

Research Article

Adaptive Kalman Estimation in Target Tracking Mixed with Random One-Step Delays, Stochastic-Bias Measurements, and Missing Measurements

Sujuan Chen, Yinya Li, Guoqing Qi, and Andong Sheng

School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China

Correspondence should be addressed to Sujuan Chen; chensujuan2010@126.com

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The objective of this paper is concerned with the estimation problem for linear discrete-time stochastic systems with mixed uncertainties involving random one-step sensor delay, stochastic-bias measurements, and missing measurements. Three Bernoulli distributed random variables are employed to describe the uncertainties. All the three uncertainties in the measurement have certain probability of occurrence in the target tracking system. And then, an adaptive Kalman estimation is proposed to deal with this problem. The adaptive filter gains can be obtained in terms of solutions to a set of recursive discrete-time Riccati equations. Examples in three scenarios of target tracking are exploited to show the effectiveness of the proposed design approach.

1. Introduction

In practical target tracking, the missing phenomenon of the sensor measurement often occurs. For example, the measurement value may contain noise only due to the shelter of the obstacles, a high noise environment, a failure in the measurement, intermittent sensor failures, high maneuverability of tracked target, and so forth. Thus, the estimation for systems with missing measurements has received much attention during the few years. In general, there are two ways to describe the missing phenomenon. One way is to model the uncertainty by using a stochastic Bernoulli binary switching sequence taking on values of 0 and 1 (see, e.g., [1–18] and the references therein). The suboptimal filtering algorithm [2] in the minimum variance sense with only missing measurements has been proposed and the robust filter [14] is designed. In [3] Sinopoli et al. studied the statistical behavior of the Kalman filter error covariance with varying Kalman gain with missing measurements and the existence of a critical value has been shown for the arrival rate of the observations. The stochastic stability of the extended kalman filter with missing measurements is analyzed in [8], while stochastic stability of the unscented kalman filter with missing measurements is studied in [13]. For benchmarking

the performance of any estimation algorithm with missing measurements in advance, the modified Cramer-Rao bound (CRLB) and modified Riccati equation have been studied in [1, 15–18]. Another way is to model the uncertainty as a Markovian jumping sequence (see [19–21] and the references therein). The convergence for semi-Markov chains has been studied in [20], where the exponential filter is designed in [21].

For the case of random delayed measurements, the delays will cause performance degradation or instability with traditional Kalman filters. Therefore, a large volume of literature has been published on the topic of filtering problem with delayed measurements; see, for example, [6, 7, 10–12, 22–27]. Most of the publications assume that the delayed measurements are always deterministic. However, it is not often the case in actual target tracking, where the time delays in the measurement may occur in a random way. A stochastic Bernoulli binary switching sequence taking on values of 0 and 1 has been employed to describe the randomly delayed measurements. In such case, the minimum variance suboptimal filter [22, 26] and the robust filter with only delayed measurements [25] are obtained.

For the case of stochastic-bias measurements, a two-stage kalman estimator for state estimation in the presence

of random bias and for tracking maneuvering targets is given in [28], while an optimal two-stage kalman filter in the presence of random bias is obtained in [29]. Moreover, optimal and suboptimal separate-bias kalman estimators for a stochastic bias are studied in [30]. The biased bearing-only parameter estimation for bistatic system is studied in [31] and the Cramer-Rao lower bound for biased bearings-only maneuvering target tracking is obtained in [32], where the stochastic-bias phenomenon is described by a stochastic Bernoulli binary switching sequence taking on values of 0 and 1.

Based on the stochastic parameterized description, recently, some suboptimal and optimal filtering algorithms have been proposed for systems with both sensor delays and missing measurements [10–12, 21, 24, 33, 34]. Adaptive kalman filtering [10] is designed mixed with random sensor delays, multiple packet dropouts, and missing measurements. The problem of robust state estimation for two-dimensional stochastic time-delay systems [12] is studied with missing measurements and sensor saturation. An optimal linear filter for discrete-time systems with randomly delayed and lost measurements with/without time stamps is studied in [34]. Up to now, the state estimation problem for systems in target tracking mixed with random one-step delays, stochastic-bias measurements, and missing measurements is seldom reported, which almost exist simultaneously in target tracking. Motivated by the results above, a new mathematical model is developed to describe the random one-step delays, stochastic-bias measurements, and missing measurements using three Bernoulli distributed random variables taking on values of 0 and 1. By state augmentation, it is converted to a stochastic parameterized system. Based on the model, An adaptive kalman estimation is obtained in target tracking via projection theory. The rest of the paper is organized as follows. Section 2 establishes the new system model and introduces the augment state Kalman estimation to deal with the proposed problem. Section 3 presents an adaptive estimation algorithm. In Section 4, simulation results are analyzed and are used to evaluate the proposed algorithm. A summary of our conclusions is given in Section 5.

2. System Model and Problem Formulation

Consider the following stochastic-biased discrete time linear time-varying state-space system with missing measurements:

$$\begin{aligned} X_{k+1} &= F_k X_k + w_k, & X_k (k \leq 0) &= X_0, \\ y_k &= d_k H_k X_k + d_k (1 - l_k) \tau_k + v_k, \end{aligned} \quad (1)$$

where $X_k \in \mathbb{R}^n$ is the state vector, $y_k \in \mathbb{R}^m$ is the measured output, and $w_k \in \mathbb{R}^p$ and $v_k \in \mathbb{R}^m$ are Gaussian random vectors with zero mean and covariance matrices $\sigma_w^2 \geq 0$ and $\sigma_v^2 \geq 0$, respectively. The bias vector $\tau_k \in \mathbb{R}^m$ is treated as a zero mean Gaussian random vector with a priori known covariance σ_τ^2 . F_k and H_k are constant matrices with compatible dimensions. The d_k is a scalar binary Bernoulli distributed random variable taking values $\{1, 0\}$ which are defined as $\text{Prob}\{d_k = 1\} = \lambda$ and $\text{Prob}\{d_k = 0\} = 1 - \lambda$.

If $d_k = 1$, the measurement at time k is available; otherwise, the measurement is missing; that is, the received observation is only the measurement noise at time k . The l_k is a scalar binary Bernoulli distributed random variable taking values $\{1, 0\}$ which are defined as $\text{Prob}\{l_k = 1\} = \eta$ and $\text{Prob}\{l_k = 0\} = 1 - \eta$. If $d_k = 1$ and $l_k = 0$, the measurement at time k is available and unbiased; else if $d_k = 1$ and $l_k = 1$, the measurement at time k is available but biased.

Assume that the initial state X_0 satisfies the mean and covariance conditions:

$$E\{X_0\} = \mu_0, \quad E\{(X_0 - \mu_0)(X_0 - \mu_0)^T\} = P_0, \quad (2)$$

where $E\{\cdot\}$ is the mathematical expectation operator and the superscript T is the transpose. The initial state μ_0 is uncorrelated with w_k , v_k , and τ_k .

Next, we will establish a unified model to describe the mixed uncertainties of missing measurements, one-step delay, and stochastic biased measurements by Bernoulli distributed random variables.

Define

$$\underline{X}_{k+1} = [X_{k+1}^T, X_k^T, y_k^T]^T. \quad (3)$$

A compact representation of the system (1) with mixed uncertainties of missing measurements, one-step delay, and stochastic-biased measurements can be given in an augmented form as follows:

$$\begin{aligned} \underline{X}_{k+1} &= A_k \underline{X}_k + B_k W_k + M_k \Lambda_k, \\ y_k &= C_k \underline{X}_k + D_k W_k + N_k \Lambda_k, \end{aligned} \quad (4)$$

where

$$\begin{aligned} W_k &= [w_k^T, v_k^T, v_{k-1}^T]^T, \\ \Lambda_k &= [\tau_k^T, \tau_{k-1}^T]^T. \end{aligned} \quad (5)$$

Further, we have the following statistical information about W_k and Λ_k :

$$\begin{aligned} E\{W_k\} &= 0, \\ E\{\Lambda_k\} &= 0, \\ P_{W_k} &= E\{W_k W_k^T\} = \text{diag}\{\sigma_w^2, \sigma_v^2, \sigma_v^2\}, \\ E\{v_k v_\tau^T\} &= 0, \quad k \neq \tau, \\ P_{\Lambda_k} &= E\{\Lambda_k \Lambda_k^T\} = \text{diag}\{\sigma_\tau^2, \sigma_\tau^2\}, \\ E\{\tau_k \tau_\tau^T\} &= 0, \quad k \neq \tau. \end{aligned} \quad (6)$$

Let $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$ denote the three models corresponding, respectively, to systems with no missing measurement, one-step sensor delay, and missing

measurement. They are defined by the following system matrices:

(a) current measurement case:

$$A_k^1 = \begin{bmatrix} F_k & 0 & 0 \\ I & 0 & 0 \\ H_k & 0 & 0 \end{bmatrix}, \quad B_k^1 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix},$$

$$M_k^1 = (1 - l_k) \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I & 0 \end{bmatrix} = (1 - l_k) \underline{M}_k^1, \quad C_k^1 = [H_k \ 0 \ 0],$$

$$D_k^1 = [0 \ I \ 0], \quad N_k^1 = (1 - l_k) [I \ 0] = (1 - l_k) \underline{N}_k^1, \quad (7)$$

(b) one-step random sensor delay case:

$$A_k^2 = \begin{bmatrix} F_k & 0 & 0 \\ I & 0 & 0 \\ 0 & H_k & 0 \end{bmatrix}, \quad B_k^2 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$M_k^2 = (1 - l_k) \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} = (1 - l_k) \underline{M}_k^2,$$

$$C_k^2 = [0 \ H_{k-1} \ 0],$$

$$D_k^2 = [0 \ 0 \ I], \quad N_k^2 = (1 - l_k) [0 \ I] = (1 - l_k) \underline{N}_k^2, \quad (8)$$

(c) missing measurement case:

$$A_k^3 = \begin{bmatrix} F_k & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_k^3 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix},$$

$$M_k^3 = (1 - l_k) \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = (1 - l_k) \underline{M}_k^3,$$

$$C_k^3 = [0 \ 0 \ 0], \quad D_k^3 = [0 \ 0 \ 0],$$

$$N_k^3 = (1 - l_k) [0 \ 0] = (1 - l_k) \underline{N}_k^3. \quad (9)$$

The systems $\{A_k, B_k, C_k, D_k, M_k, N_k\}$ can be represented as follows:

$$\{A_k, B_k, C_k, D_k, M_k, N_k\} := \sum_{q=1}^3 \beta_q \{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q\}, \quad (10)$$

where β_q satisfies

$$\sum_{q=1}^3 \beta_q = 1, \quad \beta_q = 0 \text{ or } 1. \quad (11)$$

Assume that we are able to model the real system at time k by analyzing the system. The probability of the system $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$ at time k is ρ_q . Obviously, we have $\sum_{q=1}^3 \rho_q = 1$.

Remark 1. In this paper, the uncertain model subject to missing measurements, one-step delay, and stochastic-biased measurements is considered; that is, the probabilities of the stochastic uncertainties are known. Here the missing measurements are replaced by zeros; that is, the measurement containing only noise is not a valid value. We can examine the "error detection" bits before sending them to the estimator. The delay may be much longer and the receiver (estimator side) cannot know the values without time stamps. Therefore, we cannot have an exact model $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$; that is, the value of q is unknown when the measurement arrives in an estimator. In this case, we need to know the probability ρ_q ($q = 1, 2, 3$) of each model $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$.

Define

$$\widehat{X}_{k+1} = E \left\{ [X_{k+1}^T, X_k^T]^T \right\}. \quad (12)$$

That is, \widehat{X}_{k+1} is an estimator vector of $[X_{k+1}^T, X_k^T]^T$.

We can construct an estimator of the following structure:

$$\widehat{X}_{k+1} = \widehat{A}_k \widehat{X}_k + G_k (y_k - \widehat{C}_k^q \widehat{X}_k). \quad (13)$$

Define

$$S = [I \ 0]. \quad (14)$$

From (12), we know that \widehat{X}_{k+1} is an estimate vector for $[X_{k+1}^T, X_k^T]^T$. Therefore, it is obvious that we can choose \widehat{A}_k and \widehat{C}_k^q as follows:

$$\widehat{A}_k = \begin{bmatrix} F_k & 0 \\ I & 0 \end{bmatrix}, \quad \widehat{C}_k^q = C_k^q S^T. \quad (15)$$

The state estimation error covariance is defined by

$$P_{e_k} = E \{ e_k e_k^T \}, \quad (16)$$

where

$$e_k = X_k - \widehat{X}_k. \quad (17)$$

In order to obtain the optimal G_k , we can minimize P_{e_k} .

3. Estimation Algorithm Mixed with One-Step Sensor Delays, Missing Measurements, and Stochastic-Biased Measurements

In this section, based on the stochastic system models $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$, we will derive the optimal G_k before giving one-step predictions.

From (3) and (14), we can readily have that

$$S \underline{X}_k = [X_k^T, X_{k-1}^T]^T. \quad (18)$$

Define

$$\xi_{k+1} = S \underline{X}_{k+1} - \widehat{X}_{k+1}. \quad (19)$$

Substituting (4) and (13) into (19) yields

$$\begin{aligned} \xi_{k+1} = & \sum_{q=1}^3 \beta_q \left\{ SA_k^q \underline{X}_k + (\widehat{A}_k - G_k \widehat{C}_k^q) \widehat{\underline{X}}_k \right. \\ & + (SB_k^q - G_k D_k^q) W_k + (SM_k^q - G_k N_k^q) \Lambda_k \\ & \left. - G_k C_k^q \underline{X}_k \right\}. \end{aligned} \quad (20)$$

Observing (7)–(10) and (14), we have the following results:

$$\begin{aligned} SA_k^q \underline{X}_k &= \widehat{A}_k S \underline{X}_k, \\ C_k^q \underline{X}_k &= C_k^q S^T S \underline{X}_k, \\ C_k^q \underline{X}_k &= \widehat{C}_k^q S \underline{X}_k. \end{aligned} \quad (21)$$

Furthermore, substituting (21) into (20), we can obtain the following equation:

$$\begin{aligned} \xi_{k+1} = & \sum_{q=1}^3 \beta_q \left\{ (\widehat{A}_k - G_k \widehat{C}_k^q) \xi_k \right. \\ & + (SB_k^q - G_k D_k^q) W_k + (SM_k^q - G_k N_k^q) \Lambda_k \\ & \left. + G_k (C_k^q S^T - \widehat{C}_k^q) S \underline{X}_k \right\}. \end{aligned} \quad (22)$$

Noting that $\widehat{C}_k^q = C_k^q S^T$, (22) can be rewritten as

$$\begin{aligned} \xi_{k+1} = & \sum_{q=1}^3 \beta_q \left\{ (\widehat{A}_k - G_k \widehat{C}_k^q) \xi_k + (SB_k^q - G_k D_k^q) W_k \right. \\ & \left. + (SM_k^q - G_k N_k^q) \Lambda_k \right\}. \end{aligned} \quad (23)$$

Using the statistics of W_k and Λ_k in (6), the covariance matrix of ξ_k can be computed by

$$\begin{aligned} P_{\xi_{k+1}} = & \sum_{q=1}^3 \beta_q \left\{ (\widehat{A}_k - G_k \widehat{C}_k^q) P_{\xi_k} (\widehat{A}_k - G_k \widehat{C}_k^q)^T \right. \\ & + (SB_k^q - G_k D_k^q) P_{W_k} (SB_k^q - G_k D_k^q)^T \\ & + (1 - \eta) (SM_k^q - G_k N_k^q) \\ & \left. \times P_{\Lambda_k} (SM_k^q - G_k N_k^q)^T \right\}. \end{aligned} \quad (24)$$

From (17) and (19), we can easily have

$$e_{k+1} = \Phi \xi_{k+1}, \quad \Phi = [I \ 0]. \quad (25)$$

Therefore, the problem of minimizing P_{e_k} is equivalent to

$$\min_{G_k} E \left\{ \text{Tr} \left\{ \Phi P_{\xi_{k+1}} \Phi^T \right\} \right\} \quad \text{subject to (24)}. \quad (26)$$

Let $K_k = \Phi G_k$ and left- and right-multiply (24) by Φ and Φ^T , respectively. The estimation error covariance matrix can be computed by

$$\begin{aligned} P_{e_{k+1}} = & \sum_{q=1}^3 \beta_q \left\{ (\Phi \widehat{A}_k - K_k \widehat{C}_k^q) P_{\xi_{k+1}} (\Phi \widehat{A}_k - K_k \widehat{C}_k^q)^T \right\} \\ & + \sum_{q=1}^3 \beta_q \left\{ (\Phi SB_k^q - K_k D_k^q) P_{W_k} (\Phi SB_k^q - K_k D_k^q)^T \right\} \end{aligned}$$

$$\begin{aligned} & + \sum_{q=1}^3 \beta_q \left\{ (1 - \eta) (\Phi SM_k^q - K_k N_k^q) \right. \\ & \left. \times P_{\Lambda_k} (\Phi SM_k^q - K_k N_k^q)^T \right\}. \end{aligned} \quad (27)$$

Differentiate $E\{\text{Tr} P_{e_{k+1}}\}$ with respect to K_k and set the derivative to zero. We can obtain the optimal K_k as follows:

$$\begin{aligned} K_k = & \left[\sum_{q=1}^3 \beta_q \Phi \widehat{A}_k P_{\xi_{k+1}} (\widehat{C}_k^q)^T + \sum_{q=1}^3 \beta_q \Phi SB_k^q P_{W_k} (D_k^q)^T \right. \\ & \left. + \sum_{q=1}^3 \beta_q (1 - \eta) \Phi SM_k^q P_{\Lambda_k} (N_k^q)^T \right] \\ & \times \left[\sum_{q=1}^3 \beta_q \widehat{C}_k^q P_{\xi_{k+1}} (\widehat{C}_k^q)^T + \sum_{q=1}^3 \beta_q D_k^q P_{W_k} (D_k^q)^T \right. \\ & \left. + \sum_{q=1}^3 \beta_q (1 - \eta) N_k^q P_{\Lambda_k} (N_k^q)^T \right]^{-1}. \end{aligned} \quad (28)$$

Therefore the covariance matrices of the estimation error for any (i.e., not necessarily optimal) K_k can be computed by a set of recursive discrete-time Riccati-like equations. The corresponding optimal G_k can be computed by

$$G_k = [K_k^T \ 0]^T. \quad (29)$$

Let $\widehat{A}_k = I$, $B_k^q = 0$, and $L_k = K_k$. We may obtain the relevant formula for measurement update as follows:

$$\begin{aligned} P_{e_{k+1|k+1}} = & \sum_{q=1}^3 \beta_q \left\{ (\Phi - M_k \widehat{C}_k^q) P_{\xi_{k+1}} (\Phi - M_k \widehat{C}_k^q)^T \right. \\ & \left. + L_k D_k^q P_{W_k} (D_k^q)^T M_k^T \right\} \\ & + (1 - \eta) \sum_{q=1}^3 \beta_q L_k N_k^q P_{\Lambda_k} (N_k^q)^T, \\ L_k = & \left[\sum_{q=1}^3 \beta_q \Phi P_{\xi_{k+1}} (\widehat{C}_k^q)^T + (1 - \eta) \Phi SM_k^q P_{\Lambda_k} (N_k^q)^T \right] \\ & \times \left[\sum_{q=1}^3 \beta_q \widehat{C}_k^q P_{\xi_{k+1}} (\widehat{C}_k^q)^T + \sum_{q=1}^3 \beta_q D_k^q P_{W_k} (D_k^q)^T \right. \\ & \left. + \sum_{q=1}^3 \beta_q (1 - \eta) N_k^q P_{\Lambda_k} (N_k^q)^T \right]^{-1}. \end{aligned} \quad (30)$$

We may propose the following adaptive filtering scheme. Consider the conceptual algorithm.

Step 1. The required input parameters are

$$F_k, H_k, \sigma_w^2, \sigma_v^2, \sigma_\tau^2, P_0, \rho_q \quad (q = 1, 2, 3), \quad E\{X_0\} = \mu_0. \quad (31)$$

Step 2. Initialization is as follows:

$$\begin{aligned} & \{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}, \\ & P_{W_k} = \text{diag}\{\sigma_w^2, \sigma_v^2, \sigma_\tau^2\}, \\ & P_{\Lambda_k} = \text{diag}\{\sigma_\tau^2, \sigma_\tau^2\}, \\ & P_{\xi_0} = \begin{bmatrix} P_0 & P_0 \\ P_0 & P_0 \end{bmatrix}, \end{aligned} \quad (32)$$

where we have assumed $X_{-1} = X_0$ to derive the above initialization.

Perform the following equations:

$$\begin{aligned} \widehat{X}_0 &= E\left\{\begin{bmatrix} X_0^T & X_0^T \end{bmatrix}\right\} = \begin{bmatrix} \mu_0^T & \mu_0^T \end{bmatrix}^T, \\ \widehat{C}_0 &= \sum_{q=1}^3 \rho_0^q C_0^q S^T, \\ K_0 &= \left(\sum_{q=1}^3 \beta_0^q \Phi P_{\xi_0} (\widehat{C}_0^q)^T \right) \\ & \times \left(\sum_{q=1}^3 \beta_0^q \left\{ \widehat{C}_0^q P_{\xi_0} (\widehat{C}_0^q)^T + D_0^q P_{W_0} (D_0^q)^T \right. \right. \\ & \left. \left. + (1 - \eta) \underline{N}_0^q P_{\Lambda_0} (\underline{N}_0^q)^T \right\} \right)^{-1}, \\ \widehat{X}_{0|0} &= X_0 + K_0 (y_0 - \widehat{C}_0 \widehat{X}_0), \\ P_{e_{0|0}} &= \sum_{q=1}^3 \rho_0^q (I - K_0 \widehat{C}_0^q) P_{\xi_0} (I - K_0 \widehat{C}_0^q)^T \\ & \times \sum_{q=1}^3 \rho_k^q K_0 D_0^q P_{W_0} (D_0^q)^T K_0^T \\ & + \sum_{q=1}^3 \rho_0^q (1 - \eta) K_0 \underline{N}_0^q P_{\Lambda_0} (\underline{N}_0^q)^T K_0^T. \end{aligned} \quad (33)$$

Step 3. Consider the state prediction

$$\begin{aligned} \widehat{X}_{k+1} &= F \widehat{X}_{k|k}, \\ \widehat{\underline{X}}_{k+1} &= \begin{bmatrix} \widehat{X}_{k+1}^T & \widehat{X}_k^T \end{bmatrix}^T \end{aligned} \quad (34)$$

and the prediction covariance computation

$$\begin{aligned} P_{\xi_{k+1}} &= \widehat{A}_k P_{\xi_{k|k}} \widehat{A}_k^T \\ & + \sum_{q=1}^3 \beta_k^q \left\{ S B_k^q P_{W_k} (B_k^q)^T S^T \right. \\ & \left. + (1 - \eta) S \underline{M}_k^q P_{\Lambda_k} (\underline{M}_k^q)^T S^T \right\}. \end{aligned} \quad (35)$$

Step 4. Innovation update is as follows.

If $d_k = 0$, then $K_k = 0$. Else if $d_k = 1$, then

$$\begin{aligned} K_k &= \left(\sum_{q=1}^3 \beta_k^q \Phi P_{\xi_{k+1}} (\widehat{C}_k^q)^T \right) \\ & \times \left(\sum_{q=1}^3 \beta_k^q \left\{ \widehat{C}_k^q P_{\xi_{k+1}} (\widehat{C}_k^q)^T + D_k^q P_{W_k} (D_k^q)^T \right. \right. \\ & \left. \left. + (1 - \eta) \underline{N}_k^q P_{\Lambda_k} (\underline{N}_k^q)^T \right\} \right)^{-1}. \end{aligned} \quad (36)$$

Step 5. Update $k = k + 1$:

$$\widehat{X}_{k|k} = \widehat{X}_k + K_k (y_k - \widehat{C}_k \widehat{X}_k). \quad (37)$$

Considering error covariance matrix update, perform the following equations:

$$\begin{aligned} G_k &= \begin{bmatrix} K_k^T & 0 \end{bmatrix}^T, \\ P_{\xi_{k+1|k+1}} &= \sum_{q=1}^3 \beta_k^q \left\{ (I - G_k \widehat{C}_k^q) P_{\xi_k} (I - G_k \widehat{C}_k^q)^T \right. \\ & \left. + G_k D_k^q P_{W_k} (D_k^q)^T G_k^T \right. \\ & \left. + (1 - \eta) G_k \underline{N}_k^q P_{\Lambda_k} (\underline{N}_k^q)^T G_k^T \right\}. \end{aligned} \quad (38)$$

Remark 2. We consider that the model $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$ is unknown when the measurement arrives in an estimator. In this case, we need to know the probability ρ_q ($q = 1, 2, 3$) of each model. However, we can get an exact model $\{A_k^q, B_k^q, C_k^q, D_k^q, M_k^q, N_k^q \mid q = 1, 2, 3\}$ when the measurement with time stamp arrives in an estimator, in which case, we may have an improved adaptive algorithm with time stamp.

4. Simulation Results

In this section, Monte Carlo simulations were conducted to verify the performance of the algorithms in three scenarios of target tracking: uniform rectilinear motion scenario, uniform circular motion scenario, and uniformly accelerated motion scenario. A total of 100 runs were executed in each case.

(A) *Uniform Rectilinear Motion Scenario.* Consider the system (1) with parameters

$$\begin{aligned} F_k &= I_{3 \times 3} \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \\ \sigma_w^2 &= I_{3 \times 3} \otimes \begin{bmatrix} \frac{T^3}{2} & \frac{T^2}{2} \\ \frac{3}{2} & T \end{bmatrix}, \quad \sigma_v^2 = 25 \text{ m}^2, \quad \sigma_\tau^2 = 25 \text{ m}^2, \end{aligned} \quad (39)$$

where T is the sampling period, \otimes is the Kronecker product, and $I_{3 \times 3}$ denotes the unity matrix of 3 orders.

Assume that the initial state of target is $X_0 = [-8000, 200, 0, 500, 0, 0, 1000, 0, 0]^T$, the sampling period is set to be

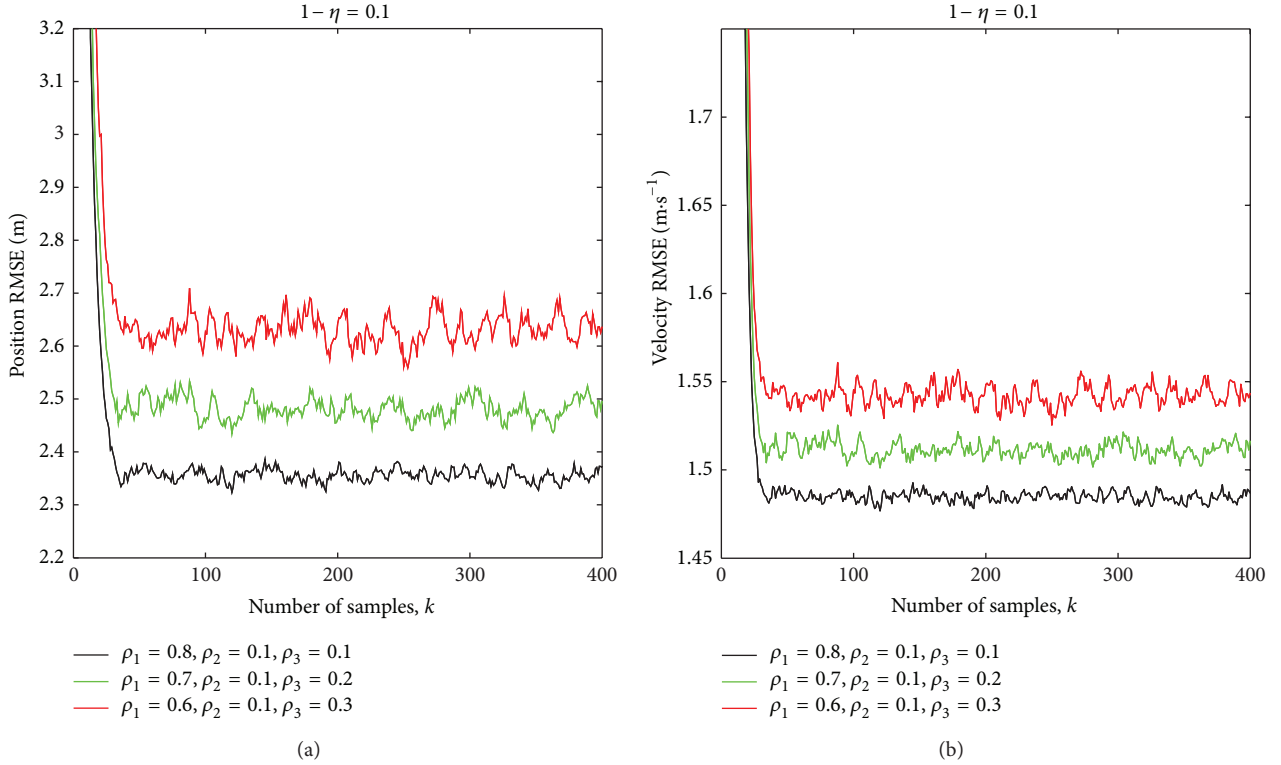


FIGURE 1: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

$T = 0.2$ s, the probability of stochastic-bias measurement is $1 - \eta = 0.1$. The simulation results are obtained by the proposed adaptive estimation algorithm in Section 4. The root mean square error (RMSE) for the states of position and velocity with different ρ_q ($q = 1, 2, 3$) is given in Figures 1–4. Seen from Figures 1–4, the proposed adaptive estimation algorithm is effective. Besides, it can be seen from Figure 1 that the tracking performance is better with higher probability when the delay probability is constant, where the black curves denote position and velocity RMSEs, respectively, with detection probability $\rho_1 = 0.8$, delay probability $\rho_2 = 0.1$, and the probability of missing probability $\rho_3 = 0.1$, the green curve with $\rho_1 = 0.7$, $\rho_2 = 0.1$, and $\rho_3 = 0.2$, and the red curve with $\rho_1 = 0.6$, $\rho_2 = 0.1$, and $\rho_3 = 0.3$. Figure 2 gives comparison curves of position RMSE and velocity RMSE, respectively, where the black curves denote position RMSE with $\rho_1 = 0.8$, $\rho_2 = 0.1$, and $\rho_3 = 0.1$, the green curve with $\rho_1 = 0.8$, $\rho_2 = 0.05$, and $\rho_3 = 0.15$, and the red curve with $\rho_1 = 0.8$, $\rho_2 = 0.15$, and $\rho_3 = 0.05$. From Figure 3, it is obvious that the estimation performance is better with lower missing measurement when the detection probability is constant. From Figure 4, we can see that the estimation algorithm has worse performance with a higher delay probability when the probability of missing measurement is constant.

Next, we consider the tracking performances with stochastic-bias probability $1 - \eta$. Figure 4 gives comparison curves of position RMSE and velocity RMSE with $\rho_1 = 0.8$, $\rho_2 = 0.1$, and $\rho_3 = 0.1$, respectively, where the black curves denote position RMSE and velocity RMSE with $1 - \eta = 0.1$,

the green curve with $1 - \eta = 0.5$, and the red curve with $1 - \eta = 0.8$. From Figure 4, we can obtain that the estimation performance is better with lower probability of stochastic-bias measurement.

To further demonstrate the effectiveness of our proposed algorithm, simulation scenarios of uniform rectilinear motion and uniform accelerated motion are given in the following.

(B) *Uniform Rectilinear Motion Scenario.* Consider the system (1) with parameters.

$$F_k = I_{3 \times 3} \otimes \begin{bmatrix} 1 & \frac{\sin(\omega T)}{\omega} & \frac{[1 - \cos(\omega T)]}{\omega^2} \\ 0 & \cos(\omega T) & \frac{\sin(\omega T)}{\omega} \\ 0 & -\omega \sin(\omega T) & \cos(\omega T) \end{bmatrix},$$

$$\sigma_w^2 = I_{3 \times 3} \otimes S, \quad \sigma_v^2 = 25 \text{ m}^2, \quad \sigma_\tau^2 = 25 \text{ m}^2,$$

$$S = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix},$$

$$A_{11} = 6\omega T - 8 \sin(\omega T) + \frac{\sin(2\omega T)}{4\omega^5},$$

$$A_{12} = \frac{[2 \sin(\omega T/2)]^4}{\omega^4},$$

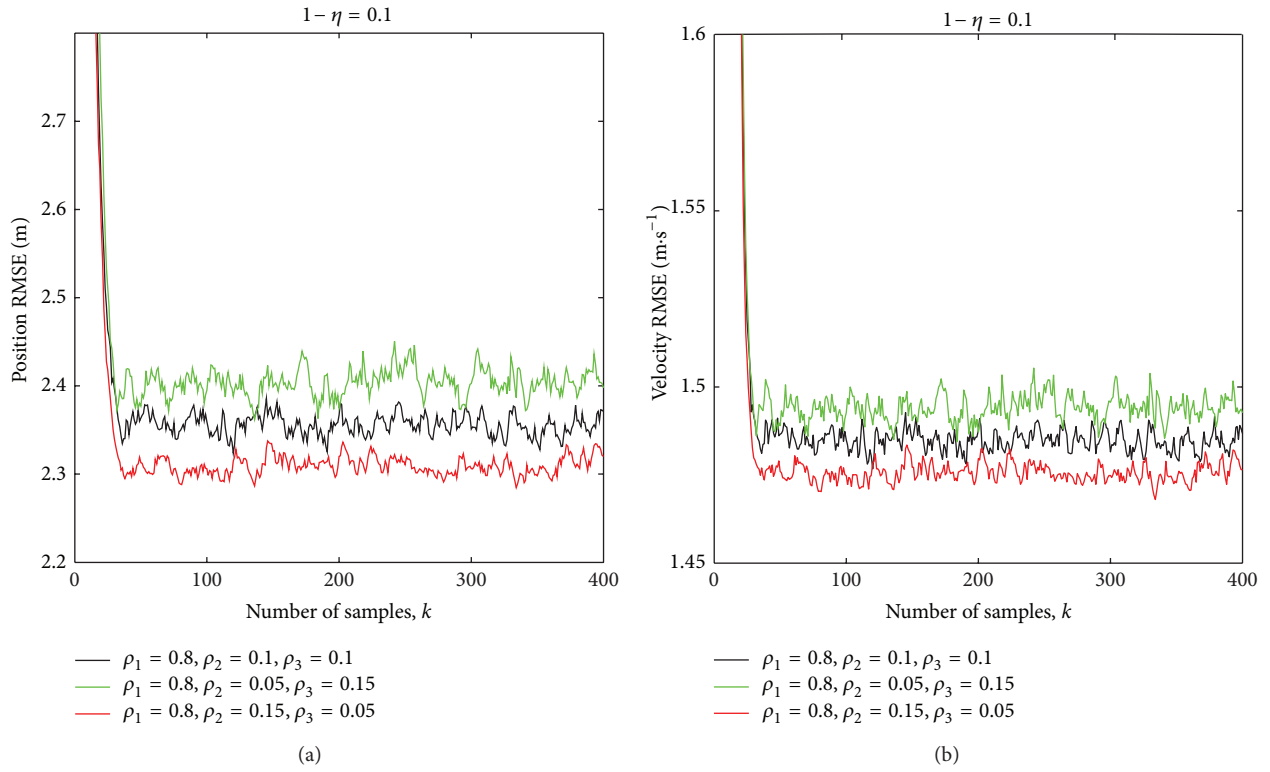


FIGURE 2: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

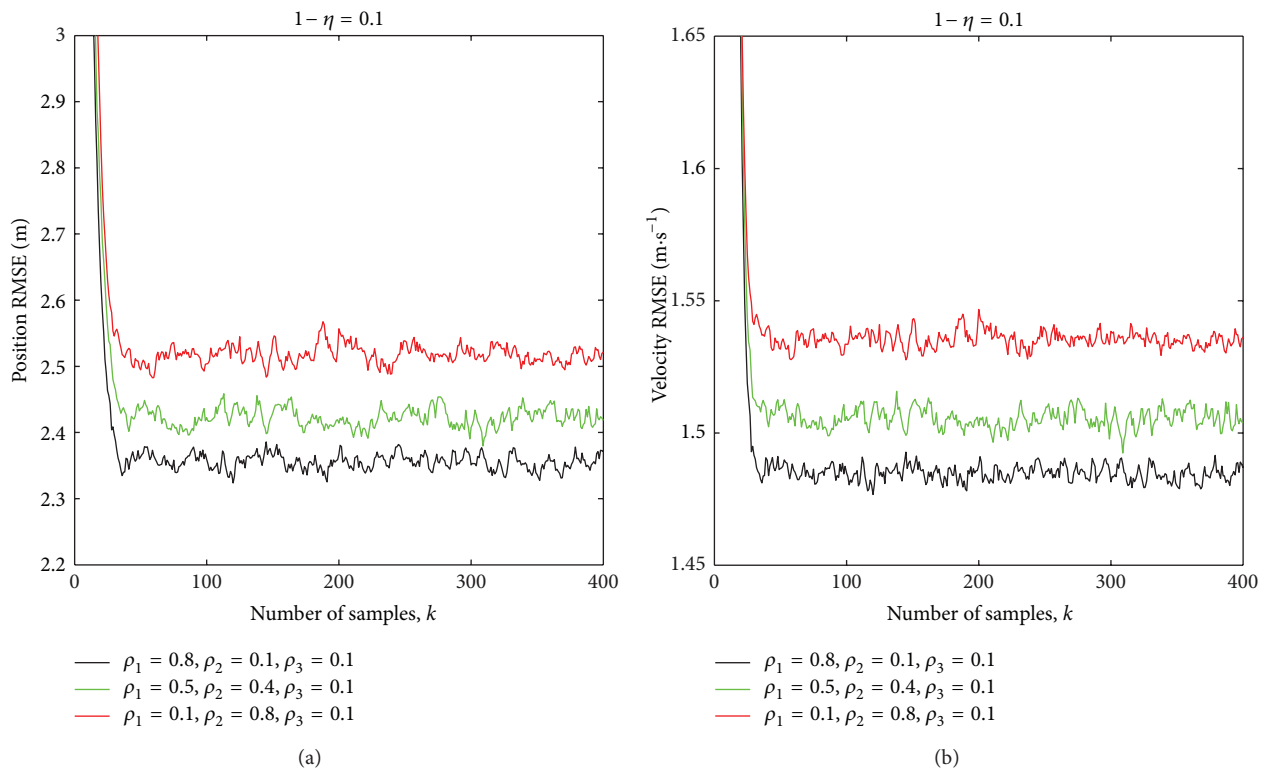


FIGURE 3: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

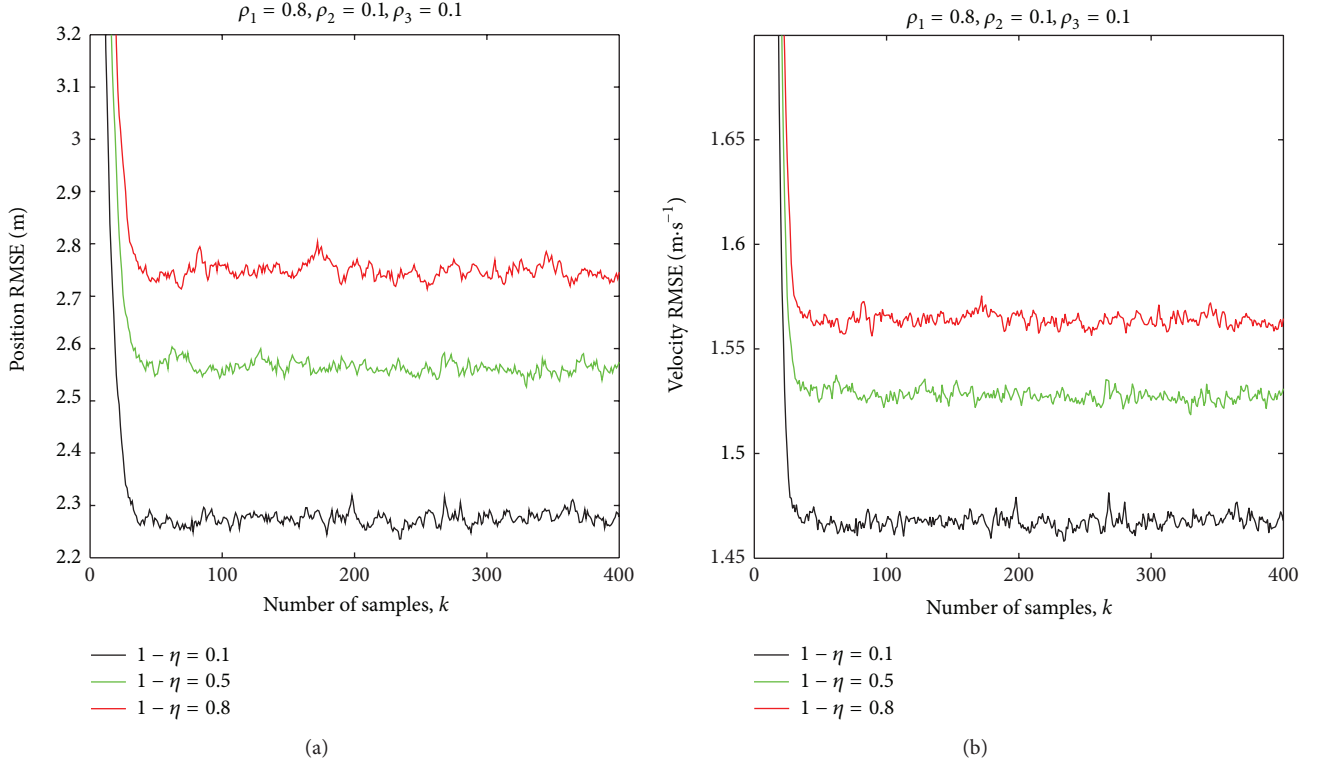


FIGURE 4: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

$$\begin{aligned}
 A_{13} &= \frac{[-2\omega T + 4 \sin(\omega T) - \sin(2\omega T)]}{4\omega^3}, \\
 A_{21} &= \frac{2[\sin(\omega T/2)]^4}{\omega^4}, \\
 A_{22} &= \frac{[2\omega T - \sin(2\omega T)]}{4\omega^3}, \\
 A_{23} &= \frac{[\sin(\omega T)]^2}{2\omega^2}, \\
 A_{31} &= \frac{[-2\omega T + 4 \sin(\omega T) - \sin(2\omega T)]}{4\omega^3}, \\
 A_{32} &= \frac{[\sin(\omega T)]^2}{2\omega^2}, \\
 A_{33} &= \frac{[2\omega T + \sin(2\omega T)]}{4\omega},
 \end{aligned} \tag{40}$$

where T is the sampling period, \otimes is the Kronecker product, and $I_{3 \times 3}$ denotes the unity matrix of 3 orders.

Assume that the initial state of target is $X_0 = [-5000, 200, -1, 500, 0, 0, 1000, 0, 0]^T$, the sampling period is set to be $T = 0.2$ s, the angular velocity is set to be $\omega = 0.05$ rad/s, and the probability of stochastic-bias measurement is $1 - \eta = 0.1$. The simulation results can be obtained as in Figures 5, 6, 7, and 8 in this scenario.

(C) *Uniform Accelerated Motion Scenario.* Consider the system (1) with parameters

$$\begin{aligned}
 F_k &= I_{3 \times 3} \otimes \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \\
 \sigma_w^2 &= I_{3 \times 3} \otimes \begin{bmatrix} \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ \frac{T^4}{8} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & \frac{T^2}{2} & T \end{bmatrix}, \\
 \sigma_v^2 &= 25 \text{ m}^2, \quad \sigma_\tau^2 = 25 \text{ m}^2,
 \end{aligned} \tag{41}$$

where T is the sampling period, \otimes is the Kronecker product, and $I_{3 \times 3}$ denotes the unity matrix of 3 orders. Assume that the initial state of target is $X_0 = [-5000, 100, -5, 500, 0, 0, 1000, 0, 0]^T$, the sampling period is set to be $T = 0.2$ s, and the probability of stochastic-bias measurement is $1 - \eta = 0.1$. The simulation results can be obtained as in Figures 9, 10, 11, and 12 in this scenario.

From Figures 5–8 in scenario B and Figures 9–12 in scenario C, it is clear that the proposed algorithm is effective. On the other hand, the estimation performance can be influenced

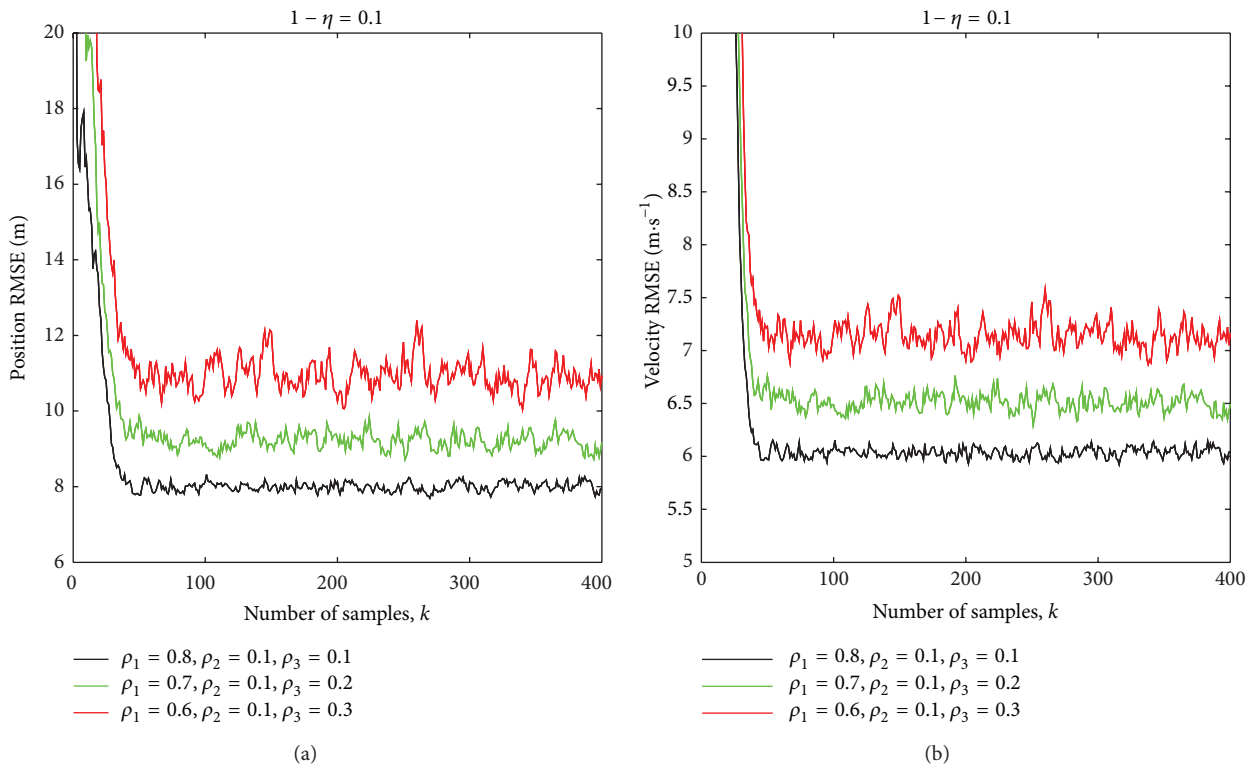


FIGURE 5: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

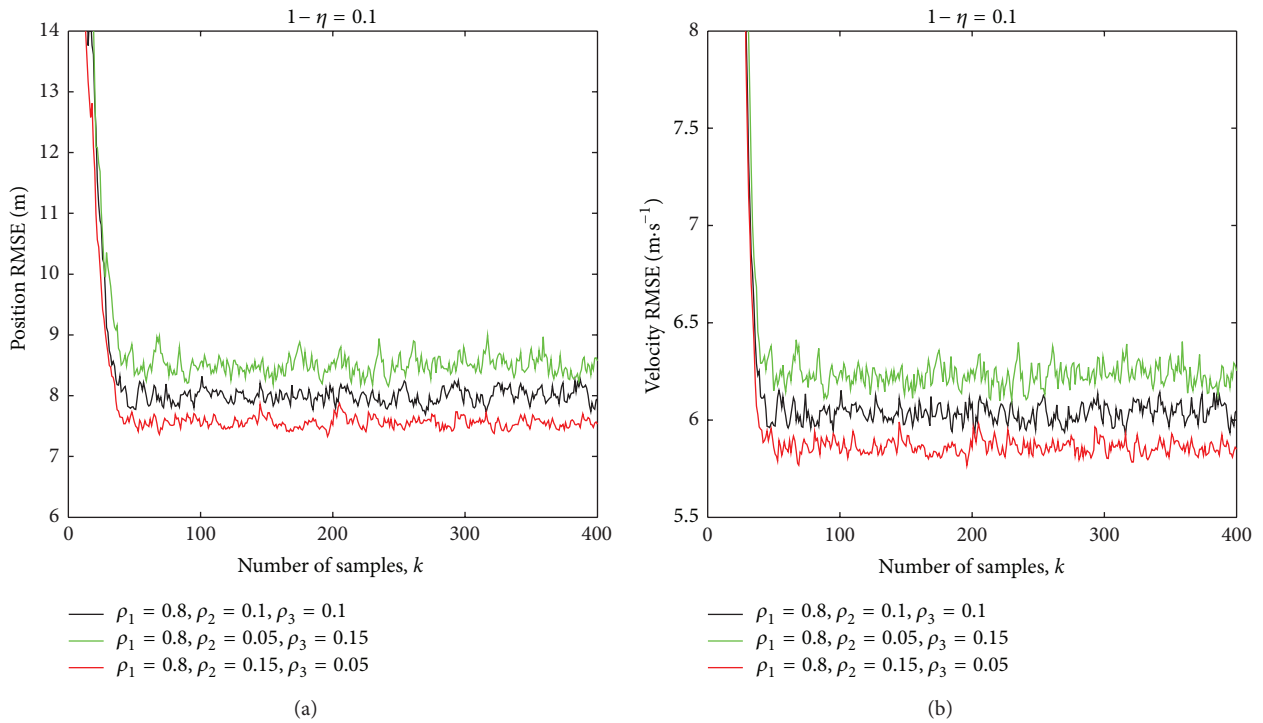


FIGURE 6: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

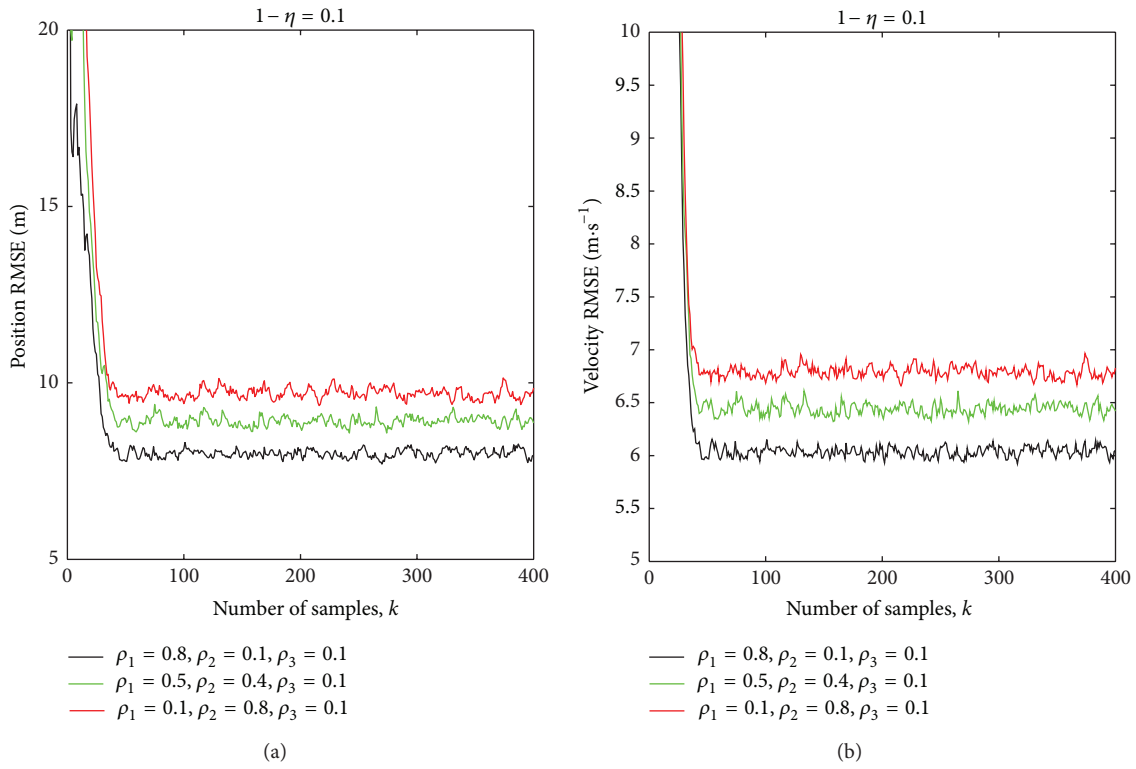


FIGURE 7: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

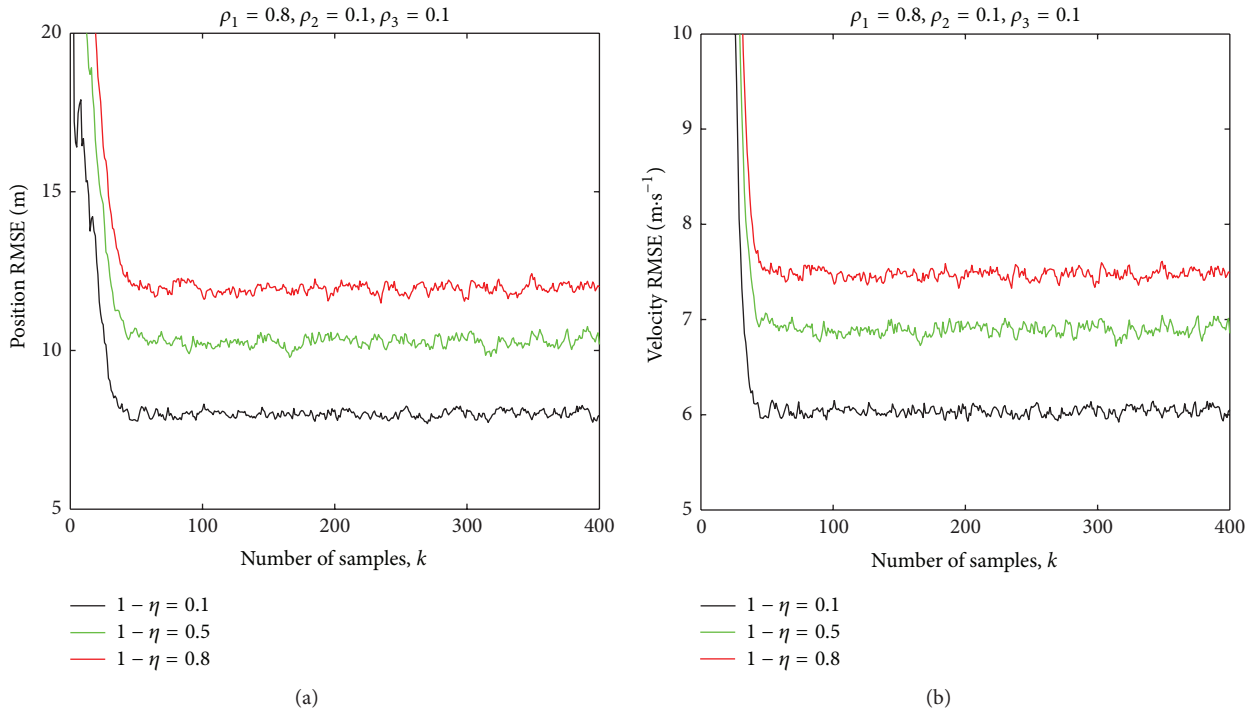


FIGURE 8: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

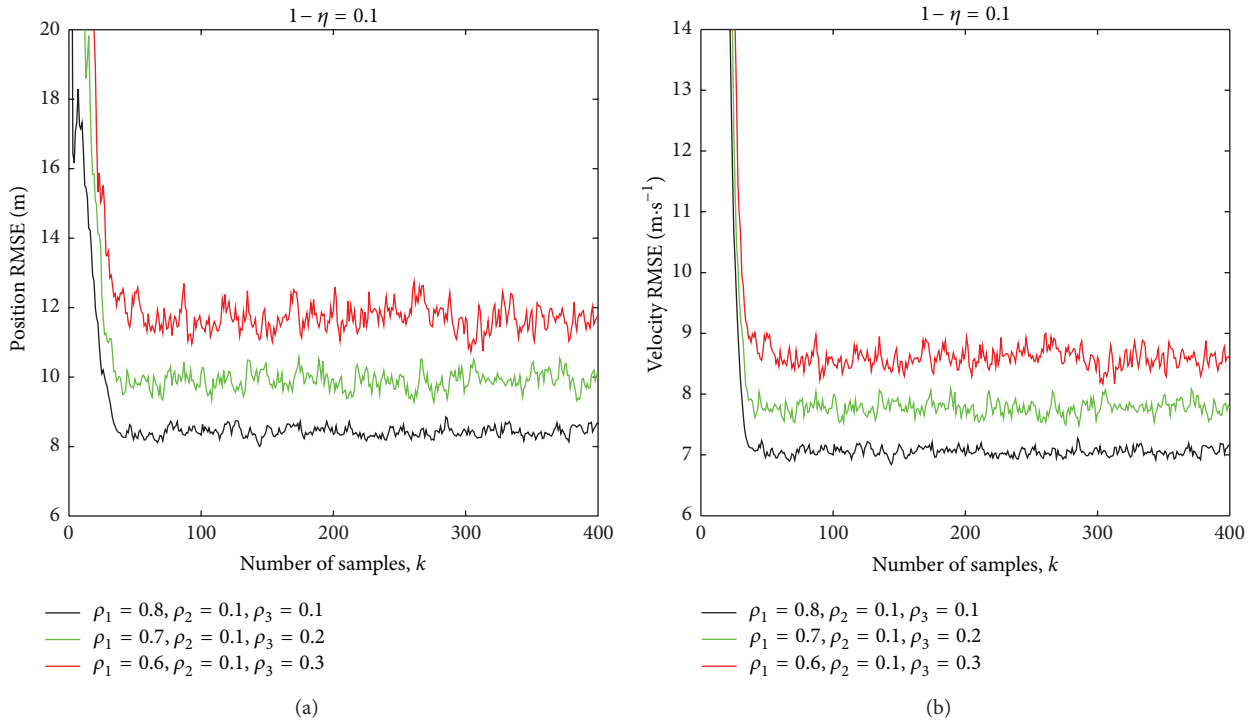


FIGURE 9: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

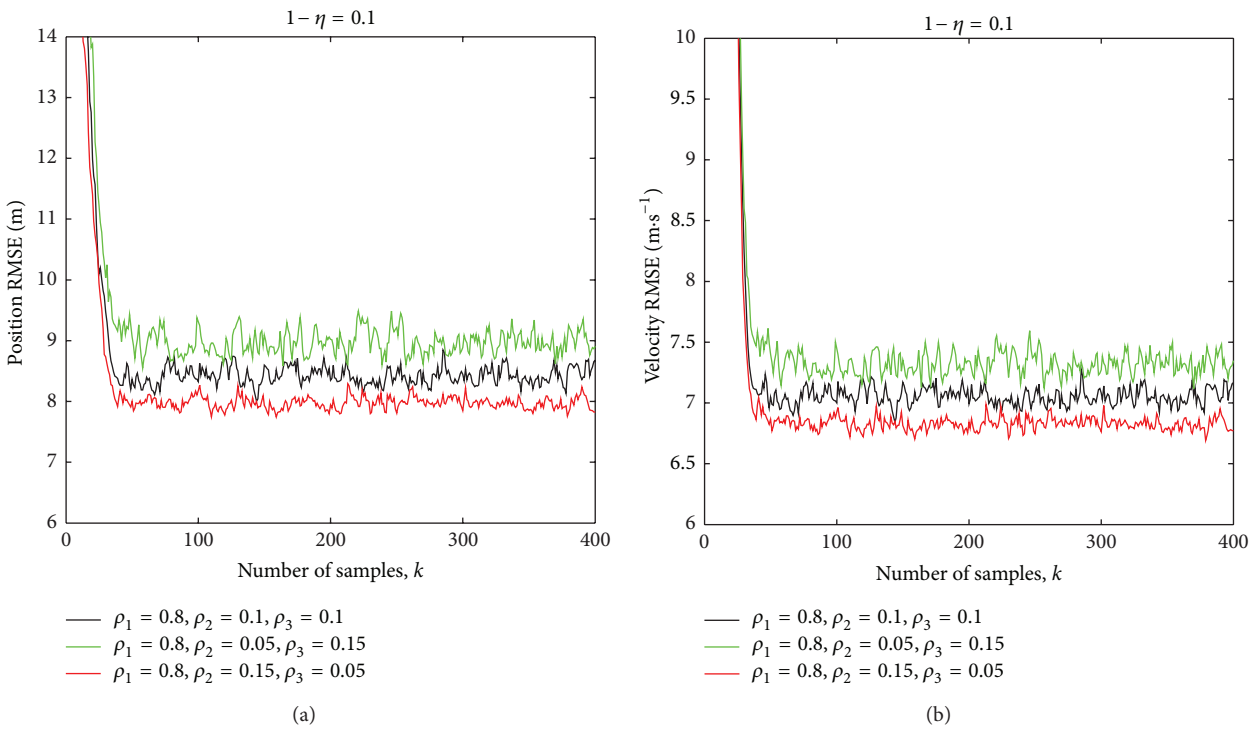


FIGURE 10: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

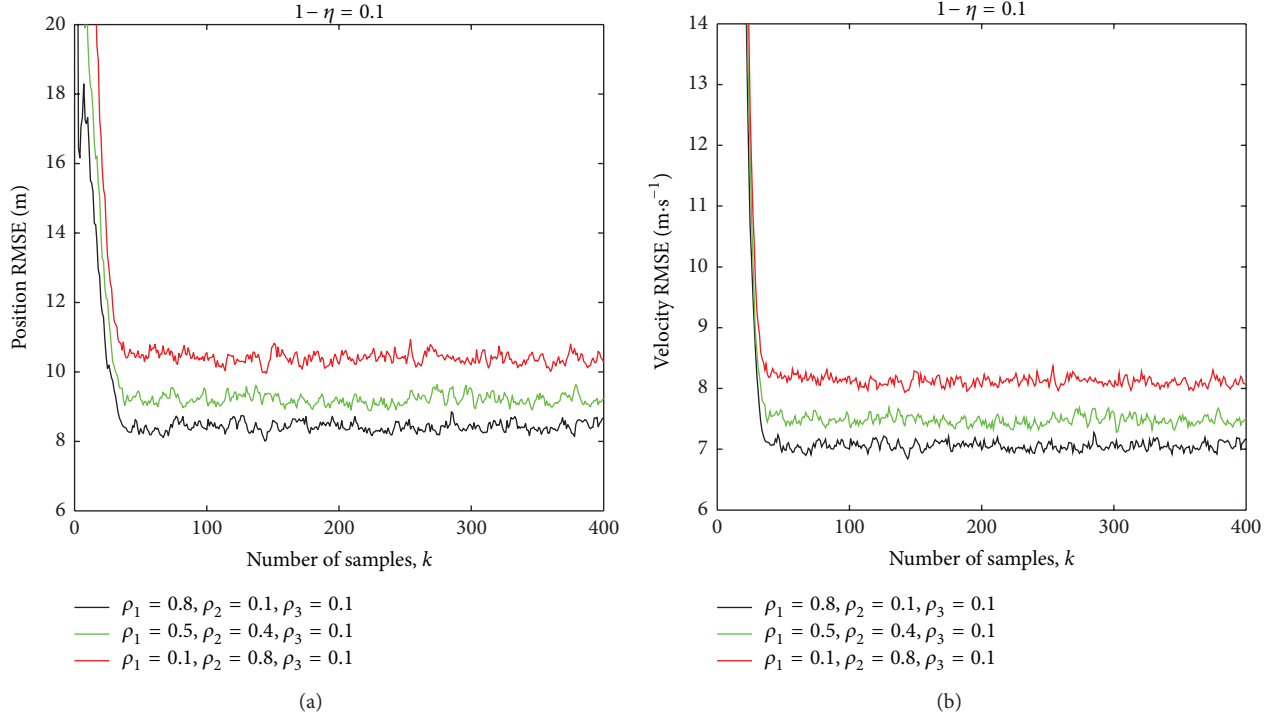


FIGURE 11: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

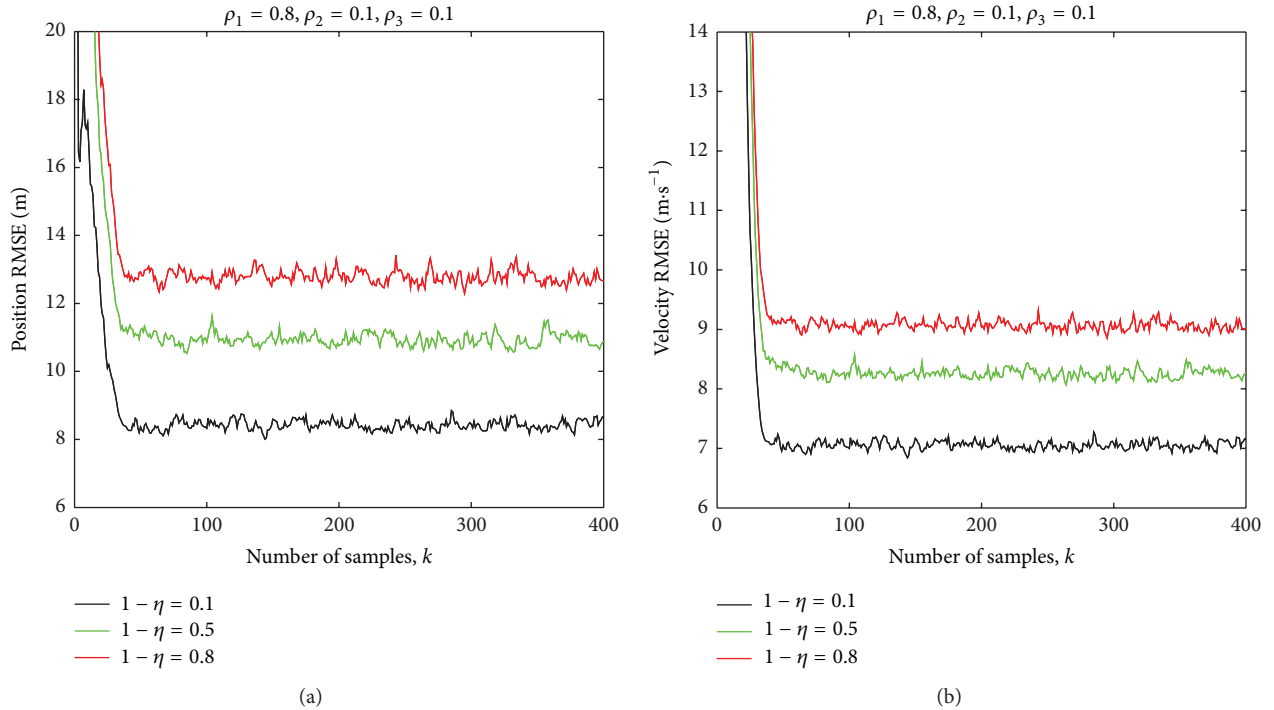


FIGURE 12: Comparison of the RMSEs for the proposed algorithm with different ρ_q ($q = 1, 2, 3$): (a) position RMSE; (b) velocity RMSE.

by detection probability, one-step delay probability, missing probability, and stochastic-bias probability. Moreover, we have the corresponding results in scenarios B and C as well as in scenario A.

5. Conclusions

This paper presented an adaptive estimation algorithm in target tracking mixed with one-step delayed measurements,

stochastic-bias measurements, and missing measurements. The contributions of this paper are as follows. First, the stochastic-bias measurement was incorporated into the proposed adaptive estimation algorithm. Second, the adaptive estimation algorithm mixed with one-step delayed measurements, stochastic-bias measurements, and missing measurements was implemented by augmenting the state vector. Examples and simulations are given to demonstrate the performances of the designed estimator. Through Monte Carlo simulations, the proposed algorithm was shown to be effective. Besides that, all the probabilities of missing measurements, stochastic-bias measurements, and one-step randomly delayed measurements have an influence on the estimation performance. Moreover, the estimation performance can be influenced by stochastic-bias probability. In this paper, we have only considered one-step sensor delays; furthermore, we can obtain the corresponding approach with multiple-step sensor delays by enlarging the state vectors.

Acknowledgments

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