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Research Letter

Flow Characteristics of Gas and Liquid through a Cell Porous Disk

Yasuyuki Hirano, Koichi Takeda, Testuya Yano, Seiji Kumagai, and Seiichi Sudo

Department of Machine Intelligence and System Engineering, Akita Prefectural University, Yurihonjo 015-0055, Japan

Correspondence should be addressed to Yasuyuki Hirano, d07s004@akita-pu.ac.jp

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This paper describes applications of cell porous materials. The authors investigated the flow characteristics of a gas-liquid mixture in a rotating porous disk. For theoretical analyses, the gas is assumed to permeate the entire disk surface. A simple one-dimensional model illustrates that the residence time of the liquid is much greater than that of the gas. Violent interaction in small cells is likely to enhance the chemical reaction between gas and liquid. Cell porous materials might also be exploited for chemical reaction purposes.

1. INTRODUCTION

Applications of porous materials have been investigated in various fields. The Institute of Theoretical and Applied Mechanics investigated gas flow around and inside the cell porous rotating disk [1, 2]. Simple one-dimensional (1D) stationary model of the flow based on Darcy's model [3] revealed that highly permeable cell porous rotors made of metal or ceramics are promising for gas transport use [4].

This study examines the use of rotating porous materials for chemical reaction between a gas and a liquid. Chemical reactions are enhanced by increasing interface and mixing. If a liquid is introduced into a rotating porous disk, it moves to the periphery, splitting into small droplets [5]. The gas and the small droplets interact violently in cell porous materials. These peculiarities may be effectively exploited in a chemical reaction between a gas and a liquid.

Theoretical analyses of gas-liquid mixture flow characteristics in the porous rotating disk are carried out using a simplified 1D stationary model. The aim of the present work is to estimate the residence times of the gas and the liquid in the disk.

2. MATHEMATICAL MODELS

2.1. Scheme of a model and assumptions

The configuration considered in this study is axisymmetric, as shown in Figure 1. In this figure, r_o and D, respectively, represent the disk radius and its width. The rotating disk is sealed from the bottom faceplate. The ambient gas enters the porous disk through its upper surface. At the periphery, the gas moves in a radial direction with nonzero angular velocity. Near the rotation axis, the liquid is supplied at a certain flow rate to a small hole with radius r_i.

In this model, the packed density of the porous disk material is assumed to be large. Under that assumption, it might be reasonable that the tangential velocity u is nearly equal to the rotating velocity $r\omega$.

The 1D stationary momentum equation inside the rotating porous disk [4] can be approximated as

$$v\frac{\partial v}{\partial r} - r\omega^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} - kv^2, \qquad (1)$$

where v is the radial velocity component, p is the pressure, ρ is the fluid density, ω is the angular velocity of disk rotation, and r is the radius in cylindrical coordinates. The last term is presumably proportional to the square of the gas velocity relative to the disk with drag force coefficient k [6].

It is known that the difference between tangential velocities of the gas and the solid can be neglected when $k \gg 1$ (this condition is satisfied in our model as shown in Section 3.2) [4].

The characteristic flow velocities are smaller than the sound velocity, which allows for neglecting the gas compressibility effects. In this model, the gas velocity is known to be

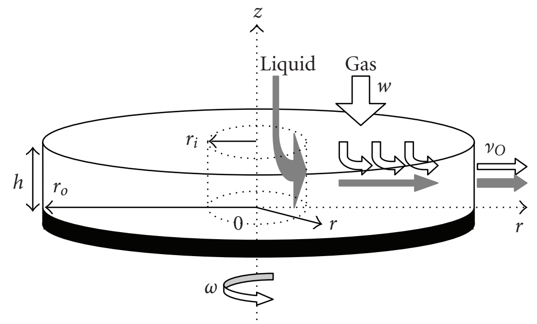

FIGURE 1: Scheme showing gas and liquid flows inside the rotating porous disk.

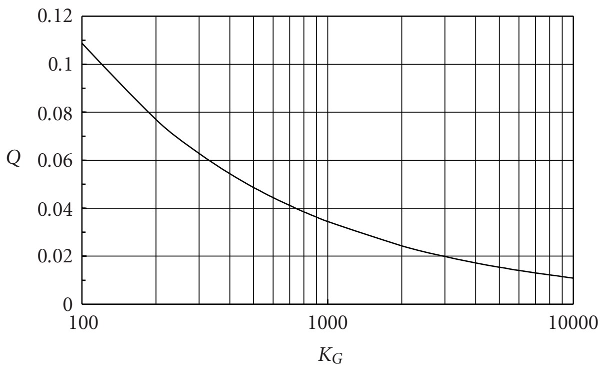

FIGURE 2: Nondimensional relation between the drag force coefficient of gas, K_Gd, and the flow rate, Q, for $H = 0.1$.

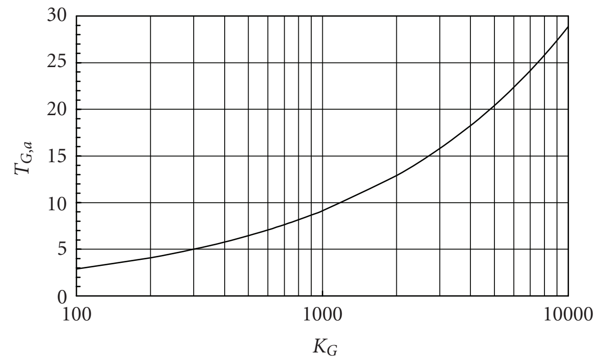

FIGURE 3: Relation between the drag force coefficient of gas, K_G, and the mean residence time of gas, $T_{G,a}$, for $H = 0.1$.

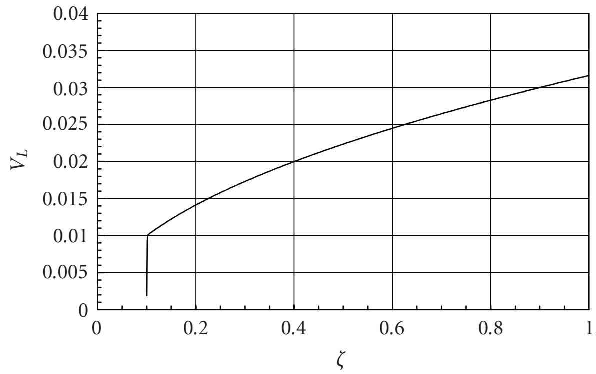

FIGURE 4: Variation of radial component of liquid velocity, V_L, versus nondimensional radius, ζ.

uniform everywhere at the upper surface of the rotating disk [4].

Within the framework of the 1D model, the continuity equation of the gas is written as

$$\frac{1}{r}\frac{\partial}{\partial r} r v_G - \frac{w}{h} = 0. \tag{2}$$

On the other hand, the liquid is treated as a cloud of small droplets. The mass density of the cloud ($\phi\rho_L$) is governed by the following equation:

$$\frac{\partial}{\partial r}\left(\phi\rho_L r v_L\right) = 0. \tag{3}$$

2.2. Mathematical model for the gas

The residence time of the gas is studied in this section.

When $r_i \approx 0$, in Figure 1, (2) can be simplified to

$$v_G(r) = \frac{rw}{2h}. \tag{4}$$

The equation of pressure at $r = r_o$ is

$$p_o + \frac{\rho_G}{2}\left(v_o^2 + u_o^2\right) = p_\infty, \quad p_o = p(r_o). \tag{5}$$

In reality, the sum of kinetic energy and static pressure in the left-hand side of (5) exceeds the static pressure of stagnating gas far from the disc p_∞ [2, 4]. In this model, equality in (5) is assumed; that is, pressure loss is neglected outside the disk.

By solving (1), (4), and (5), $p(r)$ is written as

$$p(r) = \frac{\rho_G k_G}{3}\left(\frac{w}{2h}\right)^2\left(r_o^3 - r^3\right) - \frac{\rho_G}{2}\left\{\left(\frac{w}{2h}\right)^2 - \omega^2\right\}r^2 - \rho_G\omega^2 r_o^2 + p_\infty. \tag{6}$$

The equation of pressure at $r = r_i$ can be written as follows:

$$p_i + \frac{\rho_G w^2}{2} = p_\infty, \quad p_i = p(r_i). \tag{7}$$

Substituting (7) into (6) gives

$$w = 2\omega h\sqrt{\frac{3\left(2r_o^2 - r_i^2\right)}{2k_G\left(r_o^3 - r_i^3\right) + 3\left(4h^2 - r_i^2\right)}} \approx 2\omega h r_o\sqrt{\frac{3}{k_G r_o^3 + 6h^2}} \quad (\text{for } r_i \longrightarrow 0). \tag{8}$$

When r_i is small, the gas flow rate is represented as

$$q = 2\pi r_o^3\omega h\sqrt{\frac{3}{k_G r_o^3 + 6h^2}}. \tag{9}$$

Using the following relation:

$$v_G(r) = \frac{dr}{dt}, \tag{10}$$

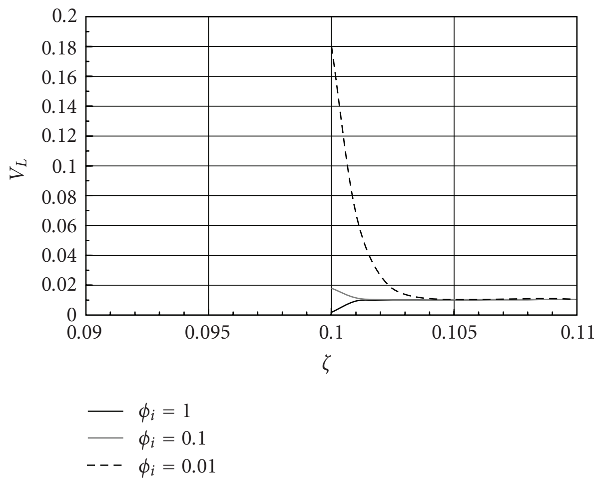

FIGURE 5: Velocity variations for various values of ϕ_i with enlarged ζ.

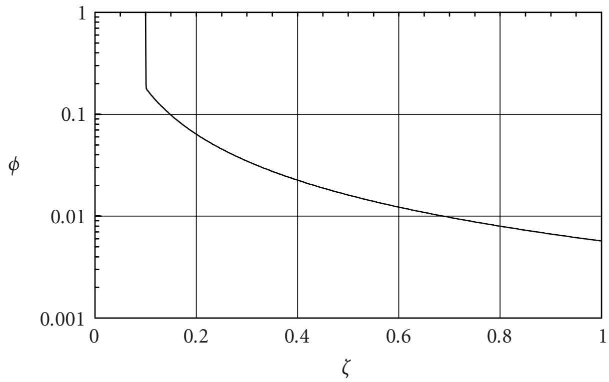

FIGURE 6: Nondimensional density of cloud, ϕ, versus nondimensional radius, ζ, for $K_L = 1000$, $H = 0.1$, $M = 10^{-4}$, $\zeta_i = 0.1$, and $\phi_i = 1$.

with (4), the residencetime of the moving gas from r to r_o is written as

$$\tau_G(r) = \int_r^{r_o} \frac{1}{\nu_G} dr = \frac{h}{w} \log \frac{r_o}{r}. \tag{11}$$

The volume element of gas coming to the disk between r and $r + dr$ is expressed as

$$dq = 2\pi w r dr. \tag{12}$$

Therefore, the average residence time of the gas is represented as

$$\tau_{G,a} = \frac{1}{q} \int \tau_G(r) dq, \tag{13}$$

where q is the total flow rate as

$$q = \int_r^{r_o} dq. \tag{14}$$

Equation (13) can be rewritten by

$$\tau_{G,a} = \frac{\pi h r_o^2}{q}. \tag{15}$$

2.3. Mathematical model for liquid

In this section, the residence time of the liquid is considered. The liquid is assumed to be a cloud of droplets with volume fraction ϕ. The continuity of mass is represented as

$$m = 2\pi h \phi \rho_L r \nu_L. \tag{16}$$

where m, ρ_L, and ν_L denote the feeding rate of the liquid, the density of the liquid, and the radial component of the liquid velocity, respectively.

The momentum equation can be expressed by the following relationship:

$$\nu_L \frac{\partial \nu_L}{\partial r} = -k_L \nu_L^2 + r\omega^2. \tag{17}$$

In (17), liquid motion is assumed to be affected only by the interactive force with the rotating disk because the force from the surrounding gas is small.

Using the boundary condition, we obtain the following:

$$m = 2\pi h \phi_i \rho_L r_i \nu_L(r_i). \tag{18}$$

Equation (19) is obtained by solving (17) which is a linear equation in ν^2. The liquid velocity is obtained as

$$\nu_L(r) = \left[\frac{\omega^2}{k_L}\left(r - \frac{1}{2k_L}\right) + \left\{ \left(\frac{m}{2\pi r_i h \phi \rho_L}\right)^2 - \frac{\omega^2}{k_L}\left(r_i - \frac{1}{2k_L}\right) \right\} e^{-2k_L(r-r_i)} \right]^{1/2}. \tag{19}$$

Similarly, the residence time of the liquid is represented as

$$\tau_L = \int_{r_i}^{r_o} \frac{1}{\nu_L(r)} dr. \tag{20}$$

It is convenient to introduce nondimensional variables:

$$\zeta = \frac{r}{r_o}, \quad \zeta_i = \frac{r_i}{r_o}, \quad W = \frac{w}{\omega r_o}, \quad V = \frac{\nu}{\omega r_o}, \quad H = \frac{h}{r_o}, \quad K = k r_o, \quad Q = \frac{q}{\omega r_o^3}, \quad T = \omega \tau, \quad M = \frac{m}{\omega \rho_L r_o^3}. \tag{21}$$

Equations (9), (15), (16), (19), and (20) can then be expressed as (22), (25), (26), (23), and (24), respectively:

$$Q = 2\pi H \sqrt{\frac{3}{K_G + 6H^2}}, \tag{22}$$

$$T_{G,a} = \frac{\pi H}{Q}, \tag{23}$$

$$\phi = \frac{M}{2\pi H \zeta V_L}, \tag{24}$$

$$V_L = \left[\frac{1}{K_L}\left(\zeta - \frac{1}{2K_L}\right) + \left\{ \left(\frac{M}{2\pi \zeta_i H \phi}\right)^2 - \frac{1}{K_L}\left(\zeta_i - \frac{1}{2K_L}\right) \right\} e^{-2K_L(\zeta-\zeta_i)} \right]^{1/2}, \tag{25}$$

$$T_L = \int_{\zeta_i}^{1} \frac{1}{V_L(\zeta)} d\zeta. \tag{26}$$

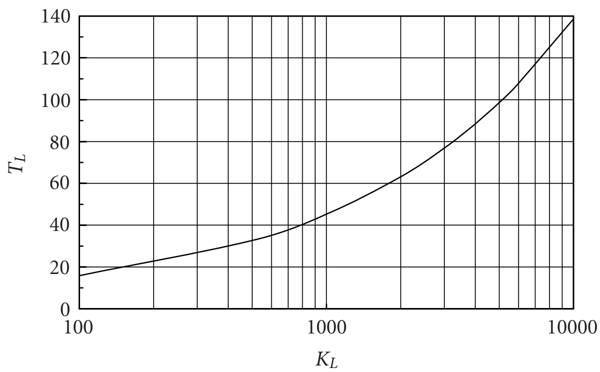

FIGURE 7: Residence time of liquid, T_L, versus the drag force coefficient of water, K_L, evaluated for $H = 0.1$, $M = 10^{-4}$, and $\phi_i = 1$. The inlet position of liquid $\zeta_i = 0.1$.

3. NUMERICAL SOLUTIONS, EXPERIMENTS, AND DISCUSSION

3.1. Numerical solutions

Figure 2 shows the nondimensional relation between the drag force coefficient of gas and the gas flow rate for $H = 0.1$. The flow rate is determined by K_G. As shown in (9), the flow rate is proportional to the angular velocity ω. The residence time is directly related to $1/\omega$.

The variation of nondimensional residence time is illustrated as a function of K_G in Figure 3 for $H = 0.1$.

Figure 4 depicts the variation of the radial component of liquid velocity with the nondimensional radius evaluated for $K_L = 1000$, $H = 0.1$, $M = 10^{-4}$, $\zeta_i = 0.1$, and $\phi_i = 1$.

Velocity change occurs steeply in a narrow range near ζ_i.

Figure 5 portrays velocity variations for various values of ϕ_i with enlarged ζ; the figure is greatly exaggerated for convenient representation. The right-hand side of (17) is positive; the liquid is accelerated there. On the contrary, if it is negative, the liquid is decelerated. Variation of the volume fraction along ζ is shown in Figure 6.

As portrayed in Figure 5, the change of initial ϕ_i has no influence, except in a very narrow region near ζ_i.

Equation (26) is calculated as a function of K_L; its results are shown in Figure 7.

Comparison of Figures 2 and 7 reveals that the residence time of liquid is approximately five times longer than that of gas for $K_L = 100 - 1000$.

3.2. Experiments

Experiments were carried out using the cell porous disk of stainless steel as shown Figure 1. Its packed density was 440 kg m^{-3}. The outer and inner diameters were 0.15 m and 0.02 m, respectively. The width was 0.027 m. The frequency of disk rotation was stable and equal to 3000 rpm.

The gas flow rate of 0.0297 m^{3}s^{-1} was obtained by the method of JIS (Japanese industrial standard no. B8330). Drag force coefficient of the gas and its resistance time were estimated to be $K_G = 300$ and $T_{G,a} = 5$ from (22) and (23), respectively.

The measurement of the residence time for liquid was conducted. The water was injected at $r_i = 0.01$ m instantaneously and it came out at the edge of the disk after certain time delay. The residence time of liquid, $T_L = 200$, was obtained from the observation of the time delay by high-speed video camera (FASTCAM ultima SE, Photron). The value of the drag force coefficient for the liquid was estimated to be $K_L = 23000$ from (25).

Results of experiments revealed that K_L is much larger than K_G. Therefore, the ratio of $T_L/T_{G,a}$ becomes much larger in a real rotating disk system.

4. CONCLUSION

Considering the application of cell porous material to a chemical reactor, the authors investigated gas-liquid mixture flow characteristics in a rotating porous disk for use in reactors.

The gas is assumed to be soaked through the entire disk surface. The theoretical results for the simplified 1D model indicate that the residence time of the gas in the disk is inversely related to the angular velocity.

On the other hand, the residence time of liquid is much longer than that of gas. The residence time of liquid injected into the disk near the center axis is also inversely related to the angular velocity, but it is almost independent of the initial volume fraction.

The residence time of the liquid is much longer than that of the gas. The relation of $T_L/T_G \gg 1$ indicates that a rotating cell porous disk system is suitable for the reaction of a small-volume liquid with a large-volume gas.

NOMENCLATURE

h: Porous disk width (m)
H: Nondimensional porous disk width
k: Drag force coefficient (m^{-1})
K: Nondimensional drag force coefficient
m: Feeding rate of liquid (m s^{-1})
M: Nondimensional feeding rate of liquid
q: Gas flow rate (m^{3}s^{-1})
Q: Nondimensional gas flow rate
r: Radial position coordinate (m)
T: Nondimensional time
u: Tangential velocity component (m s^{-1})
v: Radial velocity component (m s^{-1})
V: Nondimensional radial velocity component
w: Axial component of gas velocity at the upper surface of the porous disk (m s^{-1})
z: Axial coordinate

GREEK SYMBOLS

ζ: Nondimensional radial position coordinate
ρ: Density (kg m^{-3})
τ: Residence time (s)
ω: Angular velocity of porous disk (rad s^{-1})
ϕ: Volume fraction of liquid

SUBSCRIPTS

a: Average
G: Gas
i: Value at inner radius
L: Liquid
O: Value at outer radius

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