Hindawi
Advances in High Energy Physics
Volume 2017, Article ID 9152437, 2 pages
https://doi.org/10.1155/2017/9152437

Corrigendum

Corrigendum to "Comparative Aspects of Spin-Dependent Interaction Potentials for Spin-1/2 and Spin-1 Matter Fields"

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Received 20 October 2016; Accepted 12 February 2017; Published 19 April 2017

1. Introduction

In the article titled "Comparative Aspects of Spin-Dependent Interaction Potentials for Spin-1/2 and Spin-1 Matter Fields" [1], calculations for some interparticle potentials for sources with spin-1/2 and spin-1 in the nonrelativistic (NR) regime contained inadvertent errors which should be corrected as follows. The main conclusions are unaltered.

We note further that we have included contact terms coming from Dirac delta contributions of the following Fourier transforms:

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\frac{\vec{q}_i\vec{q}_j}{\vec{q}^2+m^2}e^{i\vec{q}\cdot\vec{r}} = \frac{\delta_{ij}}{3}\delta^{(3)}(\vec{r}) + \frac{e^{-mr}}{4\pi r^3}\left[\delta_{ij}(1+mr)-\hat{r}_i\hat{r}_j\left(m^2r^2+3mr+3\right)\right], \quad (1)$$

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\frac{\vec{q}^2}{\vec{q}^2+m^2}e^{i\vec{q}\cdot\vec{r}} = \delta^{(3)}(\vec{r}) - \frac{m^2e^{-mr}}{4\pi r}, \quad (2)$$

which are the massive generalizations of the "massless" results from [2].

In what follows we list the corrections, indicated by ΔV, that must be added to some of the potentials given in [1]. The mass of the mediator, m, corresponds to either m_ϕ or m_A, as the case may be. We also highlight that not all the potentials from [1] are affected and in this note we only indicate those which are modified (Moreover, we need to use $(S_i)_{jk} = -i\epsilon_{ijk}$ in all potentials. In [1], we had the opposite sign, so one must make $\vec{S} \to -\vec{S}$ throughout.).

2. Correcting the Potentials

2.1. Potentials (Spin-1/2). The $S-PS$ and $V-PV$ potentials from [1] are not affected. The other potentials receive partial corrections. For convenience, we define the function:

$$I(r) \equiv \frac{1}{4}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\left(\vec{p}^2-\frac{m^2}{4}\right)\frac{e^{-mr}}{4\pi r} \quad (3)$$

so that the potentials in equations (25), (30) (As mentioned in equation (30) in [1], the *PS-PS* potential for $s=1/2$ has the same functional form as the *PS-PS* potential for $s=1$.), (44), and (45) in [1] receive the respective corrections:

$$\Delta V_{S\text{-}S}^{s=1/2} = g_S^1 g_S^2 \delta_1\delta_2 I(r), \quad (4)$$

$$\Delta V_{PS\text{-}PS}^{s=1/2} = -g_{PS}^1 g_{PS}^2 \frac{\langle\vec{\sigma}\rangle_1\cdot\langle\vec{\sigma}\rangle_2}{12m_1m_2}\delta^{(3)}(\vec{r}), \quad (5)$$

$$\Delta V_{V\text{-}V}^{s=1/2} = -e_1e_2\left\{\delta_1\delta_2 I(r) + \left[\frac{\delta_1\delta_2}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)+\frac{\langle\vec{\sigma}\rangle_1\cdot\langle\vec{\sigma}\rangle_2}{6m_1m_2}\right]\delta^{(3)}(\vec{r})\right\}, \quad (6)$$

$$\Delta V^{s=1/2}_{PV\text{-}PV} = +g^1_{PV} g^2_{PV} \langle \vec{\sigma} \rangle_1 \cdot \langle \vec{\sigma} \rangle_2 \left\{ I(r) + \left[\frac{1}{8} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) - \frac{1}{m^2} \right] \frac{\delta^{(3)}(\vec{r})}{3} \right\}. \tag{7}$$

Equation (4) modifies the full potential by eliminating a momentum-independent monopole-monopole term $\sim m^2(1/m_1^2 + 1/m_2^2)(e^{-mr}/4\pi r)$. A similar modification, but momentum-dependent, is induced by (6). For the other potentials, the modifications involve spin-spin interactions, mostly coupled to contact terms.

2.2. Potentials (Spin-1)

2.2.1. Vector Representation. The corrections to equations (28), (30), (47), and (48) in [1] are, respectively:

$$\Delta V^{s=1}_{S\text{-}S} = \frac{g^1_S g^2_S}{2} \frac{\delta_1 \delta_2}{m_1 m_2} \left\{ I(r) - \frac{1}{48} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \cdot \delta^{(3)}(\vec{r}) \right\}, \tag{8}$$

$$\Delta V^{s=1}_{PS\text{-}PS} = -g^1_{PS} g^2_{PS} \frac{\langle \vec{S} \rangle_1 \cdot \langle \vec{S} \rangle_2}{3} \delta^{(3)}(\vec{r}), \tag{9}$$

$$\Delta V^{s=1}_{V\text{-}V} = -e_1 e_2 \left\{ 2\delta_1 \delta_2 I(r) + \left[\frac{2}{3} \frac{\langle \vec{S} \rangle_1 \cdot \langle \vec{S} \rangle_2}{m_1 m_2} + \frac{\delta_1 \delta_2}{6} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \right] \cdot \delta^{(3)}(\vec{r}) \right\}, \tag{10}$$

$$\Delta V^{s=1}_{PV\text{-}PV} = g^1_{PV} g^2_{PV} \frac{\langle \vec{S} \rangle_1 \cdot \langle \vec{S} \rangle_2}{12 m_1 m_2} \delta^{(3)}(\vec{r}) \tag{11}$$

whereby, in (10), $I(r)$ cancels another term in (47) from [1].

2.2.2. Tensor Representation. The correction to the *PS*-*PS* potential in this representation is $\Delta V^{s=1}_{PS\text{-}PS}/4m_1^2 m_2^2$ (cf. (9)). Also, the correction to the *PV*-*PV* potential in this representation is the same as in (11).

In [1], equations (65) receive the respective corrections:

$$\Delta V^{s=1}_{S\text{-}S} = \frac{g^1_S g^2_S}{2} \frac{\delta_1 \delta_2}{m_1 m_2} \left\{ I(r) - \frac{5}{48} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \cdot \delta^{(3)}(\vec{r}) \right\}, \tag{12}$$

$$\Delta V^{s=1}_{V\text{-}V} = -e_1 e_2 \left\{ 2\delta_1 \delta_2 I(r) + \left[\frac{2}{3} \frac{\langle \vec{S} \rangle_1 \cdot \langle \vec{S} \rangle_2}{m_1 m_2} + \frac{\delta_1 \delta_2}{3} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \right] \delta^{(3)}(\vec{r}) \right\}. \tag{13}$$

We note that the correction in (13) excludes one momentum-dependent term associated with $\vec{p}^2(1/m_1^2 + 1/m_2^2)(e^{-mr}/4\pi r)$ in the *V*-*V* potential in equation (65) from [1].

3. Concluding Remarks

We have presented the corrections to some of the results given in [1]. The corrections introduced affect mostly the monopole-monopole sectors (e.g., (4), (8), and (12)).

The conclusions presented in [1] remain unaltered: in general, the interparticle potentials associated with NR spin-1/2 and spin-1 sources present similarities, especially, in the spin-spin sectors of the *S*-*S*, *S*-*PS*, *PS*-*PS*, and *V*-*V* interactions. The most important modifications are due to the presence of contact terms, which do not play a role in macroscopic interactions.

References

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