Quantum state reconstruction in the presence of dissipation

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Abstract

We propose a realistic scheme to determine the quantum state of a single mode cavity field even after it has started to decay due to the coupling with an environment. Although dissipation destroys quantum coherences, we show that at zero temperature enough information about the initial state remains, in an observable quantity, to allow the reconstruction of its Wigner function. 42.50.-p, 03.65.Bz, 42.50.Dv

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Methods to reconstruct quantum states of light are of great importance in quantum optics. There have been several proposals using different techniques to achieve such reconstructions [1], amongst them, the direct sampling of the density matrix of a signal mode in optical homodyne tomography [2], the tomographic reconstruction by unbalanced homodyning [3]; the direct measurement (quantum endoscopy), of the Wigner function of the electromagnetic field in a cavity or the vibrational state of an ion in a trap [4,5]. It is well known that dissipation has a destructive effect in most of these schemes, and issues such as compensation of losses in quantum-state measuments have already been discussed in the literature [6].

In this contribution we present a novel method of how to reconstruct a quantum state even after the action of dissipation. We consider a single mode high-Q cavity where a nonclassical field state $\hat{\rho}(0)$ is prepared and subsequently driven by a coherent pulse. Both processes are assumed to occur in a time scale much shorter than the decay time of the cavity. Then the field is allowed to decay. We will show below that by displacing the initial state we make its quantum coherences robust enough to allow its experimental determination despite the existence of dissipation.

The master equation in the interaction picture for a damped cavity mode at zero temperature and under the Born-Markov approximation is given by [7]

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{\gamma}{2} \left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a} \right), \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators and γ the decay constant. We can define the superoperators $\hat{J}$ and $\hat{L}$ by their action on the density operator [8]

$$\hat{J}\hat{\rho} = \gamma\hat{a}\hat{\rho}\hat{a}^\dagger, \qquad \hat{L}\hat{\rho} = -\frac{\gamma}{2} \left(\hat{a}^\dagger\hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger\hat{a} \right). \tag{2}$$

The formal solution of (1) can be written as [9]

$$\hat{\rho}(t) = \exp\left[(\hat{J} + \hat{L})t \right] \hat{\rho}(0) = \exp(\hat{L}t) \exp\left[\frac{\hat{J}}{\gamma}(1 - e^{-\gamma t}) \right] \hat{\rho}(0). \tag{3}$$

We assume that the initial field $\hat{\rho}(0)$ is prepared in a time scale much shorter than the decay time of the cavity γ^{-1}. As soon as the field is generated, a coherent field $|\alpha\rangle$

is injected inside the cavity (also in a short time scale) displacing then the initial state $\hat{\rho}_\alpha = \hat{D}(\alpha)\hat{\rho}(0)\hat{D}^\dagger(\alpha)$. This procedure will enable us to obtain information about all the elements of the initial density matrix from the diagonal elements of the time-evolved displaced density matrix only. As diagonal elements decay much slower than off-diagonal ones, information about the initial state stored this way becomes robust enough to withstand the decoherence process. We will now show how this robustness can be used to obtain the Wigner function of the initial state after it has started to decay.

The diagonal matrix elements of $\hat{\rho}_\alpha(t) = \exp\left[(\hat{J} + \hat{L})t\right]\hat{\rho}_\alpha$ in the number state basis are

$$\langle m|\hat{\rho}_\alpha(t)|m\rangle = \frac{e^{-m\gamma t}}{q^m}\sum_{n=0}^{\infty} q^n \binom{n}{m} \langle n|\hat{\rho}_\alpha|n\rangle, \tag{4}$$

where $q = 1 - e^{-\gamma t}$.

We note that if we multiply those elements by the function

$$\chi(t) = 1 - 2e^{\gamma t} \tag{5}$$

and sum over m we obtain

$$F = \frac{2}{\pi}\sum_{m=0}^{\infty} \chi^m(t)\langle m|\hat{\rho}_\alpha(t)|m\rangle = \frac{2}{\pi}\sum_{n=0}^{\infty}(-1)^n\langle n|\hat{D}(\alpha)\hat{\rho}(0)\hat{D}^\dagger(\alpha)|n\rangle. \tag{6}$$

The expression above is exactly the Wigner function corresponding to $\hat{\rho}$ (the initial field state) [12] at the point specified by the complex amplitude α. Therefore if we measure the diagonal elements of the dissipated displaced cavity field $P_m(\alpha;t) = \langle m|\hat{\rho}_\alpha(t)|m\rangle$ for a range of α's, the transformation in Eq. (6) will give us the Wigner function F for this range. This is the main result of our paper; the reconstruction is made possible even under the normally destructive action of dissipation. We would like to stress that the identity in Eq. (6) means that the time-dependence is completely cancelled, bringing out the Wigner function of the initial state.

One way of determining $P_m(\alpha;t)$ is by injecting atoms into the cavity and measuring their population inversion as they exit after an interaction time τ much shorter than the

cavity decay time. We may use three-level atoms in a cascade configuration with the upper and the lower level having the same parity. In this case the population inversion is given by [10]

$$W(\alpha; t + \tau) = \sum_{n=0}^{\infty} P_n(\alpha; t) \left[\frac{\Gamma_n}{\delta_n^2} + \frac{(n+1)(n+2)}{\delta_n^2} \cos(2\delta_n \lambda \tau) \right], \quad (7)$$

where $\Gamma_n = [\Delta + \chi(n+1)]/2$, $\delta_n^2 = \Gamma_n^2 + \lambda^2(n+1)(n+2)$, Δ is the atom-field detuning, χ is the Stark shift coefficient, and λ is the coupling constant. In the case of having $\Delta = 0$ (two-photon resonance condition), $\chi = 0$, and for strong enough fields, for which it is valid the approximation $[(n+1)(n+2)]^{1/2} \approx n + 3/2$, the population inversion reduces to

$$W(\alpha; t + \tau) = \sum_{n=0}^{\infty} P_n(\alpha; t) \cos([2n+3]\lambda\tau), \quad (8)$$

By inverting the Fourier series in Eq. (8) we obtain for $P_n(\alpha; t)$

$$P_n(\alpha; t) = \frac{2\lambda}{\pi} \int_0^{\frac{\pi}{\lambda}} d\tau \, W(t + \tau) \cos([2n+3]\lambda\tau). \quad (9)$$

We need a maximum interaction time $\tau_{max} = \pi/\lambda$ much shorter than the cavity decay time. This condition implies that we must be in the strong-coupling regime, i.e. $\lambda \gg \gamma$.

Our scheme is easily generalized to other (s-parametrized [11]) quasi-probability distributions given by [12],

$$F(\alpha; s) = -\frac{2}{\pi(s-1)} \sum_{n=0}^{\infty} \left(\frac{s+1}{s-1} \right)^n \langle n | \hat{\rho}_\alpha | n \rangle, \quad (10)$$

by choosing

$$\chi(s; t) = 1 + \frac{2e^{\gamma t}}{s-1}. \quad (11)$$

In conclusion, we have presented a novel technique to reconstruct the Wigner function of an initial nonclassical state at times when the field would have normally lost its quantum coherence [13]. Reconstruction approaches do not usually take into account the effect of losses. The crucial point of our method is the driving of the initial field immediately after preparation, that is not only used to cover a region in phase space but also to store quantum

coherences in the diagonal elements of the time evolved displaced density matrix, making them robust. In other words, we have shown that the initial displacement transfers to any initial state the robustness of a coherent state [14] against dissipation.

The possibility of reconstructing quantum states at any time opens up potential applications in quantum computing. For instance, this method could be used in a scheme to refresh the state of a quantum computer in order to avoid dissipation-induced errors.

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