Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 769125, 12 pages
http://dx.doi.org/10.1155/2013/769125

Research Article

First-Order ARMA Type Fuzzy Time Series Method Based on Fuzzy Logic Relation Tables

Cem Kocak

School of Health, Hitit University, 19000 Corum, Turkey

Correspondence should be addressed to Cem Kocak; cemkocak@hotmail.com

Received 2 July 2013; Revised 1 August 2013; Accepted 3 August 2013

Academic Editor: Ming Li

Fuzzy time series approaches have an important deficiency according to classical time series approaches. This deficiency comes from the fact that all of the fuzzy time series models developed in the literature use autoregressive (AR) variables, without any studies that also make use of moving averages (MAs) variables with the exception of only one study (Egrioglu et al. (2013)). In order to eliminate this deficiency, it is necessary to have many of daily life time series be expressed with Autoregressive Moving Averages (ARMAs) models that are based not only on the lagged values of the time series (AR variables) but also on the lagged values of the error series (MA variables). To that end, a new first-order fuzzy ARMA(1,1) time series forecasting method solution algorithm based on fuzzy logic group relation tables has been developed. The new method proposed has been compared against some methods in the literature by applying them on Istanbul Stock Exchange national 100 index (IMKB) and Gold Prices time series in regards to forecasting performance.

1. Introduction

Fuzzy time series approaches not necessitating many of the limitations seen in classical time series approaches such as linearity, stationarity, and number of observations have increased the interest towards these approaches. Fuzzy time series concept first mentioned in the literature by Song and Chissom [1] was based on the fuzzy set theory of Zadeh [2]. Song and Chissom [3, 4] divided fuzzy time series into two groups, namely, time variant and time invariant. A vast majority of the studies in the literature are methods proposed for solving time invariant fuzzy time series. Because models are significantly effective on the forecasting performances during the determination of fuzzy relations stage, different approaches have been proposed in the literature. In the studies of Song and Chissom [1, 3, 4], relations are determined with complex matrix operations. In order to eliminate this complexity, a new first-order fuzzy time series model has been proposed where fuzzy logic group relation tables are used in Chen's [5] study with simplified operations not necessitating complex matrix processes. This approach of Chen [5] is used in many studies due to its positive effect on forecasting performance. Therefore, Chen [6] developed a new approach by using the fuzzy logic relation tables also in high-order fuzzy time series models. Because the methods proposed in the studies of Chen [5, 6] necessitate obtainment of many fuzzy logic group relation tables, they require numerous operations. Thus, studies where fuzzy relations are determined with artificial neural networks are commonly seen. Some Studies where artificial neural networks are used for determining fuzzy relations may be listed as the studies of Huarng and Yu [7], Aladag et al. [8], Yu and Huarng [9], and Yolcu et al. [10].

In majority of the studies in the literature, interval lengths are specified intuitively. In his study, Huarng [11] has proposed two separate approaches based on average and distribution to specify optimal interval length. The optimal interval lengths determined by the approach of Huarng [11] may be obtained to be very large values. Thus, Egrioglu et al. [12] proposed approaches based on the optimisation of interval length. Different from these studies, Huarng and Yu [13] proposed an approach based on ratio with interval length varying exponentially instead of determining a fixed interval length in the solution of first-order fuzzy time series.

Determining the ratio in the study of Huarng and Yu [13] requires many complex calculations. Therefore, Yolcu et al. [14] proposed a new approach that improves the approach of Huarng and Yu [13], based on the optimisation of ratio.

There are 3 most commonly used models in the analysis of single variable time series in classical time series approach. These are autoregressive (AR), moving averages (MAs), and mixed autoregressive moving averages (ARMAs) models. However, fuzzy time series methods developed in the literature focus on the AR model of the classical time series theory, without any study conducted on the utilisation of MA and ARMA models except Uslu et al. [15], Alpaslan et al. [16], and Aladag et al. [17]. These studies [15–17] on the other hand are methods that have been proposed for the solution of seasonal time series. No study has been made on the inclusion of error variable to the model for nonseasonal fuzzy time series with the exception of Egrioglu et al. [18] study. This study [18] is the first study developed as first order fuzzy ARMA type model based on particle swarm optimization for the solution of nonseasonal time series in the literature. All of the models in the literature have mentioned issues such as the use of universe of discourse partitioning, membership order, model order, and artificial intelligence approaches. However, the fuzzy time series models proposed in the literature only including AR variables which may lead to a model specification error. In many modelling of the real life time series, MA variables are also required. In this sense, use of only AR variables for the solution of fuzzy time series requiring also MA variables for modelling becomes insufficient regarding the forecasting performance.

For the purpose of eliminating the adverse effects mentioned, a solution algorithm for a new first-order fuzzy ARMA(1,1) time series forecast model where fuzzy relations are determined based on fuzzy logic group relations has been proposed in this study. The logic for determining fuzzy relations in the proposed method is an approach similar to that of the study of Chen [5], aiming to show that the forecasting performance can be significantly improved when the model specification error in the method of Chen [5], accepted as a fundamental approach in literature, is eliminated. For many real life time series, performance can be increased through the use of high-order fuzzy time series models due to the ability to realise solution with more information. However, because the proposed fuzzy ARMA(1,1) model uses a second variable (error variable), it utilises more information to the extent it eliminates the model error, and although just a first order model, it occurs to have a better forecasting performance than that of high order fuzzy time series methods.

In this paper, the time series which were used in application are examined according to long-range dependence. Time series can be classified short range or long-range dependence time series. Many time series have long-range dependence. The long-range dependence time series are forecasted different methods like autoregressive fractionally integrated moving average (ARFIMA). ARFIMA models have fractionally differencing parameter. First studies were concerned with estimation of fractional differencing parameter in fractional white noise processes. R/S statistic was proposed in Hurst [19]. The other important studies about fractional differenced processes are Li and Zhao [20, 21], Li [22], Stanley et al. [23], Werner [24], Beran [25], Ivanov et al. [26], Podobnik et al. [27], Zevallos and Palma [28], and Bhansali and Kokoszka [29].

In the second section of the study, basic definitions regarding fuzzy time series have been provided. In the third section, the proposed fuzzy time series forecasting model has been defined and the solution algorithm has been provided. In the fourth section, the proposed method has been applied to Istanbul Stock Exchange (IMKB) national 100 index time series and gold prices in 2009 taken from Turkish Republic Central Bank (TCMB) website, comparing it to some other methods in the literature regarding forecasting performance. An in-depth comparison has been made in this section. In the fifth section, the study has been summed up by discussing the results obtained.

2. Definition of Fuzzy Time Series

Fuzzy time series concepts and definitions have been developed in accordance to the lagged variables of times series (AR, autoregressive) in all studies conducted in the literature. Main time series definitions developed using AR variables are listed below.

Definition 1. Let $X(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \ldots$ then $F(t)$ is called a fuzzy time series defined on $X(t)$ [1, 3, 4].

Definition 2. Let us consider the fuzzy relation between $R(t, t-1)$, $F(t-1)$, and $F(t)$. For any t value, if $R(t, t-1)$ is independent from t, then $R(t, t-1) = R(t-1, t-2)$. In this case, $F(t)$ is called the time invariant fuzzy time series, while otherwise called as time variant fuzzy time series [1].

Definition 3. If the $F(t)$ fuzzy time series is only affected by one lagged $F(t-1)$ fuzzy time series, then the fuzzy relation between $F(t-1)$ and $F(t)$ is expressed as

$$F(t-1) \longrightarrow F(t). \tag{1}$$

This is called as a first-order fuzzy time series forecasting model. Then this relation can be expressed as

$$F(t) = F(t-1) \circ R(t, t-1). \tag{2}$$

The "$\circ$" operator in (2) had been determined as the max-min operator by Song and Chissom [1, 3, 4].

Definition 4. If $F(t)$ fuzzy time series is affected by the lagged fuzzy time series of $F(t-1), F(t-2), \ldots, F(t-p)$, then the fuzzy relation between $F(t)$ fuzzy time series and $F(t-1), F(t-2), \ldots, F(t-p)$ fuzzy time series may be expressed as

$$F(t-p), \ldots, F(t-2), F(t-1) \longrightarrow F(t) \tag{3}$$

and is called the pth order fuzzy time series forecasting model [6].

3. The Proposed Method

Based on the definition of fuzzy AR(1) given in Definition 3, the definition of the main fuzzy time series to represent the fuzzy ARMA(1,1) model is expressed as follows.

Definition 5. Let $F(t)$ be a fuzzy time series and let $\varepsilon(t)$ be the fuzzy error series obtained from $F(t)$ fuzzy time series. If $F(t)$ is affected by one lagged $F(t-1)$ and one lagged $\varepsilon(t-1)$ fuzzy time series, then the relationship can be expressed as

$$F(t-1), \varepsilon(t-1) \longrightarrow F(t). \tag{4}$$

This is called as first-order fuzzy autoregressive moving averages (ARMA(1,1)) time series forecasting model [18].

In this study, an algorithm has been proposed for solving the ARMA(1,1) fuzzy time series forecasting model defined in (4). In the algorithm proposed, initially the AR(1) fuzzy time series model defined in (1) is estimated. Later on, errors are calculated by taking the differences between the observed values of the times series and the forecasts obtained through the solution of fuzzy AR(1). By using these errors, the fuzzy ARMA(1,1) model defined in (4) is estimated. The algorithm of the proposed approach is given below.

Algorithm 6. The proposed method's algorithm.

Step 1 (the universe of discourse (U) and subintervals (u_i, $i = 1, 2, \dots b$) are defined). The beginning and the ending points of the universe of discourse for time series are determined. Then U is divided into subintervals according to appropriate interval length. Definition of interval length is up to the researcher. It should not be forgotten that the interval length to be determined affects the number of subintervals. If the smallest value of the time series is taken as $X_{\min}$, the largest value as $X_{\max}$, and two arbitrary values as D_1 and D_2, the universal set may be defined as the closed interval of

$$U = [X_{\min} - D_1, X_{\max} + D_2]. \tag{5}$$

u_i subintervals determined for $i = 1, 2, \dots b$ are the subintervals of the universal set U, which is defined as

$$U = \{u_1, u_2, \dots, u_b\}. \tag{6}$$

For example, for $X_{\min} = 43$ and $X_{\max} = 96$ when U is selected as [40, 100] and interval length is selected as 10, subintervals are specified as $u_1 = [40, 50]$, $u_2 = [50, 60]$, $u_3 = [60, 70]$, $u_4 = [70, 80]$, $u_5 = [80, 90]$, and $u_6 = [90, 100]$.

Step 2. For the time series, fuzzy sets are defined according to the universal set (U) and the divisions of (u_i). These fuzzy sets are expressed as

$$A_i = \frac{f_{A_i}(u_1)}{u_1} + \frac{f_{A_i}(u_2)}{u_2} + \cdots + \frac{f_{A_i}(u_b)}{u_b}, \quad \text{for } i = 1, 2, \dots, b. \tag{7}$$

For $i = 1, 2, \dots b$,

$$f_{A_i}(u_i) = \begin{cases} 1, & k = i \\ 0.5, & k = i-1,\ i+1 \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

For example, according to (7) and (8), the fuzzy set of A_3 can be expressed as $0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6$.

Step 3 (observations are fuzzified). Subintervals (u_i) where each observation occurs are defined. Then the fuzzy set A_i where the defined sub-interval has the highest membership value is determined. The fuzzy value of the observation is this A_i fuzzy set defined.

Step 4. For the purpose of determining fuzzy relations, fuzzy logic relations are identified and a fuzzy logic group relation table is formed.

For example, where the fuzzy logic relations are as $A_2 \to A_3$, $A_2 \to A_3$, $A_2 \to A_5$, according to Chen method [5], the fuzzy logic group relation for A_2 fuzzy value is as $A_2 \to A_3, A_5$. In the proposed method, this relation occurs to be as $A_2 \to A_3, A_3, A_5$. Thus, a little improvement has been realised in the fuzzy AR(1) model of Chen [5] with the proposed method.

Step 5 (fuzzy forecasts are obtained). As $F(t-1) = A_i$ and $F(t) = A_j$, 3 possible situations regarding forecast obtainment are as follows.

Situation 1. If a relation of $A_i \to A_j, \dots, A_j$ is valid on the fuzzy group relation table where A_i affects only a pieces of A_j, then the fuzzy forecast is A_j. For example, if the group relation for A_1 is as $A_1 \to A_2$ that this relation is repeated a few times in the time series, then the fuzzy forecast is specified as A_2.

Situation 2. If a relation of $A_i \to A_j, \dots, A_j, A_k, \dots, A_k, A_l, \dots, A_l$ is valid on the fuzzy group relation table where A_i affects a pieces of A_j, $b$pieces A_k and c pieces A_l, then the fuzzy forecast is $A_j, \dots, A_j, A_k, \dots, A_k, A_l, \dots, A_l$, comprising of $a+b+c$ pieces of fuzzy values. For example; if the group relation for A_2 is as $A_2 \to A_2, A_3, A_3, A_5, A_5, A_5$, then the fuzzy forecast is specified as $A_2, A_3, A_3, A_5, A_5, A_5$.

Situation 3. If $A_i \to$ *empty* on the fuzzy group relation table, the fuzzy forecast occurs to be A_i. For example, if the group relation for A_3 is $A_3 \to$ *empty*, then the fuzzy forecast is A_3.

Step 6 (defuzzification process is executed). In this step, centralisation method is used. When the fuzzy forecast for Situation 1 and Situation 3 defined in Step 5 is A_j, then the defuzzy forecast should be the middle point of the u_j subinterval that has the highest membership value within the fuzzy set A_j. For Situation 2, the defuzzy forecast is calculated with the weighted average formula below, by using the middle points ($m_j, m_k, \dots, m_l$) of the $u_j, u_k, \dots, u_l$ intervals that have

Table 1: An example to the fuzzy AR(1) solution of the proposed method.

Years	Data	Fuzzy value	Fuzzy forecast	Defuzzy forecast
2001	43	A_1	—	—
2002	47	A_1	A_1, A_1, A_1, A_2	47.5
2003	45	A_1	A_1, A_1, A_1, A_2	47.5
2004	46	A_1	A_1, A_1, A_1, A_2	47.5
2005	53	A_2	A_1, A_1, A_1, A_2	47.5
2006	65	A_3	A_3, A_3	65
2007	57	A_2	A_2, A_2	55
2008	62	A_3	A_3, A_3	65
2009	51	A_2	A_2, A_2	55

the highest membership value of each of the $A_j, A_k, \ldots, A_l$ fuzzy sets. Consider

$$\hat{x}(t) = \frac{a \times m_j + b \times m_k + c \times m_l}{a + b + c}. \quad (9)$$

On an example time series, when solutions are made according to the proposed method from Step 1 to Step 6 as per the determined subintervals of $u_1 = [40, 50]$, $u_2 = [50, 60]$, and $u_3 = [60, 70]$, the solutions displayed in Table 1 are obtained.

Step 7. Errors are calculated by taking the differences between the observed time series values and the defuzzified forecast values obtained in Step 6. Real values of the time series are $x(t)$ and the defuzzified forecast values obtained in Step 6 are $\hat{x}(t)$; the error series $e(t)$ is calculated as follows:

$$e(t) = x(t) - \hat{x}(t). \quad (10)$$

Step 8. For the errors, the universe of discourse set is defined as (V) and subintervals are defined as $(v_i,\ i = 1, 2, \ldots b)$. The same partition of the universe of discourse processes done in Step 1 is made for the error series.

Step 9. Fuzzy sets based on the universal set (V) and partitions (v_i) are defined for the errors. The fuzzy sets are expressed as

$$B_j = \frac{f_{B_j}(v_1)}{v_1} + \frac{f_{B_j}(v_2)}{v_2} + \cdots + \frac{f_{B_j}(v_c)}{v_c}, \quad \text{for } j = 1, 2, \ldots, c. \quad (11)$$

For $j = 1, 2, \ldots c$,

$$f_{B_j}(v_j) = \begin{cases} 1, & k = j \\ 0.5, & k = j-1,\ j+1 \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

For example, according to (11) and (12), the fuzzy set of B_3 can be specified as $0/v_1 + 0.5/v_2 + 1/v_3 + 0.5/v_4 + 0.5/v_5$.

Step 10 (error series $e(t)$ is fuzzified). Subintervals (v_j) for each observation are determined. Then the fuzzy set B_j where the determined sub-interval has the highest membership value is defined. The fuzzy value of the observation is this fuzzy set B_j.

Step 11 (fuzzy relations are determined and fuzzy logic group relation table is formed). The fuzzy value A_i of the fuzzy relations time series and the fuzzy value B_j of the error series are determined by taking the fuzzy values into consideration together. For the one lagged fuzzy value of the tth observation being $F(t-1) = A_i$, one lagged error value being $\varepsilon(t-1) = B_j$, and the fuzzy value being $F(t) = A_k$, the fuzzy relation given in (4) occurs to be $(A_i, B_j) \rightarrow A_k$. Thus, the fuzzy values are formed of (A_i, B_j) ordered pairs, and a relation in the manner of one lagged time series and error affecting the time series is mentionable. For example, when the fuzzy logic relations are as $(A_2, B_3) \rightarrow A_2$, $(A_2, B_3) \rightarrow A_2$, $(A_2, B_3) \rightarrow A_4$, the fuzzy logic group relation for (A_2, B_3) fuzzy value occurs to be $(A_2, B_3) \rightarrow A_2, A_2, A_4$.

Step 12 (fuzzy forecasts are obtained). As $F(t-1) = A_i$ ve $\varepsilon(t-1) = B_j$ and $F(t) = A_k$, 3 possible situations regarding forecast obtainment are as follows.

Situation 1. If a relation of $(A_i, B_j) \rightarrow A_k, \ldots, A_k$ is valid on the fuzzy group relation table where (A_i, B_j) affects only a pieces of A_k, then the fuzzy forecast is A_k. For example, if the group relation for (A_1, B_2) is as $(A_1, B_2) \rightarrow A_2$ that this relation is repeated within a few times in the time series, then the fuzzy forecast is determined as A_2.

Situation 2. If a relation of $(A_i, B_j) \rightarrow A_k, \ldots, A_k, A_l, \ldots, A_l, A_m, \ldots, A_m$ is valid on the fuzzy group relation table where (A_i, B_j) affects a pieces of A_k, b pieces A_l and c pieces A_m, then the fuzzy forecast is $A_k, \ldots, A_k, A_l, \ldots, A_l, A_m, \ldots, A_m$, comprising of $a + b + c$ pieces of fuzzy values. For example, if the group relation for (A_1, B_2) is as $(A_1, B_2) \rightarrow A_2, A_3, A_3, A_5, A_5, A_5$, then the fuzzy forecast is determined as $A_2, A_3, A_3, A_5, A_5, A_5$.

Situation 3. If $(A_i, B_j) \rightarrow$ *empty* on the fuzzy group relation table, the fuzzy forecast occurs to be A_i. For example, if the group relation for (A_1, B_2) is $(A_1, B_2) \rightarrow$ *empty*, then the fuzzy forecast is A_1.

Step 13 (defuzzification process is made). In this step, centralisation method is used. When the fuzzy forecast for Situation 1 and Situation 3 defined in Step 5 is A_j, then the defuzzy forecast should be the middle point of the u_j sub-interval that has the highest membership value within the fuzzy set

Table 2: An application of the proposed method on an example time series.

Years	Data	Error	Fuzzy value	Fuzzy forecas	Defuzzy forecast
2001	43	0	(A_1, B_2)	—	—
2002	47	−0.5	(A_1, B_1)	A_1	45
2003	45	−2.5	(A_1, B_1)	A_1, A_1, A_2	48.33
2004	46	−1.5	(A_1, B_1)	A_1, A_1, A_2	48.33
2005	53	5.5	(A_2, B_3)	A_1, A_1, A_2	48.33
2006	65	0	(A_3, B_2)	A_3	65
2007	57	2	(A_2, B_2)	A_2	55
2008	62	−3	(A_3, B_1)	A_3	65
2009	51	−4	(A_2, B_1)	A_2	55

A_j. For Situation 2, the defuzzy forecast is calculated with the weighted average formula given in (9).

On an example time series, when solutions are made according to the proposed method as per the determined time series subintervals of $u_1 = [40, 50]$, $u_2 = [50, 60]$, and $u_3 = [60, 70]$ and error series subintervals of $v_1 = [-5, 0]$, $v_2 = [0, 5]$, and $v_3 = [5, 10]$, solutions displayed in Table 2 are obtained.

4. Application

The performance indicators of the root mean square error (RMSE), mean average percentage error (MAPE), and direction accuracy (DA) values used for comparison of the results obtained are as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} \left(Y_t - \widehat{Y}_t\right)^2}{n}},$$
$$\text{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{Y_t - \widehat{Y}_t}{Y_t} \right|, \quad (13)$$
$$\text{DA} = \frac{1}{n-1} \sum_{t=1}^{n-1} \begin{cases} 1, & \left(Y_{t+1} - Y_t\right)\left(\widehat{Y}_{t+1} - Y_t\right) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Data is divided into two within the applications, assigning the first part as training set and the second part as test set obtained through taking the last observations into consideration of which number was predetermined. By looking up the fuzzy relation table obtained for the training set as per the Steps 5 and 12 of the proposed method, the fuzzy forecast of the test set and from there the defuzzy forecasts of the test set are calculated by utilizing Steps 6 and 13. After, the RMSE, MAPE, and DA values for the test set are calculated. Therefore, the future performances of the methods are determined. As the forecasts with the lowest RMSE value calculated for the test set provide the best result of the used method, the future performances of the forecasts are obtained along with the aid of MAPE and DA values.

Solution of vast majority of some fuzzy forecasting methods in the literature is realised according to the specified number of fuzzy sets, and some are realised according to interval lengths. For the purpose of maintaining consistency during the comparison of forecast performances, the interval length to be used at fuzzification stage is determined as to have the number of fuzzy sets 5 as the lowest and 35 as the highest for each application data and for all methods. Therefore, in case the universal set division is realised in accordance to interval length, the interval lengths to be tried have been specified by calculation with the formula below:

$$\text{Interval Length} = \frac{\max(\text{data}) - \min(\text{data})}{\text{Number of Fuzzy Sets}}. \quad (14)$$

The operations below have been realised when making solution via the proposed method.

(i) The last k number of data has been specified as test set, aiming to increase the future performances.

(ii) During the fuzzification of time series stage of the proposed method, different time series interval lengths for the division of universal set U have been tried. Data have been solved from Step 1 to Step 6 according to these intervals lengths. And so, a lot of forecasts have been obtained. The test set forecast with the smallest RMSE value among these forecasts has been determined as the best result of the fuzzy AR(1) model.

(iii) The error value of the first observation of data has been assumed as 0, while the error values of other observations have been calculated with the formula (9) by using the training set and test set forecasts obtained through the best result of the fuzzy AR(1) model. Thus the error series has been obtained.

(iv) Different time series interval lengths and different error series interval lengths have been tried by solving data from Step 1 to Step 13. Among these trials, the test set forecast with the smallest RMSE value has been determined as the best result of the fuzzy ARMA(1,1) model.

For the purpose of comparing the proposed method with the other fuzzy time series methods in the literature, 2 different data sets comprising of less observations (smaller sample size) and more observations (larger sample size) have been used. One of these data sets is the IMKB time series seen in Figure 1 comprising of 53 observations between

Figure 1: Graph of IMKB time series.

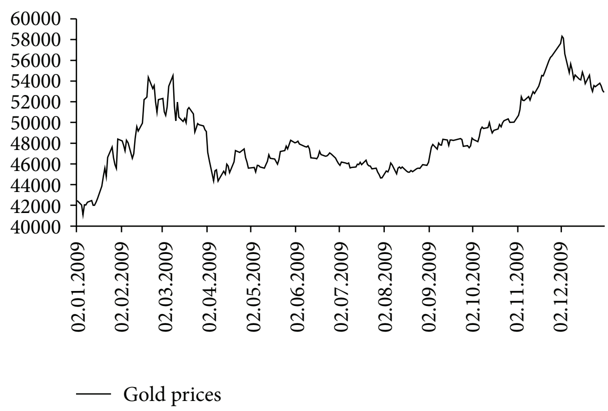

Figure 2: Graph of gold prices.

the dates of 01.10.2010 and 23.12.2010. The second data set is the gold prices time series seen in Figure 2 comprising of 248 observations received from Turkish Republic Central Bank (TCMB) website between the dates of 02.01.2009 and 31.12.2009.

In the solution of the IMKB data given in Figure 1 via the methods in the literature by taking the last 7 observations and the last 15 observations as test set, one has the following.

(i) During the fuzzification stage of the Song and Chissom [1] first-order fuzzy time series method, among the 31 different results obtained from increasing the fuzzy set number between 5 and 35, the test set forecast that has the minimum RMSE value has been determined as the best result of the method from this results.

(ii) For each of the 2nd-, 3rd-, 4th-, and 5th-order models of Chen [5] first-order fuzzy time series forecast method, Chen [6] high order fuzzy time series forecast method, and Aladag et al. [8] high order fuzzy time series forecast method, the RMSE values have been found for different intervals lengths being increased 100 units between 300 and 2300. The test set forecasts that have the minimum RMSE values among these 21 trials have been determined as the best results of the methods.

(iii) By using the optimal interval lengths calculated with the distribution-based approach of Huarng [11] and the average-based approaches, solution has been executed as per the first-order fuzzy time series forecast method of Chen [5]. Thus, the best results of the test set from the distribution based approach and the average-based approaches have been obtained via a single trial.

(iv) In the application of the ratio-based approach of Huarng and Yu [13], alpha parameter has been taken as 0.50, obtaining the best result for the test set of the method in one trial.

In the solution of the IMKB time series seen in Figure 1 via the proposed method by taking the number of test sets 7 and 15, one has the following.

(i) The division of universal set U have been taken different values as increasing the interval length between 300 and 2300 by 100 units and different forecasts have been obtained by solving from Step 1 to Step 6. The test set forecast with the smallest RMSE value among these forecasts has been determined as the best result of the fuzzy AR(1) model. The best fuzzy AR(1) results have been obtained when the interval length is 300 for 7 as the number of test sets and when the interval length is 900 for 15 as the number of test sets.

(ii) The error value of the first observation of data has been assumed as 0, while the error values of other observations have been calculated via the formula (9) by utilising data and forecasts obtained through the best result of the fuzzy AR(1) model. Thus the error series has been obtained for 53 observations.

(iii) Different trials have been made by increasing the interval length between 300 and 2300 by 100 units for the time series and by increasing the interval length between 300 and 2100 by 100 units for the error series by solving from Step 1 to Step 13. Among these trials, the test set forecast with the smallest RMSE value has been determined as the best result of the fuzzy ARMA(1,1) model.

During the application of the proposed method and the methods in the literature on IMKB time series, the parameters with which the forecasts with the best test set performance for 7 and 15 numbers of test sets occurred to be

(i) for the application of Song and Chissom [1] method, when the number of fuzzy sets is 9 for 7 and 20 for 15,

(ii) for the application of Chen [5] method, when the interval length is 300 for 7 and 900 for 15,

(iii) for the application of distribution-based Huarng [11] approach, when the interval length is 1000, and for the application of average based approach, when the interval length is 200,

(iv) for the application of ratio-based Huarng and Yu [13] approach, when the sample percentile alpha = 0.5,

Table 3: The best results obtained for 7-observation test set of IMKB data.

Date	Test set	Song and Chissom [1]	Chen [5]	Chen [6]	Huarng [11] distribution-based method	Huarng [11] average-based method	Huarng and Yu [13] rational-based method	Aladag et al. [8]	Proposed method*
15.12.2010	65499	65356	66550	65900.0	67400	66500	66784.7	67300	66266.7
16.12.2010	64429	65356	66250	65900.0	65400	66300	66178.0	64900	63700.0
17.12.2010	63524	65975	64450	64800.0	65900	64500	65878.7	64900	63700.0
20.12.2010	63502	64737	63550	64066.7	64900	63500	63514.3	64300	63700.0
21.12.2010	64820	64737	63550	63700.0	64900	63500	63514.3	63700	65900.0
22.12.2010	65440	65975	65800	64800.0	65900	65500	65878.7	65500	65900.0
23.12.2010	66219	65356	66250	65900.0	65400	66300	66178.0	66700	66266.7
	RMSE	1161.91	1001.70	928.70	1365.14	1014.73	1317.77	1034.06	606.07
	MAPE	0.01387	0.01217	0.01283	0.01773	0.01175	0.01593	0.01350	0.00762
	DA	0.50000	0.50000	0.33333	0.50000	0.50000	0.66667	0.66667	0.83333

*The best situation.

(v) for the application of Chen [6] method, when the interval length is 2200 in 3rd-order model for 7 and 1400 in 2nd-order model for 15,

(vi) For the application of Aladag et al. [8] method, when the interval length is 600 on 2nd-degree model and unit number of artificial neural network hidden layers is 5 for 7, and when the interval length is 1500 on 2nd-degree model and unit number of artificial neural network hidden layers is 6 for 15,

(vii) For the application of the proposed fuzzy ARMA(1,1) method, when the interval length of time series is 2200 and the interval length of error series is 1400 for 7, and when the interval length of time series is 1400 and the interval length of error series is 400 for 15.

Best forecasts and forecast performances of all methods in result of IMKB time series solution for 7 observation test set are summarised in Table 3.

When Table 3 is analyzed, it is seen in result of the solution of IMKB time series for 7 observation test set that the proposed method produced the best forecasting performance with a minimum RMSE value of 606.07, minimum MAPE value of 0.762%, and maximum direction accuracy of 83.33%. The graphs of the last 7 observations of IMKB time series along with the 7-observation test set forecasts obtained with the proposed method are shown together in Figure 3.

Best forecasts and forecast performances of all methods in result of IMKB time series solution for 15-observation test set are summarised in Table 4.

When Table 4 is analyzed, it is seen that the proposed method produced the best forecasting performance with a minimum RMSE value of 865.28, minimum MAPE value of 1.029%, and maximum direction accuracy of 71.43% in result of the solution of IMKB time series for 15-observation test set. The graphs of the last 15 observations of IMKB time series along with the 15-observation test set forecasts obtained with the proposed method are shown together in Figure 4.

In result of the solutions of IMKB time series, it has been observed that the proposed method significantly increased

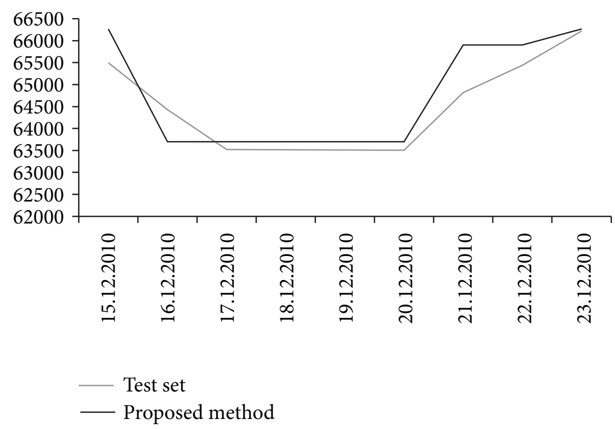

Figure 3: The graphs of the 7-observation test set of IMKB data and the forecasts of the test set obtained with the proposed method.

the future forecasting performance compared to other methods. Also in the graphs within Figures 3 and 4, the results of the proposed method are seen to be considerably similar to the test set values.

In the solution of the gold prices data given in Figure 2 via the methods in the literature by taking the last 30 observations and the last 45 observations as test set, one has the following.

(i) Gold prices solution of Song and Chissom [1], Huarng [11], and Huarng and Yu [13] methods has been conducted just as previously done on the abovementioned IMKB time series.

(ii) For each of the 2nd-, 3rd-, 4th-, and 5th-order models of Chen [5] first-order fuzzy time series forecast method, Chen [6] high-order fuzzy time series forecast method, and Aladag et al. [8] high order fuzzy time series forecast method, the RMSE values have been found for different lengths being increased 100 units between 500 and 3500. The test set forecasts

Table 4: The best results obtained for 15-observation test set of IMKB data.

Date	Test set	Song and Chissom [1]	Chen [5]	Chen [6]	Huarng [11] distribution-based method	Huarng [11] average-based method	Huarng and Yu [13] rational-based method	Aladag et al. [8]	Proposed method*
01.12.2010	66156	65695.5	66250	64833.3	65900.0	65300	66328.8	65650	66000
02.12.2010	66939	66438.2	65800	66700.0	65566.7	64100	66164.2	67150	67633
03.12.2010	66860	67088.0	67450	66700.0	67233.3	66700	67143.8	67150	66700
08.12.2010	67705	67088.0	67450	66700.0	67233.3	66700	67143.8	67150	66700
09.12.2010	65914	66252.5	66250	67633.3	65900.0	67700	66328.8	67150	68100
10.12.2010	64759	65695.5	65800	66233.3	65566.7	65900	65791.4	64150	65300
13.12.2010	66380	65695.5	65350	66700.0	65900.0	64700	65258.3	65650	65300
14.12.2010	66510	66438.2	65800	66700.0	65566.7	67100	66164.2	65650	66700
15.12.2010	65499	66438.2	65800	66700.0	67233.3	66500	66164.2	67150	66700
16.12.2010	64429	65695.5	66250	65766.7	65566.7	66300	66328.8	64150	65300
17.12.2010	63524	65695.5	65350	64366.7	65900.0	64500	65258.3	65650	63900
20.12.2010	63502	65138.5	63550	63900.0	64900.0	63500	63684.8	65650	63900
21.12.2010	64820	65138.5	63550	63900.0	64900.0	63500	63684.8	65650	63900
22.12.2010	65440	65695.5	65350	64833.3	65900.0	65500	65258.3	65650	65300
23.12.2010	66219	65695.5	66250	66700.0	65566.7	66300	66328.8	65650	66000
	RMSE	919.47	925.42	954.17	1052.90	1283.18	896.96	1060.12	865.28
	MAPE	0.01124	0.01081	0.01245	0.01285	0.01558	0.01085	0.01312	0.01029
	DA	0.71429	0.57143	0.64286	0.50000	0.57143	0.64286	0.64286	0.71429

*The best result.

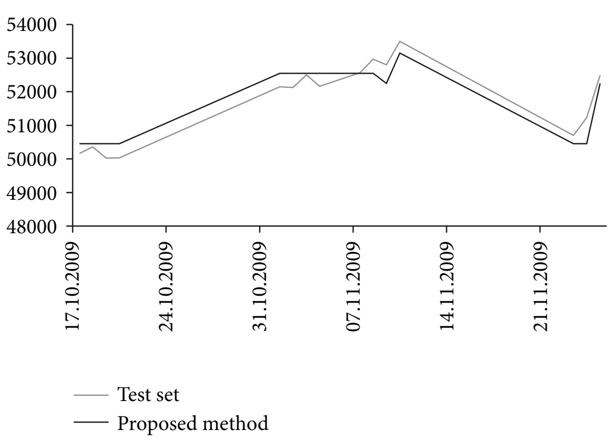

Figure 4: The graphs of the 15-observation test set of IMKB data and forecasts of the test set obtained with the proposed method.

that have the minimum RMSE values among these 31 trials have been determined as the best results of the methods.

In the solution of the gold prices data given in Figure 2 via the proposed method by taking the last 30 observations and the last 45 observations as test set, one has the following.

(i) During the fuzzification of time series of the proposed method, interval lengths of U have been tried as increasing the interval length between 500 and 3500 by 100 units by solving from Steps 1 and 6. The test set forecast with the smallest RMSE value among these forecasts has been determined as the best result of the fuzzy AR(1) model.

(ii) The error series have been obtained via the formula (9) for 248 observations with the same calculation made previously in the application of IMKB data.

(iii) Different trials have been made by increasing the interval length between 500 and 3500 by 100 units for the time series and by increasing the interval length between 100 and 1100 by 50 units for the error series by solving from Step 1 to Step 13. Among these trials, the test set forecast with the smallest RMSE value has been determined as the best result of the fuzzy ARMA(1,1) model.

During the application of the proposed method and the methods in the literature on gold prices time series, the parameters with which the forecasts with the best test set performance for 30 and 45 test sets occurred to be the following:

(i) for the application of Song and Chissom [1] method, when the number of fuzzy sets is 10 for both 30 and 45 test sets,

(ii) for the application of Chen [5] method, when the interval length is 600 for 30 and 1700 for 45,

(iii) for the application of distribution-based Huarng approach [11], when the interval length is 400, and for the application of average based approach, when the interval length is 200,

Table 5: The best results obtained for 30-observation test set of gold prices.

Date	Test set	Song and Chissom [1]	Chen [5]	Chen [6]	Huarng [11] distribution-based method	Huarng [11] average-based method	Huarng and Yu [13] rational-based method	Aladag et al. [8]	Proposed method*
17.11.2009	53935	53157	53300	53000.0	53200	53200	53356.4	53300	53400
18.11.2009	54550	53157	53900	53666.7	54000	53900	53894.0	53900	54200
19.11.2009	54495	53157	52400	54066.7	52400	51500	51715.7	54500	54200
20.11.2009	54830	53157	52400	54200.0	52400	51500	51715.7	53900	54200
23.11.2009	55950	53157	55100	54600.0	54800	54900	55017.3	54500	55000
24.11.2009	56285	52294	55700	55266.7	56000	55900	55779.2	55100	55800
25.11.2009	56430	52294	56300	56066.7	56400	56300	56164.1	55700	56600
01.12.2009	57635	52294	56300	56466.7	56400	56500	56551.6	55100	56600
02.12.2009	58330	53157	57500	57000.0	57600	57700	57730.3	55700	57400
03.12.2009	58150	53157	58100	57666.7	58400	58300	58529.8	56300	58200
04.12.2009	56630	53157	58100	58066.7	58000	58100	58128.7	56300	58200
07.12.2009	54820	53157	56900	57400.0	56800	56700	56551.6	55100	56600
08.12.2009	55660	53157	55100	56066.7	54800	54900	51715.7	55100	55000
09.12.2009	55110	52294	55700	55666.7	55600	55700	55779.2	55100	55800
10.12.2009	54180	52294	55100	55266.7	55200	55100	55017.3	54500	55000
11.12.2009	54580	53157	53900	54733.3	54000	54100	53157.9	53900	54200
14.12.2009	54190	53157	52400	54333.3	52400	51500	51715.7	54500	54200
15.12.2009	54120	53157	53900	54200.0	54000	54100	53157.9	53300	54200
16.12.2009	54855	53157	53900	54200.0	54000	54100	53157.9	53900	54200
17.12.2009	54430	53157	55100	54600.0	54800	54900	55017.3	54500	55000
18.12.2009	53750	53157	52400	54466.7	52400	51500	53157.9	54500	54200
21.12.2009	54570	53157	53900	53933.3	53200	53700	53894.0	53300	53400
22.12.2009	53400	53157	52400	53933.3	52400	51500	51715.7	54500	54200
23.12.2009	52990	53157	53300	53666.7	53200	53200	53356.4	53300	53400
24.12.2009	53575	52294	53150	53133.3	53200	53200	53524.7	52700	53400
25.12.2009	53450	53157	53300	53133.3	53200	53200	53356.4	53300	53400
28.12.2009	53795	53157	53300	53266.7	53200	53200	53356.4	53300	53400
29.12.2009	53515	53157	53900	53400.0	53200	53700	53894.0	53900	53400
30.12.2009	53095	53157	53300	53400.0	53200	53200	53356.4	52700	53400
31.12.2009	52920	52294	53300	53400.0	53600	53100	53524.7	53300	53400
	RMSE	2410.96	1031.12	857.34	1045.23	1288.80	1412.66	1003.50	707.71
	MAPE	0.03339	0.01512	0.01245	0.01530	0.01713	0.01935	0.01382	0.01028
	DA	0.55172	0.55172	0.51724	0.55172	0.62069	0.55172	0.48276	0.62069

*The best result.

(iv) for the application of ratio-based Huarng and Yu [13] approach, when the sample percentile alpha = 0.5,

(v) for the application of Chen method [6], when the interval length is 1900 in 5th order model for 30 and 800 in 3rd-order model for 45,

(vi) for the application of Aladag et al. [8], when the interval length is 600 on 5th-degree model and unit number of artificial neural network hidden layers is 5 for 30, and when the interval length is 800 on 3rd degree model and unit number of artificial neural network hidden layers is 4 for 45,

(vii) for the application of the proposed fuzzy ARMA(1,1) method, when the interval length of time series is 800 and the interval length of error series is 2500 for 30, and when the interval length of time series is 900 and the interval length of error series is 1000 for 45.

Best forecasts and forecast performances of all methods in result of gold prices time series solution for 30-observation test set are summarised in Table 5.

When Table 5 is observed, it is seen in result of the solution of gold prices time series for 30 observation test set

Table 6: The best results obtained for 45-observation test set of gold prices.

Date	Test set	Song and Chissom [1]	Chen [5]	Chen [6]	Huarng [11] distribution-based method	Huarng [11] average-based method	Huarng and Yu [13] rational-based method	Aladag et al. [8]	Proposed method*
17.10.2009	50162	50568	50350	49933.3	50800.0	50420.0	50829.1	50200	50450
18.10.2009	50355	50568	50350	50066.7	50800.0	51100.0	50829.1	50200	50450
19.10.2009	50020	50568	50350	50120.0	50000.0	49900.0	50051.5	50200	50450
20.10.2009	50030	50568	50350	51000.0	50800.0	51100.0	50829.1	50200	50450
23.11.2009	50700	50568	50350	51000.0	50800.0	51100.0	50829.1	50200	50450
24.11.2009	51230	50568	50350	51800.0	51066.7	51500.0	50824.1	51000	50450
25.11.2009	52490	50568	52050	50866.7	51200.0	51500.0	51128.5	51000	52250
01.11.2009	52150	52294	52050	51800.0	52666.7	54300.0	54499.6	52600	52550
02.11.2009	52125	52294	52050	51933.3	50600.0	52100.0	51916.0	51800	52550
03.11.2009	52500	52294	52050	51933.3	50600.0	52100.0	51916.0	51800	52550
04.11.2009	52165	52294	52050	52200.0	52666.7	54300.0	54499.6	52600	52550
07.11.2009	52560	52294	52050	52066.7	50600.0	52100.0	51916.0	51800	52550
08.11.2009	52972	52294	52050	52333.3	52666.7	54300.0	54499.6	52600	52550
09.11.2009	52800	52294	52900	52466.7	52800.0	52900.0	52867.3	52600	52250
10.11.2009	53500	52294	52050	54200.0	52800.0	52900.0	52867.3	52600	53150
17.11.2009	53935	53157	52900	53000.0	53200.0	53200.0	53205.5	53400	54950
18.11.2009	54550	53157	52900	53666.7	54000.0	53900.0	53840.7	53400	54050
19.11.2009	54495	53157	52900	54066.7	52400.0	51500.0	52393.3	54200	54950
20.11.2009	54830	53157	52900	54200.0	52400.0	51500.0	52393.3	54200	54050
23.11.2009	55950	53157	55450	54600.0	54800.0	54900.0	54832.0	54200	54950
24.11.2009	56285	52294	55450	55266.7	56000.0	55900.0	55841.6	54200	55850
25.11.2009	56430	52294	55450	56066.7	56400.0	56300.0	56182.2	55000	55850
01.12.2009	57635	52294	57150	56466.7	56400.0	56500.0	56524.9	55000	56750
02.12.2009	58330	53157	57150	57000.0	57600.0	57700.0	57565.6	55000	57650
03.12.2009	58150	53157	58850	57666.7	58400.0	58300.0	58270.1	55000	58550
04.12.2009	56630	53157	58850	58066.7	58000.0	58100.0	58270.1	55800	58550
07.12.2009	54820	53157	57150	57400.0	56800.0	56700.0	56524.9	55800	56750
08.12.2009	55660	53157	55450	56066.7	54800.0	54900.0	54832.0	55000	54950
09.12.2009	55110	52294	55450	55666.7	55600.0	55700.0	55503.0	54200	55850
10.12.2009	54180	52294	55450	55266.7	55200.0	55100.0	55166.5	54200	54950
11.12.2009	54580	53157	52900	54733.3	54000.0	54100.0	54169.1	54200	54050
14.12.2009	54190	53157	52900	54333.3	52400.0	51500.0	52393.3	54200	54950
15.12.2009	54120	53157	52900	54200.0	54000.0	54100.0	54169.1	54200	54050
16.12.2009	54855	53157	52900	54200.0	54000.0	54100.0	54169.1	54200	54050
17.12.2009	54430	53157	55450	54600.0	54800.0	54900.0	54832.0	54200	54950
18.12.2009	53750	53157	52900	54466.7	52400.0	51500.0	52393.3	54200	54050
21.12.2009	54570	53157	52900	53933.3	53200.0	53700.0	53840.7	53400	54050
22.12.2009	53400	53157	52900	53933.3	52400.0	51500.0	52393.3	53400	54950
23.12.2009	52990	53157	52900	53666.7	53200.0	53200.0	53205.5	53400	53150
24.12.2009	53575	52294	52900	53133.3	52800.0	52900.0	52867.3	52600	53150
25.12.2009	53450	53157	52900	53133.3	53200.0	53200.0	53205.5	53400	52250
28.12.2009	53795	53157	52900	53266.7	53200.0	53200.0	53205.5	53400	53150
29.12.2009	53515	53157	52900	53400.0	53200.0	53700.0	53840.7	53400	54050
30.12.2009	53095	53157	52900	53400.0	53200.0	53200.0	53205.5	53400	53150
31.12.2009	52920	52294	52900	53400.0	53600.0	53100.0	53514.3	53400	53150
	RMSE	2009.32	1028.39	787.71	1022.13	1200.24	1041.74	1068.71	719.69
	MAPE	0.02556	0.01499	0.01146	0.01499	0.01628	0.01517	0.01328	0.01072
	DA	0.54545	0.52273	0.54545	0.52273	0.61364	0.59091	0.54545	0.50000

*The best result.

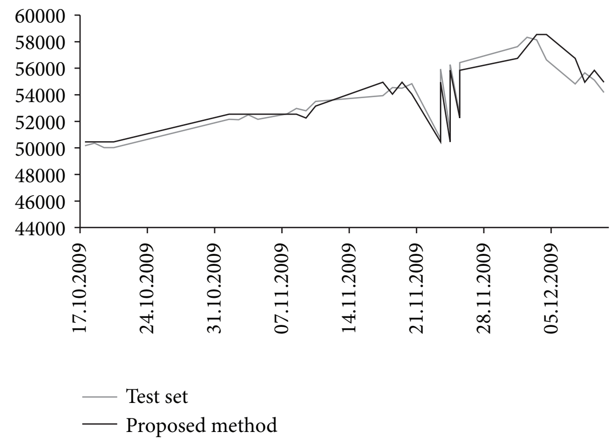

Figure 5: The graphs of the 30-observation test set of gold prices and forecasts of the test set obtained with the proposed method.

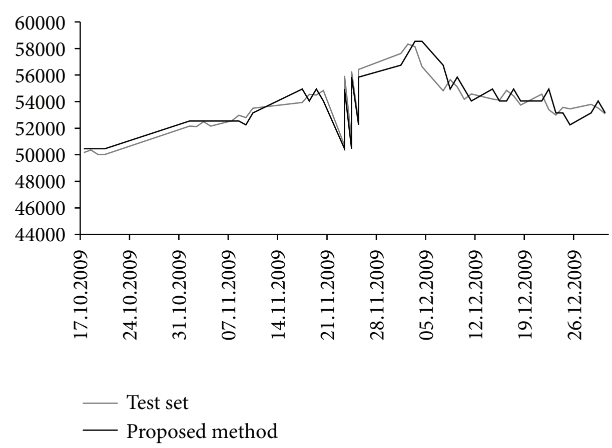

Figure 6: The graphs of the 45-observation test set of gold prices and forecasts of the test set obtained with the proposed method.

that the proposed method produced the best forecasting performance with a minimum RMSE value of 707.71, minimum MAPE value of 1.028%, and maximum direction accuracy of 62.07%. The graphs of the last 30 observations of gold prices time series along with the 30-observation test set forecasts obtained with the proposed method are shown together in Figure 5.

Best forecasts and forecast performances of all methods in result of gold prices time series solution for 45-observation test set are summarised in Table 6. Furthermore, the graphs of the last 45 observations of gold prices time series along with the 45-observation test set forecasts obtained with the proposed method are shown together in Figure 6.

When Table 6 is evaluated, it is seen in result of the solution of gold prices time series for 45 observation test set that the proposed method produced the best forecasting performance with a minimum RMSE value of 719.69, minimum MAPE value of 1.072%.

In result of the solutions of gold prices time series, it has been observed that the proposed method significantly increased the future forecasting performance compared to other methods. Also in the graphs within Figures 5 and 6, the results of the proposed method are seen to be considerably similar to the test set values.

Table 7: R/S test results.

Data	R/S statistics	$P < 0.01$
IMKB	1.6523	No
Gold price	1.9560	No

5. Discussion and Conclusion

MA variables are not included in the fuzzy time series forecast models proposed in the literature whereas real life time series are also influenced from MA variables in addition to AR variables. Therefore, redefining fuzzy times series methods as models including also MA variables are a more realistic act. In this study, a solution algorithm for a new first-order fuzzy ARMA(1,1) time series forecast model containing not only AR but also MA variables is proposed based on group relation tables. The method proposed is a basic algorithm similar to Chen [5] approach, aiming at eliminating the model specification error formed due to the exclusion of MA variables. In conclusion of the applications, it has been observed that the proposed method has higher forecasting performance than many of the fuzzy time series forecasting methods commonly used in the literature. It is an important finding that although the proposed method is a basic method based on group relation tables, it may have a higher forecasting performance even than the high order fuzzy time series methods based on artificial neural networks. Therefore, it is obvious that forecasting performance is going to increase significantly when fuzzy ARMA models are developed where fuzzy relations are specified with artificial neural networks and artificial intelligence methods or where membership values are used for specification of fuzzy relations. Thus, the proposed method may be provided with a more systematic structure and higher forecasting performance with improvements that may be done on various stages of the method during future studies. Moreover, there is no linear model assumption in the proposed fuzzy time series method. Thus, the proposed method and other fuzzy time series methods can be applied to nonlinear time series. In the application, proposed method is applied to short range dependent time series. In the future studies, it will be researched about the performance of the proposed method for long range dependent time series.

Appendix

The R/S test was applied to two time series which are used in the application. R/S test was applied by using "FinMetrics module of S-PLUS package program". The obtained results are shown in Table 7. It is obtained that both of time series have short range dependence.

References

[1] Q. Song and B. S. Chissom, "Fuzzy time series and its models," *Fuzzy Sets and Systems*, vol. 54, no. 3, pp. 269–277, 1993.

[2] L. A. Zadeh, "Fuzzy sets," *Information and Computation*, vol. 8, pp. 338–353, 1965.

[3] Q. Song and B. S. Chissom, "Forecasting enrollments with fuzzy time series. Part I," *Fuzzy Sets and Systems*, vol. 54, pp. 1–10, 1993.

[4] Q. Song and B. S. Chissom, "Forecasting enrollments with fuzzy time series. Part II," *Fuzzy Sets and Systems*, vol. 62, no. 1, pp. 1–8, 1994.

[5] S.-M. Chen, "Forecasting enrollments based on fuzzy time series," *Fuzzy Sets and Systems*, vol. 81, no. 3, pp. 311–319, 1996.

[6] S.-M. Chen, "Forecasting enrollments based on high-order fuzzy time series," *Cybernetics and Systems*, vol. 33, no. 1, pp. 1–16, 2002.

[7] K. Huarng and H.-K. Yu, "The application of neural networks to forecast fuzzy time series," *Physica A*, vol. 363, no. 2, pp. 481–491, 2006.

[8] C. H. Aladag, M. A. Basaran, E. Egrioglu, U. Yolcu, and V. R. Uslu, "Forecasting in high order fuzzy times series by using neural networks to define fuzzy relations," *Expert Systems with Applications*, vol. 36, no. 3, pp. 4228–4231, 2009.

[9] T. H.-K. Yu and K.-H. Huarng, "A neural network-based fuzzy time series model to improve forecasting," *Expert Systems with Applications*, vol. 37, no. 4, pp. 3366–3372, 2010.

[10] U. Yolcu, C. H. Aladag, E. Egrioglu, and V. R. Uslu, "Time series forecasting with a novel fuzzy time series approach: an example for Istanbul stock market," *Journal of Statistical Computation and Simulation*, vol. 83, no. 4, pp. 599–612, 2013.

[11] K. Huarng, "Effective lengths of intervals to improve forecasting in fuzzy time series," *Fuzzy Sets and Systems*, vol. 123, no. 3, pp. 387–394, 2001.

[12] E. Egrioglu, C. H. Aladag, U. Yolcu, V. R. Uslu, and M. A. Basaran, "Finding an optimal interval length in high order fuzzy time series," *Expert Systems with Applications*, vol. 37, no. 7, pp. 5052–5055, 2010.

[13] K. Huarng and T. H.-K. Yu, "Ratio-based lengths of intervals to improve fuzzy time series forecasting," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 36, no. 2, pp. 328–340, 2006.

[14] U. Yolcu, E. Egrioglu, V. R. Uslu, M. A. Basaran, and C. H. Aladag, "A new approach for determining the length of intervals for fuzzy time series," *Applied Soft Computing Journal*, vol. 9, no. 2, pp. 647–651, 2009.

[15] V. R. Uslu, C. H. Aladag, U. Yolcu, and E. Egrioğlu, "A new hybrid approach for forecasting a seasonal fuzzy time series," in *Proceedings of the International Symposium Computing Science and Engineering Proceeding Book*, pp. 1152–1158, 2010.

[16] F. Alpaslan, O. Cagcag, C. H. Aladag, U. Yolcu, and E. Egrioglu, "A novel seasonal fuzzy time series method," *Hacettepe Journal of Mathematics and Statistics*, vol. 41, no. 3, pp. 375–385, 2012.

[17] S. Aladag, C. H. Aladag, T. Mentes, and E. Egrioglu, "A new seasonal fuzzy time series method based on the multiplicative neuron model and SARIMA," *Hacettepe Journal of Mathematics and Statistics*, vol. 41, no. 3, pp. 337–345, 2012.

[18] E. Egrioglu, U. Yolcu, C. H. Aladag, and C. Kocak, "An ARMA type fuzzy time series forecasting method based on particle swarm optimization," *Mathematical Problems in Engineering*, vol. 2013, Article ID 935815, 12 pages, 2013.

[19] H. E. Hurst, "Long-term storage capacity of reservoirs," *Transactions of the American Society of Civil Engineers*, vol. 116, pp. 770–779, 1951.

[20] M. Li and W. Zhao, "On $1/f$ noise," *Mathematical Problems in Engineering*, vol. 2012, Article ID 673648, 22 pages, 2012.

[21] M. Li and W. Zhao, "Visiting power laws in cyber-physical networking systems," *Mathematical Problems in Engineering*, vol. 2012, Article ID 302786, 13 pages, 2012.

[22] M. Li, "Fractal time series: a tutorial review," *Mathematical Problems in Engineering*, vol. 2010, Article ID 157264, 26 pages, 2010.

[23] H. E. Stanley, S. V. Buldyrev, A. L. Goldberger, S. Havlin, C.-K. Peng, and M. Simons, "Long-range power-law correlations in condensed matter physics and biophysics," *Physica A*, vol. 200, no. 1 4, pp. 4–24, 1993.

[24] G. Werner, "Fractals in the nervous system: conceptual implications for theoretical neuroscience," *Frontiers in Fractal Physiology*, vol. 1, article 15, 28 pages, 2010.

[25] J. Beran, "Statistical methods for data with long-range dependence," *Statisticla Science*, vol. 7, no. 4, pp. 404–416, 1992.

[26] P. C. Ivanov, L. A. Nunes Amaral, A. L. Goldberger et al., "From 1/f noise to multifractal cascades in heartbeat dynamics," *Chaos*, vol. 11, no. 3, pp. 641–652, 2001.

[27] B. Podobnik, D. Horvatic, A. L. Ng, H. E. Stanley, and P. Ch. Ivanov, "Modeling long-range cross-correlations in two-component ARFIMA and FIARCH processes," *Physica A*, vol. 387, no. 15, pp. 3954–3959, 2008.

[28] M. Zevallos and W. Palma, "Minimum distance estimation of ARFIMA processes," *Computational Statistics & Data Analysis*, vol. 58, pp. 242–256, 2013.

[29] R. J. Bhansali and P. S. Kokoszka, "Prediction of long-memory time series: a tutorial review," in *Processes With Long-Range Correlations*, vol. 621 of *Lecture Notes in Physics*, pp. 3–21, 2003.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
Hindawi
Submit your manuscripts at http://www.hindawi.com
International Journal of Differential Equations
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization