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Research Article

Cooperative Communication in Cognitive Radio Networks under Asymmetric Information: A Contract-Theory Based Approach

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By exploiting the spatial diversity of multiple wireless nodes, cooperative communication technique is a promising technique for spectrum sharing to improve spectrum efficiency. In this paper, the incentive issue between relay nodes' (RNs') service and source's relay selection is investigated in the presence of the asymmetric information scenario. Multiuser cooperative communication is modelled as a labour market, where the source designs a contract and each relay node decides to select a contract item according to hidden information in order to obtain the best profit. The optimal contract design under both symmetric information and asymmetric information is presented based on contract theory. The contract-theoretic model for ability discrimination relay selection is formulated as an optimization problem to maximize the source's utility. A sequential optimization algorithm is proposed to obtain the optimal *relay-reward* strategy. Simulation results show that the optimal contract design scheme is effective in improving system performance for cooperative communication. This paper establishes a valuable cooperative communication mechanism in cognitive radio networks.

1. Introduction

Due to the steadily increasing number of wireless devices and applications, the demand for wireless spectrum has increased dramatically. However, a great number of licensed spectra are not effectively utilized, resulting in spectrum wastage [1]. To cope with such a dilemma, cognitive radio [2, 3] has been introduced by enabling the secondary users to opportunistically use the vacant spectrum unharmfully, which is assigned to the primary users. By obtaining spatial diversity and combating detrimental effects of wireless channels, cooperative communications technique [4–6] is considered as an effective method to improve spectrum efficiency.

Designing a cooperative communication mechanism in cognitive radio networks (CRNs) is considerably challenging. First, relay nodes (RNs) are selfish [7] and may compete for the limited spectrum resources (e.g., battery, power, and bandwidth) and only aim at maximizing their own benefit [8]. Thus, the potential relays may not be willing to cooperate without any additional incentives, which bring about a much more challenging problem to cooperative communication. Secondly, various relay selection algorithms require near-complete network information to select potential RNs effectively. However, due to the mobility of wireless users and the effects of shadowing and fading of the wireless channels, network information (e.g., locations, channel conditions, and QoS requirements) may not be available to all users [9]. Moreover, this network information may belong to users privately and users may not be willing to share this information, which results in asymmetry information between the source and RNs [10]. In this paper, we intentionally concentrate on robust cooperative communication mechanism to address these challenging issues.

The above cooperative incentive issues in relay selection have been investigated recently, the most often used being the auction mechanism [11–13]. However, when PUs own spectrum demands are high or the condition of the source's wireless channel is poor, there will be hardly any spectrum left for auction. Therefore, in this study, we intentionally concentrate on an alternative approach, based on contracts, towards the cooperative communication between one source and multiple RNs under asymmetric information scenario. Contract theory [14, 15] investigates how to design the mutually agreeable contract among economic players in the presence of asymmetric or incomplete information scenarios [16]. A principal-agent model for the source and RNs is utilized, where the source acts as the principal and each SU is an agent [17]. Contract-based solutions have been suggested for cooperative systems that are either resource exchange based, integrated contraction based, profit incentive based, or dynamic trading based [9, 18–20]. Unlike the existing literature, in this paper, considering the different ability of RNs, the source pays different basic wage to RNs for their different relay efforts. Moreover, the source offers RNs a fixed bonus coefficient related to relay performance in order to motivate them to work hard. Furthermore, potential RNs are classified into multiple user types according to their hidden information (e.g., channel condition, battery technology). We refer to this as ability discrimination relay selection.

In this paper, the incentive issue between RNs' relay service and source's relay selection is exploited in CRNs and an efficient contract-theoretic model for ability discrimination relay selection is developed under asymmetric information scenario. The main contributions of this paper are as follows:

(i) By exploiting the *cooperation mechanisms* and design challenges in cooperative communication, the contract-theoretic model for ability discrimination relay selection is proposed to describe collaborative schemes in CRNs. A parameter named *bonus ratio* is introduced in this model to motivate RNs to work hard. RNs' basic wage paid by the source is various with their different relay efforts. And multiple RNs are classified into different types according to their hidden information.

(ii) On the shoulder of contract theory, the optimal contract design in the presence of both symmetric information and asymmetric information is presented. Under symmetric information, the optimal contract is feasible if and only if it is individually rational (IR) for each RN. And, under asymmetric information, the necessary and sufficient conditions for a contract to be incentive compatible (IC) and IR are systematically characterized.

(iii) To effectively select potential RNs to participate in cooperative communication, an optimization problem is formulated, which maximizes the source's utility while meeting the IC and IR conditions of each RN. A sequential optimization algorithm is proposed to obtain the optimal *relay-reward* strategy. The performance of optimal contract-based cooperative communication mechanism is demonstrated through simulations.

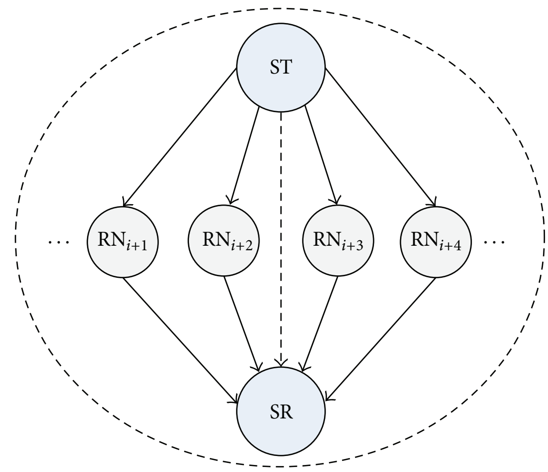

Figure 1: Cognitive radio network.

The remainder of the paper is organized as follows. The system model and problem formulation for contract-based CSS is introduced in Section 2. Then, the optimal contract designs under both symmetric information and asymmetric information are presented in Sections 3 and 4, respectively. Numerical simulation results are given and discussed in Section 5, and Section 6 concludes the paper.

2. System Model and Problem Formulation

Figure 1 shows a typical CRN with a particular wireless node acting as a source and multiple RNs. The source consists of a source transmitter (ST) and a source receiver (SR). In this cooperative communication scheme, the distributed space-time-coded protocol [21] is considered and relay selection is conceptually like the labor market. The employer, the source, recruits some RNs to cooperatively relay the traffic at high power levels, which is against RNs interests. And the employee, RN, chooses one of the contract items to maximize his/her utility. To deal with the problem of conflicting objectives between the source and RNs, a contract with several different items related to different combinations of effort level (e.g., relay power) and salary is utilized. With proper choice of space-time codes, RNs' simultaneous relay signals do not interfere with each other at the source receiver (SR). By motivating RNs to truthfully reveal their hidden information, not only can the source enjoy a significant throughput gain due to the cooperation, but also the RNs obtain certain reward, resulting in a win-win situation.

2.1. Source Modeling. In this subsection, the source model related to RNs' relay powers p_i, $i \in I$, and source's reward allocations is considered. The source's total achievable rate due to cooperative relay of the RNs [21] can be written as

$$R_S = \log\left(1 + \sum_{i\in I} \frac{p_i}{n_0}\right), \quad (1)$$

where p_i is the ith RN's transmitting power at SR side and n_0 is the noise power.

Without loss of generality, the payment w_i to the ith RN with a linear sharing scheme [22] is defined as

$$w_i = \alpha_i + \beta p_i, \quad (2)$$

where α_i is the basic wage of the ith RN and β ($\beta \in [0,1]$) is the performance-based bonus coefficient. RNs obtain the different basic wage due to the different ability.

Then, the source's objective is to design a contract to maximize its utility as follows:

$$\begin{aligned} U_S &= \rho R_S - \sum_{i=1}^{N} w_i \\ &= \rho \log\left(1 + \sum_{i\in I} \frac{p_i}{n_0}\right) - \sum_{i\in I} (\alpha_i + \beta p_i), \end{aligned} \quad (3)$$

where $\rho > 0$ is the equivalent profit per unit channel capacity, which is identical for all the RNs.

2.2. Relay Node Modeling. Considering that the ith RN has the relay channel gain ($h_{\mathrm{ST}_i,\mathrm{SR}}$) between its transmitter ST_i and the source's receiver SR, if the ith RN wants to reach received power p_i at SR, RN needs to transmit with power $p_i/h_{\mathrm{ST,SR}}$. Then, the relay communication cost of the ith RN can be represented as

$$C_i(p_i) = \frac{p_i}{h_{\mathrm{ST,SR}}}\gamma_i, \quad (4)$$

where γ_i is the relay cost per unit transmission power of the ith RN.

To facilitate the following discussions, the ith RN's type is defined as

$$\theta_i = \frac{\gamma_i}{h_{\mathrm{ST,SR}}}, \quad (5)$$

which describes all the hidden information of this RN. Lower θ_i means that the RN has a better relay channel condition (a larger channel gain $h_{\mathrm{ST,SR}}$), or it has a lower relay cost (smaller γ_i).

Then, the ith RN's utility can simply be given by

$$U_{\mathrm{RN}_i} = w_i - C_i(p_i) = \alpha_i + \beta p_i - \theta_i p_i. \quad (6)$$

To facilitate later discussions, by making $V(\theta_i, p_i) = (\theta_i - \beta)p_i$, the ith RN's utility can be defined as

$$U_{\mathrm{RN}_i} = \alpha_i - V(\theta_i, p_i), \quad (7)$$

where $V(\theta_i, p_i)$ is increasing in the private type θ_i and the relay power p_i (received at SR).

2.3. Contract Formulation. In this subsection, the contract mechanism is investigated to resolve the conflicting objectives between the source and RNs in the presence of hidden information. Due to the hidden information of RN's private type, the source needs to design a contract to incentivize the RNs to participate in relay communications to improve the source's utility. The contract items describe the RNs' relay performance and source's relay reward.

Essentially, RN's private type can be divided into two categories: continuous and discrete. Considering the practical application, the contracts can be easily and efficiently broadcasted as a finite number of values. Therefore, in this paper, N RN types are considered, which are denoted by set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$. Without loss of generality, it is assumed that $0 \le \theta_1 < \theta_2 < \cdots < \theta_N$. The total number of type-θ_i RNs is N_i. According to the revelation principle [23], in order to attract the RNs to truthfully reveal their types, it is necessary to design a contract comprised of N contract items, one for each RN type. Then, the contract can be written as $\Phi = \{p_i, \alpha_i, \forall i \in \Omega\}$, where $\Omega = 1, 2, \ldots, N$.

The optimal contract design for the symmetric and asymmetric information scenario is investigated in Sections 3 and 4, respectively. And the symmetric information scenario is considered as a benchmark. Without loss of generality, the values of n_0 are normalized to be 1 for simplification in the following analysis.

3. Optimal Contract Design under Symmetric Information

In the symmetric information scenario, the source knows precisely each RN's type information. The source only needs to make sure that each RN accepts only the contract item designed for its type with nonnegative utility. In particular, the contract needs to satisfy the following IR constraint to ensure that each type-θ_i RN obtains nonnegative utility by accepting the contract item for θ_i:

$$(\mathrm{IR:})\ \ \alpha_i - V(\theta_i, p_i) \ge 0, \quad \forall i \in \Omega. \quad (8)$$

Then, to maximize the source's utility, an optimal contract under symmetric information can be designed as follows:

$$\begin{aligned} \max_{\{\{p_i,\alpha_i\}\ge 0, \forall i\in\Omega\}} \quad & \rho \log\left(1 + \sum_{i\in\Omega} N_i p_i\right) - \sum_{i\in\Omega} N_i (\alpha_i + \beta p_i), \\ \text{s.t.} \quad & \alpha_i - V(\theta_i, p_i) \ge 0, \quad \forall i \in \Omega. \end{aligned} \quad (9)$$

Lemma 1. *To obtain the source's maximum utility, each RN achieves zero utility; that is,* $\alpha_i = V(\theta_i, p_i)$, $\forall i \in \Omega$.

Proof. By contradiction, suppose there exists an optimal contract item (p_i, α_i) with $\alpha_i - V(\theta_i, p_i) > 0$. Since the source's utility in (3) is increasing in p_i and decreasing in α_i, the source can obtain its maximum utility by decreasing α_i until

$\alpha_i - V(\theta_i, p_i) = 0$. This contradicts the above assumption and thus completes the proof. □

Based on Lemma 1, the source's utility maximization problem in (9) can be simplified as

$$\max_{\{p_i \geq 0, \forall i \in \Omega\}} \rho \log\left(1 + \sum_{i\in\Omega} N_i p_i\right) - \sum_{i\in\Omega} N_i \theta_i p_i. \quad (10)$$

Since N_i and p_i always appear together in (10), we can redefine the optimization variable as $\widetilde{p}_i = N_i p_i$ and rewrite (10) as

$$\max_{\{\widetilde{p}_i \geq 0, \forall i \in \Omega\}} \rho \log\left(1 + \sum_{i\in\Omega} \widetilde{p}_i\right) - \sum_{i\in\Omega} \theta_i \widetilde{p}_i. \quad (11)$$

Lemma 2. *To obtain the source's maximum utility, only the contract item for the lowest type θ_1 is positive and all other contract items are zero; that is, $(\widetilde{p}_1, \alpha_1) > 0$ and $(\widetilde{p}_i, \alpha_i) = 0$, $1 < i \leq N$.*

Proof. This theorem can be proved by contradiction. Suppose there exists an optimal contract item with $\widetilde{p}_i > 0$, for $i > 1$ (for type-θ_i RNs).

The source's utility achieved by allocating positive relay power only to the lowest type RNs can be denoted by

$$U_1 = \rho \log(1 + \widetilde{p}_1) - \theta_1 \widetilde{p}_1. \quad (12)$$

Since $0 \leq \theta_1 < \theta_2 < \cdots < \theta_N$, we have $\sum_{i\in\Omega} \theta_i \widetilde{p}_i > \theta_1 \sum_{i\in\Omega} \widetilde{p}_i$; then the source's utility is

$$\begin{aligned} U_2 &= \rho \log\left(1 + \sum_{i\in\Omega} \widetilde{p}_i\right) - \sum_{i\in\Omega} \theta_i \widetilde{p}_i \\ &< \rho \log\left(1 + \sum_{i\in\Omega} \widetilde{p}_i\right) - \theta_1 \sum_{i\in\Omega} \widetilde{p}_i. \end{aligned} \quad (13)$$

By setting $P' = \sum_{i\in\Omega} \widetilde{p}_i$, (13) can be simplified as

$$U_2 < \rho \log(1 + P') - \theta_1 P'. \quad (14)$$

Obviously, the right inequality of (14) is exactly equal to $U1$; then we have

$$U_2 < \rho \log(1 + P') - \theta_1 P' = U_1. \quad (15)$$

This contradicts the above assumption and thus completes the proof. □

Using Lemma 2, the optimization problem in (11) can be further simplified as

$$\max_{\widetilde{p}_1 \geq 0} \rho \log(1 + \widetilde{p}_1) - \theta_1 \widetilde{p}_1. \quad (16)$$

At this point, the source's optimization problem from involving $2N$ variables $(p_i, \alpha_i), \forall i \in \Omega$, in (9) is simplified to a single variable $\widetilde{p}_1$ in (16). Any local optimal solution (denoted by $\widehat{p}_1$) to problem (16) satisfies

$$\left.\frac{dU_S(\widetilde{p}_1)}{d\widetilde{p}_1}\right|_{\widetilde{p}_1 = \widehat{p}_1} = \frac{\rho}{1 + \widehat{p}_1} - \theta_1 = 0. \quad (17)$$

Then, the second-order derivative of problem (16) is

$$\left.\frac{\partial^2 U_S(\widetilde{p}_1)}{\partial \widetilde{p}_1^2}\right|_{\widetilde{p}_1 = \widehat{p}_1} = \frac{-\rho}{(1 + \widehat{p}_1)^2} < 0, \quad (18)$$

which means that the local optimal solution to (16) is unique and globally optimal. Thus, $p_1^* = \max((\rho - \theta_1)/\theta_1, 0)$ and $\alpha_1^* = \max((\rho - \theta_1)(\theta_1 - \beta)/\theta_1, 0)$.

4. Optimal Contract Design under Asymmetric Information

In this section, the optimal contract design under asymmetric information scenario is presented. Assume that the types of RNs are discrete and belong to a set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_N\}$. And, due to the asymmetry of network information, we assume the source has some statistical information about the RN's private type, for example, the total number of RNs K and the prior probability distribution q_i of type-θ_i RN. Obviously $q_i \in [0, 1]$ and $\sum_{i\in\Omega} q_i = 1$. A feasible contract should satisfy both the IR constraint in (8) and the incentive compatibility (IC) constraint defined as follows:

$$\alpha_i - V(\theta_i, p_i) \geq \alpha_j - V(\theta_i, p_j), \quad \forall i, j \in \Omega. \quad (19)$$

IC constraint ensures that each type-θ_i RN gets the maximum utility by choosing the contract item (p_i, α_i) designed for its type. In other words, based on the IC constraint, the source attracts RNs to truthfully reveal their private types.

Since the source knows the total number of RNs K, the probability density function of the number of RNs N_i can be written as

$$\begin{aligned} &Q(n_2, \ldots, n_N) \\ &= \Pr\left(N_2 = n_2, \ldots, N_N = n_N,\ N_1 = K - \sum_{i=2}^{N} n_i\right) \\ &= \frac{K!}{n_2! \cdots n_N! \left(K - \sum_{i=2}^{N} n_i\right)!} q_1^{K - \sum_{i=2}^{N} n_i} q_2^{n_2} \cdots q_N^{n_N}. \end{aligned} \quad (20)$$

Then, the contract design optimization problem under asymmetric information is to maximize the source's expected utility subject to the IC and IR constraints; that is,

$$\max_{\{\{p_i,\alpha_i\}\geq 0\}} \sum_{n_N=0}^{K}\sum_{n_{N-1}=0}^{K-n_N}\cdots\sum_{n_2=0}^{K-\sum_{k=3}^{N}n_k} Q(n_2,\ldots,n_N)\left\{\rho\log\left[1+\sum_{i=2}^{N}n_ip_i+\left(K-\sum_{i=2}^{N}n_i\right)p_1\right]-\left[\sum_{i=2}^{N}n_i\alpha_i+\left(K-\sum_{i=2}^{N}n_i\right)\alpha_1\right]-\beta\left[\sum_{i=2}^{N}n_ip_i+\left(K-\sum_{i=2}^{N}n_i\right)p_1\right]\right\}$$

$$\text{s.t.}\quad \text{(IC:)}\ \alpha_i - V(\theta_i,p_i)\geq \alpha_j - V(\theta_i,p_j),\quad \forall i,j\in\Omega$$
$$\text{(IR:)}\ \alpha_i - V(\theta_i,p_i)\geq 0. \tag{21}$$

4.1. Feasibility Conditions for Optimal Contract Design. In this subsection, various feasibility conditions for optimal contract design are presented. Let $\Phi = \{p_i, \alpha_i, \forall i \in \Omega\}$ be a feasible contract.

Proposition 3. *For any k, j, one has $p_k > p_j$ if and only if $\alpha_k > \alpha_j$.*

Proof. First, we prove that if $p_k > p_j$, then $\alpha_k > \alpha_j$.

Due to the IC constraint in (19), we have

$$\alpha_k - \alpha_j \geq V(\theta_k,p_k) - V(\theta_k,p_j) > 0. \tag{22}$$

Thus, $\alpha_k > \alpha_j$.

Next, we prove that if $\alpha_k > \alpha_j$, then $p_k > p_j$.

Due to the IC constraint in (19), we have

$$\alpha_j - V(\theta_j,p_j) \geq \alpha_k - V(\theta_j,p_k), \tag{23}$$

which can be transformed to be

$$V(\theta_j,p_k) - V(\theta_j,p_j) \geq \alpha_k - \alpha_j > 0. \tag{24}$$

Since $V(\theta_i, p_i)$ is increasing in p_i, thus $p_k > p_j$. □

Proposition 3 indicates that the RN offering more relay power should be given with more reward by the source, and vice versa.

Proposition 4. *For any k, j, if $\theta_k > \theta_j$, then $p_k < p_j$.*

Proof. This proposition can be proved by contradiction. Suppose there exists $p_k > p_j$ with $\theta_k > \theta_j$. Then, we have

$$\begin{aligned} V(\theta_k,p_k) - V(\theta_k,p_j) &= \theta_k(p_k-p_j),\\ V(\theta_j,p_k) - V(\theta_j,p_j) &= \theta_j(p_k-p_j). \end{aligned} \tag{25}$$

By subtracting the last two equalities, we can have

$$\begin{aligned} &V(\theta_k,p_k) - V(\theta_k,p_j) - V(\theta_j,p_k) + V(\theta_j,p_j)\\ &\quad = (\theta_k-\theta_j)(p_k-p_j) > 0. \end{aligned} \tag{26}$$

Thus,

$$V(\theta_k,p_k) + V(\theta_j,p_j) > V(\theta_k,p_j) + V(\theta_j,p_k). \tag{27}$$

Next, considering the IC constraints for both type-θ_k and type-θ_j RNs, we have

$$\begin{aligned} \alpha_k - V(\theta_k,p_k) &\geq \alpha_j - V(\theta_k,p_j),\\ \alpha_j - V(\theta_j,p_j) &\geq \alpha_k - V(\theta_j,p_k). \end{aligned} \tag{28}$$

By combining the last two inequalities, we have

$$V(\theta_k,p_j) + V(\theta_j,p_k) \geq V(\theta_k,p_k) + V(\theta_j,p_j), \tag{29}$$

which contradicts (27). This completes the proof. □

This proposition indicates that, in a feasible contract, a lower type RN should be given with more reward. Thus, combining Propositions 3 and 4, we can conclude that, for a feasible contract, all *relay-reward* contract items should satisfy

$$\begin{aligned} &0 \leq p_N \leq p_{N-1} \leq \cdots \leq p_1\\ &0 \leq \alpha_N \leq \alpha_{N-1} \leq \cdots \leq \alpha_1, \end{aligned} \tag{30}$$

with $\alpha_k = \alpha_{k+1}$ if and only if $p_k = p_{k+1}$.

Based on the previous two propositions, we obtain the following theorem, which is essential to the optimal contract design under asymmetric information.

Theorem 5. *For a contract $\Phi = \{p_i, \alpha_i, \forall i \in \Omega\}$, it is feasible if and only if all the following conditions hold:*

$$\begin{aligned} &\text{(a)}\ \textit{for}\ 1\leq i<N,\quad \alpha_{i+1}+V(\theta_i,p_i)-V(\theta_i,p_{i+1})\leq\alpha_i\leq\alpha_{i+1}+V(\theta_{i+1},p_i)-V(\theta_{i+1},p_{i+1})\\ &\text{(b)}\ \alpha_N - V(\theta_N,p_N)\geq 0\\ &\text{(c)}\ 0\leq p_N\leq p_{N-1}\leq\cdots\leq p_1\\ &\text{(d)}\ 0\leq\alpha_N\leq\alpha_{N-1}\leq\cdots\leq\alpha_1. \end{aligned} \tag{31}$$

Proof. Please refer to Appendix A. □

4.2. Optimal Contract Design. In this section, the optimal contract design is investigated. The optimal problem with complicated constraints in (21) is generally nonconvex, making it difficult to efficiently solve for the global optimum [24]. In this paper, a sequential optimization approach is adopted. Firstly, we derive the best reward allocations ($\alpha_i^*(p_i)$) given fixed feasible relay powers p_i; then, we can obtain the best relay powers (α_i^*) for the optimal contract; finally, we show that there is no gap between the solution (α_i^*, p_i^*) obtained from this sequential optimization approach and the one obtained by directly solving (21).

Theorem 6. *Let $\Phi = \{(\alpha_i, p_i), \forall i\}$ be a feasible contract with fixed relay powers $\{p_i, \forall i\}$. The optimal unique relay powers satisfy*

$$\alpha_i^* = \begin{cases} V(\theta_N, p_N), & i = N \\ V(\theta_N, p_N) + \sum_{k=i}^{N-1} [V(\theta_k, p_k) - V(\theta_k, p_{k+1})], & i \neq N. \end{cases} \tag{32}$$

Proof. Please refer to Appendix B. □

Based on Theorem 6, the optimal contract design problem in (21) can be simplified as

$$\begin{aligned} \max_{\{p_i \geq 0\}} \quad & \sum_{n_N=0}^{K} \sum_{n_{N-1}=0}^{K-n_N} \cdots \sum_{n_2=0}^{K-\sum_{k=3}^{N} n_k} Q(n_2, \dots, n_N) \left\{ \rho \log\left[1 + \sum_{i=2}^{N} n_i p_i + \left(K - \sum_{i=2}^{N} n_i\right) p_1\right] - \left[\sum_{i=2}^{N} n_i \alpha_i^*(p_i) + \left(K - \sum_{i=2}^{N} n_i\right) \alpha_1^*(p_1)\right] - \beta \left[\sum_{i=2}^{N} n_i p_i + \left(K - \sum_{i=2}^{N} n_i\right) p_1\right] \right\} \\ \text{s.t.} \quad & 0 \leq \alpha_N \leq \alpha_{N-1} \leq \cdots \leq \alpha_1. \end{aligned} \tag{33}$$

Note that (33) is a nonconvex optimization problem, making it difficult to solve efficiently. Here, a low computation complexity sequential optimization algorithm is proposed to obtain an approximate optimal solution. First, construct N candidate contracts and then select the one with the largest utility from N candidate contracts as the optimal design strategy. The sequential optimization algorithm is described as follows.

Algorithm 7 (sequential optimization algorithm for contract design under asymmetric information).

(i) Step 1: initiate N, K. Construct N candidate contracts.

(ii) Step 2: offer the same contract item $p_i > 0$ to RNs with a type equal to or smaller than type θ_i and zero for the RNs above type θ_i. That is, $p_1 = p_2 = \cdots = p_i$ and $p_{i+1} = p_{i+2} = \cdots = p_N = 0$.

(iii) Step 3: obtain the optimal p_i to maximize the source's expected utility in (33) under the above constraints by using the interior point method. The corresponding optimal reward allocation α_i^* satisfies (32) as specified in the contract.

(iv) Step 4: select the best contract out of N candidate contracts to maximize the source's expected utility.

Compared with the exhaustive search algorithm, the computational complexity of the proposed sequential optimization algorithm is much lower, for the optimization of each candidate contract only involves a scalar optimization. Assume that the possible range of p_i is denoted by $[0, P]$ and all possibilities of any p_i are spaced equally by Δ on the interval $[0, P]$. Then, in the case of the exhaustive search algorithm, it requires searching over all the possible relay power ranges for N types jointly; thus, the computational complexity is $O(\Delta^N)$. In the case of the proposed sequential optimization algorithm, the complicated problem is decomposed into N simple subproblems with one critical type in each subproblem; thus, the overall computation complexity can be reduced to $O(\Delta N)$.

5. Results and Discussion

In this section, numerical results are presented to evaluate the performance of the proposed contract-based cooperative communication method in both symmetric and asymmetric information scenarios.

5.1. Symmetric Information Scenario. The first evaluation method is to analyse the performance of relay selection in the symmetric information scenario.

In Figure 2, we plot the sources optimal utility U_S^* versus the equivalent profit ρ. The performance-based bonus coefficient β is chosen to be 0.1. As shown in Figure 2, the source's optimal utility is increasing in the equivalent profit ρ and decreasing in the lowest RNs' type θ_1. As ρ increases, the RNs have more incentives to provide relay communication with the source, and the source obtains more utility from RNs' cooperative relay. As θ_1 increases, the RNs of that type have poorer relay channel condition, or the RNs of that type have a higher relay cost for relay communication; thus, the source obtains less utility by hiring the higher RNs' type θ_1. When ρ is smaller than θ_1, the relay power p_1 of RNs is reduced to zero; thus, RNs chooses not to participate in cooperative communication, which also makes the source's utility close to zero.

In Figure 3, the RNs optimal basic wage α_1^* is plotted for different values of θ_1 and ρ with $\beta = 0.1$. The figure shows that the RNs' optimal basic wage of the lowest type θ_1, α_1^*, increases in the source's equivalent profit ρ. As the equivalent profit ρ increases, the source obtains more utility from RNs' cooperative relay; thus, the RN's basic wage paid by the source

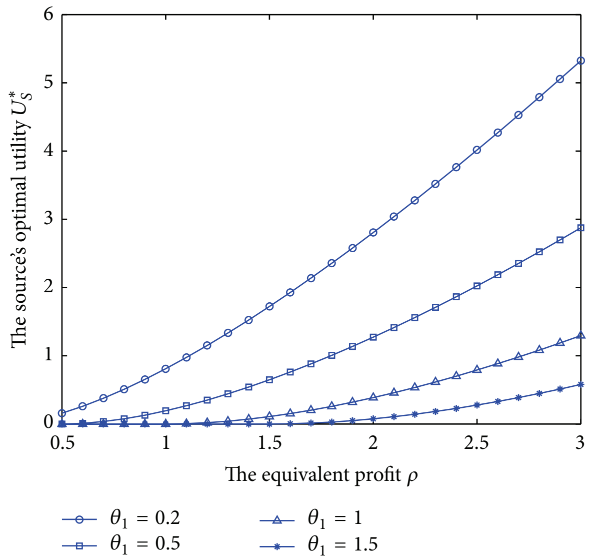

FIGURE 2: The sources optimal utility U_S^* as a function of the lowest type θ_1 and equivalent profit ρ for fixed $\beta = 0.1$.

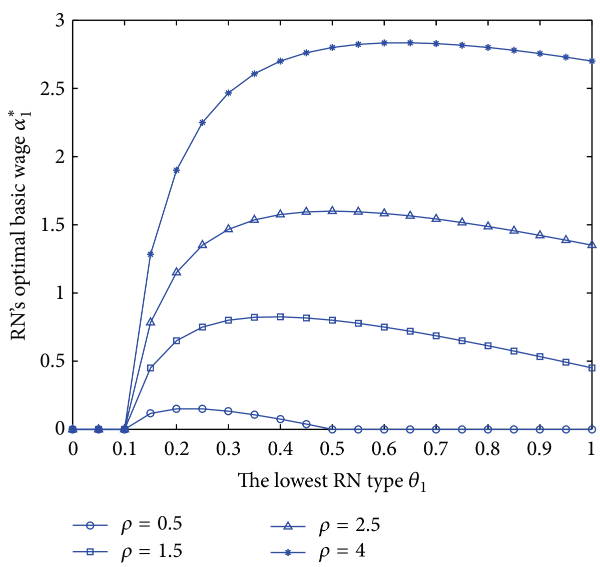

FIGURE 3: RNs optimal basic wage α_1^* at SR side as a function of the lowest type θ_1 and equivalent profit ρ for fixed $\beta = 0.1$.

increases. When $\theta_1 \leq \beta$, the performance-based bonus given by the source is higher than the relay cost of the lowest type-θ_1 RNs; thus, it is not necessary for the source to offer any more basic wage to RNs for enough relay help. This explains why the lowest curve has some zero points in certain θ_1 cases. As θ_1 becomes large, the source willingly allocates reward to obtain efficient relay help from RNs. As θ_1 becomes very large, especially $\theta_1 \geq \rho$, the source only needs to allocate a little amount of reward to RNs for enough relay help. This explains why all the curves display first increasing and then decreasing in θ_1.

Figure 4 shows the relationship between the RNs' optimal basic wage α_1^* and the performance-based bonus coefficient

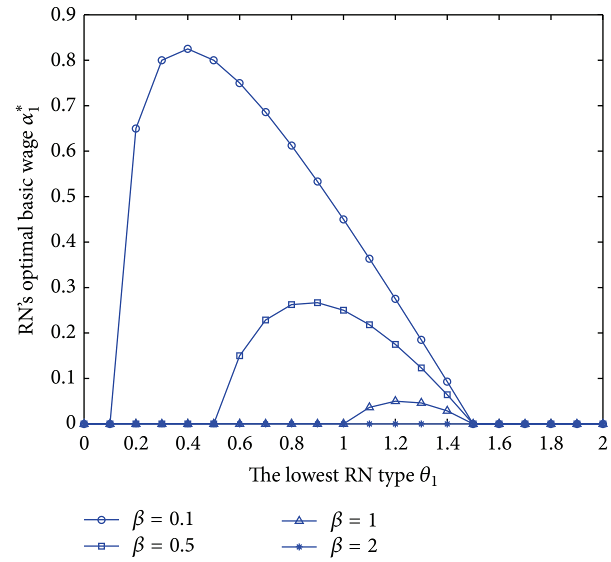

FIGURE 4: RNs optimal basic wage α_1^* at SR side as a function of the lowest type θ_1 and the performance-based bonus coefficient β for fixed $\rho = 1.5$.

β with $\rho = 1.5$. As β becomes large, the source only needs to allocate a little amount of reward to RNs for enough relay help; thus, RNs' optimal basic wage α_1^* is strictly decreasing in β. When $\theta_1 \leq \beta$, the performance-based bonus given by the source is higher than the relay cost of the lowest type-θ_1 RNs; it is not necessary for the source to offer any more basic wage to RNs for enough relay help; thus, there are some zero points in certain θ_1 cases.

5.2. Asymmetric Information Scenario. In the asymmetric information scenarios, the performance of the proposed sequential optimization algorithm is compared with that of the N-dimensional exhaustive search method [25]. The optimal solution is denoted by $E[U_S]^*$, and only two types of RNs, $\theta_1 < \theta_2$, are considered. And two candidate contracts are considered in the sequential optimization algorithm. The first candidate contract optimizes the same positive contract item $p_1 = p_2 > 0$ with the corresponding source's maximum expected utility $E[U_S]^1$. The second candidate contract optimizes the positive contract item $p_1 > 0$ with $p_2 = 0$ and the corresponding source's maximum expected utility $E[U_S]^2$. Then, the larger source's expected utility of the two candidate contracts is picked as the solution of the sequential optimization algorithm.

Figure 5 shows the source's expected utility obtained with the two candidate contracts of the sequential optimization algorithm ($E[U_S]^1$ and $E[U_S]^2$) and the optimal exhaustive search method ($E[U_S]^*$). The parameters were set as follows: $q_1 = 0.9$, $K = 2$, $\beta = 0.1$, $\theta_1 = 0.2$, and $\theta_2 = 0.5$. It is seen that the sources optimal utility with the candidate contract $E[U_S]^1$ is always lower than that of $E[U_S]^2$. And the candidate contract $E[U_S]^2$ achieves an approaching-to-optimal performance with all values of ρ simulated here. This

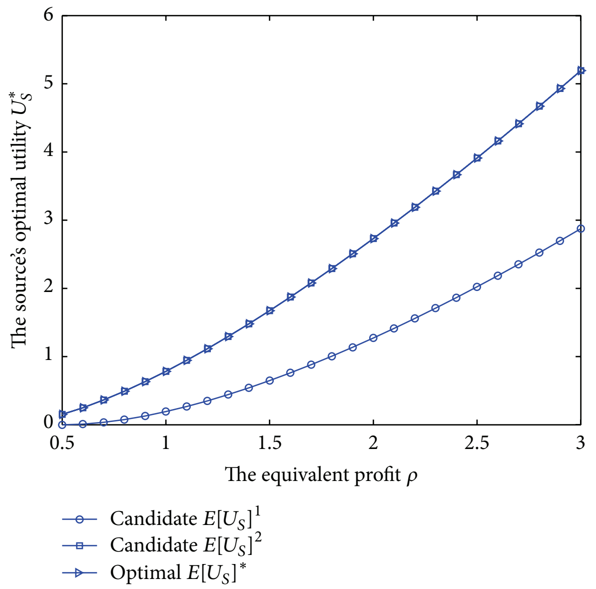

Figure 5: Comparison between the source's optimal expected utility values using various optimal search methods.

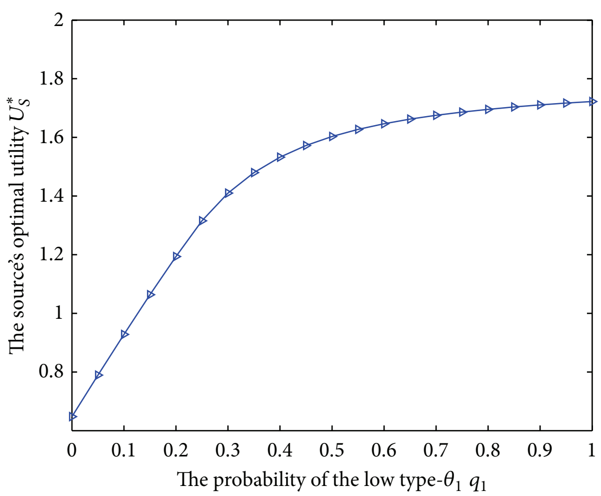

Figure 6: The source's optimal expected utility versus the probability of the low type-θ_1 q_1.

is because the source often needs the low type-θ_1 RNs relay communication for itself.

Figures 6 and 7 show the optimal contract design with various probabilities of the low type-θ_1 RNs q_1. The parameters were set as follows: $K = 6$, $\beta = 0.1$, $\theta_1 = 0.2$, and $\theta_2 = 0.5$. Since the source always hires a low type-θ_1 RN for cooperative communication, when q_1 is small, the source has to offer much more basic wage to RNs for enough relay help; thus, the source's optimal expected utility is low. As q_1 increases, the proportion of the low type-θ_1 RNs is enhanced; the source only needs to allocate a little amount of reward to RNs; thus, the source's optimal expected utility tends to increase. Moreover, when q_1 is large, it is not necessary for the source to offer any more basic wage to the type-θ_2 RNs

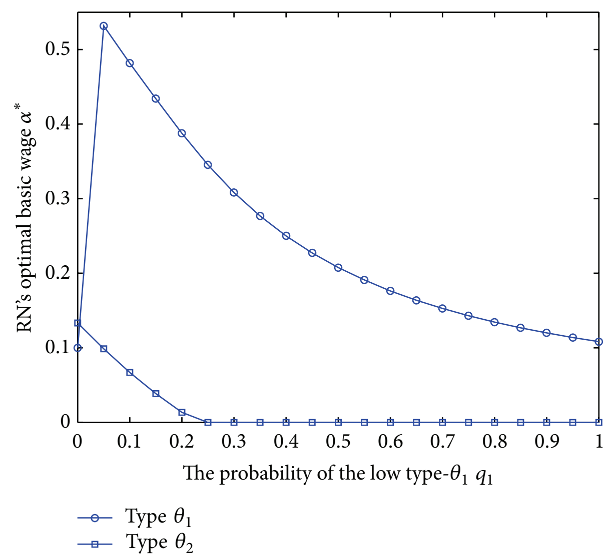

Figure 7: RNs optimal basic wage α^* versus the probability of the low type-θ_1 q_1.

for relay communication; thus, the type-θ_2 RNs' optimal basic wage tends to zero.

Figure 8 shows the source's optimal expected utility under different number of RNs and different probability of the low type-θ_1 q_1. The simulation parameters are provided underneath the figure. The increased total number of RNs K will tend to increase the number of RNs involved in relay cooperation and result in higher source's optimal expected utility. Moreover, as K increases, the growth rate of source's optimal expected utility reduces. Furthermore, increasing q_1 can enhance the proportion of the low type-θ_1 RNs; therefore, better source's optimal expected utility and faster convergence rate are expected.

5.3. Symmetric Information and Asymmetric Information Scenarios. Finally, the performance of asymmetric information scenario is considered comparing with the symmetric information benchmark. Cases 1, 2, and 3 correspond to cooperative communication scenario of *symmetric information*, *asymmetric information with large* q_1, that is, $q_1 = 0.9$, and *asymmetric information with small* q_1, that is, $q_1 = 0.5$, respectively. As shown in Figure 9, $E[U_S]^*$ under asymmetric information scenarios are often lower than the maximum utility achieved in symmetric information scenario due to the asymmetric RNs' information. And the maximum utility achieved under the asymmetric information with large q_1 is larger than that with small q_1. This is because the actual source's utility does always depend on the performance of the low type-θ_1 RN.

6. Conclusion

In this paper, the cooperative communication between one source and multiple RNs is studied. The CRN is modelled as a labour market, where the source designs a contract and each RN decides to select a contract item according

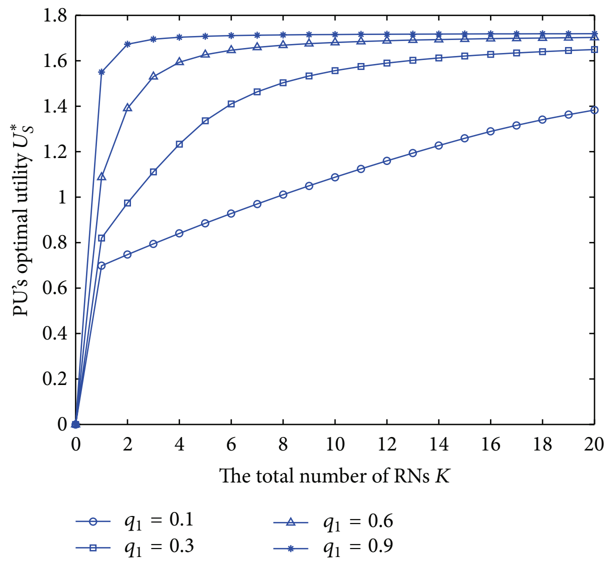

FIGURE 8: The source's optimal expected utility versus the total number of RNs K for fixed $\beta = 0.1$, $\theta_1 = 0.2$, and $\theta_2 = 0.5$.

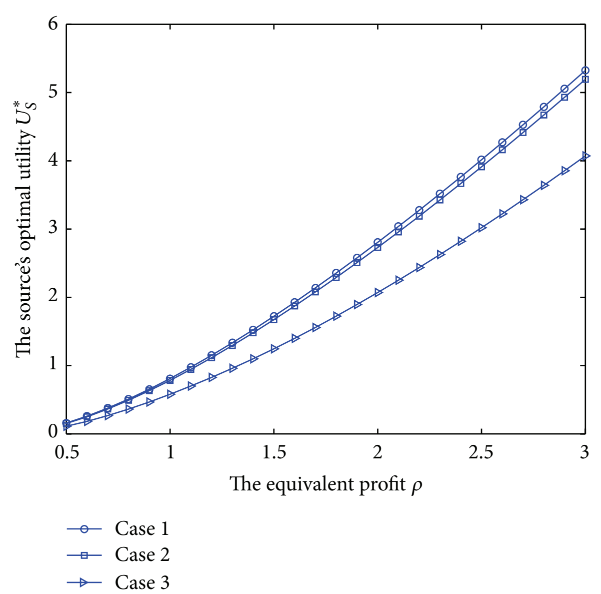

FIGURE 9: Comparison between the source's optimal expected utility values using various optimal search methods for fixed $K = 2$, $\beta = 0.1$, $\theta_1 = 0.2$, and $\theta_2 = 0.5$.

to hidden information. Under symmetric information, the optimal contract is feasible if and only if it is IR for each RN. And, under asymmetric information, the optimal contract design meeting both IC and IR conditions is systematically characterized. The contract-theoretic model for ability discrimination relay selection is formulated as an optimization problem where the source's expected utility is maximized subject to the necessary and sufficient conditions of each RN. A sequential optimization algorithm is proposed to obtain the optimal *relay-reward* strategy. Simulation results show that, due to the asymmetric information, the source's expected utility loss under asymmetric information is small compared with symmetric information. And the proposed contract-theoretic scheme can improve the system performance of cooperative communication. The overall incentive mechanism introduced in this paper is based on RNs' self-interest and fully rational hypotheses. As part of future work, we will incorporate RNs' behavioural and preference characteristics, such as fairness, equity, and reciprocity, in the framework of the standard contract design for the relay incentive mechanism.

Appendices

A. Proof of Theorem 5

A.1. Proof of Necessary Conditions. Part (b) of (31) is the same as the necessary IR constraint for the highest type θ_N in a feasible contract, and parts (c) and (d) are the same as necessary condition summarized in (30). Then, the left inequality of part (a) can be derived from the necessary IC constraint for type θ_i in a feasible contract; that is,

$$\alpha_i - V(\theta_i, p_i) \geq \alpha_{i+1} - V(\theta_i, p_{i+1}). \quad \text{(A.1)}$$

Thus,

$$\alpha_{i+1} + V(\theta_i, p_i) - V(\theta_i, p_{i+1}) \leq \alpha_i. \quad \text{(A.2)}$$

And the right inequality of part (a) can similarly be derived from the necessary IC constraint for type θ_{i+1}.

A.2. Proof of Sufficient Conditions. Let $\Phi(n) = \{(\alpha_i, p_i), | \ i = N-n+1, \ldots, N\}$ denote a subset with the last n *relay-reward* combinations.

First, we show that $\Phi(N)$ is feasible. The contract is feasible if it satisfies IR constraint in (8). This is true due to part (b) in Theorem 5.

Secondly, we prove the IC constraint for type θ_{i-1}. Since contract $\Phi(i)$ is feasible, the IC constraint for a type-θ_i RN must hold:

$$\alpha_i - V(\theta_i, p_i) \geq \alpha_k - V(\theta_i, p_k), \quad k = i, \ldots, n. \quad \text{(A.3)}$$

Also, the right inequality of (31) in part (a) can be transformed to

$$\alpha_i + V(\theta_{i-1}, p_{i-1}) - V(\theta_{i-1}, p_i) \leq \alpha_{i-1}. \quad \text{(A.4)}$$

By combining the above two inequalities, we can have

$$\begin{aligned} &\alpha_k - V(\theta_i, p_k) + V(\theta_{i-1}, p_{i-1}) - V(\theta_{i-1}, p_i) \\ &\quad \leq \alpha_{i-1} - V(\theta_i, p_i). \end{aligned} \quad \text{(A.5)}$$

Notice that $\theta_{i-1} < \theta_i$ and $k \geq i$; thus $p_i \geq p_k$. Then, we have

$$V(\theta_{i-1}, p_i) - V(\theta_{i-1}, p_k) \leq V(\theta_i, p_i) - V(\theta_i, p_k). \quad \text{(A.6)}$$

By combining the above two inequalities, we have

$$\alpha_k - V(\theta_{i-1}, p_k) \leq \alpha_{i-1} - V(\theta_{i-1}, p_{i-1}), \quad \text{(A.7)}$$

which is actually the IC constraint for type θ_{i-1}.

Thirdly, we show the IR constraint for type θ_{i-1}. Since $V(\theta_{i-1}, p_i) < V(\theta_i, p_i)$ and $\alpha_i - V(\theta_i, p_i) \geq 0$, then, we can get

$$\alpha_i - V(\theta_{i-1}, p_i) > \alpha_i - V(\theta_i, p_i). \quad \text{(A.8)}$$

Using (A.7), we also have

$$\alpha_{i-1} - V(\theta_{i-1}, p_{i-1}) \geq \alpha_i - V(\theta_{i-1}, p_i). \quad \text{(A.9)}$$

By combining the last two inequalities, we have

$$\alpha_{i-1} - V(\theta_{i-1}, p_{i-1}) \geq 0, \quad \text{(A.10)}$$

which is the IR constraint for type θ_{i-1}.

Finally, we show that if contract $\Phi(i)$ is feasible, then the new contract $\Phi(i-1)$ can be constructed by subtracting the item (α_i, p_i) and the new contract is also feasible.

Since contract $\Phi(i)$ is feasible, the IC constraint for type θ_i holds:

$$\alpha_i - V(\theta_k, p_i) \leq \alpha_k - V(\theta_k, p_k), \quad \forall k = i, \ldots, N. \quad \text{(A.11)}$$

Also, we can transform the right inequality of (31) in part (a) to

$$\alpha_{i-1} \leq \alpha_i + V(\theta_i, p_{i-1}) - V(\theta_i, p_i). \quad \text{(A.12)}$$

By combining the above two inequalities, we conclude

$$\alpha_{i-1} - V(\theta_k, p_i) \leq \alpha_k - V(\theta_k, p_k) + V(\theta_i, p_{i-1}) - V(\theta_i, p_i). \quad \text{(A.13)}$$

Note that $k \geq i$; thus $\theta_k \geq \theta_i$ and $p_{i-1} \geq p_i$. Then, we have

$$V(\theta_i, p_{i-1}) - V(\theta_i, p_i) \leq V(\theta_k, p_{i-1}) - V(\theta_k, p_i). \quad \text{(A.14)}$$

By combining the above two inequalities, we conclude

$$\alpha_{i-1} - V(\theta_k, p_{i-1}) \leq \alpha_k - V(\theta_k, p_k), \quad \forall k = i, \ldots, N, \quad \text{(A.15)}$$

which is actually the IC constraint for type θ_i. This completes the proof.

B. Proof of Theorem 6

First, the relay powers in (32) can be easily proved to satisfy the sufficient conditions of contract feasibility in Theorem 5. Then, we will prove the optimality and uniqueness of the solutions in (32).

B.1. Proof of Optimality. First, we show that, given fixed relay power, the reward allocation $\{\alpha_i^*\}$ in (32) maximizes the PU's utility:

$$U_{\mathrm{PU}} = \rho \log\left(1 + \sum_{i\in\Omega} N_i p_i\right) - \sum_{i\in\Omega} N_i(\alpha_i + \beta p_i). \quad \text{(B.1)}$$

Here, the proof is by contradiction. Suppose there exists another feasible reward allocation $\{\overline{\alpha}_i, \forall i\}$ which achieves a better solution than $\{\alpha_i^*, \forall i\}$ in (32). Since $U_{\mathrm{PU}}(\alpha_i^*) < U_{\mathrm{PU}}(\overline{\alpha}_i)$, and U_{PU} is decreasing in total reward allocations, we must have

$$\sum_{i\in\Omega} N_i \overline{\alpha}_i < \sum_{i\in\Omega} N_i \alpha_i^*. \quad \text{(B.2)}$$

Thus, there is at least one reward allocation $\overline{\alpha}_i < \alpha_i^*$ for one type θ_i.

If $i = N$, then $\overline{\alpha}_N < \alpha_N^*$. Since $\alpha_N^* = V(\theta_N, p_N)$, then $\overline{\alpha}_N < V(\theta_N, p_N)$. But this violates the IR constraint for type θ_i. Then, we must have $1 \leq i < N$.

Since $\{\overline{\alpha}_i, \forall i\}$ is feasible, then $\{\overline{\alpha}_i, \forall i\}$ must satisfy the left inequality of part (a) in Theorem 5. Thus, we have

$$\overline{\alpha}_{i+1} + V(\theta_i, p_i) - V(\theta_i, p_{i+1}) \leq \overline{\alpha}_i. \quad \text{(B.3)}$$

Also, from (32), we can have

$$\alpha_i^* - \alpha_{i+1}^* = V(\theta_i, p_i) - V(\theta_i, p_{i+1}). \quad \text{(B.4)}$$

By substituting the above equality into (B.3), we get

$$\overline{\alpha}_{i+1} + \alpha_i^* - \alpha_{i+1}^* \leq \overline{\alpha}_i. \quad \text{(B.5)}$$

Since $\overline{\alpha}_N < \alpha_N^*$, then we have $\overline{\alpha}_{i+1} < \alpha_{i+1}^*$. Using the above argument repeatedly, we finally obtain that $\overline{\alpha}_N < \alpha_N^*$, which violates the IR constraint for type θ_N again.

B.2. Proof of Uniqueness. We next prove that the relay power in (32) is the unique solution that maximizes (B.1). We also prove this by contradiction. Suppose there exists another $\{\tilde{\alpha}_i, \forall i\} \neq \{\alpha_i^*, \forall i\}$ such that $\sum_{i\in\Omega} N_i \tilde{\alpha}_i = \sum_{i\in\Omega} N_i \alpha_i^*$ in (B.1). Then, there is at least one reward allocation $\tilde{\alpha}_k > \alpha_k^*$ and one reward allocation $\tilde{\alpha}_l < \alpha_l^*$. We can focus on type θ_l and $\tilde{\alpha}_l < \alpha_l^*$. By using the same argument before, we have $\tilde{\alpha}_N < \alpha_N^* < V(\theta_N, p_N)$. But this violates the IR constraint for type θ_N.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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