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Research Article

Coupled Fixed Point Theorems under Weak Contractions

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Cho et al. [Comput. Math. Appl. 61(2011), 1254–1260] studied common fixed point theorems on cone metric spaces by using the concept of c-distance. In this paper, we prove some coupled fixed point theorems in ordered cone metric spaces by using the concept of c-distance in cone metric spaces.

1. Introduction

Many fixed point theorems have been proved for mappings on cone metric spaces in the sense of Huang and Zhang [1]. For some more results on fixed point theory and applications in cone metric spaces, we refer the readers to [2–15]. Recently, Bhaskar and Lakshmikantham [16] introduced the concept of a coupled coincidence point of a mapping F from $X \times X$ into X and a mapping g from X into X and studied fixed point theorems in partially ordered metric spaces. For some more results on couple fixed point theorems, refer to [17–23].

Recently, Cho et al. [7] introduced a new concept of c-distance in cone metric spaces, which is a cone version of w-distance of Kada et al. [24] (see also [25]) and proved some fixed point theorems for some contractive type mappings in partially ordered cone metric spaces using the c-distance.

In this paper, we prove some coupled fixed point theorems in ordered cone metric spaces by using the concept of c-distance.

2. Preliminaries

In this paper, assume that E is a real Banach space. Let P be a subset of E with int $(P) \neq \emptyset$. Then P is called a *cone* if the following conditions are satisfied:

(1) P is closed and $P \neq \{\theta\}$;

(2) $a, b \in \mathbf{R}^+$, $x, y \in P$ implies $ax + by \in P$;

(3) $x \in P \cap -P$ implies $x = \theta$.

For a cone P, define the *partial ordering* $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. We write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, while $x \ll y$ stand for $y - x \in \text{int } P$.

It can be easily shown that λ int $(P) \subseteq \text{int}(P)$ for all positive scalars λ.

Definition 2.1 (see [1]). Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies the following conditions:

(1) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, y) \preceq d(x, z) + d(y, z)$ for all $x, y, z \in X$.

Then d is called a *cone metric* on X, and (X, d) is called a *cone metric space*.

Definition 2.2 (see [1]). Let (X, d) be a cone metric space. Let (x_n) be a sequence in X and $x \in X$.

(1) If, for any $c \in X$ with $\theta \ll c$, there exists $N \in \mathbf{N}$ such that $d(x_n, x) \ll c$ for all $n \geq N$, then (x_n) is said to be *convergent* to a point $x \in X$ and x is the *limit* of (x_n). We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

(2) If, for any $c \in E$ with $\theta \ll c$, there exists $N \in \mathbf{N}$ such that $d(x_n, x_m) \ll c$ for all $n, m \geq N$, then (x_n) is called a *Cauchy sequence* in X.

(3) The space (X, d) is called a complete cone metric space if every Cauchy sequence is convergent.

Definition 2.3 (see [7]). Let $(X, \sqsubseteq)$ be a partially ordered set, and let $F : X \times X \to X$ be a function. Then the mapping F is said to have the *mixed monotone property* if $F(x, y)$ is monotone nondecreasing in x and is monotone nonincreasing in y; that is,

$$x_1 \sqsubseteq x_2 \text{ implies } F(x_1, y) \sqsubseteq F(x_2, y) \tag{2.1}$$

for all $y \in X$ and

$$y_1 \sqsubseteq y_2 \text{ implies } F(x, y_2) \sqsubseteq F(x, y_1) \tag{2.2}$$

for all $x \in X$.

Definition 2.4 (see [7]). An element $(x, y) \in X \times X$ is called a *coupled fixed point* of a mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Recently, Cho et al. [7] introduced the concept of c-distance on cone metric space (X, d) which is a generalization of w-distance of Kada et al. [24].

Definition 2.5 (see [7]). Let (X, d) be a cone metric space. Then a function $q : X \times X \to E$ is called a *c-distance* on X if the following are satisfied:

(q1) $\theta \preceq q(x, y)$ for all $x, y \in X$;

(q2) $q(x, z) \preceq q(x, y) + q(y, z)$ for all $x, y, z \in X$;

(q3) for any $x \in X$, if there exists $u = u_x \in P$ such that $q(x, y_n) \preceq u$ for each $n \geq 1$, then $q(x, y) \preceq u$ whenever (y_n) is a sequence in X converging to a point $y \in X$;

(q4) for any $c \in E$ with $\theta \ll c$, there exists $e \in E$ with $0 \preceq e$ such that $q(z, x) \ll e$ and $q(z, y) \ll c$ imply $d(x, y) \ll c$.

Cho et al. [7] noticed the following important remark in the concept of c-distance on cone metric spaces.

Remark 2.6 (see [7]). Let q be a c-distance on a cone metric space (X, d). Then

(1) $q(x, y) = q(y, x)$ does not necessarily hold for all $x, y \in X$,

(2) $q(x, y) = \theta$ is not necessarily equivalent to $x = y$ for all $x, y \in X$.

The following lemma is crucial in proving our results.

Lemma 2.7 (see [7]). *Let (X, d) be a cone metric space, and let q be a c-distance on X. Let (x_n) and (y_n) be sequences in X and $x, y, z \in X$. Suppose that (u_n) is a sequence in P converging to θ. Then the following hold:*

(1) *if $q(x_n, y) \preceq u_n$ and $q(x_n, z) \preceq u_n$, then $y = z$;*

(2) *if $q(x_n, y_n) \preceq u_n$ and $q(x_n, z) \preceq u_n$, then (y_n) converges to a point $z \in X$;*

(3) *if $q(x_n, x_m) \preceq u_n$ for each $m > n$, then (x_n) is a Cauchy sequence in X;*

(4) *If $q(y, x_n) \preceq u_n$, then (x_n) is a Cauchy sequence in X.*

3. Main Results

In this section, we prove some coupled fixed point theorems by using c-distance in partially ordered cone metric spaces.

Theorem 3.1. *Let $(X, \sqsubseteq)$ be a partially ordered set, and suppose that (X, d) is a complete cone metric space. Let q be a c-distance on X, and let $F : X \times X \to X$ be a continuous function having the mixed monotone property such that*

$$q(F(x, y), F(x^*, y^*)) \preceq \frac{k}{2}(q(x, x^*) + q(y, y^*)) \tag{3.1}$$

for some $k \in [0, 1)$ and all $x, y, x^*, y^* \in X$ with $(x \sqsubseteq x^*) \wedge (y \sqsupseteq y^*)$ or $(x \sqsupseteq x^*) \wedge (y \sqsubseteq y^*)$. If there exist $x_0, y_0 \in X$ such that $x_0 \sqsubseteq F(x_0, y_0)$ and $F(y_0, x_0) \sqsubseteq y_0$, then F has a coupled fixed point (u, v). Moreover, one has $q(v, v) = \theta$ and $q(u, u) = \theta$.

Proof. Let $x_0, y_0 \in X$ be such that $x_0 \sqsubseteq F(x_0, y_0)$ and $F(y_0, x_0) \sqsubseteq y_0$. Let $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$. Since F has the mixed monotone property, we have $x_0 \sqsubseteq x_1$ and $y_1 \sqsubseteq y_0$. Continuing this process, we can construct two sequences (x_n) and (y_n) in X such that

$$\begin{aligned} x_n &= F(x_{n-1}, y_{n-1}) \sqsubseteq x_{n+1} = F(x_n, y_n), \\ y_{n+1} &= F(y_n, x_n) \sqsubseteq y_n = F(y_{n-1}, x_{n-1}). \end{aligned} \tag{3.2}$$

Let $n \in \mathbf{N}$. Now, by (3.1), we have

$$\begin{aligned} q(x_n, x_{n+1}) &= q(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ &\leq \frac{k}{2}(q(x_{n-1}, x_n) + q(y_{n-1}, y_n)), \\ q(x_{n+1}, x_n) &= q(F(x_n, y_n), F(x_{n-1}, y_{n-1})) \\ &\leq \frac{k}{2}(q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \end{aligned} \tag{3.3}$$

From (3.3), it follows that

$$q(x_n, x_{n+1}) + q(x_{n+1}, x_n) \leq \frac{k}{2}(q(x_{n-1}, x_n) + q(y_{n-1}, y_n) + q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \tag{3.4}$$

Similarly, we have

$$q(y_n, y_{n+1}) + q(y_{n+1}, y_n) \leq \frac{k}{2}(q(x_{n-1}, x_n) + q(y_{n-1}, y_n) + q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \tag{3.5}$$

Thus it follows from (3.4) and (3.5) that

$$\begin{aligned} &q(x_n, x_{n+1}) + q(x_{n+1}, x_n) + q(y_n, y_{n+1}) + q(y_{n+1}, y_n) \\ &\quad \leq k(q(x_{n-1}, x_n) + q(y_{n-1}, y_n) + q(x_n, x_{n-1}) + q(y_n, y_{n-1})). \end{aligned} \tag{3.6}$$

Repeating (3.6) n-times, we get

$$\begin{aligned} &q(x_n, x_{n+1}) + q(x_{n+1}, x_n) + q(y_n, y_{n+1}) + d(y_{n+1}, y_n) \\ &\quad \leq k^n(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)). \end{aligned} \tag{3.7}$$

Thus we have

$$\begin{aligned} q(x_n, x_{n+1}) &\leq k^n(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)), \\ q(y_n, y_{n+1}) &\leq k^n(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)). \end{aligned} \tag{3.8}$$

Let $m, n \in \mathbf{N}$ with $m > n$. Since

$$q(x_n, x_m) \preceq \sum_{i=n}^{m-1} q(x_i, x_{i+1}),$$
$$q(y_n, y_m) \preceq \sum_{i=n}^{m-1} q(y_i, y_{i+1}), \tag{3.9}$$

and $k < 1$, we have

$$q(x_n, x_m) \preceq \frac{k^n}{1-k}\big(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\big),$$
$$q(y_n, y_m) \preceq \frac{k^n}{1-k}\big(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\big). \tag{3.10}$$

From Lemma 2.7 (3), it follows that (x_n) and (y_n) are Cauchy sequences in (X, d). Since X is complete, there exist $u, v \in X$ such that $x_n \to u$ and $y_n \to v$. Since F is continuous, we have

$$x_{n+1} = F(x_n, y_n) \longrightarrow F(u, v),$$
$$y_{n+1} = F(y_n, x_n) \longrightarrow F(v, u). \tag{3.11}$$

By the uniqueness of the limits, we get $u = f(u, v)$ and $v = F(v, u)$. Thus (u, v) is a coupled fixed point of F.

Moreover, by (3.1), we have

$$q(u, u) = q(F(u, v), F(u, v)) \preceq \frac{k}{2}\big(q(u, u) + q(v, v)\big),$$
$$q(v, v) = q(F(v, u), F(v, u)) \preceq \frac{k}{2}\big(q(v, v) + q(u, u)\big). \tag{3.12}$$

Therefore, we get

$$q(u, u) + q(v, v) \preceq k\big(q(v, v) + q(u, u)\big). \tag{3.13}$$

Since $k < 1$, we conclude that $q(u, u) + q(v, v) = \theta$, and hence $q(u, u) = \theta$ and $q(v, v) = \theta$. This completes the proof. □

Theorem 3.2. *In addition to the hypotheses of Theorem 3.1, suppose that any two elements x and y in X are comparable. Then the coupled fixed point has the form (u, u), where $u \in X$.*

Proof. As in the proof of Theorem 3.1, there exists a coupled fixed point $(u, v) \in X \times X$. Here $u = F(u, v)$ and $v = F(v, u)$. By the additional assumption and (3.1), we have

$$q(u, v) = q(F(u, v), F(v, u)) \preceq \frac{k}{2}\big(q(u, v) + q(v, u)\big),$$
$$q(v, u) = q(F(v, u), F(u, v)) \preceq \frac{k}{2}\big(q(v, u) + q(u, v)\big). \tag{3.14}$$

Thus we have

$$q(u,v) + q(v,u) \preceq k\big(q(v,u) + q(u,v)\big). \tag{3.15}$$

Since $k < 1$, we get $q(u,v) + q(v,u) = \theta$. Hence $q(u,v) = \theta$ and $q(v,u) = \theta$. Let $u_n = \theta$ and $x_n = u$. Then

$$\begin{aligned} q(x_n, u) &\preceq u_n, \\ q(x_n, v) &\preceq u_n. \end{aligned} \tag{3.16}$$

From Lemma 2.7 (1), we have $u = v$. Hence the coupled fixed point of F has the form (u,u). This completes the proof. □

Theorem 3.3. *Let $(X, \sqsubseteq)$ be a partially ordered set, and suppose that (X, d) is a complete cone metric space. Let q be a c-distance on X, and let $F : X \times X \to X$ be a function having the mixed monotone property such that*

$$q\big(F(x,y), F(x^*,y^*)\big) \preceq \frac{k}{4}\big(q(x,x^*) + q(y,y^*)\big) \tag{3.17}$$

for some $k \in (0,1)$ and all $x, y, x^*, y^* \in X$ with $(x \sqsubseteq x^*) \wedge (y \sqsupseteq y^*)$ or $(x \sqsupseteq x^*) \wedge (y \sqsubseteq y^*)$. Also, suppose that X has the following properties:

(a) *if (x_n) is a nondecreasing sequence in X with $x_n \to x$, then $x_n \sqsubseteq x$ for all $n \geq 1$;*

(b) *if (x_n) is a nonincreasing sequence in X with $x_n \to x$, then $x \sqsubseteq x_n$ for all $n \geq 1$.*

Assume there exist $x_0, y_0 \in X$ such that $x_0 \sqsubseteq F(x_0, y_0)$ and $F(y_0, x_0) \sqsubseteq y_0$. If $y_0 \sqsubseteq x_0$, then F has a coupled fixed point.

Proof. As in the proof of Theorem 3.1, we can construct two Cauchy sequences (x_n) and (y_n) in X such that

$$\begin{aligned} &x_0 \sqsubseteq x_1 \sqsubseteq \cdots \sqsubseteq x_n \sqsubseteq \cdots, \\ &y_0 \sqsupseteq y_1 \sqsupseteq \cdots y_n \sqsupseteq \cdots. \end{aligned} \tag{3.18}$$

Moreover, we have that (x_n) converges to a point $u \in X$ and (y_n) converges to $v \in X$,

$$\begin{aligned} q(x_n, x_m) &\preceq \frac{k^n}{1-k}\big(q(x_1,x_0) + q(y_1,y_0) + q(x_0,x_1) + q(y_0,y_1)\big), \\ q(y_n, y_m) &\preceq \frac{k^n}{1-k}\big(q(x_1,x_0) + q(y_1,y_0) + q(x_0,x_1) + q(y_0,y_1)\big) \end{aligned} \tag{3.19}$$

for each $n > m \geq 1$. By (q3), we have

$$q(x_n, u) \preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right),$$
$$q(y_n, v) \preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right), \tag{3.20}$$

and so

$$q(x_n, u) + q(y_n, v) \preceq \frac{2k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right). \tag{3.21}$$

By the properties (a) and (b), we have

$$v \sqsubseteq y_n \sqsubseteq y_0 \sqsubseteq x_0 \sqsubseteq x_n \sqsubseteq u. \tag{3.22}$$

By (3.17), we have

$$\begin{aligned} q(x_n, F(u,v)) &= q(F(x_{n-1}, y_{n-1}), F(u,v)) \\ &\preceq \frac{k}{4}\left(q(x_{n-1}, u) + q(y_{n-1}, v)\right), \\ q(y_n, F(v,u)) &= q(F(y_{n-1}, x_{n-1}), F(v,u)) \\ &\preceq \frac{k}{4}\left(q(y_{n-1}, v) + q(x_{n-1}, u)\right). \end{aligned} \tag{3.23}$$

Thus we have

$$q(x_n, F(u,v)) + q(y_n, F(v,u)) \preceq \frac{k}{2}\left(q(x_{n-1}, u) + q(y_{n-1}, v)\right). \tag{3.24}$$

By (3.21), we get

$$\begin{aligned} q(x_n, F(u,v)) + q(y_n, F(v,u)) &\preceq \frac{k}{2} \cdot \frac{2k^{n-1}}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right) \\ &= \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right). \end{aligned} \tag{3.25}$$

Therefore, we have

$$q(x_n, F(u,v)) \preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right),$$
$$q(y_n, F(v,u)) \preceq \frac{k^n}{1-k}\left(q(x_1, x_0) + q(y_1, y_0) + q(x_0, x_1) + q(y_0, y_1)\right). \tag{3.26}$$

By using (3.20) and (3.26), Lemma 2.7 (1) shows that $u = F(u, v)$ and $v = F(v, u)$. Therefore, (u, v) is a coupled fixed point of F. This completes the proof. □

Example 3.4. Let $E = C^1_{\mathbf{R}}[0,1]$ with $\|x\| = \|x\|_\infty + \|x'\|_\infty$ and $P = \{x \in E : x(t) \geq 0, t \in [0,1]\}$. Let $X = [0, +\infty)$ (with usual order), and let $d : X \times X \to E$ be defined by $d(x, y)(t) = |x - y|e^t$. Then (X, d) is an ordered cone metric space (see [7, Example 2.9]). Further, let $q : X \times X \to E$ be defined by $q(x, y)(t) = ye^t$. It is easy to check that q is a c-distance. Consider now the function $F : X \times X \to X$ defined by

$$F(x, y) = \begin{cases} \frac{1}{8}(x - y), & x \geq y, \\ 0, & x < y. \end{cases} \tag{3.27}$$

Then it is easy to see that

$$q(F(x, y), F(u, v)) \preceq \frac{1}{6}(q(x, u) + q(y, v)) \tag{3.28}$$

for all $x, y, u, v \in X$ with $(x \leq u) \wedge (y \geq v)$ or $(x \geq u) \wedge (y \leq v)$. Note that $0 \leq F(0, 1)$ and $1 \geq F(1, 0)$. Thus, by Theorem 3.1, it follows that F has a coupled fixed point in E. Here $(0, 0)$ is a coupled fixed point of F.

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