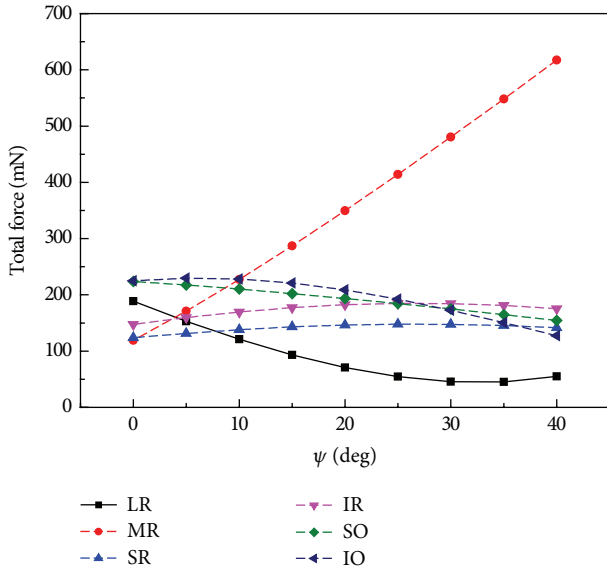
FIGURE 3: Total forces of EOMs with eye passively rotated temporal-ward in the primary innervations.
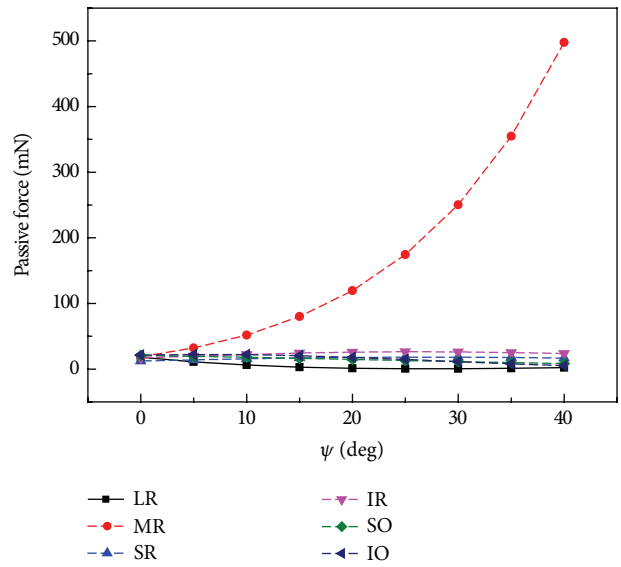

FIGURE 5: Passive forces of EOMs with eye passively rotated temporalward in the primary innervations.
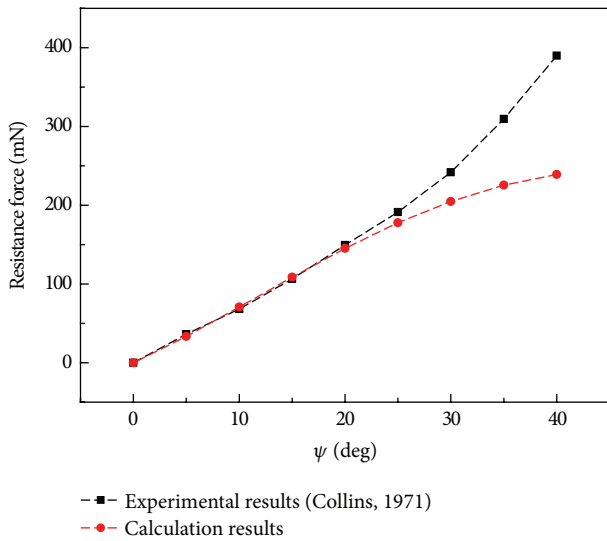

FIGURE 4: Comparison of resistance forces from the other four EOMs, except for the horizontal recti, during passive rotation of the eye in the primary innervations (note: the experimental data are adopted from Figure 27 of reference [12]).

FIGURE 6: Active forces of EOMs with eye passively rotated tempo-ralward in the primary innervations.

TABLE 1: Values of the objective function in different positions.

ψ (deg)	0	5	10	15	20	25	30	35	40
Φ (—)	0.19	0.12	0.09	0.07	0.06	0.05	0.04	0.04	0.04

enough, and they gradually decrease with the eye temporally rotating from 0° to 40°.

3.4. Passive Force. Figure 5 shows the calculation results of the passive forces. The passive force of the MR muscles sharply increases from 18.74 mN in the primary position to
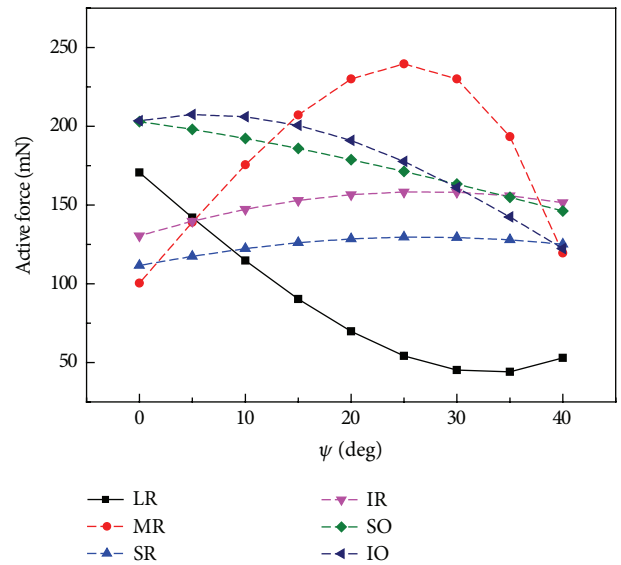
497.66 mN in the temporal 40°, whereas that of the other muscles seems to stabilize within the range of approximately 1–26 mN.

3.5. Active Force. The calculation results of the active forces are shown in Figure 6. The active force of the MR muscle increases from 100.45 mN in the primary position to 239.66 mN in the temporal 25° and then decreases to 119.58 mN in the temporal 40°. By contrast, that of the LR muscle decreases from 170.69 mN in the primary position to 44.15 mN in the temporal 35° and then rises to 52.97 mN in the temporal 40°. Variations in the vertical recti are very small; that is, the active force of the SR muscle increases

from 111.64 mN in the primary position to 129.69 mN in the temporal 25° and then decreases to 125.08 mN in the temporal 40°, whereas that of the IR muscle increases from 130.47 mN in the primary position to 158.33 mN in the temporal 25° and then decreases to 151.56 mN in the temporal 40°. For the obliques, the active force of the SO muscle decreases from 202.97 mN in the primary position to 146.17 mN in the temporal 40°, whereas that of the IO muscle slightly increases from 203.56 mN in the primary position to 207.38 mN in the temporal 5° and then decreases to 122.33 mN in the temporal 40°.

4. Discussion

In this study, the traditional model is used to simulate the eye movement for simplicity, in which the pulleys of recti are not included. Although many investigators [25, 26] have realized the importance of pulleys, the main function of pulleys is that they change the path of the force of recti. The constitutive relationships of EOMs mainly depend on the innervations and the length change of the EOMs. Obviously, pulley plays insignificant role in determining the constitutive relationships of EOMs.

An optimization method is proposed to calculate the actively contractile forces of EOMs in the primary innervations based on the theory of the mechanical equilibrium. The optimizing function is established on the assumption that all six EOMs have similar stress-strain relationships with the eye suspended in primary position. Accordingly, the values of the objective function Φ should be minimal; these values are shown in Table 1.

The errors between the calculated resistance force and the experimental data apparently and increasingly expand with eye rotation beyond 25° in Figure 4. The possible reason is that individual differences among human beings cannot be omitted; that is, the subjects differ in establishing the coordinate of the orbit in this paper [18, 21] and performing the previous experiment [12].

Physiologically, the optimal length of a muscle is defined as its length produced at the maximal active force. The optimal length of the skeletal muscle is around its normal resting length [7]. The Hill-type model, a three-element constitutive model of a skeletal muscle, suggests that different innervations lead to varied optimal active forces [2, 27]. Robinson et al. [28] determined the active forces of the LR muscle. Their measurements revealed that LR has peak active forces when the eye rotated to the nasal by 32° in different innervations. This result suggests that LR obtains its optimal length with the eye suspended in the nasal by 32°. Interestingly, the present simulation results show that LR has the lowest active force when the eye rotates temporally between 30° and 35° (Figure 6). In addition, in the primary innervation with the eye rotating temporally by 25°, the length may be the optimal length of MR because MR achieves its peak value of the active force (Figure 6). The above analysis reveals that the optimal length of the MR is not around its normal resting length, which differs from that of the skeletal muscle. Several investigators have also reported

many differences between EOMs and the skeletal muscle, for example, fiber types and biomechanical properties [6, 7, 29, 30].

However, this study cannot determine the optimal lengths of the other four EOMs, whose simulation results reveal fairly ambiguous peak values (Figure 6). To determine the optimal lengths of the SR and IR muscles, eye movement in the vertical plane should be further investigated. The optimal lengths of the SO and IO muscles are also difficult to determine because little is known on simple eye movement as controlled by the oblique.

In the process of clinical strabismus correction, the primary position is crucial. The purpose of strabismus surgery is to make the binocular optical axes parallel and forward in the primary position. The calculation of the surgical amount is also based on the mechanical constitutive relationships of EOMs in the primary position. The abnormal length of some EOMs may lead to strabismus [31–33], and numerous patients suffer from strabismus surgery [1, 34–36]. The constitutive relationships of the EOMs in primary innervations ((9a)–(9f) and (14a)–(14f)) can contribute to the treatment of EOM disorders, such as strabismus. During strabismus surgery, the length and the location of insertion of some EOM may need to be adjusted in the primary innervation, but the operational decisions of the strabismus surgery, such as the adjustment of the length and the location of an EOM, still lack well-defined clinical guidelines [37]. And after the adjustment of the specified EOM, how many forces the other EOMs generate to maintain the eye movement is important. All of the above depend on the mechanical constitutive relationships or length-tension relationships of the six EOMs in primary innervations.

In the existing eye movement models, only the length-tension relationships of the horizontal recti have been obtained by experiment [12–16]. The length-tension relationships of the other four EOMs were mainly empirical, deduced by that of the horizontal recti [2, 17]. For example, in the eye movement model proposed by Wei et al. [2], length-tension relationships of EOMs are obtained from the length-tension relationships of LR and scaled by their cross sections. The stiffness of the EOMs is occasionally defined as constant [38, 39] or the active forces of the EOMs are assumed to be linearly proportional to their innervations [40, 41]. The eye movement model of Pascolo and Carniel [5] modifies Hill's muscle model and skillfully avoids the test difficulty of the real EOM innervations, which can be used to examine healthy case and pathological case such as strabismus based on the dynamical equilibrium. The present simulation is based on the static equilibrium in the primary position. All these examples show that the mechanical constitutive relationships are essential to eye movement modeling. Compared with previous descriptions of the mechanical properties of EOMs, the constitutive relationships of EOMs presented in this study ((9a)–(9f) and (14a)–(14f)) are obtained based on previous experimental data and verified by previous experiments. Evidently, the relationships are closer to reality. The obtained constitutive relationships can be integrated into the eye movement model, which can be used to calculate the surgical amounts of EOMs in strabismus surgery more precisely.

The ophthalmic diseases, such as the mechanically vertical strabismus [42] and cyclotropia [43] introduced by the congenitally anatomic abnormality, were evaluated qualitatively in the strabismus clinic, but the quantitative analysis will be needed. Therefore, the length-tension relationships of the other EOMs except for the recti are necessary. Because of the difficulty of the human EOMs anatomy and the complicated ethics and morals, the length-tension relationships of the SR, IR, SO, and IO are obviously difficult to measure experimentally. In this paper, the assumption that the six EOMs have the similar stress-strain relationships in the primary position is proposed based on the experiment of strabismus surgery [12]. And the mechanical constitutive relationships of the six EOMs in the primary innervations are deduced. Although the experiment is not conducted and the assumption may be slightly different from the reality, the simulated resistance forces coincide well with the experimental results [12]. Therefore, the reliability of the assumption is verified indirectly.

5. Conclusion

To improve eye movement modeling and predict the surgical amount of clinical EOM surgery, this work aimed to determine the mechanical constitutive relationships of six EOMs in primary innervations, particularly the contractile constitutive relationships. Based on previous experimental data of the LR and the assumption that six EOMs have similar strain-stress relationships in primary position, the contractile constitutive relationships of the six EOMs in primary innervations were obtained by combining the equilibrium equations and optimizing functions. The rationality of the proposed assumption and the obtained constitutive relationships were verified through comparisons with previous experimental data. Results indicated that the optimal length of the EOM was not around its normal resting length, which differed from that of the skeletal muscle. Given that the mechanical model of the eye movement system proposed by Pascolo and Carniel would become a routine diagnostic tool of the disturbances of nystagmic movements and strabismus [5], the obtained constitutive relationships of EOMs could be integrated into the eye movement model to predict the surgical amounts of EOMs before EOM surgery.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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