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Research Article

Convergence Analysis for a Modified SP Iterative Method

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We consider a new iterative method due to Kadioglu and Yildirim (2014) for further investigation. We study convergence analysis of this iterative method when applied to class of contraction mappings. Furthermore, we give a data dependence result for fixed point of contraction mappings with the help of the new iteration method.

1. Introduction

Recent progress in nonlinear science reveals that iterative methods are most powerful tools which are used to approximate solutions of nonlinear problems whose solutions are inaccessible analytically. Therefore, in recent years, an intensive interest has been devoted to developing faster and more effective iterative methods for solving nonlinear problems arising from diverse branches in science and engineering.

Very recently the following iterative methods are introduced in [1] and [2], respectively:

$$\begin{aligned} x_0 &\in D, \\ x_{n+1} &= T y_n, \\ y_n &= (1-\alpha_n) z_n + \alpha_n T z_n, \\ z_n &= (1-\beta_n) x_n + \beta_n T x_n, \quad n \in \mathbb{N}, \end{aligned} \tag{1}$$

$$\begin{aligned} u_0 &\in D, \\ u_{n+1} &= T v_n, \\ v_n &= (1-\alpha_n) T u_n + \alpha_n T w_n, \\ w_n &= (1-\beta_n) u_n + \beta_n T u_n, \quad n \in \mathbb{N}, \end{aligned} \tag{2}$$

where D is a nonempty convex subset of a Banach space B, T is a self map of D, and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in $[0, 1]$.

While the iterative method (1) fails to be named in [1], the iterative method (2) is called Picard-S iteration method in [2]. Since iterative method (1) is a special case of SP iterative method of Phuengrattana and Suantai [3], we will call it here Modified SP iterative method.

It was shown in [1] that Modified SP iterative method (1) is faster than all Picard [4], Mann [5], Ishikawa [6], and S [7] iterative methods in the sense of Definitions 1 and 2 given below for the class of contraction mappings satisfying

$$\|Tx - Ty\| \le \delta \|x - y\|, \quad \delta \in (0,1), \forall x, y \in B. \tag{3}$$

Using the same class of contraction mappings (3), Gürsoy and Karakaya [2] showed that Picard-S iteration method (2) is also faster than all Picard [4], Mann [5], Ishikawa [6], S [7], and some other iterative methods in the existing literature.

In this paper, we show that Modified SP iterative method converges to the fixed point of contraction mappings (3). Also, we establish an equivalence between convergence of iterative methods (1) and (2). For the sake of completness, we give a comparison result between the rate of convergences of iterative methods (1) and (2), and it thus will be shown that Picard-S iteration method is still the fastest method. Finally, a data dependence result for the fixed point of the contraction mappings (3) is proven.

The following definitions and lemmas will be needed in order to obtain the main results of this paper.

Definition 1 (see [8]). Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be two sequences of real numbers with limits a and b, respectively. Suppose that

$$\lim_{n\to\infty}\frac{|a_n-a|}{|b_n-b|}=l \tag{4}$$

exists.

(i) If $l=0$, then we say that $\{a_n\}_{n=0}^{\infty}$ converges faster to a than $\{b_n\}_{n=0}^{\infty}$ to b.

(ii) If $0<l<\infty$, then we say that $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ have the same rate of convergence.

Definition 2 (see [8]). Assume that for two fixed point iteration processes $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ both converging to the same fixed point p, the following error estimates,

$$\begin{aligned}\|u_n-p\|&\le a_n\quad\forall n\in\mathbb{N},\\ \|v_n-p\|&\le b_n\quad\forall n\in\mathbb{N},\end{aligned}\tag{5}$$

are available where $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ are two sequences of positive numbers (converging to zero). If $\{a_n\}_{n=0}^{\infty}$ converges faster than $\{b_n\}_{n=0}^{\infty}$, then $\{u_n\}_{n=0}^{\infty}$ converges faster than $\{v_n\}_{n=0}^{\infty}$ to p.

Definition 3 (see [9]). Let $T,\widetilde{T}:B\to B$ be two operators. We say that $\widetilde{T}$ is an approximate operator of T if for all $x\in B$ and for a fixed $\varepsilon>0$ we have

$$\|Tx-\widetilde{T}x\|\le\varepsilon. \tag{6}$$

Lemma 4 (see [10]). *Let $\{\sigma_n\}_{n=0}^{\infty}$ and $\{\rho_n\}_{n=0}^{\infty}$ be nonnegative real sequences and suppose that for all $n\ge n_0$, $\tau_n\in(0,1)$, $\sum_{n=1}^{\infty}\tau_n=\infty$, and $\rho_n/\tau_n\to 0$ as $n\to\infty$*

$$\sigma_{n+1}\le(1-\tau_n)\sigma_n+\rho_n \tag{7}$$

holds. Then $\lim_{n\to\infty}\sigma_n=0$.

Lemma 5 (see [11]). *Let $\{\sigma_n\}_{n=0}^{\infty}$ be a nonnegative sequence such that there exists $n_0\in\mathbb{N}$, for all $n\ge n_0$; the following inequality holds. Consider*

$$\sigma_{n+1}\le(1-\tau_n)\sigma_n+\tau_n\mu_n, \tag{8}$$

where $\tau_n\in(0,1)$, for all $n\in\mathbb{N}$, $\sum_{n=0}^{\infty}\tau_n=\infty$ and $\eta_n\ge 0$, $\forall n\in\mathbb{N}$. Then

$$0\le\limsup_{n\to\infty}\sigma_n\le\limsup_{n\to\infty}\mu_n. \tag{9}$$

2. Main Results

Theorem 6. *Let D be a nonempty closed convex subset of a Banach space B and $T:D\to D$ a contraction map satisfying condition (3). Let $\{x_n\}_{n=0}^{\infty}$ be an iterative sequence generated by (1) with real sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{k=0}^{\infty}\alpha_k=\infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges to a unique fixed point of T, say x_*.*

Proof. The well-known Picard-Banach theorem guarantees the existence and uniqueness of x_*. We will show that $x_n\to x_*$ as $n\to\infty$. From (3) and (1) we have

$$\begin{aligned}\|x_{n+1}-x_*\|&=\|Ty_n-Tx_*\|\\ &\le\delta\|y_n-x_*\|\\ &\le\delta\{(1-\alpha_n)\|z_n-x_*\|+\alpha_n\delta\|z_n-x_*\|\}\\ &\le\delta[1-\alpha_n(1-\delta)]\|z_n-x_*\|\\ &\le\delta[1-\alpha_n(1-\delta)]\\ &\quad\times\{(1-\beta_n)\|x_n-x_*\|+\beta_n\delta\|x_n-x_*\|\}\\ &\le\delta[1-\alpha_n(1-\delta)][1-\beta_n(1-\delta)]\|x_n-x_*\|\\ &\le\delta[1-\alpha_n(1-\delta)]\|x_n-x_*\|.\end{aligned}\tag{10}$$

By induction on the inequality (10), we derive

$$\begin{aligned}\|x_{n+1}-x_*\|&\le\|x_0-x_*\|\delta^{n+1}\prod_{k=0}^{n}[1-\alpha_k(1-\delta)]\\ &\le\|x_0-x_*\|\delta^{n+1}e^{-(1-\delta)\sum_{k=0}^{n}\alpha_k}.\end{aligned}\tag{11}$$

Since $\sum_{k=0}^{\infty}\alpha_k=\infty$, taking the limit of both sides of inequality (11) yields $\lim_{n\to\infty}\|x_n-x_*\|=0$; that is, $x_n\to x_*$ as $n\to\infty$. □

Theorem 7. *Let D, B, and T with fixed point x_* be as in Theorem 6. Let $\{x_n\}_{n=0}^{\infty}$, $\{u_n\}_{n=0}^{\infty}$ be two iterative sequences defined by (1) and (2), respectively, with real sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{k=0}^{\infty}\alpha_k\beta_k=\infty$. Then the following are equivalent:*

(i) *$\{x_n\}_{n=0}^{\infty}$ converges to x_*;*

(ii) *$\{u_n\}_{n=0}^{\infty}$ converges to x_*.*

Proof. We will prove (i)⇒(ii). Now by using (1), (2), and condition (3), we have

$$\begin{aligned}\|x_{n+1}-u_{n+1}\|&=\|Ty_n-Tv_n\|\\ &\le\delta\|y_n-v_n\|\\ &=\delta\|(1-\alpha_n)z_n+\alpha_nTz_n-(1-\alpha_n)Tu_n\\ &\qquad-\alpha_nTw_n\|\\ &\le\delta\{(1-\alpha_n)\|z_n-Tz_n\|+(1-\alpha_n)\delta\|z_n-u_n\|\\ &\qquad+\alpha_n\delta\|z_n-w_n\|\}\end{aligned}$$

$$\leq (1-\alpha_n)\,\delta\,\|x_n-u_n\| + (1-\alpha_n)\,\delta\beta_n\,\|Tx_n-x_n\| + \alpha_n\delta\,\|z_n-w_n\| + (1-\alpha_n)\,\|z_n-Tz_n\|$$
$$\leq (1-\alpha_n)\,\delta\,\|x_n-u_n\| + (1-\alpha_n)\,\delta\beta_n\,\|Tx_n-x_n\| + \alpha_n\delta\,[1-\beta_n(1-\delta)]\,\|x_n-u_n\| + (1-\alpha_n)\,\|z_n-Tz_n\|$$
$$\leq [1-\alpha_n(1-\delta)]\,\|x_n-u_n\| + (1-\alpha_n)\,\delta\beta_n\,\|Tx_n-x_n\| + (1-\alpha_n)\,\|z_n-Tz_n\|. \tag{12}$$

Define

$$\sigma_n := \|x_n-u_n\|,$$
$$\tau_n := \alpha_n(1-\delta) \in (0,1), \tag{13}$$
$$\rho_n := (1-\alpha_n)\,\delta\beta_n\,\|x_n-Tx_n\| + (1-\alpha_n)\,\|z_n-Tz_n\|.$$

Since $\lim_{n\to\infty}\|x_n-x_*\| = 0$ and $Tx_* = x_*$, $\lim_{n\to\infty}\|x_n-Tx_n\| = \lim_{n\to\infty}\|z_n-Tz_n\| = 0$ which implies $\rho_n/\tau_n \to 0$ as $n\to\infty$. Since also $\alpha_n, \beta_n \in [0,1]$ for all $n\in\mathbb{N}$

$$\alpha_n\beta_n < \alpha_n; \tag{14}$$

hence the assumption $\sum_{k=0}^{\infty}\alpha_k\beta_k = \infty$ leads to

$$\sum_{k=0}^{\infty}\alpha_k = \infty. \tag{15}$$

Thus all conditions of Lemma 4 are fulfilled by (12), and so $\lim_{n\to\infty}\|x_n-u_n\| = 0$. Since

$$\|u_n-x_*\| \leq \|x_n-u_n\| + \|x_n-x_*\|,$$
$$\lim_{n\to\infty}\|u_n-x_*\| = 0. \tag{16}$$

Using the same argument as above one can easily show the implication (ii)⇒(i); thus it is omitted here. □

Theorem 8. *Let D, B, and T with fixed point x_* be as in Theorem 6. Let $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ be real sequences in $(0,1)$ satisfying*

(i) $\lim_{n\to\infty}\alpha_n = \lim_{n\to\infty}\beta_n = 0$.

For given $x_0 = u_0 \in D$, consider iterative sequences $\{x_n\}_{n=0}^{\infty}$ and $\{u_n\}_{n=0}^{\infty}$ defined by (1) and (2), respectively. Then $\{u_n\}_{n=0}^{\infty}$ converges to x_* faster than $\{x_n\}_{n=0}^{\infty}$ does.

Proof. The following inequality comes from inequality (10) of Theorem 6:

$$\|x_{n+1}-x_*\| \leq \|x_0-x_*\|\,\delta^{n+1} \times \prod_{k=0}^{n}[1-\alpha_k(1-\delta)]\,[1-\beta_k(1-\delta)]. \tag{17}$$

The following inequality is due to ([2], inequality (2.5) of Theorem 1):

$$\|u_{n+1}-x_*\| \leq \|u_0-x_*\|\,\delta^{2(n+1)} \times \prod_{k=0}^{n}[1-\alpha_k\beta_k(1-\delta)]. \tag{18}$$

Define

$$a_n := \|u_0-x_*\|\,\delta^{2(n+1)}\prod_{k=0}^{n}[1-\alpha_k\beta_k(1-\delta)],$$
$$b_n := \|x_0-x_*\|\,\delta^{n+1}\prod_{k=0}^{n}[1-\alpha_k(1-\delta)]\,[1-\beta_k(1-\delta)]. \tag{19}$$

Since $x_0 = u_0$

$$\theta_n := \frac{a_n}{b_n} = \frac{\delta^{n+1}\prod_{k=0}^{n}[1-\alpha_k\beta_k(1-\delta)]}{\prod_{k=0}^{n}[1-\alpha_k(1-\delta)]\,[1-\beta_k(1-\delta)]}. \tag{20}$$

Therefore, taking into account assumption (i), we obtain

$$\lim_{n\to\infty}\frac{\theta_{n+1}}{\theta_n} = \lim_{n\to\infty}\frac{\delta\,[1-\alpha_{n+1}\beta_{n+1}(1-\delta)]}{[1-\alpha_{n+1}(1-\delta)]\,[1-\beta_{n+1}(1-\delta)]} = \delta < 1. \tag{21}$$

It thus follows from well-known ratio test that $\sum_{n=0}^{\infty}\theta_n < \infty$. Hence, we have $\lim_{n\to\infty}\theta_n = 0$ which implies that $\{u_n\}_{n=0}^{\infty}$ is faster than $\{x_n\}_{n=0}^{\infty}$. □

In order to support analytical proof of Theorem 8 and to illustrate the efficiency of Picard-S iteration method (2), we will use a numerical example provided by Sahu [12] for the sake of consistent comparison.

Example 9. Let $B = \mathbb{R}$ and $D = [0,\infty)$. Let $T: D \to D$ be a mapping and for all $x \in D$, $Tx = \sqrt[3]{3x+18}$. T is a contraction with contractivity factor $\delta = 1/\sqrt[3]{18}$ and $x_* = 3$; see [12]. Take $\alpha_n = \beta_n = \gamma_n = 1/(n+1)$ with initial value $x_0 = 1000$. Tables 1, 2, and 3 show that Picard-S iteration method (2) converges faster than all SP [3], Picard [4], Mann [5], Ishikawa [6], S [7], CR [13], S^* [14], Noor [15], and Normal-S [16] iteration methods including a new three-step iteration method due to Abbas and Nazir [17].

We are now able to establish the following data dependence result.

Theorem 10. *Let $\widetilde{T}$ be an approximate operator of T satisfying condition (3). Let $\{x_n\}_{n=0}^{\infty}$ be an iterative sequence generated by (1) for T and define an iterative sequence $\{\widetilde{x}_n\}_{n=0}^{\infty}$ as follows:*

$$\widetilde{x}_0 \in D,$$
$$\widetilde{x}_{n+1} = \widetilde{T}\widetilde{y}_n,$$
$$\widetilde{y}_n = (1-\alpha_n)\,\widetilde{z}_n + \alpha_n\widetilde{T}\widetilde{z}_n, \tag{22}$$
$$\widetilde{z}_n = (1-\beta_n)\,\widetilde{x}_n + \beta_n\widetilde{T}\widetilde{x}_n, \quad n\in\mathbb{N},$$

Table 1: Comparison speed of convergence among various iteration methods.

Number of iterations	Picard-S	Abbas and Nazir	Modified SP	S^*
1	3.101431265	3.944094141	3.101431265	3.101431265
2	3.000970459	3.032885422	3.003472396	3.006099262
3	3.000010797	3.001099931	3.000191044	3.000474311
4	3.000000126	3.000033381	3.000012841	3.000040908
5	3.000000001	3.000000928	3.000000964	3.000003733
6	3.000000000	3.000000024	3.000000078	3.000000354
7	3.000000000	3.000000000	3.000000007	3.000000034
8	3.000000000	3.000000000	3.000000001	3.000000004
9	3.000000000	3.000000000	3.000000000	3.000000000
⋮	⋮	⋮	⋮	⋮

Table 2: Comparison speed of convergence among various iteration methods.

Number of iterations	CR	Normal S	S	Picard
1	3.101431265	3.944094141	3.944094141	14.45128320
2	3.004853706	3.056995075	3.079213170	3.944094141
3	3.000341967	3.004449310	3.007910488	3.101431265
4	3.000027911	3.000384457	3.000829879	3.011228065
5	3.000002459	3.000035123	3.000088928	3.001247045
6	3.000000227	3.000003324	3.000009637	3.000138554
7	3.000000022	3.000000323	3.000001051	3.000015395
8	3.000000003	3.000000032	3.000000115	3.000001710
9	3.000000000	3.000000003	3.000000013	3.000000190
10	3.000000000	3.000000000	3.000000001	3.000000021
11	3.000000000	3.000000000	3.000000000	3.000000002
12	3.000000000	3.000000000	3.000000000	3.000000000
⋮	⋮	⋮	⋮	⋮

Table 3: Comparison speed of convergence among various iteration methods.

Number of iterations	SP	Noor	Ishikawa	Mann
1	3.101431265	3.101431265	3.944094141	14.45128320
2	3.017380074	3.053700718	3.500544608	9.197688670
3	3.006056041	3.037176288	3.346563527	7.322609407
4	3.002849358	3.028678163	3.267333303	6.346715746
5	3.001583841	3.023463545	3.218710750	5.744057924
6	3.000979045	3.019921623	3.185684999	5.333095485
7	3.000651430	3.017350921	3.161716071	5.034023149
8	3.000457519	3.015395770	3.143487338	4.806124994
9	3.000334906	3.013856108	3.129133091	4.626397579
10	3.000253300	3.012610550	3.117521325	4.480838008
11	3.000196722	3.011581068	3.107924338	4.360421594
12	3.000156165	3.010715155	3.099852480	4.259063398
13	3.000126272	3.009976148	3.092963854	4.172507477
⋮	⋮	⋮	⋮	⋮

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$ satisfying (i) $1/2 \le \alpha_n$, (ii) $\beta_n \le \alpha_n$ for all $n \in \mathbb{N}$, and (iii) $\sum_{n=0}^{\infty} \alpha_n = \infty$. If $Tx_* = x_*$ and $\widetilde{T}\widetilde{x}_* = \widetilde{x}_*$ such that $\widetilde{x}_n \to \widetilde{x}_*$ as $n \to \infty$, then we have

$$\left\| x_* - \widetilde{x}_* \right\| \le \frac{4\varepsilon}{1-\delta}, \tag{23}$$

where $\varepsilon > 0$ is a fixed number and $\delta \in (0,1)$.

Proof. It follows from (1), (3), (22), and assumption (ii) that

$$\begin{aligned}
\left\| x_{n+1} - \widetilde{x}_{n+1} \right\| &= \left\| Ty_n - T\widetilde{y}_n + T\widetilde{y}_n - \widetilde{T}\widetilde{y}_n \right\| \\
&\le \left\| Ty_n - T\widetilde{y}_n \right\| + \left\| T\widetilde{y}_n - \widetilde{T}\widetilde{y}_n \right\| \\
&\le \delta \left\| y_n - \widetilde{y}_n \right\| + \varepsilon \\
&\le \delta \left(1-\alpha_n\right) \left\| z_n - \widetilde{z}_n \right\| \\
&\quad + \delta\alpha_n \left\| Tz_n - T\widetilde{z}_n \right\| + \delta\alpha_n \left\| T\widetilde{z}_n - \widetilde{T}\widetilde{z}_n \right\| + \varepsilon \\
&\le \delta \left[1-\alpha_n\left(1-\delta\right)\right] \left\| z_n - \widetilde{z}_n \right\| + \delta\alpha_n\varepsilon + \varepsilon \\
&\le \delta \left[1-\alpha_n\left(1-\delta\right)\right] \left[1-\beta_n\left(1-\delta\right)\right] \left\| x_n - \widetilde{x}_n \right\|
\end{aligned}$$

$$+\delta\left[1-\alpha_n(1-\delta)\right]\beta_n\varepsilon+\delta\alpha_n\varepsilon+\varepsilon$$
$$\leq\left[1-\alpha_n(1-\delta)\right]\left\|x_n-\widetilde{x}_n\right\|+2\alpha_n\varepsilon+\varepsilon. \tag{24}$$

From assumption (i) we have

$$1\leq 2\alpha_n, \tag{25}$$

and thus, inequality (24) becomes

$$\begin{aligned}\left\|x_{n+1}-\widetilde{x}_{n+1}\right\| &\leq\left[1-\alpha_n(1-\delta)\right]\left\|x_n-\widetilde{x}_n\right\|+4\alpha_n\varepsilon\\ &\leq\left[1-\alpha_n(1-\delta)\right]\left\|x_n-\widetilde{x}_n\right\|\\ &\quad+\alpha_n(1-\delta)\frac{4\varepsilon}{1-\delta}.\end{aligned} \tag{26}$$

Denote that

$$\sigma_n:=\left\|x_n-\widetilde{x}_n\right\|,\qquad \tau_n:=\alpha_n(1-\delta)\in(0,1),\qquad \mu_n:=\frac{4\varepsilon}{1-\delta}. \tag{27}$$

It follows from Lemma 5 that

$$0\leq\limsup_{n\to\infty}\left\|x_n-\widetilde{x}_n\right\|\leq\limsup_{n\to\infty}\frac{4\varepsilon}{1-\delta}. \tag{28}$$

From Theorem 6 we know that $\lim_{n\to\infty}x_n=x_*$. Thus, using this fact together with the assumption $\lim_{n\to\infty}\widetilde{x}_n=\widetilde{x}_*$ we obtain

$$\left\|x_*-\widetilde{x}_*\right\|\leq\frac{4\varepsilon}{1-\delta}. \tag{29}$$

□

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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