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Research Article

Different Kinds of Singular and Nonsingular Exact Traveling Wave Solutions of the Kudryashov-Sinelshchikov Equation in the Special Parametric Conditions

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In this paper, by using the integral bifurcation method, we studied the Kudryashov-Sinelshchikov equation. In the special parametric conditions, some singular and nonsingular exact traveling wave solutions, such as periodic cusp-wave solutions, periodic loop-wave solutions, smooth loop-soliton solutions, smooth solitary wave solutions, periodic double wave solutions, periodic compacton solutions, and nonsmooth peakon solutions are obtained. Further more, the dynamic behaviors of these exact traveling wave solutions are investigated. It is found that the waveforms of some traveling wave solutions vary with the changes of parameters.

1. Introduction

The investigation of the traveling wave solutions for nonlinear evolution equations plays an important role in mathematical physics. For example, the phenomena of wave motion observed in fluid dynamics, plasma, and elastic media are usually described by the solitary wave, kink wave, peakon, compacton, and loop soliton. In fact, lots of physical models have solitary wave solutions, kink wave solutions, peakon solutions, compacton solutions, and loop-soliton solutions. Lots of new traveling wave solutions and phenomena of wave motion are discovered continually by many researchers on investigating exact solutions of different kinds of nonlinear evolution equations. Recently, Kudryashov and Sinelshchikov [1] introduced the following equation:

$$u_t+\alpha uu_x+u_{xxx}-\epsilon(uu_{xx})_x-\beta u_xu_{xx}-\nu u_{xx}-\delta(uu_x)_x=0, \tag{1}$$

where α, ϵ, β, ν, and δ are real parameters. Equation (1) describes the pressure waves in the liquid with gas bubbles taking into account the heat transfer and viscosity [1]. We call (1) Kudryashov-Sinelshchikov equation. When $\epsilon=\beta=\delta=0$, (1) becomes classical KdV-Burgers (Burgers-KdV) equation; see [2–7] and the references cited therein. When $\epsilon=\beta=\nu=\delta=0$, (1) becomes famous KdV equation. Very recently, when parameters $\nu=\delta=0$, in [8–11], the authors studied the special case of (1) as follows:

$$u_t+\alpha uu_x+u_{xxx}-(uu_{xx})_x-\beta u_xu_{xx}=0. \tag{2}$$

In [8], Ryabov found four families of solitary wave solutions of (2) when $\beta=-3$ or $\beta=-4$. In [9], Li and Chen discussed the existence of different kinds of traveling wave solutions by using the approach of dynamical system, according to different phase orbits of the traveling system of (2); twenty-six kinds of exact traveling wave solutions are obtained under the parametric conditions $\beta=-3$, -4 and $\beta=1,2$. In [10], He et al. discussed the bifurcations of phase portraits and investigated exact traveling wave solutions of (2) under $\beta=-3,1,2$. In [11], He et al. investigated periodic loop solutions of (2) and discussed the limit forms of these periodic loop solutions focusing on the case $\beta=2$. In the paper [12], Randrüüt studied (2) under the transformation $b=2+\beta$; in the cases $b<0$, $b=0$, and $b>0$ (i.e., $\beta<-2$, $\beta=-2$, and $\beta>-2$), Randrüüt obtained some exact traveling wave solutions and discussed their dynamical behaviors; some interesting phenomena of traveling waves are successfully explained. Particularly, when $b>2$ (i.e., $\beta>0$), a kind of new periodic wave solution which is called meandering solution was obtained by Randrüüt in this paper.

However, many singular and nonsingular exact traveling wave solutions of Kudryashov-Sinelshchikov equation were not obtained yet in these references [8–12].

In this paper, we will study (2) again by using the integral bifurcation method [13, 14]; the integral bifurcation method possessed some advantages of the bifurcation theory of the planar dynamic system [15–19] and auxiliary equation method; it is easily combined with some transformations [20, 21] and useful for many nonlinear partial differential equations (PDEs) including some higher order equations of KdV type, such as the higher order KdV equation of neglecting the highest order infinitesimal term [22]. So, by using this method, we will obtain some new traveling wave solutions of (1), which are different from the results in [8–12]. Some interesting phenomena will be presented.

The rest of this paper is organized as follows: in Section 2, we will derive two-dimensional planar system which is equivalent to (2) and give its first integral equation. In Section 3, by using the integral bifurcation method, we will obtain some new traveling wave solutions of (2) and discuss their dynamic properties.

2. The 2-Dimensional Planar Dynamical System of (2)

Making a transformation $u(t,x) = \phi(\xi) + \lambda$ with $\xi = kx - ct$, (2) can be reduced to the following ordinary differential equation:

$$-c\phi' + \alpha(\phi+\lambda)\phi' + \phi''' - \left[(\phi+\lambda)\phi''\right]' - \beta\phi'\phi'' = 0. \quad (3)$$

Integrating (3) with respect to ξ, we have

$$(1-\lambda-\phi)\phi'' = g + (c-\alpha\lambda)\phi - \frac{1}{2}\alpha\phi^2 + \frac{1}{2}\beta(\phi')^2, \quad (4)$$

where g is an integral constant. Let $d\phi/d\xi = y$. Thus, (4) can be rewritten as the following 2-dimensional planar system:

$$\frac{d\phi}{d\xi} = y, \qquad \frac{dy}{d\xi} = \frac{g + (c-\alpha\lambda)\phi - (1/2)\alpha\phi^2 + (1/2)\beta y^2}{(1-\lambda-\phi)}, \quad (5)$$

which is a singular system. The system (5) has the following first integral:

$$H(\phi,y) \equiv (-1+\lambda+\phi)^{\beta}\left[y^2 + \frac{a_0 + a_1\phi + a_2\phi^2}{\beta(\beta+1)(\beta+2)}\right] = h, \quad (6)$$

where h is integral constant and $a_0 = 2g(\beta+1)(\beta+2) - 2(\lambda-1)[c(\beta+2) - \alpha\lambda(\beta+1) - \alpha]$, $a_1 = 2(c-\alpha\lambda)\beta^2 + 2\beta[2c - \alpha(\lambda+1)]$, and $a_2 = -\alpha\beta(\beta+1)$. Particularly, when $\beta_1 = 1$, (6) becomes

$$H(\phi,y) \equiv (-1+\lambda+\phi) \times \left[y^2 + \frac{1}{3}\alpha\phi^3 + (c-\alpha)\phi^2 + 2g\phi\right] = h. \quad (7)$$

And when $\beta = 2$, (6) becomes

$$H(\phi,y) \equiv (-1+\lambda+\phi)^2 y^2 - \frac{1}{4}\alpha\phi^4 + \frac{2c - 3\alpha\lambda + \alpha}{3}\phi^3 + \left[g + (c-\alpha\lambda)(-1+\lambda)\right]\phi^2 + 2g(-1+\lambda)\phi = h. \quad (8)$$

Obviously, (7) is very simpler than (8), that is, the case of $\beta = 1$ is very simpler than the case of $\beta = 2$. Thus we will only discuss the case of $\beta = 2$; the case of $\beta = 1$ can be similarly discussed, we omit the discussions of case $\beta = 1$ in this paper. It is easy to find that the derivative $dy/d\xi$ in the right side of (5) is not defined when $1 - \lambda - \phi = 0$. Therefore, we make a transformation as follows:

$$d\xi = (1-\lambda-\phi)\,d\tau, \quad (9)$$

where τ is a parameter. Under the transformation (9) and $\beta = 2$, (5) becomes the following regular system:

$$\frac{d\phi}{d\tau} = (1-\lambda-\phi)y, \qquad \frac{dy}{d\tau} = g + (c-\alpha\lambda)\phi - \frac{1}{2}\alpha\phi^2 + y^2. \quad (10)$$

Obviously, (8) can be rewritten as follows:

$$y^2 = \frac{1}{(\phi+\lambda-1)^2} \times \left[\frac{\alpha}{4}\phi^4 - \frac{2(c-\alpha\lambda) - \alpha(\lambda-1)}{3}\phi^3 - \left(g + (c-\alpha\lambda)(\lambda-1)\right)\phi^2 - 2g(\lambda-1)\phi + h\right]. \quad (11)$$

3. Exact Traveling Wave Solutions of (2)

In this section, we will investigate exact traveling wave solutions of (2) by using its first integral equation (11).

Case 1. When $\lambda = (2c+\alpha)/3\alpha$, $g = 0$, (11) can be reduced to

$$y = \pm\frac{\sqrt{P\phi^4 + Q\phi^2 + R}}{\phi+\lambda-1}, \quad (12)$$

where $P = (1/4)\alpha$, $Q = -8(c-\alpha)^2/9\alpha$, and $R = h$. Substituting (12) into the first equation of (10) yields

$$\frac{d\phi}{d\tau} = \pm\sqrt{P\phi^4 + Q\phi^2 + R}. \quad (13)$$

By using (13) and (9), we will obtain different kinds of exact traveling wave solutions of (2); see the following discussion.

(i) If $-10 < c < -4 - 2\sqrt{2}$ or $-4 + 3\sqrt{2} < c < 2$ and $\alpha = -4$, $m = (1/3)\sqrt{-(1/2)c^2 - 4c + 10}$, $h = (-1/18)(c^2 + 8c - 2)$, then $P = -1$, $Q = 2 - m^2$, and $R = m^2 - 1$.

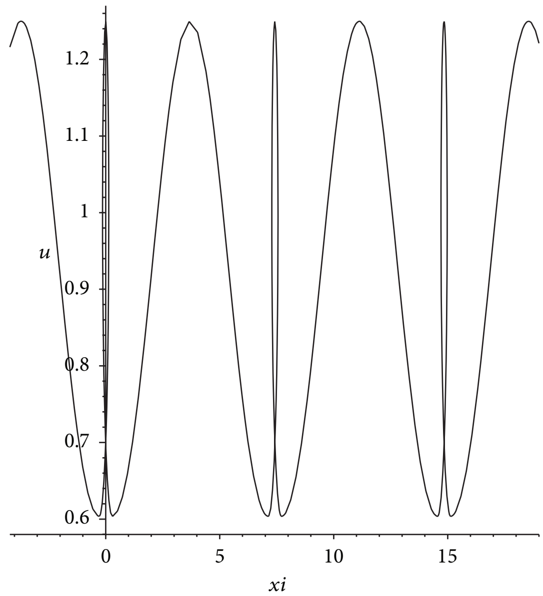


FIGURE 1: The profile of periodic loop-wave solution (14) for $\alpha = -4,\ c = 0.5$.

Under these conditions, by using (13) and (9), we obtain a periodic wave solution of (2) as follows:

$$u = \lambda + \mathrm{dn}(\tau, m), \qquad \xi = (1-\lambda)\tau - \arcsin(\mathrm{sn}(\tau, m)). \tag{14}$$

The profile of solution (14) is a periodic loop wave, which is shown in Figure 1.

(ii) If $-2 - \sqrt{34}/2 < c < -2 + \sqrt{34}/2$ and $m = (1/34)\sqrt{1462 - 136c \pm 102\sqrt{-2c^2 - 8c + 9}}$, $\alpha = (2/17)(4c - 9) \pm (6/17)\sqrt{-2c^2 - 8c + 9}$, then $P = 1 - m^2$, $Q = 2 - m^2$ and $R = 1$. Under these conditions, by using (13) and (9), we obtain a periodic wave solution of (2) as follows:

$$u = \lambda + \mathrm{sc}(\tau, m),$$
$$\xi = (1-\lambda)\tau - \frac{1}{2\sqrt{1-m^2}} \ln\left[\frac{\mathrm{dn}(\tau, m) + \sqrt{1-m^2}}{\mathrm{dn}(\tau, m) - \sqrt{1-m^2}}\right]. \tag{15}$$

(iii) If $-\infty < c < 4 - 3\sqrt{2}$ or $4 + 3\sqrt{2} < c < \infty$ and $m = (1/3)\sqrt{(1/2)c^2 - 4c - 1}$, $\alpha = 4$, $h = (1/18)(c^2 - 8c - 2)$, then $P = 1$, $Q = -(1 + m^2)$ and $R = m^2$. Under these conditions, by using (13) and (9), we obtain a periodic wave solution of (2) as follows:

$$u = \lambda + \mathrm{ns}(\tau, m),$$
$$\xi = (1-\lambda)\tau - \ln\left[\frac{\mathrm{sn}(\tau, m)}{\mathrm{cn}(\tau, m) + \mathrm{dn}(\tau, m)}\right]. \tag{16}$$

(iv) If $4 - 3\sqrt{2} < c < 4 + 3\sqrt{2}$ and $m = (1/6)\sqrt{-c^2 + 8c - 2}$, $\alpha = 4$, $h = (1/1296)(c^2 - 8c + 34)(c^2 - 8c - 2)$, then $P = 1$, $Q = 2m^2 - 1$, $R = -m^2 + m^4$. Under these conditions, by using (13) and (9), we obtain a periodic wave solution of (2) as follows:

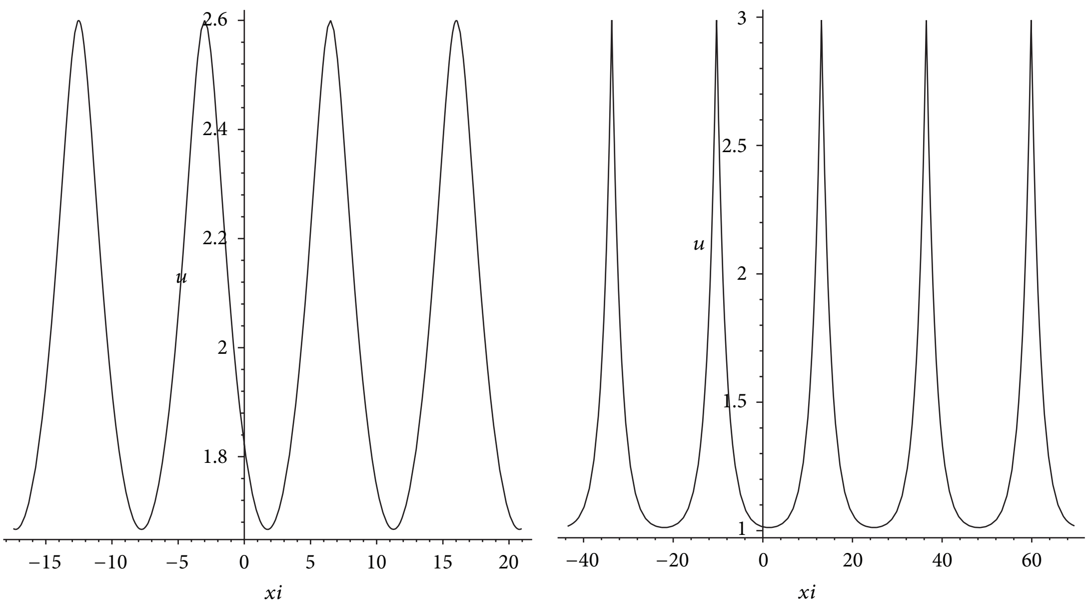
$$u = \lambda + \mathrm{ds}(\tau, m),$$
$$\xi = (1-\lambda)\tau - \ln\left[\frac{\mathrm{sn}(\tau, m)}{1 + \mathrm{cn}(\tau, m)}\right]. \tag{17}$$

(v) If $1 - 3\sqrt{2}/2 < c < -1/2$ or $5/2 < c < 1 + 3\sqrt{2}/2$ and $m = (1/3)\sqrt{-4c^2 + 8c + 14}$, $\alpha = 1$, $h = (1/81)(2c^2 - 4c - 7)^2$, then $P = 1/4$, $Q = (1/2)(m^2 - 2)$, and $R = (1/4)m^4$. Under these conditions, by using (13) and (9), we obtain two periodic wave solutions of (2) as follows:

$$u = \lambda + \mathrm{ns}(\tau, m) + \mathrm{ds}(\tau, m),$$
$$\xi = (1-\lambda)\tau - \ln\left[\frac{\mathrm{sn}(\tau, m)}{\mathrm{cn}(\tau, m) + \mathrm{dn}(\tau, m)}\right] - \ln\left[\frac{\mathrm{sn}(\tau, m)}{1 + \mathrm{cn}(\tau, m)}\right], \tag{18}$$

$$u = \lambda + \mathrm{ns}(\tau, m) - \mathrm{ds}(\tau, m),$$
$$\xi = (1-\lambda)\tau - \ln\left[\frac{\mathrm{sn}(\tau, m)}{\mathrm{cn}(\tau, m) + \mathrm{dn}(\tau, m)}\right] + \ln\left[\frac{\mathrm{sn}(\tau, m)}{1 + \mathrm{cn}(\tau, m)}\right]. \tag{19}$$

The waveforms of solution (19) are transformable; the smooth periodic wave becomes the non-smooth periodic cusp wave as parameter c varies; their profiles are shown in Figures 2(a) and 2(b). In fact, this non-smooth periodic cusp wave is a limit form of smooth periodic wave.



(a) Smooth periodic wave (b) Nonsmooth periodic cusp wave

Figure 2: The profiles of periodic wave solutions (19) for (a) $\alpha = 1,\ c = 2.7$, (b) $\alpha = 1,\ c = 2.5001$.

Case 2. When $g = 0,\ h = 0$, (11) can be reduced to

$$y = \frac{\sqrt{A\phi^2 + B\phi^3 + C\phi^4}}{\phi + \lambda - 1}, \tag{20}$$

where $A = (c - \alpha\lambda)(\lambda - 1)$, $B = \alpha\lambda - (1/3)(2c + \lambda)$, and $C = (1/4)\alpha$. Substituting (20) into the first equation of (10) yields

$$\frac{d\phi}{d\tau} = \pm\sqrt{A\phi^2 + B\phi^3 + C\phi^4}. \tag{21}$$

By using (21) and (9), we will obtain different kinds of soliton solutions; see the following discussions.

(i) When $A > 0,\ C > 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$\begin{aligned} u &= \lambda - \frac{AB\,\text{sech}^2\left(\left(\sqrt{A}/2\right)\tau\right)}{B^2 - AC\left[1 + \epsilon\tanh\left(\left(\sqrt{A}/2\right)\tau\right)\right]^2}, \\ \xi &= (1 - \lambda)\,\tau \\ &\quad + \frac{2}{\epsilon\sqrt{C}}\tanh^{-1}\left[\frac{\sqrt{AC}}{B}\left(1 + \epsilon\tanh\left(\frac{\sqrt{A}}{2}\tau\right)\right)\right], \end{aligned} \tag{22}$$

where τ is a parameter. The solution (22) contains three transformable wave-forms as the parameter c varies; these three wave-forms are dark loop soliton, dark peakon, and smooth-dark soliton, which are shown in Figures 3(a), 3(b), and 3(c).

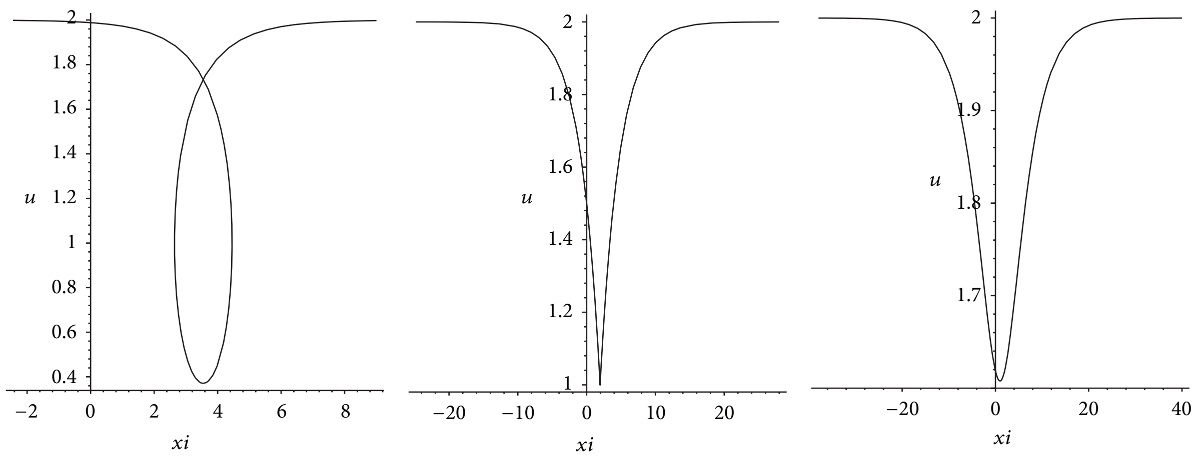
(ii) When $A > 0,\ C < 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$\begin{aligned} u &= \lambda - \frac{AB\,\text{sech}^2\left(\left(\sqrt{A}/2\right)\tau\right)}{B^2 - AC\left[1 + \epsilon\tanh\left(\left(\sqrt{A}/2\right)\tau\right)\right]^2}, \\ \xi &= (1 - \lambda)\,\tau + \frac{2}{\epsilon\sqrt{-C}} \\ &\quad \times\arctan\left[\frac{\sqrt{-AC}}{B}\left(1 + \epsilon\tanh\left(\frac{\sqrt{A}}{2}\tau\right)\right)\right], \end{aligned} \tag{23}$$

where τ is a parameter. The solution (23) contains two wave-forms, the dark loop soliton, and smooth-bright soliton which are shown in Figure 4.

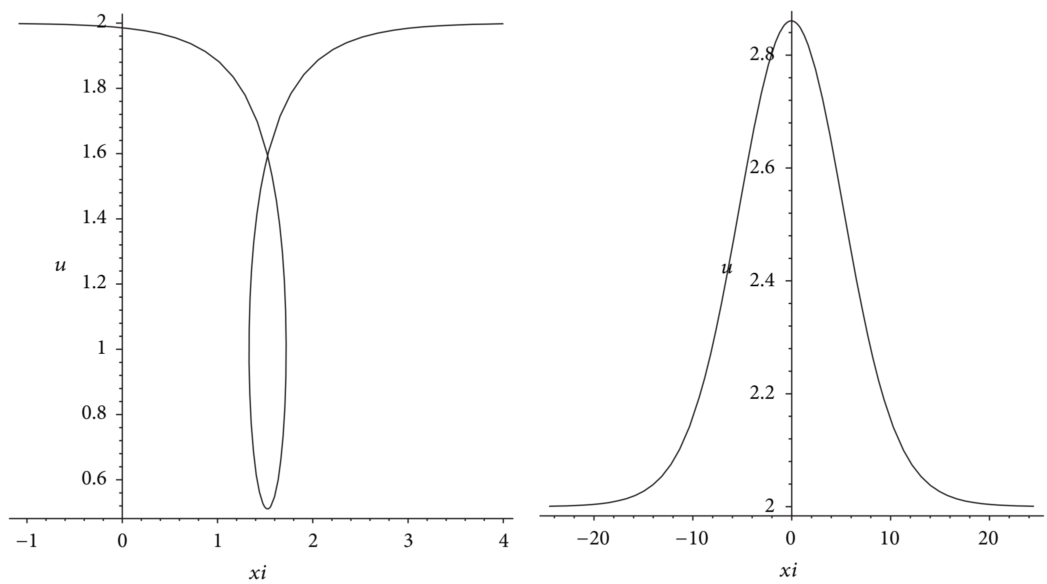
(iii) When $A < 0,\ C > 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$\begin{aligned} u &= \lambda + \frac{AB\,\text{csch}^2\left(\left(\sqrt{A}/2\right)\tau\right)}{B^2 - AC\left[1 + \epsilon\coth\left(\left(\sqrt{A}/2\right)\tau\right)\right]^2}, \\ \xi &= (1 - \lambda)\,\tau + \frac{2}{\epsilon\sqrt{C}} \\ &\quad \times\tanh^{-1}\left[\frac{\sqrt{AC}}{B}\left(1 + \epsilon\coth\left(\frac{\sqrt{A}}{2}\tau\right)\right)\right], \end{aligned} \tag{24}$$



(a) Dark loop soliton (b) Dark peakon (c) Smooth-dark soliton

FIGURE 3: The profile of soliton solutions (22) for (a) $\alpha = 0.5$, $\lambda = 2$, and $c = 0.3$, (b) $\alpha = 0.5$, $\lambda = 2$, $c = 0.875$, and (c) $\alpha = 0.5$, $\lambda = 2$, and $c = 0.9375$.



(a) Dark loop soliton (b) Smooth-bright soliton

FIGURE 4: The profile of soliton solutions (23) for (a) $\alpha = -0.5$, $\lambda = 2$, and $c = -5$ and (b) $\alpha = -0.5$, $\lambda = 2$, and $c = -1.15$.

(iv) When $A < 0$, $C < 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \frac{AB\operatorname{csch}^2\left(\left(\sqrt{A}/2\right)\tau\right)}{B^2 - AC\left[1 + \epsilon\coth\left(\left(\sqrt{A}/2\right)\tau\right)\right]^2},$$
$$\xi = (1-\lambda)\tau + \frac{2}{\epsilon\sqrt{-C}} \times \arctan\left[\frac{\sqrt{-AC}}{B}\left(1 + \epsilon\coth\left(\frac{\sqrt{A}}{2}\tau\right)\right)\right]. \tag{25}$$

The solution (25) contains four transformable waveforms as the parameter c varies; these four waveforms are broken wave, broken soliton, dark loop soliton, and smooth-bright soliton, which are shown in Figures 5(a), 5(b), 5(c), and 5(d).

(v) When $A > 0$, $\Delta > 0$, and $C > 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \frac{2A\operatorname{sech}\left(\sqrt{A}\tau\right)}{\epsilon\sqrt{\Delta} - B\operatorname{sech}\left(\sqrt{A}\tau\right)},$$
$$\xi = (1-\lambda)\tau + \frac{2}{\sqrt{C}}\tanh^{-1}\left[\frac{B + \epsilon\sqrt{\Delta}}{2\sqrt{AC}}\tanh\left(\frac{\sqrt{A}}{2}\tau\right)\right]. \tag{26}$$

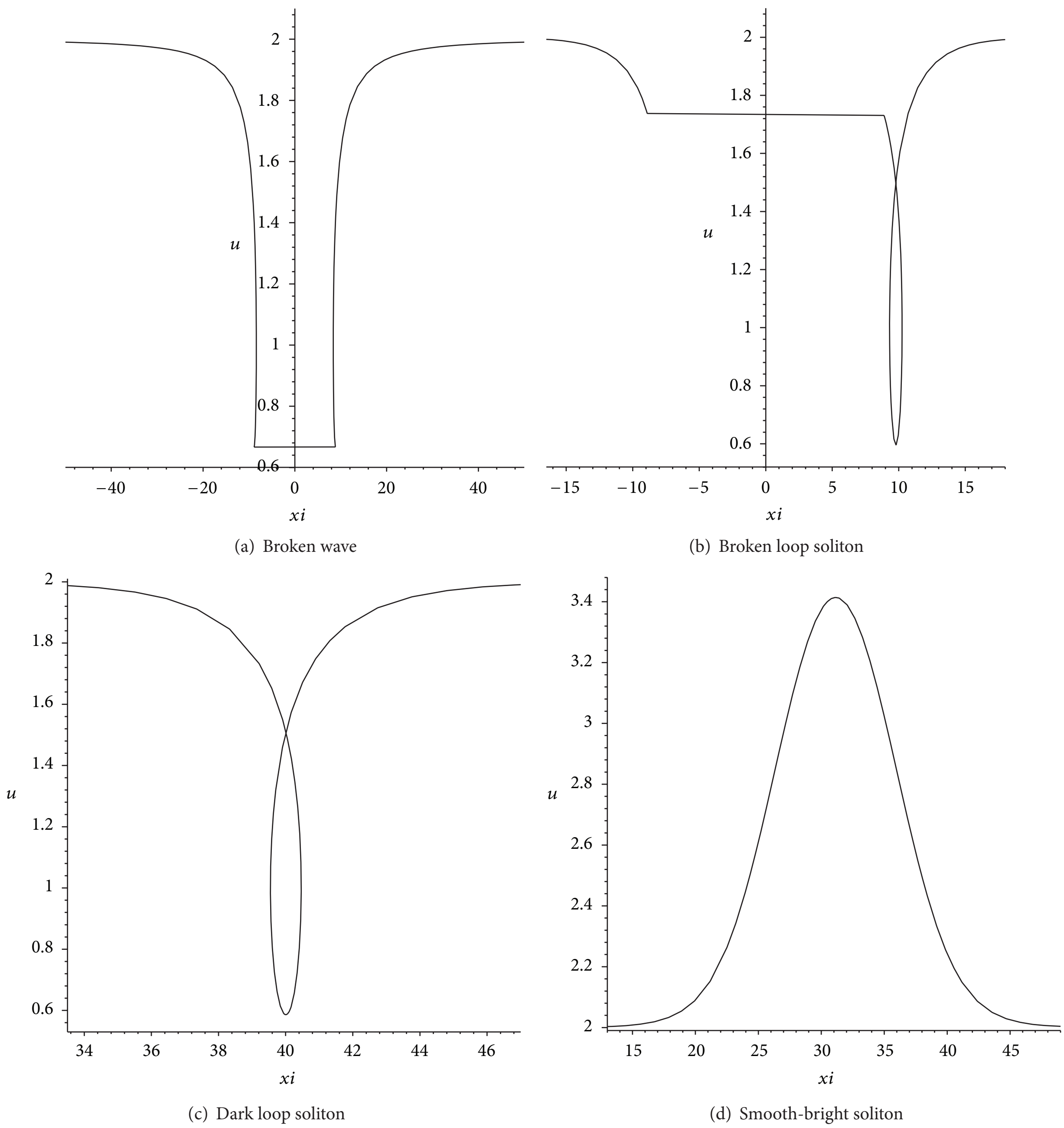


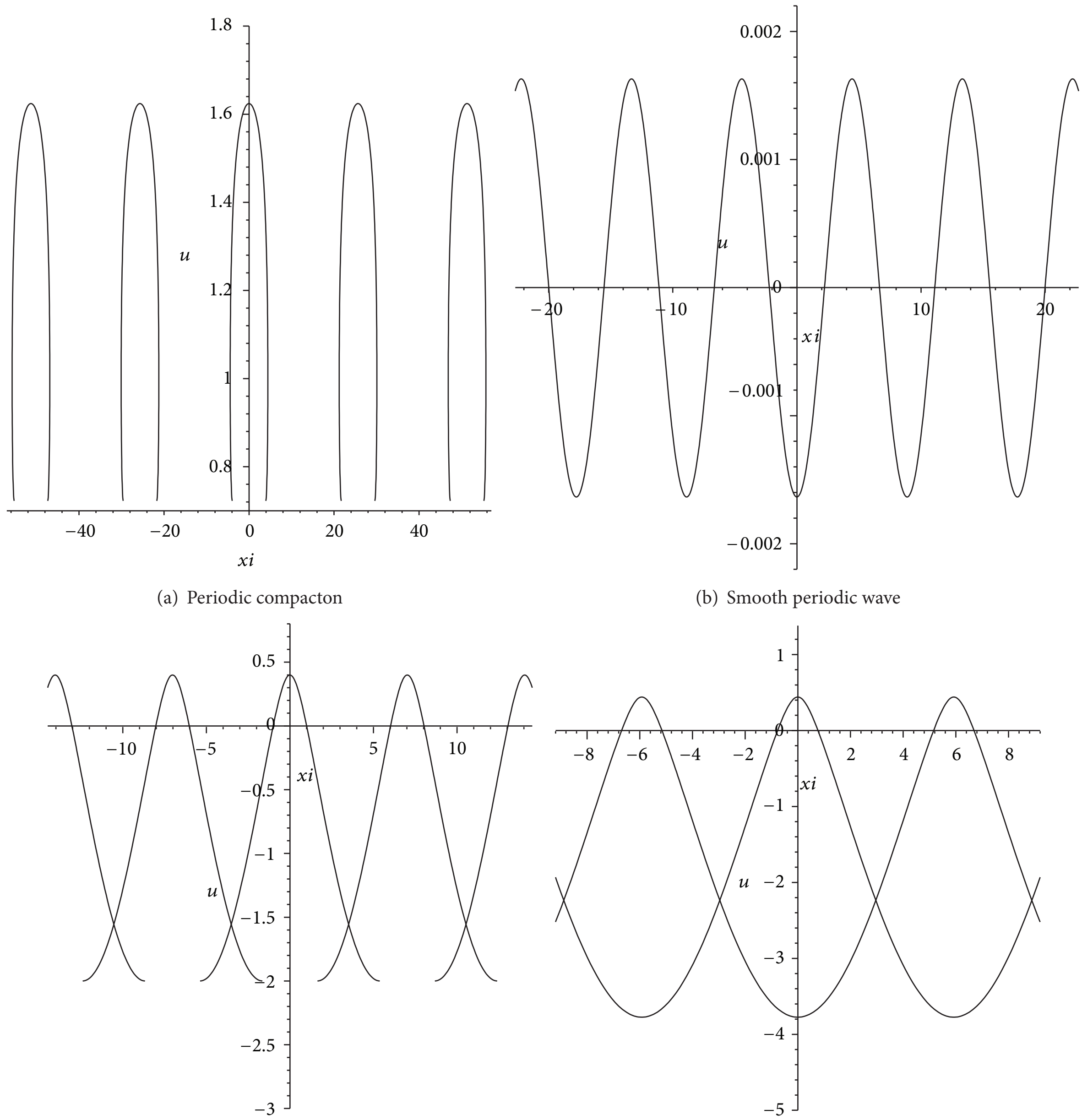
Figure 5: The profile of soliton solutions (25) for (a) $\alpha = -0.5$, $\lambda = 2$, and $c = -1.0001$, (b) $\alpha = -0.5$, $\lambda = 2$, and $c = -1.2$, and (c) $\alpha = -0.5$, $\lambda = 2$, and $c = -1.25$, and (d) $\alpha = -0.5$, $\lambda = 2$, and $c = -1.25000001$.

(vi) When $A > 0$, $\Delta > 0$, and $C < 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \frac{2A\operatorname{sech}\left(\sqrt{A}\tau\right)}{\epsilon\sqrt{\Delta} - B\operatorname{sech}\left(\sqrt{A}\tau\right)},$$
$$\xi = (1-\lambda)\tau - \frac{2}{\sqrt{-C}}\arctan\left[\frac{B+\epsilon\sqrt{\Delta}}{2\sqrt{-AC}}\tanh\left(\frac{\sqrt{A}}{2}\tau\right)\right]. \tag{27}$$

(vii) When $A < 0$, $\Delta > 0$, and $C < 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \frac{2A\sec\left(\sqrt{-A}\tau\right)}{\epsilon\sqrt{\Delta} - B\sec\left(\sqrt{-A}\tau\right)},$$
$$\xi = (1-\lambda)\tau - \frac{2}{\sqrt{-C}}\arctan\left[\frac{B+\epsilon\sqrt{\Delta}}{2\sqrt{AC}}\tan\left(\frac{\sqrt{-A}}{2}\tau\right)\right]. \tag{28}$$



(a) Periodic compacton

(b) Smooth periodic wave

(c) Periodic overlap wave

(d) Double periodic overlap wave

Figure 6: The profile of soliton solutions (28) for (a) $\alpha = -0.5$, $\lambda = 2$, and $c = -0.94$, (b) $\alpha = -0.5$, $\lambda = 2$, and $c = -0.499999$, (c) $\alpha = -0.5$, $\lambda = 2$, and $c = -0.2$, and (d) $\alpha = -0.5$, $\lambda = 2$, and $c = 0.875$.

The solution (28) contains four transformable waveforms as the parameter c varies; these four waveforms are periodic compacton, smooth periodic wave, periodic overlap wave, and double periodic overlap wave, which are shown in Figures 6(a), 6(b), 6(c), and 6(d).

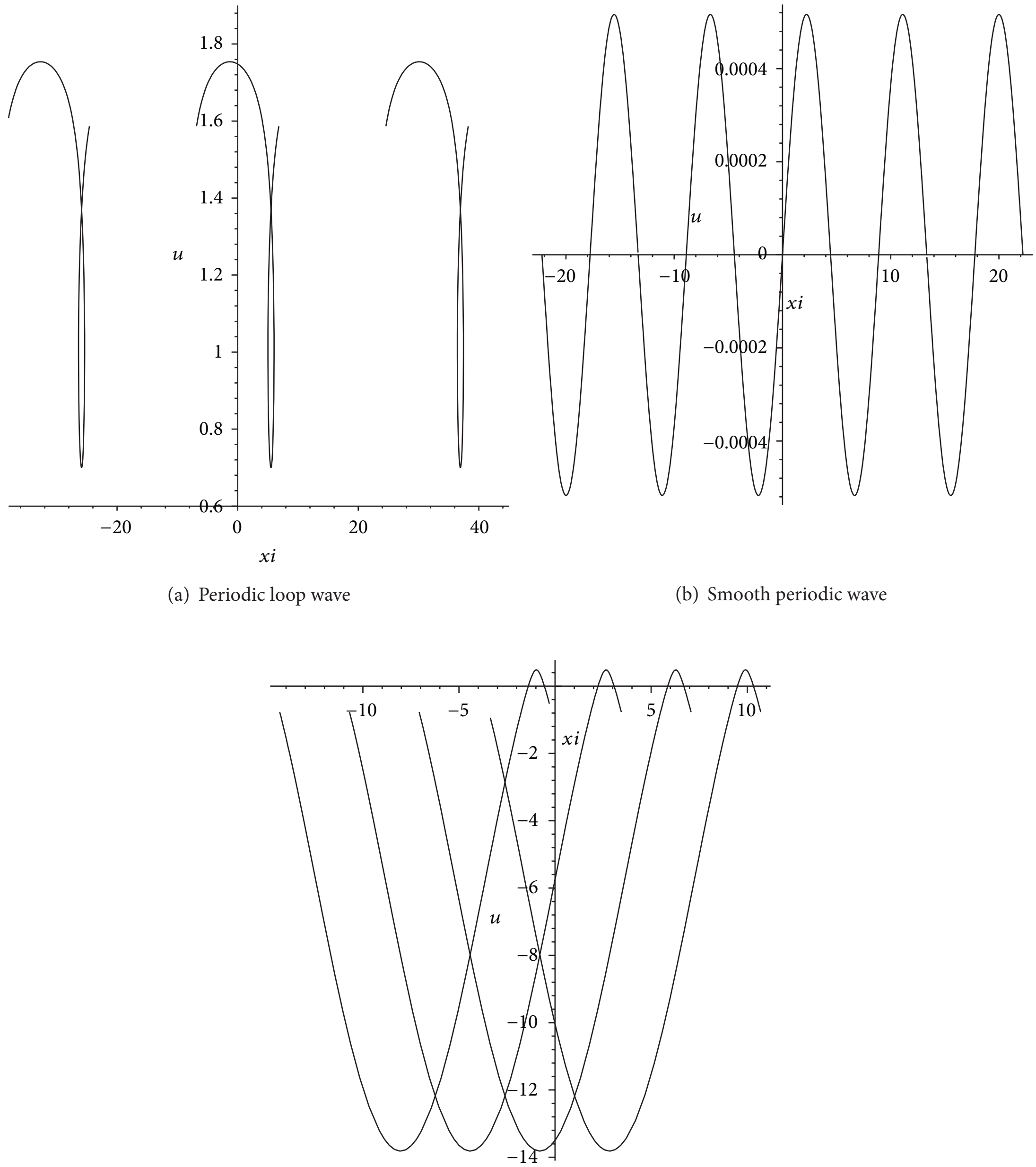
(viii) When $A > 0$, $\Delta < 0$, and $C > 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \frac{2A\operatorname{csch}\left(\sqrt{A}\tau\right)}{\epsilon\sqrt{-\Delta} - B\operatorname{csch}\left(\sqrt{A}\tau\right)},$$

$$\xi = (1-\lambda)\tau + \frac{2}{\sqrt{C}} \times \tanh^{-1}\left[\frac{B\tanh\left(\left(\sqrt{A}/2\right)\tau\right) + \epsilon\sqrt{-\Delta}}{2\sqrt{AC}}\right]. \quad (29)$$

(ix) When $A < 0$, $\Delta > 0$, and $C < 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \frac{2A\csc\left(\sqrt{-A}\tau\right)}{\epsilon\sqrt{\Delta} - B\csc\left(\sqrt{-A}\tau\right)},$$



(a) Periodic loop wave

(b) Smooth periodic wave

(c) Multiperiodic overlap wave

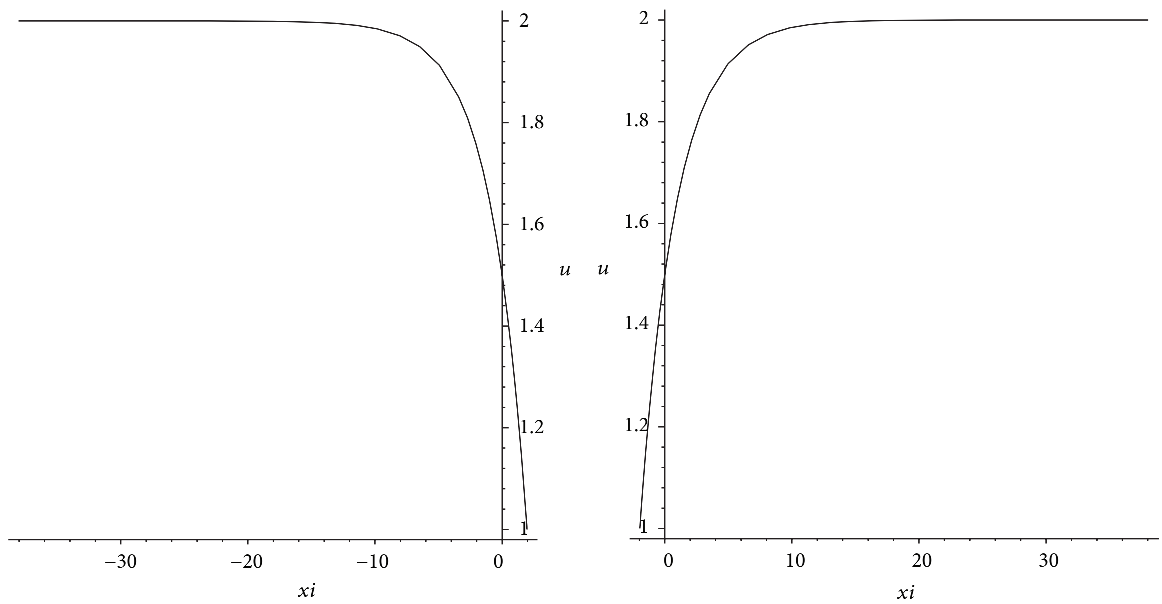
Figure 7: The profile of soliton solutions (30) for (a) $\alpha = -0.5,\ \lambda = 2$, and $c = -0.96$, (b) $\alpha = -0.5,\ \lambda = 2$, and $c = 0.4999999$, and (c) $\alpha = -0.5,\ \lambda = 2$, and $c = 2$.

$$\xi = (1-\lambda)\tau + \frac{2}{\sqrt{-C}} \times \arctan\left[\frac{-B\tan\left(\left(\sqrt{-A}/2\right)\tau\right) + \epsilon\sqrt{\Delta}}{2\sqrt{AC}}\right]. \quad (30)$$

The solution (30) contains three transformable waveforms as the parameter c varies; these three waveforms are periodic loop wave, smooth periodic wave, and multiperiodic overlap wave, which are shown in Figures 7(a), 7(b), and 7(c).

(x) When $A < 0,\ C > 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda - \frac{A\csc^2\left(\left(\sqrt{-A}/2\right)\tau\right)}{B + 2\epsilon\sqrt{-AC}\cot\left(\left(\sqrt{-A}/2\right)\tau\right)},$$
$$\xi = (1-\lambda)\tau - \frac{1}{\epsilon\sqrt{C}}\ln\left|B + 2\epsilon\sqrt{-AC}\cot\left(\frac{\sqrt{-A}}{2}\tau\right)\right|. \quad (31)$$



(a) Generalized kink wave (b) Generalized kink wave

FIGURE 8: The profiles of soliton solutions (32) for (a) $\alpha = 0.5$, $\lambda = 2$, and $c = 0.875$, and $\epsilon = -1$ and (b) $\alpha = 0.5$, $\lambda = 2$, $c = 0.875$, and $\epsilon = 1$.

(xi) When $A > 0$, $\Delta = 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda - \frac{A}{B}\left[1 + \epsilon \tanh\left(\frac{\sqrt{A}}{2}\tau\right)\right],$$
$$\xi = (1-\lambda)\tau + \frac{A}{B}\tau + \frac{2\epsilon\sqrt{A}}{B}\ \ln\left|\cosh\left(\frac{\sqrt{A}}{2}\tau\right)\right|. \quad (32)$$

The solution (32) contains two generalized kink waves when $\epsilon = \pm 1$, which are shown in Figures 8(a) and 8(b).

(xii) When $A > 0$, $B = 0$, and $C < 0$, by using (21) and (9), we obtain a soliton solution as follows:

$$u = \lambda + \sqrt{-\frac{A}{C}}\operatorname{sech}\left(\epsilon\sqrt{A}\tau\right),$$
$$\xi = (1-\lambda)\tau - \frac{1}{\epsilon\sqrt{-C}}\arctan\left[\sinh\left(\epsilon\sqrt{A}\tau\right)\right]. \quad (33)$$

If $A = 0$, $C < 0$

$$u = \lambda + \frac{4B}{(B\tau)^2 - 4C},$$
$$\xi = (1-\lambda)\tau - \frac{2}{\sqrt{-C}}\arctan\left(\frac{B\tau}{2\sqrt{-C}}\right). \quad (34)$$

4. Conclusion

In this work, by using the integral bifurcation method, we studied Kudryashov-Sinelshchikov equation in the special case $\beta = 2$. Some singular and non-singular traveling wave solutions, such as periodic compacton solutions, non-continuous periodic loop-wave solutions, peakon solutions broken wave solutions, periodic cusp-wave solutions, smooth soliton solutions, continuous periodic loop-wave solutions, and smooth periodic wave solutions are obtained. Some solutions such as (19), (22), (23), (25), (28), and (30) contain multiwave forms; their wave-forms vary accordingly as the parameter c varies. For example, the wave-forms of solution (19) show that the smooth periodic wave becomes the non-smooth periodic cusp wave as parameter c varies. The wave-forms of solution (22) show that the solitary wave becomes solitary cusp wave (peakon) when the value of c arrives at certain limit value; then solitary cusp wave becomes the loop wave when the value of c exceeds this limit value. Indeed, the non-smooth periodic cusp wave can be considered as a kind of long-wave limit of smooth periodic solutions; the loop wave also can be considered as limit wave of smooth solitary wave; these phenomena are very similar to the one in [23]. In [23], according to Braun and Randrüüt's explaination, solitary wave can be considered as the long-wave limit of periodic solutions in KdV case. By the way, these results which obtained in this paper are different from those in [8–12].

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