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Research Article

A Metric Observer for Induction Motors Control

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This paper deals with metric observer application for induction motors. Firstly, assuming that stator currents and speed are measured, a metric observer is designed to estimate the rotor fluxes. Secondly, assuming that only stator currents are measured, another metric observer is derived to estimate rotor fluxes and speed. The proposed observer validity is checked throughout simulations on a 4 kW induction motor drive.

1. Introduction

In the two last decades, the most significant developments in induction motors control have been field-oriented control [1] and nonlinear input-output and state feedback linearization techniques [2] with real-world industry application. More advanced control techniques have also been proposed, such as (1) passivity-based approach, which exploits the system energy dissipation property to solve the underlying control problem [3], (2) sliding mode-based control approaches [4–7] and the higher-order ones [8, 9], and (3) flatness-based control approaches [10].

Otherwise, in most of the above-mentioned control approaches, an observer has to be used since a part of the motor state is not measurable in industrial applications. Several observers have been proposed in the literature. The most well-known and popular ones are given in [11, 12], in which authors have proposed the model reference adaptive system (MRAS) for the estimation of the induction motor speed, from measured phase voltages and currents, based on the adaptive control theory. Some other observer has been proposed based on the context of more advanced and/or intelligent control technique such as sliding mode [13], high gain observer [14], and the mean value theorem [15].

Screening deeply the literature on observers design for nonlinear systems, specific ones have been proposed in [16, 17], namely, *metric observers*. In this particular and still challenging observer design context, it is proposed to investigate the effectiveness of metric observers for induction motors control. This paper's objective is therefore twofold: (1) assuming the stator currents and rotor speed to be measured, a reduced-order metric observer is proposed to estimate the induction motor rotor fluxes; (2) assuming that only the stator currents are measured, a nonlinear reduced-order metric observer is derived to estimate rotor fluxes and speed.

The paper is organized as follows. Section 2 deals with the induction modeling. In the first part of Section 3, a reduced-order metric observer is derived to estimate the rotor flux and, in the second part, a nonlinear reduced-order one for rotor flux and rotor speed estimation is proposed. Section 4 gives simulation results for validation purposes. Finally some concluding remarks end the paper.

2. Induction Motor Model

Assuming that we have balanced three-phase AC voltages and that stator windings are uniformly distributed, and based on the well-known two-phase equivalent machine representation, the induction motor can be described by fifth-order nonlinear differential equations with four electrical variables (currents and fluxes), one mechanical variable (rotor speed), and two control variables (stator voltages). In a fixed (a, b) frame, one has

$$\dot{X} = f(X,U) = A(\omega(t))X + BU, \quad \dot{\omega}(t) = k(\varphi_a i_b - \varphi_b i_a) - \frac{T_L}{J} - \frac{f_{\text{rot}}}{J}\omega, \tag{1}$$

where

$$U = \begin{bmatrix} u_a & u_b \end{bmatrix}^T, \quad X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} i_a & i_b & \varphi_a & \varphi_b \end{bmatrix}^T,$$
$$A(\omega(t)) = \begin{pmatrix} -\gamma & 0 & \frac{K}{T_r} & pK\omega(t) \\ 0 & -\gamma & -pK\omega(t) & \frac{K}{T_r} \\ \frac{M}{T_r} & 0 & -\frac{1}{T_r} & -p\omega(t) \\ 0 & \frac{M}{T_r} & p\omega(t) & -\frac{1}{T_r} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

with

$$\sigma = 1 - \frac{M^2}{L_s L_r}, \quad K = \frac{M}{\sigma L_s L_r}, \quad \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}, \quad k = p\frac{M}{JL_r}. \tag{3}$$

It is worth noticing that the only measured variables are stator currents so that the output state equation is

$$Y = H(X) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} X. \tag{4}$$

It can be easily shown that state X is observable from Y.

3. The Metric Observer

In this section, we propose a reduced-order metric observer for rotor fluxes estimation. Then, we will introduce a nonlinear reduced-order one for rotor fluxes and rotor speed estimations.

3.1. Reduced-Order Observer for Fluxes Estimation. Assuming that the stator currents x_1 and x_2 and the rotor speed ω are measured, we consider the induction motor fourth-order model. Consider

$$\begin{aligned} \dot{x}_1 &= -\gamma x_1 + \frac{K}{T_r}x_3 + pK\omega(t)x_4 + \frac{1}{\sigma L_s}u_a, \\ \dot{x}_2 &= -\gamma x_2 + \frac{K}{T_r}x_4 - pK\omega(t)x_3 + \frac{1}{\sigma L_s}u_b, \\ \dot{x}_3 &= \frac{M}{T_r}x_1 - \frac{1}{T_r}x_3 - p\omega(t)x_4, \\ \dot{x}_4 &= \frac{M}{T_r}x_2 - \frac{1}{T_r}x_4 + p\omega(t)x_3. \end{aligned} \tag{5}$$

A possible reduced-order observer for $\hat{x}_3$ and $\hat{x}_4$ is a simple copy of the two last equations of our dynamic model. Consider

$$\begin{aligned} \dot{\hat{x}}_3 &= \frac{M}{T_r}\hat{x}_1 - \frac{1}{T_r}\hat{x}_3 - p\omega(t)\hat{x}_4, \\ \dot{\hat{x}}_4 &= \frac{M}{T_r}\hat{x}_2 - \frac{1}{T_r}\hat{x}_4 + p\omega(t)\hat{x}_3. \end{aligned} \tag{6}$$

To ensure this observer's exponential convergence, we introduce two intermediate variables [16]:

$$\begin{aligned} \overline{x}_3 &= \hat{x}_3 + \Gamma\hat{x}_1, \\ \overline{x}_4 &= \hat{x}_4 + \Gamma\hat{x}_2. \end{aligned} \tag{7}$$

The new dynamic equations are given by

$$\begin{aligned} \dot{\overline{x}}_3 &= (\Gamma K - 1)\left(\frac{\overline{x}_3}{T_r} + p\omega\overline{x}_4\right) + f_1, \\ \dot{\overline{x}}_4 &= (1 - \Gamma K)\left(p\omega\overline{x}_3 - \frac{1}{T_r}\overline{x}_4\right) + f_2 \end{aligned} \tag{8}$$

with

$$\begin{aligned} f_1 &= \left[\left(\frac{M}{T_r} - \gamma\Gamma\right) + \frac{\Gamma}{T_r}(1 - \Gamma K)\right]\hat{x}_1 \\ &\quad - (\Gamma K - 1)\Gamma p\omega\hat{x}_2 + \frac{\Gamma}{\sigma L_s}u_a, \\ f_2 &= (\Gamma K - 1)\Gamma p\omega\hat{x}_1 \\ &\quad + \left[\left(\frac{M}{T_r} - \gamma\Gamma\right) + \frac{\Gamma}{T_r}(1 - \Gamma K)\right]\hat{x}_2 + \frac{\Gamma}{\sigma L_s}u_b. \end{aligned} \tag{9}$$

$\hat{x}_1$ and $\hat{x}_2$ are now replaced by measurements x_1 and x_2 [16]. This leads to the following observer equation with the intermediate variables:

$$\begin{aligned} \dot{\overline{x}}_3 &= (\Gamma K - 1)\left(\frac{1}{T_r}\overline{x}_3 + p\omega\overline{x}_4\right) + h_1, \\ \dot{\overline{x}} &= (1 - \Gamma K)\left(p\omega\overline{x}_3 - \frac{1}{T_r}\overline{x}_4\right) + h_2 \end{aligned} \tag{10}$$

with

$$h_1 = \left[\left(\frac{M}{T_r} - \gamma\Gamma\right) + \frac{\Gamma}{T_r}(1 - \Gamma K)\right] x_1 - (\Gamma K - 1)\Gamma p\omega x_2 + \frac{\Gamma}{\sigma L_s} u_a,$$
$$h_2 = (\Gamma K - 1)\Gamma p\omega x_1 + \left[\left(\frac{M}{T_r} - \gamma\Gamma\right) + \frac{\Gamma}{T_r}(1 - \Gamma K)\right] x_2 + \frac{\Gamma}{\sigma L_s} u_b. \quad (11)$$

The result of this computation is then mapped back to the original reduced state space with

$$\hat{x}_3 = \overline{x}_3 - \Gamma x_1,$$
$$\hat{x}_4 = \overline{x}_4 - \Gamma x_2. \quad (12)$$

This leads to the following new observer dynamics in $\hat{x}_3$ and $\hat{x}_4$ [16]:

$$\dot{\hat{x}}_3 = (\Gamma K - 1)\left(\frac{1}{T_r}\hat{x}_3 + p\omega\hat{x}_4\right) + g_1,$$
$$\dot{\hat{x}}_4 = (1 - \Gamma K)\left(p\omega\hat{x}_3 - \frac{1}{T_r}\hat{x}_4\right) + g_2 \quad (13)$$

with

$$g_1 = \frac{M}{T_r}x_1 - \frac{\Gamma K}{T_r}\hat{x}_3 - p\omega\Gamma K\hat{x}_4,$$
$$g_2 = \frac{M}{T_r}x_2 - \frac{\Gamma K}{T_r}\hat{x}_4 + p\omega\Gamma K\hat{x}_3. \quad (14)$$

The Jacobian or rate of deformation tensor of this system is

$$F = \begin{pmatrix} (\Gamma K - 1)\dfrac{1}{T_r} & (\Gamma K - 1)p\omega \\ (1 - \Gamma K)p\omega & (\Gamma K - 1)\dfrac{1}{T_r} \end{pmatrix}. \quad (15)$$

The strain tensor rate is

$$E = \begin{pmatrix} \dfrac{2}{T_r}(\Gamma K - 1) & 0 \\ 0 & \dfrac{2}{T_r}(\Gamma K - 1) \end{pmatrix}. \quad (16)$$

We follow the same procedure as in [16, 17]. The exponential convergence of the reduced-order observer is guaranteed for a strain tensor uniformly negative definite rate. The rate of the strain tensor is uniformly negative definite if and only if $\exists\beta > 0$ such that the following conditions are satisfied:

$$\frac{2}{T_r}(\Gamma K - 1) \le -\beta < 0,$$
$$\left[\frac{2}{T_r}(\Gamma K - 1)\right]^2 \ge \beta > 0. \quad (17)$$

3.2. Reduced-Order Observer for Rotor Fluxes and Speed Estimation. Assuming now that just the stator currents are available, we will design a reduced-order observer to estimate rotor fluxes and speed. Consider the fifth-order model written as

$$\dot{x}_1 = -\gamma x_1 + \frac{K}{T_r}x_3 + pK\omega x_4 + \frac{1}{\sigma L_s}u_a,$$
$$\dot{x}_2 = -\gamma x_2 + \frac{K}{T_r}x_4 - pK\omega x_3 + \frac{1}{\sigma L_s}u_b,$$
$$\dot{x}_3 = \frac{M}{T_r}x_1 - \frac{1}{T_r}x_3 - px_4\omega,$$
$$\dot{x}_4 = \frac{M}{T_r}x_2 - \frac{1}{T_r}x_4 + px_3\omega,$$
$$\dot{\omega} = k(x_2x_3 - x_1x_4) - \frac{f_{\text{rot}}}{J}\omega - \frac{T_L}{J}. \quad (18)$$

A possible reduced-order observer for x_3, x_4, and ω is a simple copy of the last three equations of our fifth-order dynamic model:

$$\dot{\hat{x}}_3 = \frac{M}{T_r}\hat{x}_1 - \frac{1}{T_r}\hat{x}_3 - p\hat{x}_4\hat{\omega},$$
$$\dot{\hat{x}}_4 = \frac{M}{T_r}\hat{x}_2 - \frac{1}{T_r}\hat{x}_4 + p\hat{x}_3\hat{\omega},$$
$$\dot{\hat{\omega}} = \text{cst}\,(\hat{x}_2\hat{x}_3 - \hat{x}_1\hat{x}_4) - \frac{f_{\text{rot}}}{J}\hat{\omega} - \frac{T_L}{J}. \quad (19)$$

To ensure the above observer's exponential convergence, we introduce three intermediate variables:

$$\overline{x}_3 = \hat{x}_3 + \widehat{\Phi}_1,$$
$$\overline{x}_4 = \hat{x}_4 + \widehat{\Phi}_2,$$
$$\overline{\omega} = \hat{\omega} + \widehat{\Phi}_3 \quad (20)$$

with

$$\widehat{\Phi}_1 = \Gamma_1\begin{bmatrix}\hat{x}_1 \\ \hat{x}_2\end{bmatrix},$$
$$\widehat{\Phi}_2 = \Gamma_2\begin{bmatrix}\hat{x}_1 \\ \hat{x}_2\end{bmatrix},$$
$$\widehat{\Phi}_3 = \Gamma_3\begin{bmatrix}\hat{x}_1 \\ \hat{x}_2\end{bmatrix},$$
$$\Gamma_i = [\Gamma_{i1} \quad \Gamma_{i2}]. \quad (21)$$

The coordinates' intermediate change leads to the following new observer equations:

$$
\begin{aligned}
\dot{\overline{x}}_3 &= \left[-\frac{1}{T_r} + \Gamma_{11}\frac{K}{T_r} - \Gamma_{12} pK\widehat{\Phi}_3\right]\overline{x}_3 \\
&\quad + \left[\Gamma_{12}\frac{K}{T_r} - p\left(\Gamma_{11}K-1\right)\widehat{\Phi}_3\right]\overline{x}_4 \\
&\quad + \left[\Gamma_{12} pK\overline{x}_3 + p\left(\Gamma_{11}K-1\right)\overline{x}_4 + \widehat{J}_1\right]\overline{\omega} + \widehat{G}_1, \\
\widehat{G}_1 &= \widehat{F}_1 - \frac{1}{T_r}\left(\Gamma_{11}K-1\right)\widehat{\Phi}_1 - \Gamma_{12}\frac{K}{T_r}\widehat{\Phi}_2 - \widehat{J}_1\widehat{\Phi}_3, \\
\widehat{F}_1 &= \left(\frac{M}{T_r} - \gamma\Gamma_{11}\right)\widehat{x}_1 - \gamma\Gamma_{12}\widehat{x}_2 + \frac{\Gamma_{11}}{\sigma L_s}u_a + \frac{\Gamma_{12}}{\sigma L_s}u_b, \\
\widehat{J}_1 &= \Gamma_{12} pK\widehat{\Phi}_1 - p\left(\Gamma_{11}K-1\right)\widehat{\Phi}_2, \\
\dot{\overline{x}}_4 &= \left[\Gamma_{21}\frac{K}{T_r} + p\left(\Gamma_{22}K-1\right)\widehat{\Phi}_3\right]\overline{x}_3 \\
&\quad + \left[-\frac{1}{T_r} + \Gamma_{22}\frac{K}{T_r} - \Gamma_{21} pK\widehat{\Phi}_3\right]\overline{x}_4 \\
&\quad + \left[p\left(1-\Gamma_{22}K\right)\overline{x}_3 + \Gamma_{21} pK\overline{x}_4 + \widehat{J}_2\right]\overline{\omega} + \widehat{G}_2, \\
\widehat{G}_2 &= \widehat{F}_2 - \frac{1}{T_r}\left(\Gamma_{22}K-1\right)\widehat{\Phi}_2 - \Gamma_{21}\frac{K}{T_r}\widehat{\Phi}_1 - \widehat{J}_2\widehat{\Phi}_3, \\
\widehat{F}_1 &= -\gamma\Gamma_{21}\widehat{x}_1 + \left(\frac{M}{T_r} - \gamma\Gamma_{22}\right)\widehat{x}_2 + \frac{\Gamma_{21}}{\sigma L_s}u_a + \frac{\Gamma_{22}}{\sigma L_s}u_b, \\
\widehat{J}_2 &= p\left(\Gamma_{22}K-1\right)\widehat{\Phi}_1 + \Gamma_{21} pK\widehat{\Phi}_2, \\
\dot{\overline{\omega}} &= \left[\mathrm{cst}\,\widehat{x}_2 + \Gamma_{31}\frac{K}{T_r} + pK\Gamma_{32}\widehat{\Phi}_3\right]\overline{x}_3 \\
&\quad + \left[-\mathrm{cst}\,\widehat{x}_1 + \Gamma_{32}\frac{K}{T_r} - pK\Gamma_{31}\widehat{\Phi}_3\right]\overline{x}_4 \\
&\quad + \left[-pK\Gamma_{32}\overline{x}_3 + pK\Gamma_{32}\overline{x}_4 + \widehat{J}_3\right]\overline{\omega} + \widehat{G}_3, \\
\widehat{G}_3 &= \widehat{F}_3 - \left(\mathrm{cst}\,\widehat{x}_2 + \Gamma_{31}\frac{K}{T_r}\right)\widehat{\Phi}_1 \\
&\quad - \left(-\mathrm{cst}\,\widehat{x}_1 + \Gamma_{32}\frac{K}{T_r}\right)\widehat{\Phi}_2 - \widehat{J}_3\widehat{\Phi}_3, \\
\widehat{F}_3 &= -\gamma\Gamma_{31}\widehat{x}_1 - \gamma\Gamma_{32}\widehat{x}_2 + \frac{\Gamma_{31}}{\sigma L_s}u_a + \frac{\Gamma_{32}}{\sigma L_s}u_b - \frac{T_L}{J}, \\
\widehat{J}_3 &= pK\Gamma_{32}\widehat{\Phi}_1 - pK\Gamma_{31}\widehat{\Phi}_2 - \frac{f_{\mathrm{rot}}}{J}.
\end{aligned}
\tag{22}
$$

$\widehat{x}_1$ and $\widehat{x}_2$ are now replaced by measurements x_1 and x_2. This leads to the following observer equation with the intermediate variables:

$$
\begin{aligned}
\dot{\overline{x}}_3 &= \left[-\frac{1}{T_r} + \Gamma_{11}\frac{K}{T_r} + \Gamma_{12} pK\Phi_3\right]\overline{x}_3 \\
&\quad + \left[\Gamma_{12}\frac{K}{T_r} - p\left(\Gamma_{11}K-1\right)\Phi_3\right]\overline{x}_4 \\
&\quad + \left[-\Gamma_{12} pK\overline{x}_3 + p\left(\Gamma_{11}K-1\right)\overline{x}_4 + J_1\right]\overline{\omega} \\
&\quad + G_1, \\
G_1 &= F_1 - \frac{1}{T_r}\left(\Gamma_{11}K-1\right)\Phi_1 - \Gamma_{12}\frac{K}{T_r}\Phi_2 - J_1\Phi_3, \\
F_1 &= \left(\frac{M}{T_r} - \gamma\Gamma_{11}\right)x_1 - \gamma\Gamma_{12}x_2 + \frac{\Gamma_{11}}{\sigma L_s}u_a + \frac{\Gamma_{12}}{\sigma L_s}u_b, \\
J_1 &= \Gamma_{12} pK\Phi_1 - p\left(\Gamma_{11}K-1\right)\Phi_2, \\
\dot{\overline{x}}_4 &= \left[\Gamma_{21}\frac{K}{T_r} + p\left(\Gamma_{22}K-1\right)\Phi_3\right]\overline{x}_3 \\
&\quad + \left[-\frac{1}{T_r} + \Gamma_{22}\frac{K}{T_r} - \Gamma_{21} pK\Phi_3\right]\overline{x}_4 \\
&\quad + \left[p\left(1-\Gamma_{22}K\right)\overline{x}_3 + \Gamma_{21} pK\overline{x}_4 + J_2\right]\overline{\omega} + G_2, \\
G_2 &= F_2 - \frac{1}{T_r}\left(\Gamma_{22}K-1\right)\Phi_2 - \Gamma_{21}\frac{K}{T_r}\Phi_1 - J_2\Phi_3, \\
F_1 &= -\gamma\Gamma_{21}x_1 + \left(\frac{M}{T_r} - \gamma\Gamma_{22}\right)x_2 + \frac{\Gamma_{21}}{\sigma L_s}u_a + \frac{\Gamma_{22}}{\sigma L_s}u_b, \\
J_2 &= p\left(\Gamma_{22}K-1\right)\Phi_1 + \Gamma_{21} pK\Phi_2, \\
\dot{\overline{\omega}} &= \left[\mathrm{cst}\,x_2 + \Gamma_{31}\frac{K}{T_r} + pK\Gamma_{32}\Phi_3\right]\overline{x}_3 \\
&\quad + \left[-\mathrm{cst}\,x_1 + \Gamma_{32}\frac{K}{T_r} - pK\Gamma_{31}\Phi_3\right]\overline{x}_4 \\
&\quad + \left[-pK\Gamma_{32}\overline{x}_3 + pK\Gamma_{32}\overline{x}_4 + J_3\right]\overline{\omega} + G_3, \\
G_3 &= F_3 - \left(\mathrm{cst}\,x_2 + \Gamma_{31}\frac{K}{T_r}\right)\Phi_1 \\
&\quad - \left(-\mathrm{cst}\,x_1 + \Gamma_{32}\frac{K}{T_r}\right)\Phi_2 - J_3\Phi_3, \\
F_3 &= -\gamma\Gamma_{31}x_1 - \gamma\Gamma_{32}x_2 + \frac{\Gamma_{31}}{\sigma L_s}u_a + \frac{\Gamma_{32}}{\sigma L_s}u_b - \frac{T_L}{J}, \\
J_3 &= pK\Gamma_{32}\Phi_1 - pK\Gamma_{31}\Phi_2 - \frac{f_{\mathrm{rot}}}{J}
\end{aligned}
\tag{23}
$$

with

$$
\begin{aligned}
\Phi_1 &= \Gamma_1\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \\
\Phi_2 &= \Gamma_2\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \\
\Phi_3 &= \Gamma_3\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.
\end{aligned}
\tag{24}
$$

The result of this computation is then mapped back to the original reduced state space with

$$\begin{aligned}\hat{x}_3 &= \bar{x}_3 - \Phi_1,\\ \hat{x}_4 &= \bar{x}_4 - \Phi_2,\\ \hat{\omega} &= \bar{\omega} - \Phi_3.\end{aligned} \tag{25}$$

As discussed in [16, 17], this leads to the following new observer dynamics in $\hat{x}_3$, $\hat{x}_4$, and $\hat{\omega}$. The estimated flux $\hat{x}_3 = \hat{\varphi}_a$ is given by

$$\begin{aligned}\dot{\hat{x}}_3 = {} & \frac{1}{T_r}\left(\Gamma_{11}K - 1\right)\hat{x}_3 + \Gamma_{12}\frac{K}{T_r}\hat{x}_4\\ & + \left(-\Gamma_{12}pK\hat{x}_3 + p\left(\Gamma_{11}K - 1\right)\hat{x}_4\right)\hat{\omega} + H_1\end{aligned} \tag{26}$$

with

$$\begin{aligned}H_1 = {} & A_1\Phi_1 + A_2\Phi_2 + G_1\\ & - \Gamma_{11}\left(-\gamma x_1 + \frac{K}{T_r}x_3 + pK\omega x_4 + \frac{1}{\sigma L_s}u_a\right)\\ & - \Gamma_{12}\left(-\gamma x_2 + \frac{K}{T_r}x_4 - pK\omega x_3 + \frac{1}{\sigma L_s}u_b\right),\\ A_1 = {} & -\frac{1}{T_r} + \Gamma_{11}\frac{K}{T_r} - \Gamma_{12}pK\Phi_3,\\ A_2 = {} & \Gamma_{12}\frac{K}{T_r} - p\left(\Gamma_{11}K - 1\right)\Phi_3.\end{aligned} \tag{27}$$

The estimated flux $\hat{x}_4 = \hat{\varphi}_b$ is given by

$$\begin{aligned}\dot{\hat{x}}_4 = {} & \Gamma_{21}\frac{K}{T_r}\hat{x}_3 + \left[-\frac{1}{T_r} + \Gamma_{22}\frac{K}{T_r}\right]\hat{x}_4\\ & + \left[p\left(1 - \Gamma_{22}K\right)\hat{x}_3 + \Gamma_{21}pK\hat{x}_4\right]\hat{\omega} + H_2\end{aligned} \tag{28}$$

with

$$\begin{aligned}H_2 = {} & B_1\Phi_1 + B_2\Phi_2 + G_1\\ & - \Gamma_{21}\left(-\gamma x_1 + \frac{K}{T_r}x_3 + pK\omega x_4 + \frac{1}{\sigma L_s}u_a\right)\\ & - \Gamma_{22}\left(-\gamma x_2 + \frac{K}{T_r}x_4 - pK\omega x_3 + \frac{1}{\sigma L_s}u_b\right),\\ B_1 = {} & \Gamma_{21}\frac{K}{T_r} + p\left(\Gamma_{22}K - 1\right)\Phi_3,\\ B_2 = {} & -\frac{1}{T_r} + \Gamma_{22}\frac{K}{T_r} - \Gamma_{21}pK\Phi_3.\end{aligned} \tag{29}$$

The estimated speed $\hat{x}_5 = \hat{\omega}$ is given by

$$\begin{aligned}\dot{\hat{\omega}} = {} & \left[\text{cst}\,x_2 + \Gamma_{31}\frac{K}{T_r}\right]\hat{x}_3 + \left[-\text{cst}\,x_1 + \Gamma_{32}\frac{K}{T_r}\right]\hat{x}_4\\ & + \left[-pK\Gamma_{32}\hat{x}_3 + pK\Gamma_{32}\hat{x}_4\right]\hat{\omega} + H_3\end{aligned} \tag{30}$$

with

$$\begin{aligned}H_3 = {} & C_1\Phi_1 + C_2\Phi_2 + G_3\\ & - \Gamma_{31}\left(-\gamma x_1 + \frac{K}{T_r}x_3 + pK\omega x_4 + \frac{1}{\sigma L_s}u_a\right)\\ & - \Gamma_{32}\left(-\gamma x_2 + \frac{K}{T_r}x_4 - pK\omega x_3 + \frac{1}{\sigma L_s}u_b\right),\\ C_1 = {} & \text{cst}\,x_2 + \Gamma_{31}\frac{K}{T_r} + pK\Gamma_{32}\Phi_3,\\ C_2 = {} & -\text{cst}\,x_1 + \Gamma_{32}\frac{K}{T_r} - pK\Gamma_{31}\Phi_3.\end{aligned} \tag{31}$$

The Jacobian or rate of deformation tensor of this system is

$$F = \begin{pmatrix} F_{11} & F_{12} & F_{13}\\ F_{21} & F_{22} & F_{23}\\ F_{31} & F_{32} & F_{33}\end{pmatrix} \tag{32}$$

with

$$\begin{aligned}F_{11} &= -\frac{1}{T_r} + \Gamma_{11}\frac{K}{T_r} - \Gamma_{12}pK\hat{\omega},\\ F_{12} &= \Gamma_{12}\frac{K}{T_r} + p\left(\Gamma_{11}K - 1\right)\hat{\omega},\\ F_{13} &= -\Gamma_{12}pK\hat{x}_3 + p\left(\Gamma_{11}K - 1\right)\hat{x}_4,\\ F_{21} &= \Gamma_{21}\frac{K}{T_r} - p\left(\Gamma_{22}K - 1\right)\hat{\omega},\\ F_{22} &= -\frac{1}{T_r} + \Gamma_{22}\frac{K}{T_r} + \Gamma_{21}pK\hat{\omega},\\ F_{23} &= p\left(1 - \Gamma_{22}K\right)\hat{x}_3 + \Gamma_{21}pK\hat{x}_4,\\ F_{31} &= \text{cst}\,x_2 + \Gamma_{31}\frac{K}{T_r} - pK\Gamma_{32}\hat{\omega},\\ F_{32} &= -\text{cst}\,x_1 + \Gamma_{32}\frac{K}{T_r} + pK\Gamma_{31}\hat{\omega},\\ F_{33} &= -pK\Gamma_{32}\hat{x}_3 + pK\Gamma_{32}\hat{x}_4.\end{aligned} \tag{33}$$

The strain tensor rate is

$$E = \begin{pmatrix} E_{11} & E_{12} & E_{13}\\ E_{21} & E_{22} & E_{23}\\ E_{31} & E_{32} & E_{33}\end{pmatrix} \tag{34}$$

with

$$\begin{aligned} E_{11} &= 2F_{11}, \\ E_{12} &= E_{21} = F_{12} + F_{21}, \\ E_{13} &= E_{31} = F_{13} + F_{31}, \\ E_{22} &= 2F_{22}, \\ E_{23} &= E_{32} = F_{23} + F_{32}, \\ E_{33} &= 2F_{33}. \end{aligned} \quad (35)$$

Exponential convergence of the reduced-order observer is guaranteed for a uniformly negative definite rate of strain tensor. The strain tensor rate is uniformly negative definite if and only if $\exists\beta > 0$ such that the following conditions are satisfied:

$$E_{11} \leq -\beta < 0,$$
$$\det\begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} \geq \beta > 0, \quad (36)$$
$$\det(E) \leq -\beta < 0.$$

4. Simulations Results

This section deals with simulations highlighting the proposed reduced-order metric observer's feasibility. The simulated induction motor ratings are the following: 4 kW, 4 poles, 1440 rpm, $R_s = 1.9\,\Omega$, $R_r = 1.73\,\Omega$, $L_s = 0.1157\,\text{H}$, $L_r = 0.1154\,\text{H}$, $L_m = 0.1126\,\text{H}$, and $J = 0.041\,\text{Kg}\cdot\text{N/m}^2$.

4.1. Simulation Results for the Reduced-Order Observer for Fluxes Estimation. Figures 1–3 show the real flux, its estimation, and the estimation error in a-axis ($\varphi_a - \widehat{\varphi}_a$), respectively. These results were obtained for an unloaded machine ($T_L = 0$), with an observer gain of $\Gamma = 0.05$.

Figures 4–6 illustrate the real flux, its estimation, and the estimation error in b-axis, respectively. These results are also obtained for the same values of the observer gain Γ and the load torque.

Figures 7 and 8 show flux errors in the (a, b) frame, respectively. These results were obtained for a load torque step change illustrated by Figure 9. In this case, Figure 10 shows the rotor speed. These results clearly show that the proposed reduced-order flux estimator is quite robust to external disturbances. Its robustness has also been checked versus parameter variations. Furthermore, it should be mentioned that the same observer gain Γ could be adopted for any load torque change.

4.2. Simulation Results of the Reduced-Order Observer for Rotor Fluxes and Speed Estimation. Figures 11 and 12 show the estimated rotor fluxes $\widehat{\varphi}_a$ and $\widehat{\varphi}_b$, respectively. Figures 13–15 illustrate the corresponding real rotor speed, its estimate, and the speed error, respectively. These results were obtained for $\Gamma_1 = \Gamma_2 = \Gamma_3 = [0.8710^{-5} \ \ 10^{-6}]^T$ and a load torque $T_L = 5\,\text{Nm}$.

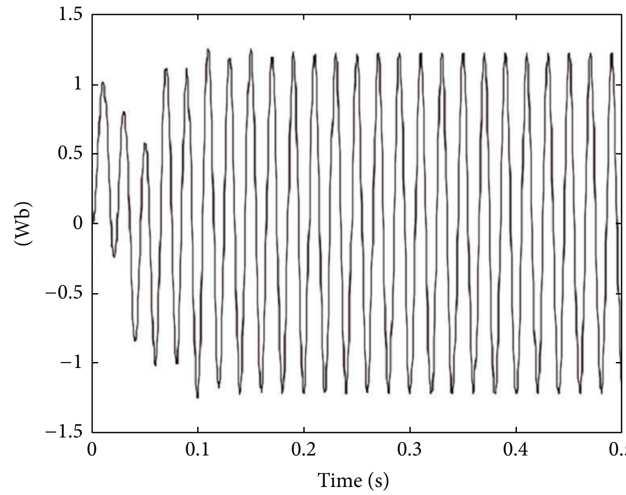

Figure 1: Real flux φ_a.

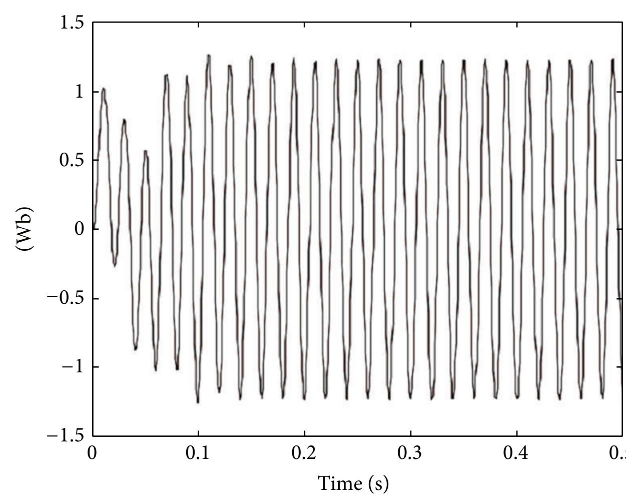

Figure 2: Estimated flux $\widehat{\varphi}_a$.

The proposed nonlinear observer obviously gives quite good estimation results. However, in case of load torque variations, the observer gains should be adjusted.

5. Conclusion

This paper has investigated the effectiveness of metric observers for induction motor control purposes. Firstly, assuming that stator currents and speed are measured, a metric observer was designed to estimate rotor fluxes. Afterward, assuming that only stator currents are measured, a metric observer was derived to estimate rotor fluxes and speed. The achieved simulation results on a 4 kW induction motor drive have clearly highlighted the effectiveness of the two proposed observers. Further investigations should be carried out for comparison purposes.

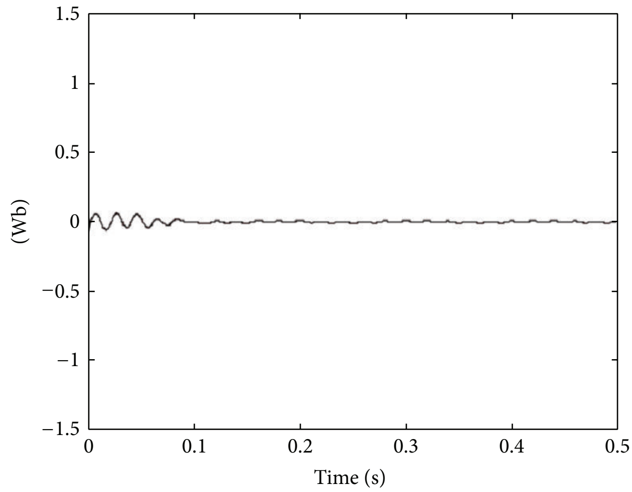

FIGURE 3: Flux error in a-axis.

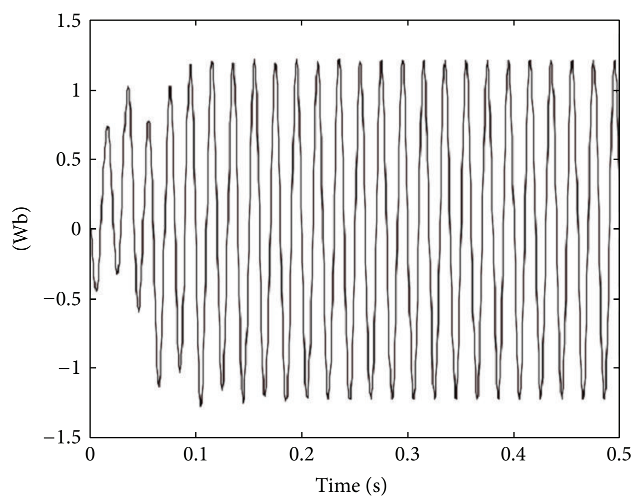

FIGURE 4: Real flux φ_b.

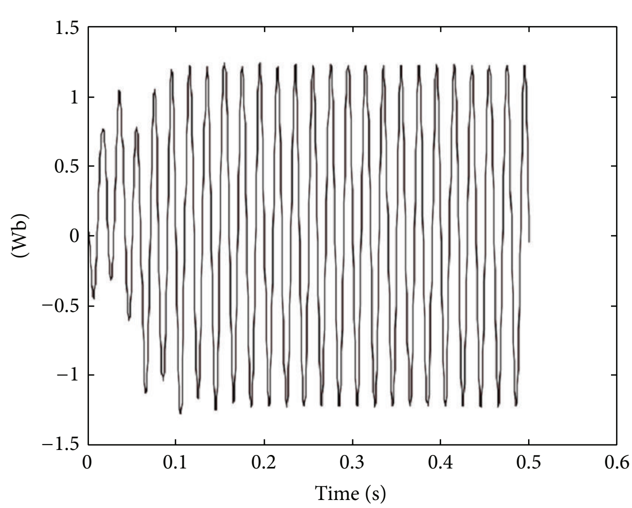

FIGURE 5: Estimated flux $\widehat{\varphi}_b$.

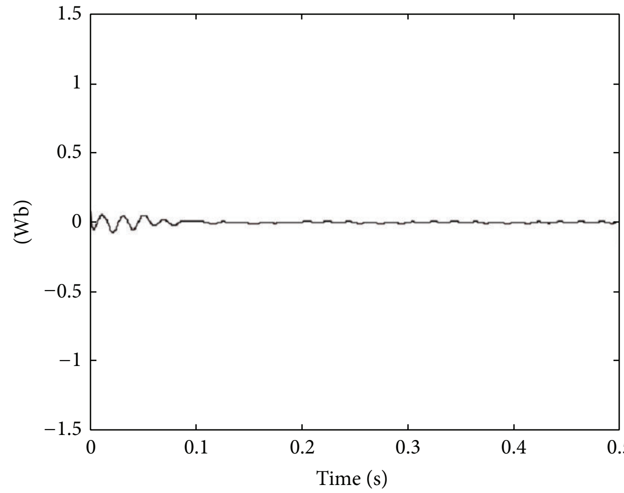

FIGURE 6: Flux error in b-axis.

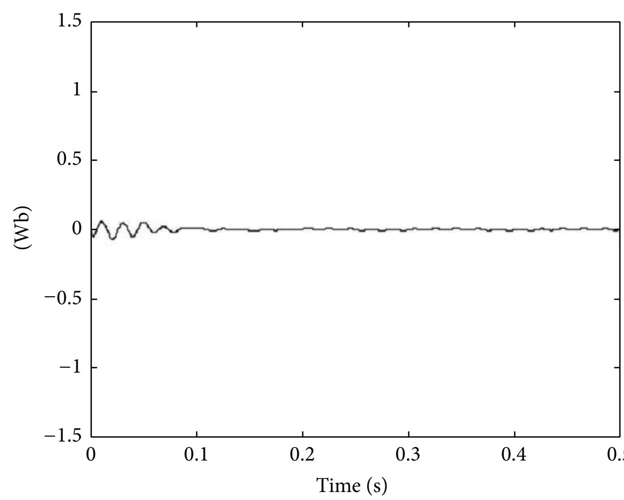

FIGURE 7: Flux estimation error in a-axis.

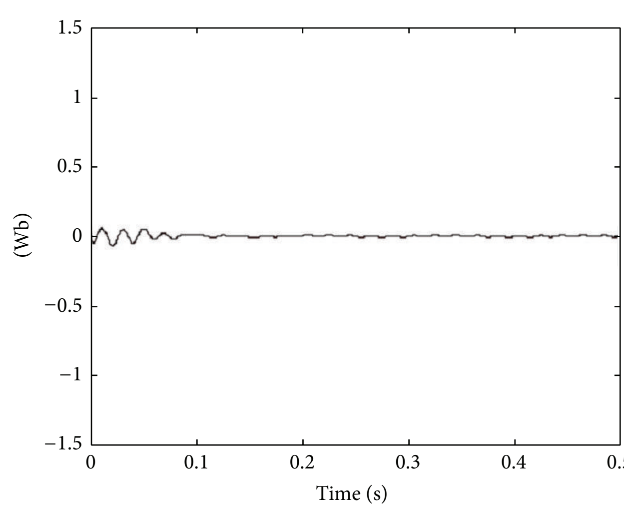

FIGURE 8: Flux estimation error in b-axis.

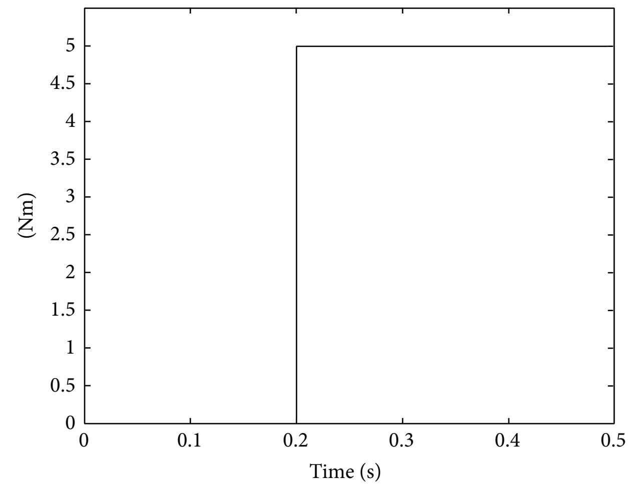

Figure 9: Load torque step change.

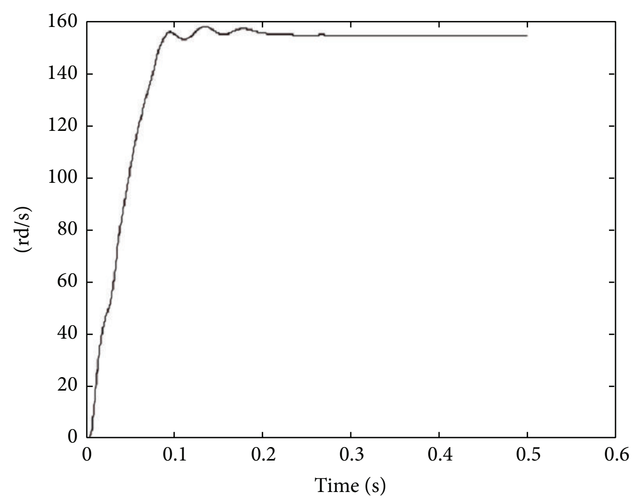

Figure 10: Rotor speed.

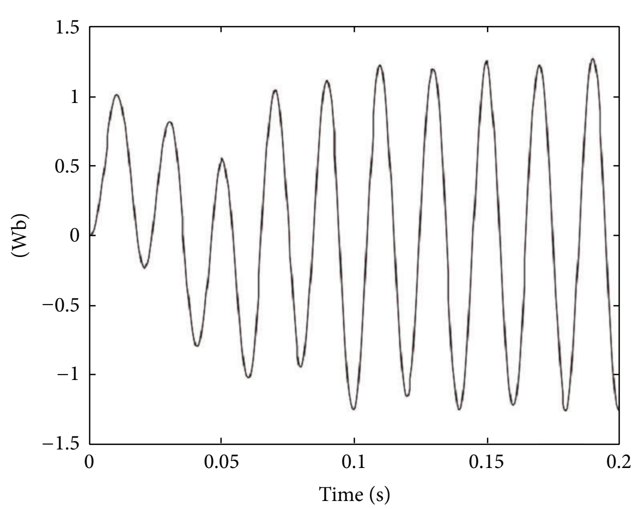

Figure 11: Estimated flux $\hat{\varphi}_a$.

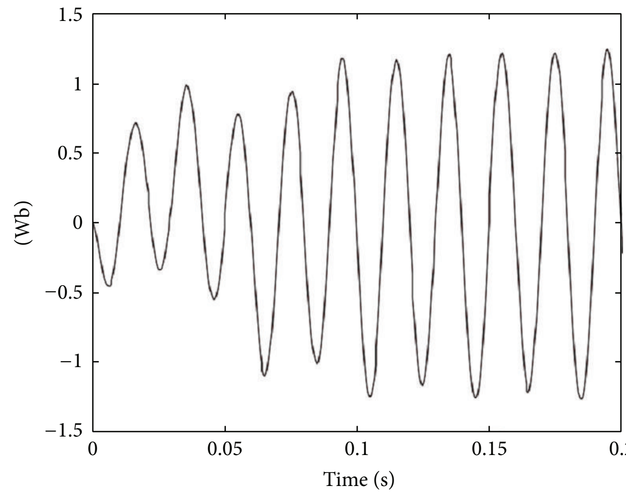

Figure 12: Estimated flux $\hat{\varphi}_b$.

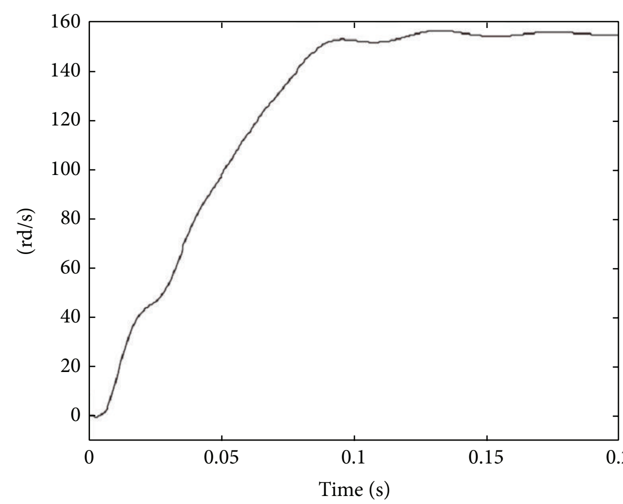

Figure 13: Real rotor speed.

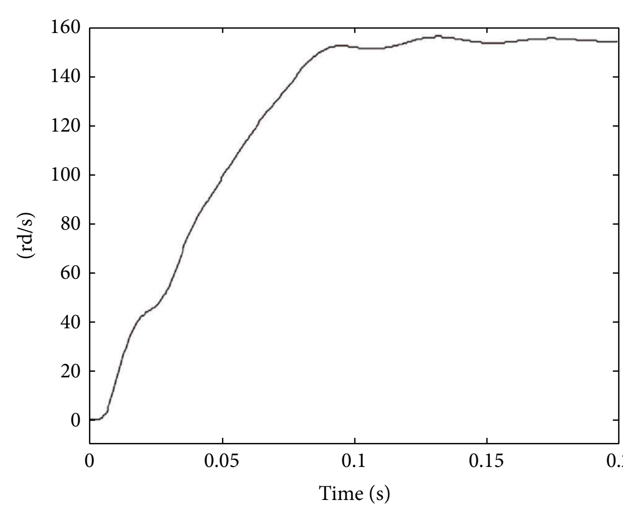

Figure 14: Estimated rotor speed.

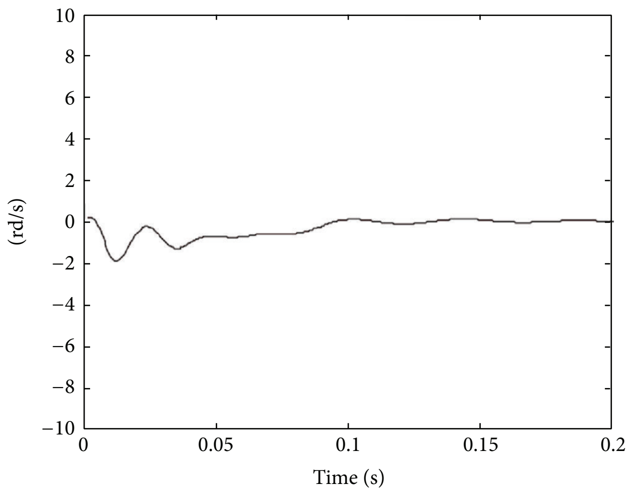

FIGURE 15: Rotor speed estimation error.

Nomenclature

i_a, i_b:	Stator currents
u_a, u_b:	Stator voltages
φ_a, φ_b:	Rotor fluxes
T_L:	Load torque
ω:	Mechanical speed
L_s, R_s:	Stator inductance and resistance
L_r, R_r:	Rotor inductance and resistance
M:	Mutual inductance
$T_r = L_r/R_r$:	Rotor time constant
p:	Pole pairs number
J:	Rotor inertia.

Competing Interests

The authors declare that they have no competing interests.

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