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Research Article

Robust Quantized Generalized H_2 Filtering for Uncertain Discrete-Time Fuzzy Systems

Jidong Wang,[1] Xiaoping Si,[1] and Kezhen Han[2]

[1] *School of Electrical Engineering, North China University of Water Resources and Electric Power, Zhengzhou 450011, China*
[2] *School of Information Science and Engineering, Northeastern University, Shenyang 110819, China*

Correspondence should be addressed to Jidong Wang; kfwjd@ncwu.edu.cn

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This paper deals with the problem of robust generalized H_2 filter design for uncertain discrete-time fuzzy systems with output quantization. Firstly, the outputs of the system are quantized by a memoryless logarithmic quantizer before being transmitted to a filter. Then, attention is focused on the design of a generalized H_2 filter to mitigate quantization effects, such that the filtering error systems ensure the robust stability with a prescribed generalized H_2 noise attenuation level. Via applying Finsler lemma to introduce some slack variables and using the fuzzy Lyapunov function, sufficient conditions for the existence of a robust generalized H_2 filter are expressed in terms of linear matrix inequalities (LMIs). Finally, a numerical example is provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

Quantization in feedback control systems has received much attention in recent years [1–8]. This is mainly due to the wide application of digital computers in control systems. For example, it usually arises in a distributed or network based control system, where information should be quantized before being transmitted through a communication channel under limited bandwidth. However, quantization of a stabilizing controller may lead to limiting cycles and chaotic behavior as described in [1]. And there are substantially two reasons to account for these changes in the system's behavior. One is saturation and the other is deterioration of performance near the system equilibrium. Therefore, considerable efforts have been devoted to develop tools for better analysis and design of quantized feedback systems [2–6]. In order to find out quantized characteristic, [6] investigated the quantization model from statistical perspective and found that quantization is inherently a nonlinear feature. For stabilization of discrete-time SISO linear systems, [2] proposed that the coarsest quantizer that quadratically stabilizes such a linear system is logarithmic, which can be computed by solving a special linear quadratic regulator problem. Unfortunately, the results are difficult to extend to the multiple-input case. By noting that the quantization error can be treated as uncertainty or nonlinearity and can be bounded by a sector bound, [9] proposed a sector bound approach to deal with the quantized feedback control problem. Recently, by recognizing that only a quadratic Lyapunov function is used in the sector bound approach, [5] designed a quantization-dependent Lyapunov function approach which can lead to less conservative results.

On the other hand, as it is well known, Takagi-Sugeno (T-S) model has proved its effectiveness in the study of nonlinear systems. Indeed, it gives a simpler formulation from mathematical point of view to represent the behavior of nonlinear systems [10]. On the whole, they are composed of linear models blended together with nonlinear functions. Then, for the stability and stabilization of such T-S fuzzy model, some tools inspired from the study of linear systems are proposed. In particular, there have been a lot of results to study the stability problem, such as in ([11–14] and the references therein). Nevertheless, the use of the quadratic Lyapunov function leads to conservative results and reaches quickly its limits. To overcome the drawback, different Lyapunov functions have been proposed ([15–18] and the references therein).

For the purpose of analysis and synthesis, estimating the state variables of a dynamic system is important in helping to improve our knowledge about the system concerned. In this meaning, H_2 filtering design arises as an efficient strategy whenever the noise input is assumed to have a known power spectral density. The problem has been faced using Riccati-based approaches [19] and by means of LMIs methods [20]. In the case where there exists insufficient statistical information about the noise input, the well-known H_∞ filtering design and peak-to-peak filtering method can be employed [21]. From another point of view, when the closed-loop system is described in the term of mapping between the space of time-domain input disturbances in l_2 and the space of time-domain controlled outputs in l_∞, the generalized H_2 control problem is considered, where the conventional H_2 norm is replaced by an operator norm. Due to the fact that the generalized H_2 performance is useful for handling stochastic aspects such as measurement noise and random disturbances, it has been received much attention ([22–26] and the references therein). More recently, based on piecewise Lyapunov functions, [27–29] have done some remarkable works on generalized H_2 synthesis of fuzzy systems. Reference [30] provides complete results on the induced l_2 and generalized H_2 filtering for a class of discrete-time systems with repeated scalar nonlinearities. Taking quantization and packet loss into consideration, a generalized H_2 filter has been designed in [31]. For discrete-time fuzzy systems, [32] has proposed a robust generalized H_2 controller via basis-dependent Lyapunov functions.

In this paper, we are to tackle the robust quantized generalized H_2 filtering problem for a class of nonlinear discrete-time systems with norm-bounded uncertainties, which is different from the contents and research techniques in [31]. Via applying Finsler lemma in [33] to introduce some slack variables to provide extra free dimensions in the solution space and using the variable definition method in [34] to relax the structural constraints of these introduced elements, the robust quantized generalized H_2 filter can be designed by solving a set of linear matrix inequalities and can be easily checked through utilizing available numerical software, such as Matlab and Scilab. The main contribution of this paper is that it deals with the quantized error and model additive uncertainty simultaneously for nonlinear discrete-time systems with prescribed generalized H_2 performance, which has little related literature on this multiobjective filtering problem to the best of our knowledge.

The paper is organized as follows. The next section provides some useful notations and lemmas. In Section 3, generalized H_2 performance is firstly considered, and then, a sufficient condition to mitigate quantization error effects is deduced. According to the previous results, a LMI-based approach is established with the systems uncertainties taken into consideration in the end of Section 3. Section 4 gives the standard robust quantized generalized H_2 filter design method. Finally, a numerical example is provided to illustrate the effectiveness of our main results.

Notations. Throughout this paper, the symbol $*$ induces a symmetric structure in LMIs. For a matrix A, A^T and A^{-1} denote its transpose and inverse if it exists, respectively. The matrix inequality $A > 0$ $(A < 0)$ means that A is square symmetric and A is positive (negative) definite. The notation $l_2[0,\infty)$ represents the space of square-integrable vector functions over $[0,\infty)$. And $He\{A\}$ denotes $(A^T + A)$ for simplicity. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

2. Problem Statement and Preliminaries

Consider an uncertain nonlinear discrete-time system represented by the following uncertain T-S fuzzy model, where the ith rule is described as follows:

R^i: if $\upsilon_1(k)$ is M_{1i}, and ..., and $\upsilon_\eta(k)$ is $M_{\eta i}$, then,

$$\begin{aligned} x(k+1) &= \overline{A}_i x(k) + \overline{B}_i \omega(k), \\ y(k) &= \overline{C}_i x(k) + \overline{D}_i \omega(k), \\ z(k) &= \overline{L}_i x(k) + \overline{J}_i \omega(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathscr{R}^n$ is the state variable, $\omega(k) \in \mathscr{R}^m$ is the disturbance input and $\omega(k) \in l_2[0,\infty)$, $z(k) \in \mathscr{R}^q$ is the signal to be estimated, and $y(k) \in \mathscr{R}^f$ is the measurement output. $\{\upsilon_1(k), \upsilon_2(k), \dots, \upsilon_\eta(k)\}$ are premise variables. $M_{di}, i = 1, 2, \dots, r, d = 1, 2, \dots, \eta$ are fuzzy sets, and r is the number of fuzzy rules. The matrices $\overline{A}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i, \overline{L}_i$, and $\overline{J}_i$ are system matrices of appropriate dimensions with the following parametric uncertainties:

$$\begin{bmatrix} \overline{A}_i & \overline{B}_i \\ \overline{C}_i & \overline{D}_i \\ \overline{L}_i & \overline{J}_i \end{bmatrix} = \begin{bmatrix} A_i & B_i \\ C_i & D_i \\ L_i & J_i \end{bmatrix} + \begin{bmatrix} H_i \\ T_i \\ K_i \end{bmatrix} \Delta_P(k) \begin{bmatrix} F_i & E_i \end{bmatrix}, \tag{2}$$

where $A_i, B_i, C_i, D_i, L_i, J_i, H_i, F_i, E_i, T_i$, and K_i for $i = 1, 2, \dots, r$ are known constant matrices with appropriate dimensions. And $\Delta_P(k)$ is an uncertain matrix satisfying $\Delta_P^T(k)\Delta_P(k) \le I$. It is easy to note that such parameter uncertainty is treated as a particular perturbation of linear fractional form, where the linear fractional representation $\Delta_{\mathscr{P}}(k) = (I - \Delta_P(k)\mathscr{J})^{-1}\Delta_P(k)$ reduces to norm-bounded one $\Delta_{\mathscr{P}}(k) = \Delta_P(k)$ when $\mathscr{J} = 0$.

Using a standard singleton fuzzifier, product inference, and centre weighted average defuzzifier, a compact presentation of the overall fuzzy model is given by

$$\begin{aligned} x(k+1) &= \overline{A}(\theta) x(k) + \overline{B}(\theta) \omega(k), \\ y(k) &= \overline{C}(\theta) x(k) + \overline{D}(\theta) \omega(k), \\ z(k) &= \overline{L}(\theta) x(k) + \overline{J}(\theta) \omega(k), \end{aligned} \tag{3}$$

with

$$\overline{A}(\theta) = A(\theta) + \Delta A(\theta) = \sum_{i=1}^{r} h_i(v(k))\left(A_i + H_i\Delta_P(k)F_i\right),$$
$$\overline{B}(\theta) = B(\theta) + \Delta B(\theta) = \sum_{i=1}^{r} h_i(v(k))\left(B_i + H_i\Delta_P(k)E_i\right),$$
$$\overline{C}(\theta) = C(\theta) + \Delta C(\theta) = \sum_{i=1}^{r} h_i(v(k))\left(C_i + T_i\Delta_P(k)F_i\right),$$
$$\overline{D}(\theta) = D(\theta) + \Delta D(\theta) = \sum_{i=1}^{r} h_i(v(k))\left(D_i + T_i\Delta_P(k)E_i\right),$$
$$\overline{L}(\theta) = L(\theta) + \Delta L(\theta) = \sum_{i=1}^{r} h_i(v(k))\left(L_i + K_i\Delta_P(k)F_i\right),$$
$$\overline{J}(\theta) = J(\theta) + \Delta J(\theta) = \sum_{i=1}^{r} h_i(v(k))\left(J_i + K_i\Delta_P(k)E_i\right), \quad (4)$$

where $v(k) = [v_1, v_2, \ldots, v_\eta]$ and $h_i(v(k)) = \tau_i(v(k))/\sum_{j=1}^{r}\tau_j(v(k)) \geq 0$ is the normalized weight for each rule with $\tau_i(v(k)) = \Pi_{d=1}^{\eta} M_{di}(v_d(k)) \geq 0$ and $\sum_{i=1}^{r} h_i(v(k)) = 1$.

Then, the filter considered here is given as follows:

$$\begin{aligned} x_F(k+1) &= A_F x_F(k) + B_F y_q(k), \\ z_F(k) &= C_F x_F(k) + D_F y_q(k), \\ y_q(k) &= Q(y(k)), \end{aligned} \quad (5)$$

where $x_F(k) \in \mathscr{R}^n$ and $z_F(k) \in \mathscr{R}^q$ are the state and output of the filter, respectively. The matrices A_F, B_F, C_F, and D_F are filter parameters to be determined. $y_q(k) \in \mathscr{R}^f$ is the input of the filter and $Q(\cdot) = [Q_1(\cdot)\ \ Q_2(\cdot)\cdots Q_f(\cdot)]^T$ is a quantizer which is assumed to be symmetric (i.e., $Q(-v) = -Q(v)$). Note that filter (5) is not the fuzzy type. The reason for applying the basis-independent filter is to avoid the design difficulty in the presence of quantization error and systems uncertainties.

Here, we employ the static time-invariant logarithmic quantizer. According to [4], the set of quantized levels is given as

$$\mathscr{V}_j = \left\{\pm v_i^{(j)}, v_i^{(j)} = \rho_j^i v_0^{(j)}, i = 0, \pm 1, \pm 2, \ldots\right\} \cup \{0\},$$
$$v_0^{(j)} > 0, \quad 0 < \rho_j < 1,$$
$$Q_j(y) = \begin{cases} v_i^{(j)}: & 0 < \dfrac{1}{1+\delta_j} v_i^{(j)} < y \leq \dfrac{1}{1-\delta_j} v_i^{(j)} \\ 0: & y = 0 \\ -Q_j(-y): & y < 0, \end{cases} \quad (6)$$

where

$$\delta_j = \frac{1-\rho_j}{1+\rho_j}. \quad (7)$$

From (6)-(7), for the designed filter (5), it can be concluded that, for any $y(k) \in \mathscr{R}^f$,

$$\left|Q(y(k)) - y(k)\right| \leq \delta y(k) \quad (8)$$

holds, where $\delta = \operatorname{diag}(\delta_1, \delta_2, \ldots, \delta_f)$. Therefore, $y_q(k)$ can be depicted as

$$y_q(k) = Q(y(k)) = (I + \Delta(k))\, y(k), \quad |\Delta(k)| \leq \delta, \quad (9)$$

where $\Delta(k) = \operatorname{diag}(\Delta_1(k), \Delta_2(k), \ldots, \Delta_f(k))$.

Defining an augmented state vector $\tilde{x}(k) = [x^T(k)\ x_F^T(k)]^T$ and $e(k) = z(k) - z_F(k)$, we can obtain the following filtering error system:

$$\tilde{x}(k+1) = \widetilde{A}(\theta)\tilde{x}(k) + \widetilde{B}(\theta)\omega(k), \quad (10)$$
$$e(k) = \widetilde{C}(\theta)\tilde{x}(k) + \widetilde{D}(\theta)\omega(k), \quad (11)$$

where

$$\begin{aligned} \widetilde{A}(\theta) &= \widehat{A}(\theta) + \Delta\widehat{A}(\theta) \\ &= \begin{bmatrix} \overline{A}(\theta) & 0 \\ B_F\overline{C}(\theta) & A_F \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_F\Delta(k)\overline{C}(\theta) & 0 \end{bmatrix}, \\ \widetilde{B}(\theta) &= \widehat{B}(\theta) + \Delta\widehat{B}(\theta) \\ &= \begin{bmatrix} \overline{B}(\theta) \\ B_F\overline{D}(\theta) \end{bmatrix} + \begin{bmatrix} 0 \\ B_F\Delta(k)\overline{D}(\theta) \end{bmatrix}, \\ \widetilde{C}(\theta) &= \widehat{C}(\theta) + \Delta\widehat{C}(\theta) \\ &= \left[\overline{L}(\theta) - D_F\overline{C}(\theta)\ \ -C_F\right] + \left[-D_F\Delta(k)\overline{C}(\theta)\ \ 0\right], \\ \widetilde{D}(\theta) &= \widehat{D}(\theta) + \Delta\widehat{D}(\theta) \\ &= \overline{J}(\theta) - D_F\overline{D}(\theta) + \left(-D_F\Delta(k)\overline{D}(\theta)\right). \end{aligned} \quad (12)$$

In order to facilitate the later operations, we note

$$\begin{aligned} M_B &= \begin{bmatrix} 0 \\ B_F \end{bmatrix}, & N_C(\theta) &= \left[\overline{C}(\theta)\ \ 0\right], \\ M_D &= -D_F, & N_D(\theta) &= \overline{D}(\theta). \end{aligned} \quad (13)$$

Then, we have

$$\begin{aligned} \Delta\widehat{A}(\theta) &= M_B\Delta(k)N_C(\theta), & \Delta\widehat{D}(\theta) &= M_D\Delta(k)N_D(\theta), \\ \Delta\widehat{B}(\theta) &= M_B\Delta(k)N_D(\theta), & \Delta\widehat{C}(\theta) &= M_D\Delta(k)N_C(\theta). \end{aligned} \quad (14)$$

To this end, the objective of this paper can be summarized as follows.

Given uncertain discrete-time system (1), develop a full-order filter of the form in (5) such that

(1) the filtering error system (10)-(11) is stochastically stable with system norm-bounded uncertainties and quantization error effects when $\omega(k) \in l_2[0, \infty)$;

(2) the filtering error system (10)-(11) has a prescribed generalized H_2 disturbance attenuation level γ; that is, under the zero initial condition $\tilde{x}(0) = 0$, $\|e(k)\|_\infty < \gamma\|\omega(k)\|_2$ is satisfied for any nonzero $\omega(k) \in l_2[0, \infty)$ [11].

And,

$$\|e(k)\|_\infty := \sup_k \left\{ \sqrt{e^T(k)\, e(k)} \right\},$$
$$\|\omega(k)\|_2 := \left\{ \sqrt{\sum_{k=0}^{\infty} \omega^T(k)\, \omega(k)} \right\}. \tag{15}$$

Before ending this section, we introduce the following lemmas, which will be used subsequently.

Lemma 1 (see [35]). *Let* Γ, Λ, *and* $\Delta(k)$ *be real matrices of appropriate dimensions with* $\Delta(k)$ *satisfying* $\Delta^T(k)\Delta(k) \le I$. *Then, for any scalar* ϵ, *we have*

$$\Gamma\Delta(k)\Lambda + \Lambda^T\Delta^T(k)\Gamma^T \le \epsilon^{-1}\Gamma\Gamma^T + \epsilon\Lambda^T\Lambda. \tag{16}$$

Lemma 2 (Finsler's lemma, see [33, 36]). *Given matrices* $\Theta = \Theta^T \in \mathscr{R}^{n\times n}$ *and* $\Upsilon \in \mathscr{R}^{n\times n}$, *then*

$$v^T\Theta v < 0, \quad \forall v \in \mathscr{R}^n, \qquad \Upsilon v = 0, \quad v \neq 0, \tag{17}$$

if and only if there exists matrix $\Omega \in \mathscr{R}^{n\times n}$, *such that*

$$\Theta + \Omega\Upsilon + \Upsilon^T\Omega^T < 0. \tag{18}$$

Lemma 3 (see [37, 38]). *If the following conditions hold:*

$$X_{ii} < 0, \quad i = 1, 2, \ldots, r,$$
$$\frac{1}{r-1}X_{ii} + \frac{1}{2}\left(X_{ij} + X_{ji}\right) < 0, \quad i \neq j,\ i, j = 1, 2, \ldots, r, \tag{19}$$

then the following inequality holds:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(v(k))\, h_j(v(k))\, X_{ij} < 0,$$
$$\sum_{i=1}^{r} h_i(v(k)) = 1. \tag{20}$$

3. Generalized H_2 Filtering Analysis

The purpose of this section is to lay preliminary results regarding quantized generalized H_2 filtering with system matrices uncertainties considered. First, Lemma 4 provides a fundamental analytic condition of generalized H_2 filtering. Then, based on the condition, quantization error effects will be dealt with by Lemma 7.

Lemma 4. *Consider the system* (3) *and suppose the filter matrices in* (5) *are known. Then, the filtering error system* (10)-(11) *is asymptotically stable with a generalized* H_2 *disturbance attenuation level bound* γ *if there exist matrices* $G(\theta), N(\theta), S(\theta)$ *and symmetric matrix* $P(\theta) > 0$ *such that the following inequalities hold:*

$$\begin{bmatrix} -P(\theta) & * & * \\ 0 & -I & * \\ \widetilde{C}(\theta) & \widetilde{D}(\theta) & -\gamma^2 I \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} -P(\theta) + He\left\{G(\theta)\widetilde{A}(\theta)\right\} & * & * \\ N(\theta)\widetilde{A}(\theta) + \widetilde{B}^T(\theta)G^T(\theta) & -I + He\left\{N(\theta)\widetilde{B}(\theta)\right\} & * \\ S(\theta)\widetilde{A}(\theta) - G^T(\theta) & S(\theta)\widetilde{B}(\theta) - N^T(\theta) & P^+(\theta) - He\{S(\theta)\} \end{bmatrix} < 0. \tag{22}$$

Proof. For filtering error system, a fuzzy Lyapunov functional candidate can be constructed as follows:

$$V(\tilde{x}(k), k) = \tilde{x}^T(k)\, P(\theta)\, \tilde{x}(k),$$
$$P(\theta) = \sum_{i=1}^{r} h_i(v(k))\, P_i > 0. \tag{23}$$

Then, the difference of $V(\tilde{x}(k), k)$ along solutions of (10)-(11) is computed as

$$\Delta V(\tilde{x}(k), k) = V(\tilde{x}(k+1), k+1) - V(\tilde{x}(k), k). \tag{24}$$

Let

$$\mathscr{J} = \Delta V(\tilde{x}(k), k) - \omega^T(k)\, \omega(k). \tag{25}$$

Then,

$$\mathscr{J} = \xi^T(k)\, \Phi(\theta)\, \xi(k), \tag{26}$$

where

$$\xi(k) = \left[\tilde{x}^T(k) \ \ \omega^T(k) \ \ \tilde{x}^T(k+1)\right]^T,$$
$$\Phi(\theta) = \begin{bmatrix} -P(\theta) & * & * \\ 0 & -I & * \\ 0 & 0 & P^+(\theta) \end{bmatrix}, \tag{27}$$

and $P^+(\theta) = \sum_{i=1}^{r} h_i(v(k+1))P_i$.

For (10), it is obvious that

$$\Upsilon(\theta)\, \xi(k) = 0, \quad \Upsilon(\theta) = \left[\widetilde{A}(\theta) \ \ \widetilde{B}(\theta) \ \ -I\right]. \tag{28}$$

Now, with inequalities (26) and (28) in hand, according to Lemma 2, inequality (26) < 0 for any $\xi(k) \neq 0$ if there exist $G(\theta)$, $N(\theta)$, and $S(\theta)$ such that the following inequality

$$\Phi(\theta) + \begin{bmatrix} G(\theta) \\ N(\theta) \\ S(\theta) \end{bmatrix} \Upsilon(\theta) + \Upsilon^T(\theta) \begin{bmatrix} G(\theta) \\ N(\theta) \\ S(\theta) \end{bmatrix}^T < 0 \tag{29}$$

holds. To the end, it is obvious to get inequality (22) < 0 and $\Delta V(\tilde{x}(k), k) < \omega^T(k)\omega(k)$ if inequality (29) < 0.

Then, under the zero initial condition $\tilde{x}(0) = 0$, for any nonzero $\omega(k) \in l_2[0, \infty)$, one has

$$\begin{aligned} &\sum_{k=0}^{N-1} \left\{\Delta V(\tilde{x}(k), k) - \omega^T(k)\omega(k)\right\} \\ &\quad = V(\tilde{x}(N), N) - \sum_{k=0}^{N-1} \omega^T(k)\omega(k) < 0 \end{aligned} \tag{30}$$

for any $N \in \{1, 2, \ldots\}$. And, from (30), we can obtain that

$$\begin{aligned} V(\tilde{x}(N), N) + \omega^T(k)\omega(k) &< \sum_{k=0}^{N} \omega^T(k)\omega(k) \\ &< \sum_{k=0}^{\infty} \omega^T(k)\omega(k). \end{aligned} \tag{31}$$

Now, consider the generalized H_2 performance. By using Schur complement equivalence, one has by (21) that

$$\left[\widetilde{C}(\theta)\ \ \widetilde{D}(\theta)\right]^T \gamma^{-2} \left[\widetilde{C}(\theta)\ \ \widetilde{D}(\theta)\right] - \begin{bmatrix} P(\theta) & 0 \\ 0 & I \end{bmatrix} < 0. \tag{32}$$

Note, for any $N \in \{1, 2, \ldots\}$,

$$\begin{aligned} &e^T(N)e(N) - \gamma^2\left(V(\tilde{x}(N), N) + \omega^T(k)\omega(k)\right) \\ &\quad = \begin{bmatrix} \tilde{x}(N) \\ \omega(N) \end{bmatrix}^T \\ &\quad \times \left\{\left[\widetilde{C}(\theta)\ \ \widetilde{D}(\theta)\right]^T \left[\widetilde{C}(\theta)\ \ \widetilde{D}(\theta)\right] - \gamma^2 \begin{bmatrix} P(\theta) & 0 \\ 0 & I \end{bmatrix}\right\} \\ &\quad \times \begin{bmatrix} \tilde{x}(N) \\ \omega(N) \end{bmatrix}. \end{aligned} \tag{33}$$

It is easy to conclude that inequality (33) < 0 if condition (32) holds, which leads to

$$\|e(k)\|_\infty^2 < \gamma^2\left(V(\tilde{x}(k), k) + \omega^T(k)\omega(k)\right). \tag{34}$$

Finally, together with inequality (31), we can deduce that $\|e(k)\|_\infty < \gamma\|\omega(k)\|_2$. This completes the proof. □

Remark 5. When $\omega(k) = 0$, it is effortless to prove that the system is stable if (22) holds. And, in the derivation of the above conclusion, three slack variables $G(\theta)$, $N(\theta)$, and $S(\theta)$ are introduced, which will reduce some conservatism.

Remark 6. It should be emphasized that the existing conditions in the literature for the analysis and design of robust filtering systems are only sufficient [34]. Therefore, many efforts have been made in the direction of reducing the conservativeness of the analysis and design methods for improving the systems performance. Therein, the approach of introducing auxiliary variables is widely applied. In particular, the results derived from Lemma 2 perform well to provide extra degree of freedom by adding auxiliary variables [39]. However, as stated above, the conservatism can not be still avoided due to sufficiency of obtained conditions. Therefore, in the future work, we will be devoted to solving this topic from other perspectives.

In the following discussion, we will process the quantization output error effects. Based on Lemma 4, a significant conclusion is obtained.

Lemma 7. *Suppose uncertainty system* (3) *and corresponding filter* (5) *with output quantization are given; the filtering error system* (10)-(11) *is stable with generalized H_2 norm bound γ if there exist matrices $G(\theta)$, $N(\theta)$, and $S(\theta)$, symmetric matrix $P(\theta) > 0$, and scalars $\zeta_1(\theta) > 0, \zeta_2(\theta) > 0$ such that the following matrix inequalities hold:*

$$\begin{bmatrix} -P(\theta) & * & * & * & * \\ 0 & -I & * & * & * \\ \widehat{C}(\theta) & \widehat{D}(\theta) & -\gamma^2 I & * & * \\ 0 & 0 & M_D^T & -\zeta_2(\theta) I & * \\ \zeta_2(\theta) N_C(\theta) & \zeta_2(\theta) N_D(\theta) & 0 & 0 & -\zeta_2(\theta) I \end{bmatrix} < 0, \tag{35}$$

$$\begin{bmatrix} -P(\theta)+He\left\{G(\theta)\widehat{A}(\theta)\right\} & * & * & * & * \\ N(\theta)\widehat{A}(\theta)+\widehat{B}^T(\theta)G^T(\theta) & -I+He\left\{N(\theta)\widehat{B}(\theta)\right\} & * & * & * \\ S(\theta)\widehat{A}(\theta)-G^T(\theta) & S(\theta)\widehat{B}(\theta)-N^T(\theta) & P^+(\theta)-He\{S(\theta)\} & * & * \\ M_B^T G^T(\theta) & M_B^T N^T(\theta) & M_B^T S^T(\theta) & -\zeta_1(\theta)I & * \\ \zeta_1(\theta)\delta N_C(\theta) & \zeta_1(\theta)\delta N_D(\theta) & 0 & 0 & -\zeta_1(\theta)I \end{bmatrix} < 0. \quad (36)$$

Proof. Assume that the conditions in Lemma 4 are satisfied, and take the quantized error into account; that is,

$$\widetilde{\Lambda}(\theta)=\widehat{\Lambda}(\theta)+\Delta\widehat{\Lambda}(\theta), \quad \Lambda=A,B,C,D. \quad (37)$$

Then, with the definition of $\Delta\widehat{\Lambda}(\theta)$, inequality (22) can also be rewritten as

$$\begin{aligned} &\begin{bmatrix} -P(\theta)+He\left\{G(\theta)\widehat{A}(\theta)\right\} & * & * \\ N(\theta)\widehat{A}(\theta)+\widehat{B}^T(\theta)G^T(\theta) & -I+He\left\{N(\theta)\widehat{B}(\theta)\right\} & * \\ S(\theta)\widehat{A}(\theta)-G^T(\theta) & S(\theta)\widehat{B}(\theta)-N^T(\theta) & P^+(\theta)-S(\theta)-S^T(\theta) \end{bmatrix} \\ &+He\left\{\begin{bmatrix} G(\theta)M_B \\ N(\theta)M_B \\ S(\theta)M_B \end{bmatrix}\Delta(k)\begin{bmatrix} N_C(\theta) & N_D(\theta) & 0 \end{bmatrix}\right\}<0. \end{aligned} \quad (38)$$

By Lemma 1 and some simple matrix transformations, inequality (38) holds if there exist scalars $\zeta_1(\theta)>0$ such that

$$\begin{aligned} &\begin{bmatrix} -P(\theta)+He\left\{G(\theta)\widehat{A}(\theta)\right\} & * & * \\ N(\theta)\widehat{A}(\theta)+\widehat{B}^T(\theta)G^T(\theta) & -I+He\left\{N(\theta)\widehat{B}(\theta)\right\} & * \\ S(\theta)\widehat{A}(\theta)-G^T(\theta) & S(\theta)\widehat{B}(\theta)-N^T(\theta) & P^+(\theta)-S(\theta)-S^T(\theta) \end{bmatrix} \\ &+\begin{bmatrix} G(\theta)M_B & \delta N_C^T(\theta) \\ N(\theta)M_B & \delta N_D^T(\theta) \\ S(\theta)M_B & 0 \end{bmatrix}\begin{bmatrix} \zeta_1^{-1}(\theta)I & * \\ 0 & \zeta_1(\theta)I \end{bmatrix}\begin{bmatrix} G(\theta)M_B & \delta N_C^T(\theta) \\ N(\theta)M_B & \delta N_D^T(\theta) \\ S(\theta)M_B & 0 \end{bmatrix}^T<0. \end{aligned} \quad (39)$$

In the end, applying Schur complement to (39) and performance congruence transformation by $\mathrm{diag}\{I,I,I,I,\zeta_1(\theta)I\}$, inequality (36) can be obtained. Following a similar development as in the proof of inequality (36), inequality (35) can be established easily from inequality (21) in Lemma 4. □

Remark 8. The quantization error is modeled as a kind of norm-bounded uncertainty in Lemma 7. Notice that quantized error will always exist in a real system; therefore, without loss of generality, we assume that $\|\Delta(k)\|>0$. Then, $\Delta(k)$ can also be expressed as $(\Delta(k)/\delta)\delta$ in (38), where $\Delta(k)/\delta \le I$. Finally, by Lemma 1, it is easy to get (39).

Remark 9. In order to reduce the complexity in the process of obtaining analytic conditions, Lemma 7 has not considered the uncertainties of system matrices. This problem will be discussed in the following part.

With the aid of aforementioned captions, we will deal with the robustness of system (10)-(11) in the back of this section. Before beginning, according to (3), (10), and (11), some definitions are given to facilitate the later development. Let

$$\begin{aligned} \widehat{A}(\theta) &= A^*(\theta)+U(\theta)\Delta_P(k)V(\theta), \\ \widehat{B}(\theta) &= B^*(\theta)+U(\theta)\Delta_P(k)E(\theta), \\ \widehat{C}(\theta) &= C^*(\theta)+(K(\theta)+W(\theta))\Delta_P(k)V(\theta), \\ \widehat{D}(\theta) &= D^*(\theta)+(K(\theta)+W(\theta))\Delta_P(k)E(\theta), \\ N_C(\theta) &= N_C^*(\theta)+T(\theta)\Delta_P(k)V(\theta), \\ N_D(\theta) &= N_D^*(\theta)+T(\theta)\Delta_P(k)E(\theta), \end{aligned} \quad (40)$$

where

$$A^*(\theta) = \begin{bmatrix} A(\theta) & 0 \\ B_F C(\theta) & A_F \end{bmatrix}, \qquad B^*(\theta) = \begin{bmatrix} B(\theta) \\ B_F D(\theta) \end{bmatrix},$$
$$C^*(\theta) = \begin{bmatrix} L(\theta) - D_F C(\theta) & -C_F \end{bmatrix},$$
$$D^*(\theta) = J(\theta) - D_F D(\theta), \qquad N_C^*(\theta) = \begin{bmatrix} C(\theta) & 0 \end{bmatrix},$$
$$N_D^*(\theta) = D(\theta), \qquad U(\theta) = \begin{bmatrix} H(\theta) \\ B_F T(\theta) \end{bmatrix},$$
$$V(\theta) = \begin{bmatrix} F(\theta) & 0 \end{bmatrix}, \qquad W(\theta) = -D_F T(\theta). \tag{41}$$

Then, taking definitions (40) into inequalities (35)-(36) and following the same line as in the proof of Lemma 7, we canestablish the following solution for the robust generalized H_2 filtering analytic conditions.

Theorem 10. *Suppose that uncertain system* (3) *and filter* (5) *matrices are given. Then, the filtering error system* (10)-(11) *is stable with the generalized H_2 disturbance attenuation level bound $\gamma > 0$ and the quantization density $\rho > 0$ if there exist matrices $G(\theta), N(\theta), S(\theta)$, symmetric matrix $P(\theta) > 0$, and constant positive scalars $\zeta_1(\theta), \zeta_2(\theta), \zeta_3(\theta), \zeta_4(\theta)$ such that the following matrix inequalities hold:*

$$\begin{bmatrix} -P(\theta) & * & * & * & * & * & * \\ 0 & -I & * & * & * & * & * \\ C^*(\theta) & D^*(\theta) & -\gamma^2 I & * & * & * & * \\ 0 & 0 & M_D^T & -\zeta_2(\theta) I & * & * & * \\ \zeta_2(\theta) N_C^*(\theta) & \zeta_2(\theta) N_D^*(\theta) & 0 & 0 & -\zeta_2(\theta) I & * & * \\ 0 & 0 & K^T(\theta) + W^T(\theta) & 0 & \zeta_2(\theta) T^T(\theta) & -\zeta_4(\theta) I & * \\ \zeta_4(\theta) V(\theta) & \zeta_4(\theta) E(\theta) & 0 & 0 & 0 & 0 & -\zeta_4(\theta) I \end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} \Psi_{11}(\theta) & * & * & * & * & * & * \\ \Psi_{21}(\theta) & \Psi_{22}(\theta) & * & * & * & * & * \\ \Psi_{31}(\theta) & \Psi_{32}(\theta) & \Psi_{33}(\theta) & * & * & * & * \\ M_B^T G^T(\theta) & M_B^T N^T(\theta) & M_B^T S^T(\theta) & -\zeta_1(\theta) I & * & * & * \\ \zeta_1(\theta) \delta N_C^*(\theta) & \zeta_1(\theta) \delta N_D^*(\theta) & 0 & 0 & -\zeta_1(\theta) I & * & * \\ U^T(\theta) G^T(\theta) & U^T(\theta) N^T(\theta) & U^T(\theta) S^T(\theta) & 0 & \zeta_1(\theta) \delta T^T(\theta) & -\zeta_3(\theta) I & * \\ \zeta_3(\theta) V(\theta) & \zeta_3(\theta) E(\theta) & 0 & 0 & 0 & 0 & -\zeta_3(\theta) I \end{bmatrix} < 0, \tag{43}$$

where

$$\Psi_{11}(\theta) = -P(\theta) + He\{G(\theta) A^*(\theta)\},$$
$$\Psi_{21}(\theta) = N(\theta) A^*(\theta) + B^{*^T}(\theta) G^T(\theta),$$
$$\Psi_{22}(\theta) = -I + He\{N(\theta) B^*(\theta)\},$$
$$\Psi_{31}(\theta) = S(\theta) A^*(\theta) - G^T(\theta),$$
$$\Psi_{32}(\theta) = S(\theta) B^*(\theta) - N^T(\theta),$$
$$\Psi_{33}(\theta) = P^+(\theta) - He\{S(\theta)\}. \tag{44}$$

Therefore, the robust quantized generalized H_2 filter design can be given as follows.

4. Robust Quantized Generalized H_2 Filter Design

In this section, based on Theorem 10, a linearization procedure will be provided to design the generalized H_2 filter in the form of (5). Similar to the method in [5], we assume that

$$G(\theta) = \begin{bmatrix} G_1(\theta) & \lambda_1 G_2 \\ G_3(\theta) & \lambda_2 G_2 \end{bmatrix}, \qquad N(\theta) = \begin{bmatrix} N_1(\theta) & \lambda_5 N_2 G_2 \end{bmatrix},$$
$$S(\theta) = \begin{bmatrix} S_1(\theta) & \lambda_3 G_2 \\ S_3(\theta) & \lambda_4 G_2 \end{bmatrix}. \tag{45}$$

Theorem 11. *Given quantization density $\rho > 0$ and considering the uncertain discrete-time fuzzy system* (3)*, there will be a filter to mitigate the quantization output error effects and to ensure robust stability of filtering error system* (10)-(11) *with prescribed generalized H_2 performance $\gamma > 0$, if there exist matrices $P_{1i}, P_{2i}, P_{3i}, G_{1i}, G_{3i}, S_{1i}, S_{3i}, N_{1i}$, $i = 1, 2, \ldots, r$, $\mathscr{A}, \mathscr{B}, \mathscr{C}, \mathscr{D}$, nonsingular matrix G_2, and positive scalars $\zeta_{1ij}, \zeta_{2ij}, \zeta_{3ij}, \zeta_{4ij}$ for $i, j = 1, 2, \ldots, r$ such that the following inequalities hold:*

$$X_{ii} < 0,$$
$$\Xi_{ijl} < 0,$$
$$\frac{1}{r-1} X_{ii} + \frac{1}{2}\left(X_{ij} + X_{ji}\right) < 0,$$
$$\frac{1}{r-1} \Xi_{iil} + \frac{1}{2}\left(\Xi_{ijl} + \Xi_{jil}\right) < 0, \tag{46}$$

for $i, j, l = 1, 2, \ldots, r$, where

$$X_{ij} = \begin{bmatrix} -P_{1i} & * & * & * & * & * & * & * \\ -P_{2i} & -P_{3i} & * & * & * & * & * & * \\ 0 & 0 & -I & * & * & * & * & * \\ L_j - \mathscr{D}C_j & -\mathscr{C} & J_j - \mathscr{D}D_j & -\gamma^2 I & * & * & * & * \\ 0 & 0 & 0 & -\mathscr{D}^T & -\zeta_{2ij}I & * & * & * \\ \zeta_{2ij}C_j & 0 & \zeta_{2ij}D_j & 0 & 0 & -\zeta_{2ij}I & * & * \\ 0 & 0 & 0 & K_j^T - T_j^T\mathscr{D}^T & 0 & \zeta_{2ij}T_j^T & -\zeta_{4ij}I & * \\ \zeta_{4ij}F_j & 0 & \zeta_{4ij}E_j & 0 & 0 & 0 & 0 & -\zeta_{4ij}I \end{bmatrix},$$

$$\Xi_{ijl} = \begin{bmatrix} Y_{11ij} & * & * & * & * & * & * & * & * \\ Y_{21ij} & Y_{22i} & * & * & * & * & * & * & * \\ Y_{31ij} & Y_{32ij} & Y_{33ij} & * & * & * & * & * & * \\ Y_{41ij} & Y_{42i} & Y_{43ij} & Y_{44il} & * & * & * & * & * \\ Y_{51ij} & Y_{52} & Y_{53ij} & Y_{54il} & Y_{55l} & * & * & * & * \\ \lambda_1\mathscr{B}^T & \lambda_2\mathscr{B}^T & \lambda_5\mathscr{B}^T N_2^T & \lambda_3\mathscr{B}^T & \lambda_4\mathscr{B}^T & -\zeta_{1ij}I & * & * & * \\ \zeta_{1ij}\delta C_j & 0 & \zeta_{1ij}\delta D_j & 0 & 0 & 0 & -\zeta_{1ij}I & * & * \\ Y_{81ij} & Y_{82ij} & Y_{83ij} & Y_{84ij} & Y_{85ij} & 0 & \zeta_{1ij}\delta T_j^T & -\zeta_{3ij}I & * \\ \zeta_{3ij}F_j & 0 & \zeta_{3ij}E_j & 0 & 0 & 0 & 0 & 0 & -\zeta_{3ij}I \end{bmatrix}. \tag{47}$$

$$Y_{11ij} = -P_{1i} + He\left\{G_{1i}A_j + \lambda_1\mathscr{B}C_j\right\},$$

$$Y_{21ij} = -P_{2i} + G_{3i}A_j + \lambda_2\mathscr{B}C_j + \lambda_1\mathscr{A}^T,$$

$$Y_{22i} = -P_{3i} + He\left\{\lambda_2\mathscr{A}\right\},$$

$$Y_{31ij} = N_{1i}A_j + B_j^T G_{1i}^T + \lambda_1 D_j^T\mathscr{B}^T + \lambda_5 N_2\mathscr{B}C_j,$$

$$Y_{32ij} = B_j^T G_{3i}^T + \lambda_2 D_j^T\mathscr{B}^T + \lambda_5 N_2\mathscr{A},$$

$$Y_{33ij} = -I + He\left\{N_{1i}B_j + \lambda_5 N_2\mathscr{B}D_j\right\},$$

$$Y_{41ij} = S_{1i}A_j + \lambda_3\mathscr{B}C_j - G_{1i}^T,$$

$$Y_{42i} = \lambda_3\mathscr{A} - G_{3i}^T,$$
$$Y_{43ij} = S_{1i}B_j + \lambda_3\mathscr{B}D_j - N_{1i}^T,$$

$$Y_{44il} = P_{1l} - S_{1i} - S_{1i}^T,$$

$$Y_{51ij} = S_{3i}A_j + \lambda_4\mathscr{B}C_j - \lambda_1 G_2^T,$$

$$Y_{52} = \lambda_4\mathscr{A} - \lambda_2 G_2^T,$$

$$Y_{53ij} = S_{3i}B_j + \lambda_4\mathscr{B}D_j + \lambda_5 G_2^T N_2^T,$$

$$Y_{54il} = P_{2l} - S_{3i} - \lambda_3 G_2^T,$$

$$Y_{55l} = P_{3l} - \lambda_4 G_2 - \lambda_4 G_2^T,$$

$$Y_{81ij} = H_j^T G_{1i}^T + \lambda_1 T_j^T\mathscr{B}^T,$$
$$Y_{82ij} = H_j^T G_{3i}^T + \lambda_2 T_j^T\mathscr{B}^T,$$

$$Y_{83ij} = H_j^T N_{1i}^T + \lambda_5 T_j^T\mathscr{B}^T N_2^T,$$

$$Y_{84ij} = H_j^T S_{1i}^T + \lambda_3 T_j^T\mathscr{B}^T,$$

$$Y_{85ij} = H_j^T S_{3i}^T + \lambda_4 T_j^T\mathscr{B}^T, \tag{48}$$

then, a suitable filter with given generalized H_2 performance $\gamma > 0$ can be designed by

$$A_F = G_2^{-1}\mathscr{A}, \qquad B_F = G_2^{-1}\mathscr{B},$$
$$C_F = \mathscr{C}, \qquad D_F = \mathscr{D}. \tag{49}$$

Proof. From Theorem 10, we can design the robust generalized H_2 filter if inequalities (42)-(43) are solvable. To obtain the design conditions of filter (5), one assumes that the matrices variables involved in (43) have the following forms:

$$P(\theta) = \sum_{i=1}^{r} h_i(\nu(k)) P_i = \sum_{i=1}^{r} h_i(\nu(k)) \begin{bmatrix} P_{1i} & * \\ P_{2i} & P_{3i} \end{bmatrix},$$

$$G(\theta) = \sum_{i=1}^{r} h_i(\nu(k)) G_i = \sum_{i=1}^{r} h_i(\nu(k)) \begin{bmatrix} G_{1i} & \lambda_1 G_2 \\ G_{3i} & \lambda_2 G_2 \end{bmatrix},$$

$$N(\theta) = \sum_{i=1}^{r} h_i(\nu(k)) N_i = \sum_{i=1}^{r} h_i(\nu(k)) \begin{bmatrix} N_{1i} & \lambda_5 N_2 G_2 \end{bmatrix},$$

$$S(\theta) = \sum_{i=1}^{r} h_i(\nu(k)) S_i = \sum_{i=1}^{r} h_i(\nu(k)) \begin{bmatrix} S_{1i} & \lambda_3 G_2 \\ S_{3i} & \lambda_4 G_2 \end{bmatrix}, \tag{50}$$

and let

$$\mathscr{A} = G_2 A_F, \qquad \mathscr{B} = G_2 B_F,$$
$$\mathscr{C} = C_F, \qquad \mathscr{D} = D_F. \tag{51}$$

Then, with the aforementioned related matrices considered, substituting the above matrices into inequalities (42)-(43)

and utilizing Lemma 3, Theorem 11 can be obtained. The proof is completed. □

Remark 12. $\lambda_{\{1,2,3,4,5\}}, N_2$ are free slack scalar variables, which will be useful to reduce the conservatism of the special forms of $G(\theta)$, $N(\theta)$, and $S(\theta)$ by providing extra free dimensions in the solution space for Theorem 11.

Remark 13. In the practical applications, quantization error bound δ can be calculated according to the given quantization density ρ. Then, inequalities (46) are strictly LMIs that can be easily and effectively solved via LMI control toolbox [40].

5. Numerical Example

In this section, we use an example to demonstrate the applicability and effectiveness of the proposed approach. Consider a tunnel diode circuit, whose fuzzy discrete modeling was done in [41] with a sampling time $T = 0.02$. Now the discrete-time fuzzy system considered has two rules:

$$\begin{gathered}
\text{Plant rules:} \quad \text{Define } x(k) = [x_1(k) \;\; x_2(k)]^T. \\
\text{If } x_1(k) \text{ is } M_1, \\
\text{then } \begin{cases} x(k+1) = \left(A_1 + H_1 \Delta_p(k) F_1\right) x(k) \\ \qquad + \left(B_1 + H_1 \Delta_p(k) E_1\right) \omega(k), \\ y(k) = \left(C_1 + T_1 \Delta_p(k) F_1\right) x(k) \\ \qquad + \left(D_1 + T_1 \Delta_p(k) E_1\right) \omega(k), \\ z(k) = \left(L_1 + K_1 \Delta_p(k) F_1\right) x(k) \\ \qquad + \left(J_1 + K_1 \Delta_p(k) E_1\right) \omega(k). \end{cases} \\
\text{If } x_1(k) \text{ is } M_2, \\
\text{then } \begin{cases} x(k+1) = \left(A_2 + H_2 \Delta_p(k) F_2\right) x(k) \\ \qquad + \left(B_2 + H_2 \Delta_p(k) E_2\right) \omega(k), \\ y(k) = \left(C_2 + T_2 \Delta_p(k) F_2\right) x(k) \\ \qquad + \left(D_2 + T_2 \Delta_p(k) E_2\right) \omega(k), \\ z(k) = \left(L_2 + K_2 \Delta_p(k) F_2\right) x(k) \\ \qquad + \left(J_2 + K_2 \Delta_p(k) E_2\right) \omega(k), \end{cases}
\end{gathered} \tag{52}$$

where

$$\begin{gathered}
A_1 = \begin{bmatrix} 0.9887 & 0.9024 \\ -0.018 & 0.81 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.0093 \\ 0.0181 \end{bmatrix}, \\
C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D_1 = 1, \qquad L_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
J_1 = 0.3, \qquad H_1 = \begin{bmatrix} 0.54 \\ -0.15 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} -0.15 & -0.65 \end{bmatrix}, \\
E_1 = 0.38, \qquad T_1 = -0.78, \qquad K_1 = -0.79, \\
A_2 = \begin{bmatrix} 0.90337 & 0.8617 \\ -0.0172 & 0.8103 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.0091 \\ 0.0181 \end{bmatrix}, \\
C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D_2 = 1, \qquad L_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
J_2 = 0.5, \qquad H_2 = \begin{bmatrix} 0.98 \\ -0.02 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.01 & -0.45 \end{bmatrix}, \\
E_2 = 0.04, \qquad T_2 = 0.26, \qquad K_2 = 0.83,
\end{gathered} \tag{53}$$

with membership function assumed in the following:

$$h_1 = \begin{cases} \dfrac{x_1(k) + 3}{3}, & -3 \le x_1(k) \le 0 \\ 0, & x_1(k) \le -3 \\ \dfrac{3 - x_1(k)}{3}, & 0 \le x_1(k) \le 3 \\ 0, & x_1(k) > 3 \end{cases} \tag{54}$$

and $h_2 = 1 - h_1$.

Then, given quantization density $\rho_1 = \rho_2 = 0.98$, the minimum generalized H_2 performance level with fixed parameters $\lambda_1 = 0.73, \lambda_2 = 0.35, \lambda_3 = 0.78, \lambda_4 = 0.44$, $\lambda_5 = -0.004$, and $N_2 = [0.37 \;\; 0.08]$ by using Matlab LMI control toolbox to solve the convex optimization problem in Theorem 11 is $\gamma^* = 0.4424$. And the corresponding filter parameters are

$$\begin{gathered}
A_F = \begin{bmatrix} 1.0647 & 1.0365 \\ -0.0606 & 0.7741 \end{bmatrix}, \qquad B_F = \begin{bmatrix} 0.0221 \\ -0.105 \end{bmatrix}, \\
C_F = \begin{bmatrix} -0.0999 & -0.0513 \end{bmatrix}, \qquad D_F = 0.3201.
\end{gathered} \tag{55}$$

To verify the effectiveness of solved filter, the external disturbance is defined as $\omega(k) = e^{-0.05}(4k), k = 1, 2, \ldots$, and the initial conditions are chosen as $x(0) = [0 \;\; 0]^T, x_F(0) = [0 \;\; 0]^T$. By considering $\Delta_P(k) = \sin(0.1k)$, Figure 1 shows the state responses of the plant. Define $\Delta(k) = \sin(0.5k)$ for $k = 1, 2, \ldots$; the simulation results of $z(k), z_F(k)$, and $x_F(k)$ are given in Figures 2 and 3, respectively. Finally, from Figure 4, we can see that the designed filter meets the prescribed requirements.

6. Conclusions

In this paper, the problem of robust quantized generalized H_2 filtering for uncertain discrete-time fuzzy systems has been studied. Based on the fuzzy Lyapunov function, a less conservative approach is exploited to derive sufficient conditions for designing a robust filter that guarantees a generalized H_2 performance and mitigates the output quantization measurement error simultaneously. Moreover, such filter can be obtained easily by solving a set of linear matrix

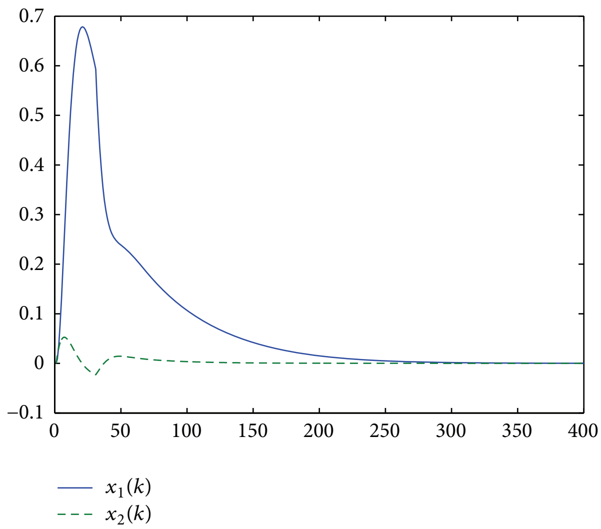

FIGURE 1: State response of $x(k)$.

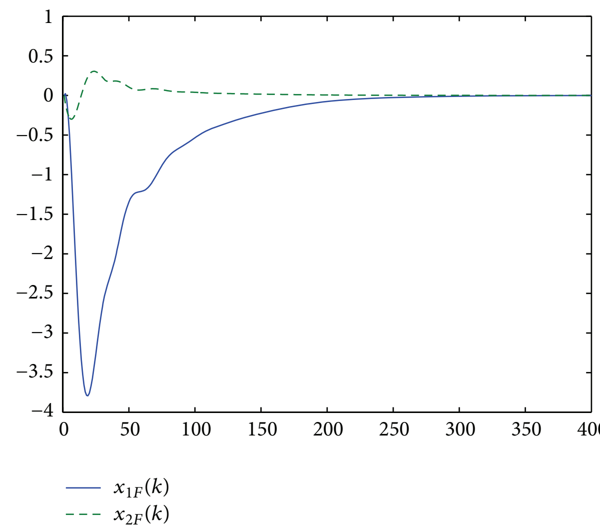

FIGURE 3: Response of $x_F(k)$.

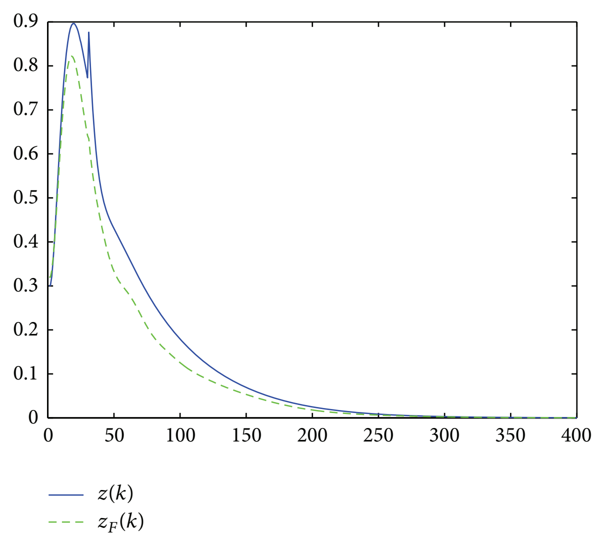

FIGURE 2: Response of $z(k)$ and $z_F(k)$.

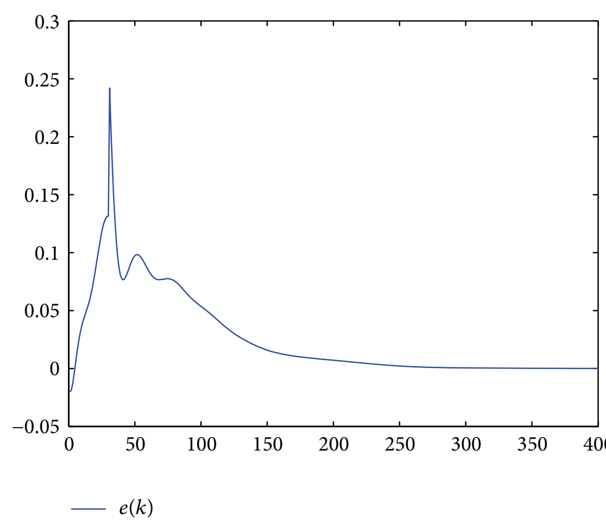

FIGURE 4: Error response of $e(k)$.

inequalities. Finally, a simulation example has been given to illustrate the successful application of the proposed method.

Conflict of Interests

The authors declared that they have no conflict of interests related to this work.

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