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Research Article

Chaotic Tolerant Synchronization Analysis with Propagation Delay and Actuator Faults

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The criteria for tolerant synchronization with a constant propagation delay and actuator faults are presented by using matrix analysis techniques. A new algorithm, which constructs the extended error systems in order to make the conservation of the stability lower, is proposed. Based on proper Lyapunov-Krasovskii functional, the novel delay-dependent fault tolerant synchronization analyses are derived. Finally, numerical examples show the effectiveness of the proposed method.

1. Introduction

The realization of OGY (Ott-Grebogi-Yorker) chaos-control method [1] and PC (Pecora and Carroll) synchronization method [2] has been attracting researchers' attention since the 1990s. Chaos synchronization [3–13] is of great practical significance and has aroused great interest in recent years.

They all focused on the design of synchronization under normal operating conditions in the above works described. But, in practical chaotic secure communication systems, sensors, actuators, and inner components may inevitably fail, which can lead to sharp performance decline of chaotic secure. For the reason, fault tolerant synchronization and control [14–20] of chaotic systems have been the hot topic of intensive researches recently.

More recent works studied the fault tolerant synchronization and control, but they were limited to construct Lyapunov-Krasovskii functional

$$V(t) = V_1(e(t)) + V_2\left(\tilde{f}(t)\right), \quad (1)$$

which separates $e(t)$ and $\tilde{f}(t)$, where $e(t)$ is the synchronization error and $\tilde{f}(t)$ is the error of fault function. Motivating the limitation and the extended transformation [21, 22], the effective new method, which does the extended transformation for the error system, to consider Lyapunov-Krasovskii functional

$$V(t) = V\left(\left(e^T(t), \tilde{f}^T(t)\right)^T\right), \quad (2)$$

which do not separate $e(t)$ and $\tilde{f}(t)$, for the chaotic fault tolerant synchronization with a constant propagation delay and actuator faults, makes the conservation descent. Finally, numerical examples are given to verify the above method.

Notations. In this paper, R, R^n, and $R^{n\times m}$ denote, respectively, the real number, the real n-vectors, and the real $n\times m$ matrices. The superscript "T" stands for the transpose of a matrix. The symbol $X > Y$ $(X \geq Y)$, where X and Y are symmetric matrices, means that X-Y is positive definite (positive semidefinite). I is the identity matrix of appropriate dimensions. "$*$" denotes the matrix entries implied by symmetry.

2. Preliminaries and Systems Description

Consider a chaotic master system with the actuator faults item $f(t)$ in the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + g(x(t)) + h(x(t-\theta)) + Ef(t), \\ y(t) &= Cx(t) + Df(t), \end{aligned} \quad (3)$$

where $x(t) \in R^n$ is the measurable state vector. $y(t) \in R^p$ is the output vector, and A, C, D, and E are proper dimension constant matrices. $g(x(t)), h(x(t-\theta))$ are known continuous nonlinear functions. $\theta > 0$ is the delay.

Assumption 1. There exist the matrices $U_1, U_2, M_1, M_2, W_1, W_2, V_1, V_2 \in R^{n\times n}$, and the nonlinear functions $g(\cdot)$, and $h(\cdot)$ satisfy

$$(U_1 (g(x) - g(y)) - U_2 (x - y))^T \cdot (M_1 (g(x) - g(y)) - M_2 (x - y)) \le 0, \quad (4)$$

$$(W_1 (h(x) - h(y)) - W_2 (x - y))^T \cdot (V_1 (h(x) - h(y)) - V_2 (x - y)) \le 0 \quad (5)$$

for all $x, y \in R^n$.

The slave system linked with the chaotic master system (3) is described by

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + g(\hat{x}(t)) + h(\hat{x}(t-\theta)) + E\hat{f} \\ &\quad - L(\hat{y}(t-\tau) - y(t-\tau)), \\ \hat{y}(t) &= C\hat{x}(t) + D\hat{f}(t),\end{aligned} \quad (6)$$

where $\hat{x}(t) \in R^n$ is the measurable state vector and $\tau > 0$ is a constant propagation delay.

Let $e(t) = \hat{x}(t) - x(t)$, $\tilde{f}(t) = \hat{f}(t) - f(t)$, and $\varphi(t) = \hat{y}(t) - y(t) = Ce(t) + D\tilde{f}(t)$, and then

$$\begin{aligned}\dot{e}(t) &= Ae(t) + g(\hat{x}(t)) - g(x(t)) + h(\hat{x}(t-\theta)) \\ &\quad - h(x(t-\theta)) + E\tilde{f}(t) - LCe(t-\tau) \\ &\quad - LD\tilde{f}(t-\tau), \\ \varphi(t) &= Ce(t) + D\tilde{f}(t).\end{aligned} \quad (7)$$

Lemma 2 (see [23, 24]). *For any constant matrix $W \in R^{n\times n}$, $W > 0$, scalar $0 < h(t) < h$, and vector function $w(t) : [0, h] \to R^n$ such that the integrations concerned are well defined, and then*

$$\left(\int_0^{h(t)} w(s)\,ds\right)^T W \left(\int_0^{h(t)} w(s)\,ds\right) \le h(t) \int_0^{h(t)} w^T(s) W w(s)\,ds. \quad (8)$$

Lemma 3 (see [25]). *Let $Q > 0$, H, $F(t)$, and E be real matrices of appropriate dimensions, with $F(t)$ satisfying $F^T(t)F(t) \le I$. Then, the following inequalities are equivalent:*

(1) $Q + HF(t)E + E^T F^T(t) H^T < 0$;

(2) *there exists a scalar $\varepsilon > 0$ such that $Q + \varepsilon^{-1} HH^T + \varepsilon E^T E < 0$.*

Lemma 4 (see [26–28]). *Let A, L, E, and $F(t)$ be real matrices of appropriate dimensions, with $F(t)$ satisfying $F^T(t)F(t) \le I$. Then, one has the following:*

(1) *for any scalar $\varepsilon > 0$,*

$$LFE + E^T F^T L^T < \varepsilon^{-1} LL^T + \varepsilon E^T E; \quad (9)$$

(2) *for any matrix $P > 0$ and scalar $\varepsilon > 0$, such that $\varepsilon I - EFE^T > 0$,*

$$\begin{aligned}&(A + LFE)^T P (A + LFE) \\ &\quad \le A^T PA + A^T PE(\varepsilon I - E^T PE)^{-1} E^T PA + \varepsilon L^T L.\end{aligned} \quad (10)$$

3. Fault Tolerant Synchronization Analysis

3.1. Fault Tolerant Synchronization Analysis When $f(t)$ and $\hat{f}(t)$ Are Derivable on t. Let $\dot{\tilde{f}}(t) = -G\varepsilon(t) = -GCe(t) - GD\tilde{f}(t)$, and we have

$$\begin{aligned}\dot{e}(t) &= Ae(t) + g(\hat{x}(t)) - g(x(t)) + h(\hat{x}(t-\theta)) \\ &\quad - h(x(t-\theta)) + E\tilde{f}(t) - LCe(t) + LCe(t) \\ &\quad - LCe(t-\tau) - LD\tilde{f}(t) + LD\tilde{f}(t) \\ &\quad - LD\tilde{f}(t-\tau) \\ &= (A - LC)e(t) + g(\hat{x}(t)) - g(x(t)) \\ &\quad + h(\hat{x}(t-\theta)) - h(x(t-\theta)) + (E - LD)\tilde{f}(t) \\ &\quad + LC\int_{t-\tau}^{t} \dot{e}(s)\,ds + LD\int_{t-\tau}^{t} \dot{\tilde{f}}(s)\,ds.\end{aligned} \quad (11)$$

Suppose $\eta(t) = \begin{pmatrix} e(t) \\ \tilde{f}(t) \end{pmatrix}$, and then

$$\begin{aligned}\dot{e}(t) &= (A - LC, E - LD)\begin{pmatrix} e(t) \\ \tilde{f}(t) \end{pmatrix} + g(\hat{x}(t)) \\ &\quad - g(x(t)) + h(\hat{x}(t-\theta)) - h(x(t-\theta)) \\ &\quad + (LC, LD)\begin{pmatrix} \int_{t-\tau}^{t} \dot{e}(s)\,ds \\ \int_{t-\tau}^{t} \dot{\tilde{f}}(s)\,ds \end{pmatrix} \\ &= (A - LC, E - LD)\eta(t) + g(\hat{x}(t)) \\ &\quad - g(x(t)) + (LC, LD)\int_{t-\tau}^{t} \dot{\eta}(s)\,ds, \\ \dot{\tilde{f}}(t) &= (-GC, -GD)\begin{pmatrix} e(t) \\ \tilde{f}(t) \end{pmatrix} = (-GC, -GD)\eta(t),\end{aligned}$$

$$\dot{\eta}(t) = \begin{pmatrix} A-LC & E-LD \\ -GC & -GD \end{pmatrix}\eta(t) + \begin{pmatrix} g(\widehat{x}(t)) - g(x(t)) \\ 0 \end{pmatrix} + \begin{pmatrix} h(\widehat{x}(t-\theta)) - h(x(t-\theta)) \\ 0 \end{pmatrix} + \begin{pmatrix} LC & LD \\ 0 & 0 \end{pmatrix}\int_{t-\tau}^{t}\dot{\eta}(s)\,ds,$$
$$\dot{\eta}(t) = B\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix} + \begin{pmatrix} q(t) \\ 0 \end{pmatrix} + R\int_{t-\tau}^{t}\dot{\eta}(s)\,ds, \quad (12)$$

where

$$B = \begin{pmatrix} A-LC & E-LD \\ -GC & -GD \end{pmatrix},$$
$$p(t) = g(\widehat{x}(t)) - g(x(t)),$$
$$q(t) = h(\widehat{x}(t-\theta)) - h(x(t-\theta)), \quad (13)$$
$$R = \begin{pmatrix} LC & LD \\ 0 & 0 \end{pmatrix}.$$

From the assumption, we have

$$(g(x)-g(y))^T U_1^T M_1 (g(x)-g(y)) - (g(x)-g(y))^T U_1^T M_2 (x-y) - (x-y)^T U_2^T M_1 (g(x)-g(y)) + (x-y)^T U_2^T M_2 (x-y) \le 0. \quad (14)$$

So, we can get from assumption that

$$(p(t))^T U_1^T M_1 p(t) - (p(t))^T U_1^T M_2 e(t) - (e(t))^T U_2^T M_1 p(t) + (e(t))^T U_2^T M_2 e(t) \le 0,$$
$$\begin{pmatrix} e^T(t) & \widetilde{f}^T(t) \end{pmatrix}\begin{pmatrix} -U_2^T M_2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} e(t) \\ \widetilde{f}(t) \end{pmatrix} + \begin{pmatrix} e^T(t) & \widetilde{f}^T(t) \end{pmatrix}\begin{pmatrix} U_2^T M_1 & R_1 \\ 0 & R_2 \end{pmatrix}\begin{pmatrix} p(t) \\ 0 \end{pmatrix} + \begin{pmatrix} p^T(t) & 0 \end{pmatrix}\begin{pmatrix} U_1^T M_2 & 0 \\ R_3 & R_4 \end{pmatrix}\begin{pmatrix} e(t) \\ \widetilde{f}(t) \end{pmatrix} + \begin{pmatrix} p^T(t) & 0 \end{pmatrix}\begin{pmatrix} -U_1^T M_1 & R_5 \\ R_6 & R_7 \end{pmatrix}\begin{pmatrix} p(t) \\ 0 \end{pmatrix} \ge 0, \quad (15)$$

in which the proper dimension matrices $R_i \in R^{n\times n}$, $i = 1, 2, \ldots, 7$, are arbitrary.

That is,

$$\eta^T(t)\begin{pmatrix} -U_2^T M_2 & 0 \\ 0 & 0 \end{pmatrix}\eta(t) + \eta^T(t)\begin{pmatrix} U_2^T M_1 & R_1 \\ 0 & R_2 \end{pmatrix}\begin{pmatrix} p(t) \\ 0 \end{pmatrix} + \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\begin{pmatrix} U_1^T M_2 & 0 \\ R_3 & R_4 \end{pmatrix}\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\begin{pmatrix} -U_1^T M_1 & R_5 \\ R_6 & R_7 \end{pmatrix}\begin{pmatrix} p(t) \\ 0 \end{pmatrix} \ge 0,$$
$$\eta^T(t)\left(\frac{1}{2}\left(S_1+S_1^T\right)\right)\eta(t) + \eta^T(t)\left(\frac{1}{2}\left(S_2+S_3^T\right)\right)\begin{pmatrix} p(t) \\ 0 \end{pmatrix} + \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\left(\frac{1}{2}\left(S_3+S_2^T\right)\right)\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\left(\frac{1}{2}\left(S_4+S_4^T\right)\right)\begin{pmatrix} p(t) \\ 0 \end{pmatrix} \ge 0, \quad (16)$$

where

$$S_1 = \begin{pmatrix} -\varepsilon U_2^T M_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} \varepsilon U_2^T M_1 & \varepsilon R_1 \\ 0 & \varepsilon R_2 \end{pmatrix}, \quad S_3 = \begin{pmatrix} \varepsilon U_1^T M_2 & 0 \\ \varepsilon R_3 & \varepsilon R_4 \end{pmatrix}, \quad S_4 = \begin{pmatrix} -\varepsilon U_1^T M_1 & \varepsilon R_5 \\ \varepsilon R_6 & \varepsilon R_7 \end{pmatrix}, \quad (17)$$
$$\varepsilon > 0$$

is any constant. Consider

$$\eta^T(t)\left(S_1+S_1^T\right)\eta(t) + \eta^T(t)\left(S_2+S_3^T\right)\begin{pmatrix} p(t) \\ 0 \end{pmatrix} + \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\left(S_3+S_2^T\right)\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\left(S_4+S_4^T\right)\begin{pmatrix} p(t) \\ 0 \end{pmatrix} \ge 0, \quad (18)$$
$$\xi^T(t)\,\Lambda_1\xi(t) \ge 0,$$

where

$$\xi(t) = \left(\eta^T(t)\ \dot{\eta}^T(t)\ \left(p^T(t)\ 0\right)\ \left(q^T(t)\ 0\right)\ \int_{t-\tau}^{t}\dot{\eta}^T(s)\,ds\ \int_{t-\theta}^{t}\dot{\eta}^T(s)\,ds\right)^T,$$
$$\Lambda_1 = \begin{pmatrix} S_1+S_1^T & 0 & 0 & S_2+S_3^T & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & S_4+S_4^T & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{pmatrix}. \tag{19}$$

Imitating the above inference, we obtain from inequality (5) the following:

$$\xi^T(t)\,\Lambda_2\xi(t) \geq 0, \tag{20}$$

where

$$\Lambda_2 = \delta\begin{pmatrix} Y_1^T\Delta Y_1 & 0 & 0 & Y_1^T\Delta Y_3 & 0 & Y_1^T\Delta Y_2 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & Y_3^T\Delta Y_3 & 0 & Y_3^T\Delta Y_2 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & Y_2^T\Delta Y_2 \end{pmatrix},$$
$$\delta > 0,$$
$$Y_1 = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \tag{21}$$
$$Y_2 = \begin{pmatrix} -I & 0 \\ 0 & 0 \end{pmatrix},$$
$$Y_3 = \begin{pmatrix} 0 & 0 \\ I & R_8 \end{pmatrix},$$
$$\Delta = \begin{pmatrix} -W_2^TV_2 - V_2^TW_2 & W_2^TV_1 + V_2^TW_1 \\ * & -W_1^TV_1 - V_1^TW_1 \end{pmatrix}.$$

We choose Lyapunov-Krasovskii functional

$$V(t) = \eta^T(t)\,P\eta(t) + \int_{-\tau}^{0}\int_{t+r}^{t}\dot{\eta}^T(s)\,Q\dot{\eta}(s)\,ds\,dr + \int_{-\theta}^{0}\int_{t+r}^{t}\dot{\eta}^T(s)\,\Theta\dot{\eta}(s)\,ds\,dr. \tag{22}$$

Differentiating $V(t)$ with respect to t, the following result is yielded:

$$\begin{aligned}\dot{V}(t) &= 2\eta^T(t)\,P\dot{\eta}(t) + \tau\dot{\eta}^T(t)\,Q\dot{\eta}(t) - \int_{t-\tau}^{t}\dot{\eta}^T(s)\,Q\dot{\eta}(s)\,ds \\ &= 2\eta^T(t)\,PB\eta(t) + 2\eta^T(t)\,P\begin{pmatrix} p(t) \\ 0 \end{pmatrix} \\ &\quad + 2\eta^T(t)\,PR\int_{t-\tau}^{t}\dot{\eta}(s)\,ds + \tau\dot{\eta}^T(t)\,Q\dot{\eta}(t) \\ &\quad - \int_{t-\tau}^{t}\dot{\eta}^T(s)\,Q\dot{\eta}(s)\,ds + \theta\dot{\eta}^T(t)\,\Theta\dot{\eta}(t) \\ &\quad - \int_{t-\theta}^{t}\dot{\eta}^T(s)\,\Theta\dot{\eta}(s)\,ds.\end{aligned} \tag{23}$$

From Lemma 2, we have

$$\begin{aligned}\dot{V}(t) &\leq \eta^T(t)\left(PB + B^TP\right)\eta(t) + 2\eta^T(t)\,P\begin{pmatrix} p(t) \\ 0 \end{pmatrix} \\ &\quad + 2\eta^T(t)\,P\begin{pmatrix} q(t) \\ 0 \end{pmatrix} + 2\eta^T(t)\,PR\int_{t-\tau}^{t}\dot{\eta}(s)\,ds \\ &\quad + \tau\dot{\eta}^T(t)\,Q\dot{\eta}(t) \\ &\quad - \frac{1}{\tau}\left(\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right)^T Q\left(\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right) \\ &\quad + \theta\dot{\eta}^T(t)\,\Theta\dot{\eta}(t) \\ &\quad - \frac{1}{\theta}\left(\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right)^T \Theta\left(\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right) \\ &= \xi^T(t)\,\Pi\xi(t) \geq 0,\end{aligned} \tag{24}$$

where

$$\Pi = \begin{pmatrix} PB+B^TP & 0 & P & P & PR & 0 \\ * & \tau Q & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{\tau}Q & 0 \\ * & * & * & * & * & -\frac{1}{\theta}\Theta \end{pmatrix}. \quad (25)$$

From model (12), we have

$$\begin{aligned} 0 &= 2\left(Q_1\eta(t) + Q_2\dot{\eta}(t)\right)^T \\ &\quad \cdot \left(-\dot{\eta}(t) + B\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix} + R\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right) \\ &= \xi^T(t)\,\Xi\xi(t), \end{aligned} \quad (26)$$

where

$$\Xi = \begin{pmatrix} Q_1^TB+B^TQ_1 & -Q_1^T+B^TQ_2^T & Q_1^T & Q_1^T & Q_1^TR & 0 \\ * & -Q_2^T-Q_2 & Q_2^T & Q_2^T & Q_2^TR & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{pmatrix}. \quad (27)$$

From formulas (18), (20), (24), and (26), we get

$$\begin{aligned} \dot{V}(t) &\le \xi^T(t)\left(\Lambda_1 + \Lambda_2 + \Pi + \Xi\right)\xi(t) \\ &= \xi^T(t)\,\Omega\xi(t), \end{aligned} \quad (28)$$

where $\Omega = \Lambda + \Pi + \Xi$.

Based on the above derivation, we have the following result.

Theorem 5. *The fault tolerant synchronization (3) and (6) is achieved if there exist constants $\tau > 0$, $\theta > 0$, $\delta > 0$, and $\varepsilon > 0$, the positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, and $\Theta > 0$, and the matrices Q_j, M_j, U_j, $j = 1, 2$, L, C, D, R_i, $i = 1, 2, \ldots, 8$, such that the matrix $\Omega < 0$.*

Remark 6. After the extended transformation $e(t) \to \eta(t)$, system (7) is turned into system (12). Based on the Lyapunov functional in [15–21], we take Lyapunov functional

$$\begin{aligned} V(t) &= \eta^T(t)P\eta(t) + \int_{-\tau}^{0}\int_{t+r}^{t}\dot{\eta}^T(s)\,Q\dot{\eta}(s)\,ds\,dr \\ &\quad + \int_{-\theta}^{0}\int_{t+r}^{t}\dot{\eta}^T(s)\,\Theta\dot{\eta}(s)\,ds\,dr, \end{aligned} \quad (29)$$

and we get matrix Ω. The conservation of stability of error system (7) can be decreased by choosing the matrices R_i, $i = 1, 2, \ldots, 8$, and the constant $\varepsilon > 0$.

Corollary 7. *When $D = 0$ in system (3), result similar to Theorem 5 can be obtained.*

When $h(x(t-\theta)) = 0$, inequality (18) is transformed to

$$\sigma^T(t)\,\Lambda\sigma(t) \ge 0, \quad (30)$$

where

$$\begin{aligned} &\sigma(t) \\ &\quad = \left(\eta^T(t)\;\; \dot{\eta}^T(t)\;\; \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\;\; \left(\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right)^T\right)^T, \\ &\Lambda = \begin{pmatrix} S_1+S_1^T & 0 & S_2+S_3^T & 0 \\ * & 0 & 0 & 0 \\ * & * & S_4+S_4^T & 0 \\ * & * & * & 0 \end{pmatrix}, \\ &S_1 = \begin{pmatrix} -\varepsilon U_2^TM_2 & 0 \\ 0 & 0 \end{pmatrix}, \\ &S_2 = \begin{pmatrix} \varepsilon U_2^TM_1 & \varepsilon R_1 \\ 0 & \varepsilon R_2 \end{pmatrix}, \\ &S_3 = \begin{pmatrix} \varepsilon U_1^TM_2 & 0 \\ \varepsilon R_3 & \varepsilon R_4 \end{pmatrix}, \\ &S_4 = \begin{pmatrix} -\varepsilon U_1^TM_1 & \varepsilon R_5 \\ \varepsilon R_6 & \varepsilon R_7 \end{pmatrix}, \\ &\varepsilon > 0 \end{aligned} \quad (31)$$

is any constant. The proper dimension matrices $R_i \in R^{n\times n}$, $i = 1, 2, \ldots, 7$, are arbitrary.

We choose Lyapunov functional

$$V(t) = \eta^T(t)P\eta(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\eta}^T(s)\,Q\dot{\eta}(s)\,ds\,d\theta, \quad (32)$$

and differentiating $V(t)$ with respect to t and using Lemma 2 yield

$$\dot{V}(t) \le \sigma^T(t)\,\Pi\sigma(t), \quad (33)$$

where

$$\Pi = \begin{pmatrix} PB+B^TP & 0 & P & PR \\ * & \tau Q & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\frac{1}{\tau}Q \end{pmatrix}. \quad (34)$$

From model (12), we have

$$0 = 2\left(Q_1\eta(t) + Q_2\dot{\eta}(t)\right)^T \cdot \left(-\dot{\eta}(t) + B\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix} + R\int_{t-\tau}^{t}\dot{\eta}(s)\,ds\right) = \sigma^T(t)\,\Xi\sigma(t), \tag{35}$$

where

$$\Xi = \begin{pmatrix} Q_1^T B + B^T Q_1 & -Q_1^T + B^T Q_2^T & Q_1^T & Q_1^T R \\ * & -Q_2^T - Q_2 & Q_2^T & Q_2^T R \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}. \tag{36}$$

Based on the above derivation, we have the following result.

Corollary 8. *The fault tolerant synchronization (3) and (5) is achieved if there exist constants $\tau, \varepsilon > 0$, the positive definite matrices $P = P^T > 0$, $Q = Q^T > 0$, and the matrices Q_j, M_j, U_j, $j = 1, 2$, Q_j, M_j, U_j, $j = 1, 2$, L, C, D, R_i, $i = 1, 2, \ldots, 7$, such that matrix $\Omega < 0$.*

3.2. Fault Tolerant Synchronization Analysis When $f(t)$ and $\hat{f}(t)$ Are Derivable on $x(t)$. Let $df(t)/dx(t) = \hbar(x(t)) = H \cdot F(x(t)) \cdot N$ and $d\hat{f}(t)/dx(t) = \hbar(\hat{x}(t)) = H \cdot F(\hat{x}(t)) \cdot N$, H, N be proper dimension matrices, with satisfying $F^T(x(t))F(x(t)) \le I$, where H, N are proper dimension matrices and I is identity matrix. Then,

$$\begin{aligned} \dot{e}(t) &= Ae(t) + g(\hat{x}(t)) - g(x(t)) + h(\hat{x}(t-\theta)) \\ &\quad - h(x(t-\theta)) + E\tilde{f}(t) - LCe(t) + LCe(t) \\ &\quad - LCe(t-\tau) - LD\tilde{f}(t) + LD\tilde{f}(t) \\ &\quad - LD\tilde{f}(t-\tau) \\ &= (A - LC)e(t) + g(\hat{x}(t)) - g(x(t)) \\ &\quad + (E - LD)\tilde{f}(t) + LC\int_{t-\tau}^{t}\dot{e}(s)\,ds \\ &\quad + LD\int_{t-\tau}^{t}\dot{\tilde{f}}(s)\,ds. \end{aligned} \tag{37}$$

Suppose $\eta(t) = \begin{pmatrix} e(t) \\ \tilde{f}(t) \end{pmatrix}$; we have

$$\begin{aligned} \dot{e}(t) &= (A - LC, E - LD)\begin{pmatrix} e(t) \\ \tilde{f}(t) \end{pmatrix} + g(\hat{x}(t)) \\ &\quad - g(x(t)) + h(\hat{x}(t-\theta)) - h(x(t-\theta)) \\ &\quad + (LC, LD)\begin{pmatrix} \int_{t-\tau}^{t}\dot{e}(s)\,ds \\ \int_{t-\tau}^{t}\dot{\tilde{f}}(s)\,ds \end{pmatrix} \\ &= (A - LC, E - LD)\eta(t) + g(\hat{x}(t)) - g(x(t)) \\ &\quad + h(\hat{x}(t-\theta)) - h(x(t-\theta)) \\ &\quad + (LC, LD)\int_{t-\tau}^{t}\dot{\eta}(s)\,ds. \end{aligned} \tag{38}$$

According to $df(t)/dx(t) = \hbar(x(t)) = H \cdot F(x(t)) \cdot N$, $d\hat{f}(t)/dx(t) = \hbar(\hat{x}(t)) = H \cdot F(\hat{x}(t)) \cdot N$, we get

$$\begin{aligned} \tilde{f}(t) &= \int_0^1 \hbar((1-\lambda)x(t) + \lambda\hat{x}(t))\,e(t)\,d\lambda \\ &= \int_0^1 H \cdot F((1-\lambda)x(t) + \lambda\hat{x}(t)) \cdot N \cdot e(t)\,d\lambda, \\ -\delta\tilde{f}(t) &+ \delta\int_0^1 H \cdot F((1-\lambda)x(t) + \lambda\hat{x}(t)) \cdot N \cdot e(t)\,d\lambda \\ &= 0, \\ \Big(\delta\int_0^1 H &\cdot F((1-\lambda)x(t) + \lambda\hat{x}(t)) \cdot N\,d\lambda, -\delta I\Big)\eta(t) \\ &= 0, \end{aligned} \tag{39}$$

where $\delta > 0$. Consider

$$\begin{aligned} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}\dot{\eta}(t) &= \begin{pmatrix} A - LC & E - LD \\ \delta\int_0^1 H \cdot F((1-\lambda)x(t) + \lambda\hat{x}(t)) \cdot N\,d\lambda & -\delta I \end{pmatrix}\eta(t) \\ &\quad + \begin{pmatrix} g(\hat{x}(t)) - g(x(t)) \\ 0 \end{pmatrix} + \begin{pmatrix} LC & LD \\ 0 & 0 \end{pmatrix}\int_{t-\tau}^{t}\dot{\eta}(s)\,ds, \\ E_1\dot{\eta}(t) &= B\eta(t) + \begin{pmatrix} p(t) \\ 0 \end{pmatrix} + \begin{pmatrix} q(t) \\ 0 \end{pmatrix} + R\int_{t-\tau}^{t}\dot{\eta}(s)\,ds, \end{aligned} \tag{40}$$

where

$$\begin{aligned} E_1 &= \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \\ B &= \begin{pmatrix} A - LC & E - LD \\ \delta\int_0^1 H \cdot F((1-\lambda)x(t) + \lambda\hat{x}(t)) \cdot N\,d\lambda & -\delta I \end{pmatrix}, \\ p(t) &= g(\hat{x}(t)) - g(x(t)), \\ R &= \begin{pmatrix} LC & LD \\ 0 & 0 \end{pmatrix}. \end{aligned} \tag{41}$$

From the assumption, we have

$$\varsigma^T(t)\Lambda_1\varsigma(t) \geq 0, \tag{42}$$

where

$$\varsigma(t) = \left(\eta^T(t)\ \ \dot{e}^T(t)\ \ \left(p^T(t)\ \ 0\right)\ \ \left(q^T(t)\ \ 0\right)\ \ \int_{t-\tau}^{t}\dot{\eta}^T(s)\,ds\ \ \int_{t-\theta}^{t}\dot{e}^T(s)\,ds\right)^T,$$

$$\Lambda_1 = \begin{pmatrix} S_1+S_1^T & 0 & 0 & S_2+S_3^T & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & S_4+S_4^T & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} -\varepsilon U_2^T M_2 & 0 \\ 0 & 0 \end{pmatrix},$$

$$S_2 = \begin{pmatrix} \varepsilon U_2^T M_1 & \varepsilon R_1 \\ 0 & \varepsilon R_2 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} \varepsilon U_1^T M_2 & 0 \\ \varepsilon R_3 & \varepsilon R_4 \end{pmatrix},$$

$$S_4 = \begin{pmatrix} -\varepsilon U_1^T M_1 & \varepsilon R_5 \\ \varepsilon R_6 & \varepsilon R_7 \end{pmatrix},$$

$$\varepsilon > 0 \tag{43}$$

is any constant. The proper dimension matrices $R_i \in R^{n\times n}$, $i = 1, 2, \ldots, 7$, are arbitrary

$$\varsigma^T(t)\Lambda_2\varsigma(t) \geq 0, \tag{44}$$

where

$$\Lambda_2 = \delta \begin{pmatrix} Y_1^T\Delta Y_1 & 0 & 0 & Y_1^T\Delta Y_3 & 0 & Y_1^T\Delta Y_2 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & Y_3^T\Delta Y_3 & 0 & Y_3^T\Delta Y_2 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & -W_2^TV_2 - V_2^TW_2 \end{pmatrix},$$

$$\delta > 0,$$

$$Y_1 = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} -I & 0 \\ 0 & 0 \end{pmatrix},$$

$$Y_3 = \begin{pmatrix} 0 & 0 \\ I & R_8 \end{pmatrix},$$

$$\Delta = \begin{pmatrix} -W_2^TV_2 - V_2^TW_2 & W_2^TV_1 + V_2^TW_1 \\ * & -W_1^TV_1 - V_1^TW_1 \end{pmatrix}. \tag{45}$$

We choose Lyapunov function

$$V(t) = e^T(t)P_1e(t) + \int_{-\tau}^{0}\int_{t+r}^{t}\dot{e}^T(s)Q\dot{e}(s)\,ds\,dr + \int_{-\theta}^{0}\int_{t+r}^{t}\dot{e}^T(s)\Theta\dot{e}(s)\,ds\,dr$$

$$
= \eta^T(t) PE_1\eta(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s) Q\dot{e}(s)\, ds\, d\theta + \int_{-\theta}^{0}\int_{t+r}^{t} \dot{e}^T(s)\Theta\dot{e}(s)\, ds\, dr, \tag{46}
$$

where $P = \begin{pmatrix} P_1 & P_2 \\ 0 & P_3 \end{pmatrix}$, and it is easy to know that $PE_1 = E_1^T P^T$.

The derivation of $V(t)$ on t is

$$
\begin{aligned}
\dot{V}(t) &= 2\eta^T(t) PE_1\dot{\eta}(t) + \tau\dot{e}^T(t) Q\dot{e}(t) - \int_{t-\tau}^{t} \dot{e}^T(s) Q\dot{e}(s)\, ds + \theta\dot{e}^T(t)\Theta\dot{e}(t) - \int_{t-\theta}^{t}\dot{e}^T(s)\Theta\dot{e}(s)\, ds \\
&= 2\eta^T(t) PB\eta(t) + 2\eta^T(t) P\begin{pmatrix} p(t) \\ 0 \end{pmatrix} + 2\eta^T(t) PR\int_{t-\tau}^{t}\dot{\eta}(s)\, ds + \tau\dot{e}^T(t) Q\dot{e}(t) \\
&\quad - \int_{t-\tau}^{t}\dot{e}^T(s) Q\dot{e}(s)\, ds + \theta\dot{e}^T(t)\Theta\dot{e}(t) - \int_{t-\theta}^{t}\dot{e}^T(s)\Theta\dot{e}(s)\, ds.
\end{aligned} \tag{47}
$$

From Lemma 2, we have

$$
\begin{aligned}
\dot{V}(t) &\le \eta^T(t)\left(PB + B^T P^T\right)\eta(t) + 2\eta^T(t) P\begin{pmatrix} p(t) \\ 0 \end{pmatrix} + 2\eta^T(t) P\begin{pmatrix} q(t) \\ 0 \end{pmatrix} + 2\eta^T(t) PR\int_{t-\tau}^{t}\dot{\eta}(s)\, ds \\
&\quad + \tau\dot{e}^T(t) Q\dot{e}(t) - \frac{1}{\tau}\left(\int_{t-\tau}^{t}\dot{e}^T(s)\, ds\right)^T Q\left(\int_{t-\tau}^{t}\dot{e}^T(s)\, ds\right) \\
&\quad + \theta\dot{e}^T(t)\Theta\dot{e}(t) - \frac{1}{\theta}\left(\int_{t-\theta}^{t}\dot{e}^T(s)\, ds\right)^T\Theta\left(\int_{t-\theta}^{t}\dot{e}(s)\, ds\right) \\
&= \varsigma^T(t)\Pi\varsigma(t),
\end{aligned} \tag{48}
$$

where

$$
\Pi = \begin{pmatrix}
PB + B^T P^T & 0 & P & P & PR & 0 \\
* & \tau Q & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 \\
* & * & * & * & -\frac{1}{\tau}Q & 0 \\
* & * & * & * & * & -\frac{1}{\theta}\Theta
\end{pmatrix}. \tag{49}
$$

From model (3), we have

$$
\begin{aligned}
0 = 2&\left(Q_1 e(t) + Q_2\dot{e}(t) + Q_3\int_{t-\tau}^{t}\dot{\eta}(s)\, ds\right)^T\Big(-\dot{e}(t) \\
&+ (A - LC, E - LD)\eta(t) + p(t) + q(t) \\
&+ (LC, LD)\int_{t-\tau}^{t}\dot{\eta}(s)\, ds\Big) = \varsigma^T(t)\Xi\varsigma(t),
\end{aligned} \tag{50}
$$

where

$$
\Xi = \begin{pmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{13} & \Xi_{14} & 0 \\
* & \Xi_{22} & \Xi_{23} & \Xi_{23} & \Xi_{24} & 0 \\
* & * & \Xi_{33} & 0 & \Xi_{34} & 0 \\
* & * & * & \Xi_{33} & \Xi_{34} & 0 \\
* & * & * & * & \Xi_{44} & 0 \\
* & * & * & * & * & 0
\end{pmatrix},
$$

$$
\begin{aligned}
\Xi_{11} &= \begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(A - LC, E - LD) + \left(\begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(A - LC, E - LD)\right)^T, \\
\Xi_{12} &= -\begin{pmatrix} Q_1^T \\ 0 \end{pmatrix} + \left(Q_2^T(A - LC, E - LD)\right)^T, \\
\Xi_{13} &= \begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(I, R_8), \\
\Xi_{14} &= \begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(LC, LD) + \left(Q_3^T(A - LC, E - LD)\right)^T, \\
\Xi_{22} &= -\left(Q_2 + Q_2^T\right), \\
\Xi_{23} &= Q_2^T(I, R_9), \\
\Xi_{24} &= -Q_3 + Q_2^T(LC, LD)^T, \\
\Xi_{33} &= 0, \\
\Xi_{34} &= Q_3^T(I, R_9), \\
\Xi_{44} &= Q_3^T(LC, LD) + \left(Q_3^T(LC, LD)\right)^T,
\end{aligned} \tag{51}
$$

where the proper dimension matrices $R_9 \in R^{n\times n}$ is arbitrary.

From formulas (42), (44), (48), and (50), we get

$$
\dot{V}(t) \le \varsigma^T(t)(\Lambda + \Pi + \Xi)\varsigma(t) = \varsigma^T(t)\Omega\varsigma(t), \tag{52}
$$

where

$$\Omega = \Lambda + \Pi + \Xi = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{pmatrix},$$

$$\Omega_{11} = \begin{pmatrix} S_1 + S_1^T + \Xi_{11} + Y_1^T \Delta Y_1 + PB + B^T P^T & \Xi_{12} \\ * & \tau Q + \Xi_{22} \end{pmatrix},$$

$$\Omega_{12} = \begin{pmatrix} P + \Xi_{13} & S_2 + S_3^T + \Xi_{13} + P + Y_1^T \Delta Y_3 & PR + \Xi_{14} & Y_1^T \Delta Y_2 \\ \Xi_{23} & \Xi_{23} & \Xi_{24} & 0 \end{pmatrix},$$

$$\Omega_{22} = \begin{pmatrix} S_4 + S_4^T & 0 & \Xi_{34} & 0 \\ * & Y_3^T \Delta Y_3 & \Xi_{34} & Y_3^T \Delta Y_2 \\ * & * & -\tau^{-1} Q + \Xi_{44} & 0 \\ * & * & * & -W_2^T V_2 - V_2^T W_2 \end{pmatrix}, \quad (53)$$

$$PB + B^T P^T = \begin{pmatrix} P_1 (A - LC) & P_1 (E - LD) - \delta P_2 \\ 0 & -\delta P_3 \end{pmatrix} + \begin{pmatrix} P_1 (A - LC) & P_1 (E - LD) - \delta P_2 \\ 0 & -\delta P_3 \end{pmatrix}^T$$
$$+ \delta \begin{pmatrix} P_2 H \int_0^1 F((1-\lambda) x(t) + \lambda \hat{x}(t))\, d\lambda N & 0 \\ P_3 H \int_0^1 F((1-\lambda) x(t) + \lambda \hat{x}(t))\, d\lambda N & 0 \end{pmatrix} + \delta \begin{pmatrix} P_2 H \int_0^1 F((1-\lambda) x(t) + \lambda \hat{x}(t))\, d\lambda N & 0 \\ P_3 H \int_0^1 F((1-\lambda) x(t) + \lambda \hat{x}(t))\, d\lambda N & 0 \end{pmatrix}^T$$
$$= Z + Z^T + \delta \begin{pmatrix} P_2 H \\ P_3 H \end{pmatrix} \int_0^1 F((1-\lambda) x(t) + \lambda \hat{x}(t))\, d\lambda \begin{pmatrix} N & 0 \end{pmatrix}$$
$$+ \delta \left[\begin{pmatrix} P_2 H \\ P_3 H \end{pmatrix} \int_0^1 F((1-\lambda) x(t) + \lambda \hat{x}(t))\, d\lambda \begin{pmatrix} N & 0 \end{pmatrix} \right]^T,$$

where $Z = \begin{pmatrix} P_1(A-LC) & P_1(E-LD)-\delta P_2 \\ 0 & -\delta P_3 \end{pmatrix}$.

From Lemma 3, we obtain that $S_1 + S_1^T + \Xi_{11} + PB + B^T P^T < 0$ is equivalent to

$$S_1 + S_1^T + \Xi_{11} + Z + Z^T + \varepsilon^{-1} \delta^2 \begin{pmatrix} P_2 H H^T P_2^T & P_2 H H^T P_3^T \\ * & P_3 H H^T P_3^T \end{pmatrix} + \varepsilon \begin{pmatrix} N^T N & 0 \\ 0 & 0 \end{pmatrix} < 0. \quad (54)$$

Based on the above derivation, we have the following result.

Theorem 9. *The fault tolerant synchronization (3) and (6) is achieved if there exist constants* $\tau, \varepsilon > 0$, $\delta > 0$, *the positive definite matrices* $P_1 = P_1^T > 0$, $Q = Q^T > 0$, *and the matrices* Q_j, M_j, U_j, $j = 1, 2$, Q_j, M_j, U_j, $j = 1, 2$, L, C, D, R_i, $i = 1, 2, \ldots, 9$, *such that matrix* $\Sigma < 0$, *where*

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Omega_{12} \\ * & \Omega_{22} \end{pmatrix},$$

$$\Theta_{11} = S_1 + S_1^T + \Xi_{11} + Z + Z^T + \varepsilon^{-1} \delta^2 \begin{pmatrix} P_2 H H^T P_2^T & P_2 H H^T P_3^T \\ * & P_3 H H^T P_3^T \end{pmatrix} + \varepsilon \begin{pmatrix} N^T N & 0 \\ 0 & 0 \end{pmatrix}. \quad (55)$$

Remark 10. After extending system (37) to singular system (40) by the transformation $e(t) \rightarrow \eta(t)$, taking proper matrices R_i, $i = 1, 2, \ldots, 9$, and the constant $\varepsilon > 0$ can decrease the conservation of stability of error system (37) by constructing the Lyapunov functional

$$\begin{aligned} V(t) &= e^T(t) P_1 e(t) + \int_{-\tau}^{0}\int_{t+r}^{t} \dot{e}^T(s) Q \dot{e}(s)\,ds\,dr \\ &\quad + \int_{-\theta}^{0}\int_{t+r}^{t} \dot{e}^T(s) \Theta \dot{e}(s)\,ds\,dr \\ &= \eta^T(t) P E_1 \eta(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s) Q \dot{e}(s)\,ds\,d\theta \\ &\quad + \int_{-\theta}^{0}\int_{t+r}^{t} \dot{e}^T(s) \Theta \dot{e}(s)\,ds\,dr. \end{aligned} \tag{56}$$

When $h(x(t-\theta)) = 0$, inequality (26) is transformed to

$$\omega^T(t) \Lambda \omega(t) \geq 0, \tag{57}$$

where

$$\omega(t) = \left(\eta^T(t)\ \ \dot{e}^T(t)\ \ \begin{pmatrix} p(t) \\ 0 \end{pmatrix}^T\ \ \left(\int_{t-\tau}^{t} \dot{e}(s)\,ds\right)^T\right)^T,$$

$$\Lambda = \begin{pmatrix} S_1 + S_1^T & 0 & S_2 + S_3^T & 0 \\ * & 0 & 0 & 0 \\ * & * & S_4 + S_4^T & 0 \\ * & * & * & 0 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} -\varepsilon U_2^T M_2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{58}$$

$$S_2 = \begin{pmatrix} \varepsilon U_2^T M_1 & \varepsilon R_1 \\ 0 & \varepsilon R_2 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} \varepsilon U_1^T M_2 & 0 \\ \varepsilon R_3 & \varepsilon R_4 \end{pmatrix},$$

$$S_4 = \begin{pmatrix} -\varepsilon U_1^T M_1 & \varepsilon R_5 \\ \varepsilon R_6 & \varepsilon R_7 \end{pmatrix},$$

$$\varepsilon > 0$$

is any constant. The proper dimension matrices $R_i \in R^{n\times n}$, $i = 1, 2, \ldots, 7$, are arbitrary.

We choose Lyapunov function

$$\begin{aligned} V(t) &= e^T(t) P_1 e(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s) Q \dot{e}(s)\,ds\,d\theta \\ &= \eta^T(t) P E_1 \eta(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^T(s) Q \dot{e}(s)\,ds\,d\theta, \end{aligned} \tag{59}$$

where $P = \begin{pmatrix} P_1 & P_2 \\ 0 & P_3 \end{pmatrix}$, and it is easy to know that $PE_1 = E_1^T P^T$, and differentiating $V(t)$ with respect to t and using Lemma 2 yield

$$\dot{V}(t) \leq \omega^T(t) \Pi \omega(t), \tag{60}$$

where

$$\Pi = \begin{pmatrix} PB + B^T P^T & 0 & P & PR \\ * & \tau Q & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\frac{1}{\tau} Q \end{pmatrix}. \tag{61}$$

From model (12), we have

$$\begin{aligned} 0 &= 2\left(Q_1 e(t) + Q_2 \dot{e}(t) + Q_3 \int_{t-\tau}^{t} \dot{\eta}(s)\,ds\right)^T \Big(-\dot{e}(t) \\ &\quad + (A - LC, E - LD)\,\eta(t) + p(t) \\ &\quad + (LC, LD)\int_{t-\tau}^{t} \dot{\eta}(s)\,ds\Big) = \sigma^T(t)\,\Xi\,\sigma(t), \end{aligned} \tag{62}$$

where

$$\begin{aligned} \Xi &= \begin{pmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{pmatrix}, \\ \Xi_{11} &= \begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(A - LC, E - LD) + \left(\begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(A - LC, E - LD)\right)^T, \\ \Xi_{12} &= -\begin{pmatrix} Q_1^T \\ 0 \end{pmatrix} + \left(Q_2^T (A - LC, E - LD)\right)^T, \\ \Xi_{13} &= \begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(I, R_8), \\ \Xi_{14} &= \begin{pmatrix} Q_1^T \\ 0 \end{pmatrix}(LC, LD) + \left(Q_3^T (A - LC, E - LD)\right)^T, \\ \Xi_{22} &= -\left(Q_2 + Q_2^T\right), \\ \Xi_{23} &= Q_2^T (I, R_8), \\ \Xi_{24} &= -Q_3 + Q_2^T (LC, LD)^T, \\ \Xi_{33} &= 0, \\ \Xi_{34} &= Q_3^T (I, R_8), \\ \Xi_{44} &= Q_3^T (LC, LD) + \left(Q_3^T (LC, LD)\right)^T, \end{aligned} \tag{63}$$

where the proper dimension matrices $R_8 \in R^{n\times n}$ is arbitrary.

Based on the above derivation, we have the following result.

Corollary 11. *The fault tolerant synchronization (3) and (5) is achieved if there exist constants $\tau, \varepsilon > 0$, $\delta > 0$, the positive definite matrices $P_1 = P_1^T > 0$, $Q = Q^T > 0$, and*

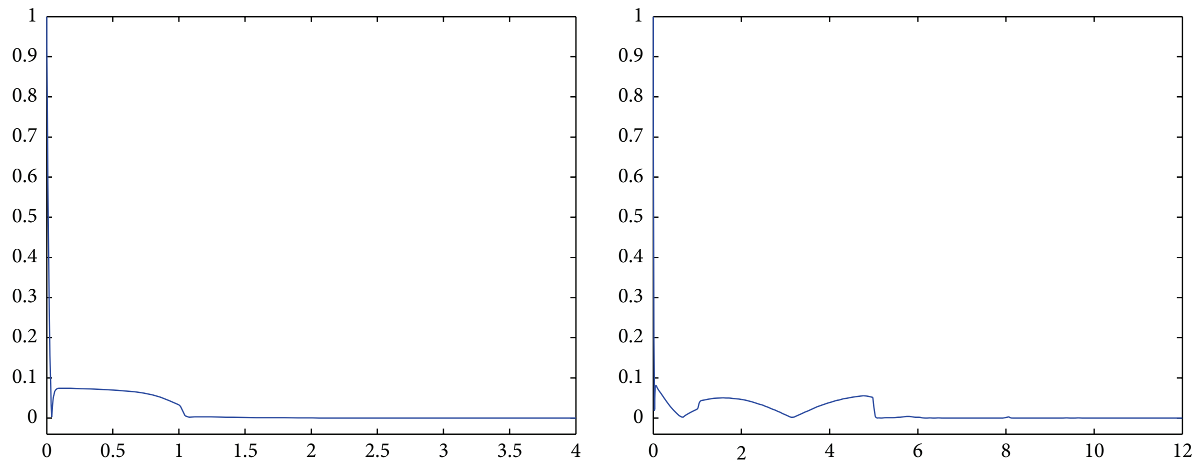

Figure 1: The state responses of $x(t)$ and $\widehat{x}(t)$ when $f(t)$ and $\widehat{f}(t)$ are derivable on t.

the matrices Q_j, M_j, U_j, $j = 1, 2$, Q_j, M_j, U_j, $j = 1, 2$, L, C, D, R_i, $i = 1, 2, \ldots, 8$, *such that matrix* $\Theta < 0$, *where*

$$\Theta = \begin{pmatrix} \Theta_{11} & \Omega_{12} \\ * & \Omega_{22} \end{pmatrix},$$

$$\begin{aligned} \Theta_{11} &= S_1 + S_1^T + \Xi_{11} + Z + Z^T \\ &\quad + \varepsilon^{-1}\delta^2 \begin{pmatrix} P_2HH^TP_2^T & P_2HH^TP_3^T \\ * & P_3HH^TP_3^T \end{pmatrix} \\ &\quad + \varepsilon \begin{pmatrix} N^TN & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned} \tag{64}$$

4. Numerical Examples

Example 1. Consider a typical delayed Hopfield neural networks [29–32] with two neurons

$$\begin{aligned} A &= -I, \\ g(x(t)) &= \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{pmatrix} \begin{pmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{pmatrix}, \\ h(x(t-\theta)) &= \begin{pmatrix} 0.15 & -0.1 \\ -0.2 & -2.5 \end{pmatrix} \begin{pmatrix} \tanh(x_1(t-\theta)) \\ \tanh(x_2(t-\theta)) \end{pmatrix}, \\ \tau &= 0.01, \\ \theta &= 1, \\ LC &= LD = -38I, \\ E &= 2I, \\ x(t) &= \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \end{aligned} \tag{65}$$

When $f(t)$ and $\widehat{f}(t)$ are derivable on t, we take

$$\begin{aligned} f(t) &= \begin{cases} (0,0)^T, & 0 \le t \le 2 \\ (0, 2\sin t)^T, & t \ge 2, \end{cases} \\ \dot{\widetilde{f}}(t) &= \begin{pmatrix} 0.2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) - x_3(t) \\ x_2(t) - x_4(t) \end{pmatrix} - 2\widetilde{f}(t). \end{aligned} \tag{66}$$

Simulation results are shown in Figure 1.

When $f(t)$ and $\widehat{f}(t)$ are derivable on $x(t)$, we take

$$f(t) = \begin{cases} (0,0)^T, & 0 \le t \le 10 \\ (0.2\sin(x_1(t)), 0.4\sin(x_2(t)))^T, & t \ge 10. \end{cases} \tag{67}$$

Simulation results are shown in Figure 2.

Example 2. Consider chaotic Lü system [31, 32]; that is,

$$\begin{aligned} A &= \begin{pmatrix} -36 & 36 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & -3 \end{pmatrix}, \\ g(x(t)) &= \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix}, \end{aligned} \tag{68}$$

where $x(t) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. We choose

$$LC = \begin{pmatrix} 0 & -38 & 0 \\ 0 & -38 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$LD = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -38 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

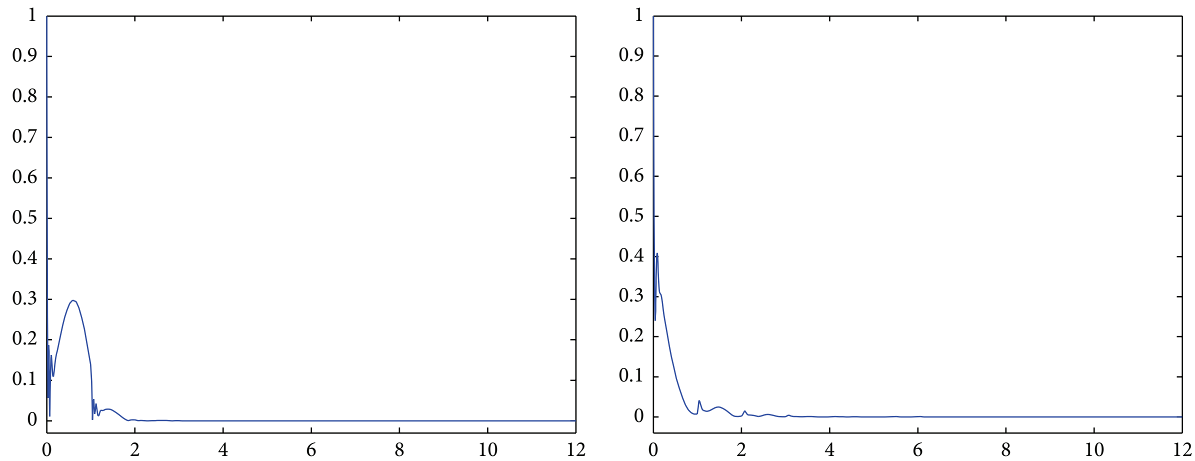

Figure 2: The state responses of $x(t)$ and $\hat{x}(t)$ when $f(t)$ and $\hat{f}(t)$ are derivable on $x(t)$.

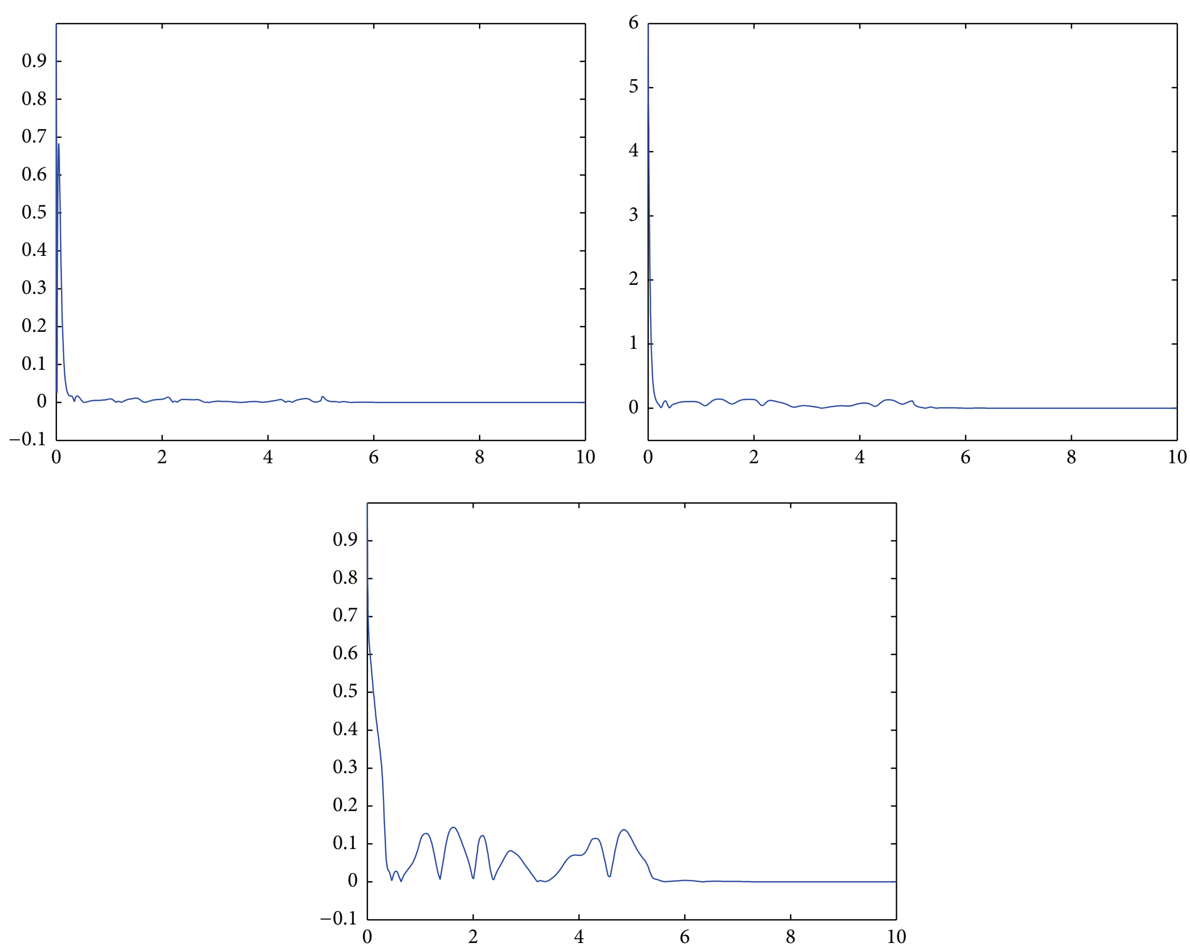

Figure 3: The state responses of $x(t)$ and $\hat{x}(t)$ when $f(t)$ and $\hat{f}(t)$ are derivable on t.

$$E = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix},$$
$$\tau = 0.01. \qquad (69)$$

When $f(t)$ and $\hat{f}(t)$ are derivable on t, we take

$$f(t) = \begin{cases} (0, 0, 0)^T, & 0 \le t \le 2 \\ (0, 2\sin t, 0)^T, & t \ge 2, \end{cases} \qquad (70)$$

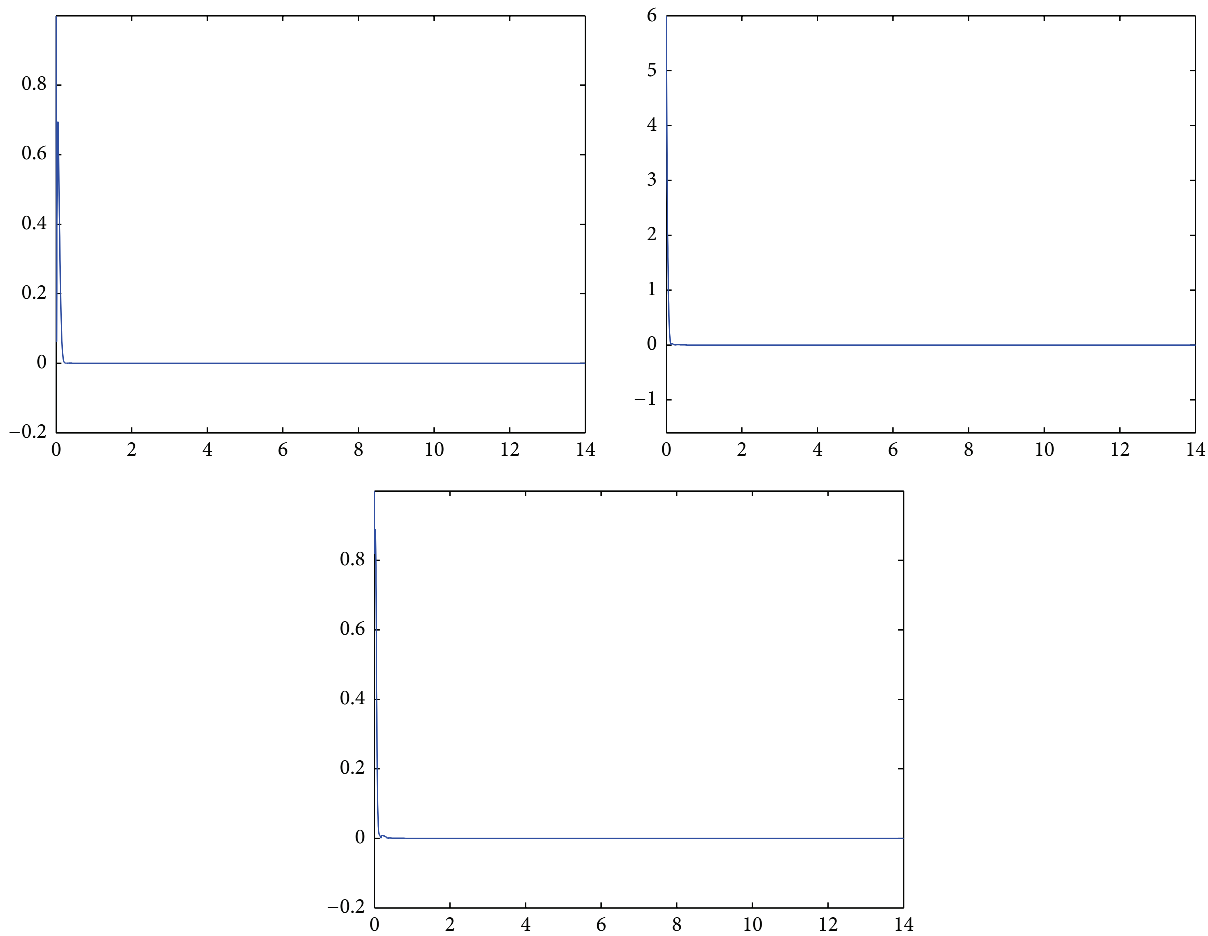

Figure 4: The state responses of $x(t)$ and $\hat{x}(t)$ when $f(t)$ and $\hat{f}(t)$ are derivable on $x(t)$.

$$\dot{\tilde{f}}(t) = \begin{pmatrix} 0.2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) - x_3(t) \\ x_2(t) - x_4(t) \end{pmatrix} - 2\tilde{f}(t). \tag{71}$$

Simulation results are shown in Figure 3.

When $f(t)$ and $\hat{f}(t)$ are derivable on $x(t)$, we take

$$f(t) = \begin{cases} (0,0,0)^T, & 0 \le t \le 5 \\ (0, 0.4\sin(x_1(t)), 0.4\sin(x_2(t)))^T, & t \ge 5. \end{cases} \tag{72}$$

Simulation results are shown in Figure 4.

5. Conclusions

In this paper, taking into account the constant propagation delay and actuator faults and the extended transformation of the error systems, the problem of the chaotic fault tolerant synchronization has been addressed. The conservation of stability of the error systems is lower than that of other existing literatures.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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