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Research Article

H_∞ Excitation Control Design for Stochastic Power Systems with Input Delay Based on Nonlinear Hamiltonian System Theory

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This paper presents H_∞ excitation control design problem for power systems with input time delay and disturbances by using nonlinear Hamiltonian system theory. The impact of time delays introduced by remote signal transmission and processing in wide-area measurement system (WAMS) is well considered. Meanwhile, the systems under investigation are disturbed by random fluctuation. First, under prefeedback technique, the power systems are described as a nonlinear Hamiltonian system. Then the H_∞ excitation controller of generators connected to distant power systems with time delay and stochasticity is designed. Based on Lyapunov functional method, some sufficient conditions are proposed to guarantee the rationality and validity of the proposed control law. The closed-loop systems under the control law are asymptotically stable in mean square independent of the time delay. And we through a simulation of a two-machine power system prove the effectiveness of the results proposed in this paper.

1. Introduction

Time delay always exists in power systems control area. It is often ignored when controller is mainly applied in local systems where the communication time delay is very small compared to the system time constants (see, e.g., [1, 2] and the references therein). Due to the further study of phase measurement unit (PMU) and WAMS, coordinated stability control has got a lot of attention. It uses remote measuring information given by WAMS/PMU. Unlike the small delay in local control, the time delay in wide-area power systems can vary from tens to several hundred milliseconds or more. Since that the large time delay will go against the stability of the system and reduce the performance of the system, so it is very necessary to consider the influence of it on the power system stability analysis and controller design. Besides, the generators are interfered with speed regulation, fluctuation of load, mechanical torsional vibration, the changes of damping coefficients, and so on in the transient process. These random fluctuations can be regarded as a kind of random process [3]. However, the application of the Itô differential formula will lead to the appearance of gravitation and the Hessian term. What is more, the stochastic disturbance (Wiener process) will cause no definition of the system states' derivative [4]. Therefore, stochastic and delay factors increase the difficulties of the analysis and synthesis [5]. Some results, which took signal transmission time delays or stochasticity in power systems into account, have been obtained. Reference [6] presented a free-weighting matrix method based on linear control design approach for the wide-area robust damping controller associated with flexible alternating current transmission system device to improve the dynamical performance of the large-scale power systems. Reference [7] proposed a delay-independent decentralized coordinated robust approach to design excitation controller in terms of H_∞ optimization method incorporating linear matrix inequality (LMI) technique. Considering the nonlinear effects of randomized torsional oscillation on the excitation regulation dynamic process of a generator rotor and exploiting Monte-Carlo principle and numerical methods, the algorithms and workflow of the proposed excitation control system's transient stability analysis approach were presented in [3]. Reference [8] presented a stochastic cost model and a solution technique for optimal scheduling of the generators in a wind integrated power system considering the demand and wind generation uncertainties.

Based on the linearization at steady state operating point, lots of the techniques are by far achieved and applied to controller design in power systems. These techniques have some disadvantages, such as ignoring some nonlinearities of the system and just expressing the partial structures of the system. What is more, the designed controllers are generally relatively complicated and not very easy to realize online operation. Therefore, some nonlinear methods should be worked out to achieve good control performance for the power systems in consideration of time-delay, stochastics, and disturbances. In recent years, energy-based Lyapunov function method has obtained numerous attention, and remarkable achievements have been reached with this method in the analysis and synthesis of nonlinear systems, as well as in the power systems (see, e.g., [9–13] and the references therein). The method can thoroughly take advantage of the internal structural properties of the systems and make the control design relatively simple. An important step in using energy-based control strategy is to transform the system into a dissipative Hamiltonian system formulation. This kind of system, proposed by [14], has great benefits for that its Hamilton function can be used as the sum of potential energy (excluding gravitational potential energy) and kinetic energy in physical systems and also can be taken as a Lyapunov function (see, e.g., [11, 15–18]). Using the energy-based Hamilton function method, [11] investigated the adaptive H_∞ excitation control of multimachine power systems with disturbances. Simulations show that the control strategy proposed in [11] was more effective than some other control schemes. Considering the impact of time delays in acquisition and transmission of key signals in power systems, [19] deals with the H_∞ excitation control problem of n-machine power system with time-delay and disturbances.

The purpose of this paper is to present a suitable controller structure for the stochastic power systems with input delay and disturbances using the nonlinear Hamiltonian system theory in order to weaken the impact of stochasticity and delay on the control performance of the power systems. Firstly, the prefeedback with delay method is to be used to describe the system as a dissipative Hamiltonian system formulation. Next, based on the obtained new system formulation, we will deal with the H_∞ control problem by using Newton-Leibniz formula, a few properties of norm and matrices. The main results will be proposed for the Hamiltonian system and the power system as well. Finally, we will test and verify the obtained results in this paper by an example of a two-machine power system with delay, stochasticity, and disturbances.

The rest of the paper is organized as follows. Section 2 provides the problem formulation, nonlinear Hamilton realization and some preliminaries. Section 3 gives the main results. Analysis of the achieved results by a two-machine power system example and the conclusion are given in Sections 4 and 5, respectively.

Notations. Throughout the paper the superscript "T" stands for matrix transposition. $\mathscr{R}$ denotes the set of real numbers, $\mathscr{R}_+$ the set of all nonnegative real numbers, $\mathscr{R}^n$ the n-dimensional Euclidean space, and $\mathscr{R}^{n\times m}$ the real matrices with dimension $n \times m$. Diag$\{\cdots\}$ stands for diagonal matrix in which the diagonal elements are the elements in $\{\cdots\}$; $\|\cdot\|$ stands for either the Euclidean vector norm or the induced matrix 2-norm. For any symmetric matrices X and Y, $X \geq Y$ (resp., $X > Y$) means that the matrix X-Y is positive semidefinite (resp., positive definite). tr$[X]$ denotes the trace for square matrix X. $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$) denotes the maximum (minimum) of eigenvalue of a real symmetric matrix P. $\mathscr{C}_{n,\tau} = \mathscr{C}([-\tau, 0], \mathscr{R}^n)$ means the Banach space of continuous functions from $[-\tau, 0]$ to $\mathscr{R}^n$. $C^b_{\mathscr{F}_0}([-\tau, 0]; \mathscr{R}^n)$ denotes the family of all $\mathscr{F}_0$-measurable bounded $C([-\tau, 0]; \mathscr{R}^n)$-valued random variables $\phi = \{\phi(t) : t \in [-\tau, 0]\}$. C^i denotes the set of all functions with continuous ith partial derivatives; $C^{2,1}$ is the family of all functions which are C^2 in the first argument and C^1 in the second argument; $C^{2,1}(\mathscr{R}^n \times [-\tau, \infty); \mathscr{R}_+)$ stands for the family of all nonnegative functions $V(x, t)$ on $\mathscr{R}^n \times [-\tau, \infty)$ which are C^2 in x and C^1 in t. What is more, for the sake of simplicity, throughout the paper, we denote $\partial H/\partial x$ by ∇H.

2. Problem Formulation and Nonlinear Hamilton Realization

Consider the following n-machine power systems, each generator of which is described by a third-order dynamic model (see [1, 20]):

$$\begin{aligned}
\dot{\delta}_i &= \omega_i - \omega_0, \\
\dot{\omega}_i &= \frac{\omega_0}{M_i} P_{mi} - \frac{D_i}{M_i}(\omega_i - \omega_0) - \frac{\omega_0}{M_i} P_{ei} + \epsilon_{i1}, \\
\dot{E}'_{qi} &= -\frac{1}{T_{d0i}} E_{qi} + \frac{1}{T_{d0i}} u_{fi}(t) + \epsilon_{i2},
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
E_{qi} &= E'_{qi} + I_{di}\left(x_{di} - x'_{di}\right), \\
I_{di} &= B_{ii} E'_{qi} - \sum_{j=1, j\neq i}^{n} B_{ij} E'_{qj} \cos\left(\delta_i - \delta_j\right), \\
P_{ei} &= G_{ii} E'^2_{qi} + E'_{qi} \sum_{j=1, j\neq i}^{n} B_{ij} E'_{qj} \sin\left(\delta_i - \delta_j\right), \\
& \qquad i = 1, 2, \ldots, n,
\end{aligned} \tag{2}$$

δ_i is the power angle of the ith generator (radians), ω_i is the rotor speed of the ith generator (rad/s), $\omega_0 = 2\pi f_0$, E'_{qi} is the q-axis internal transient voltage of the ith generator (per unit), x_{di} is the d-axis transient reactance (per unit), x'_{di} is the d-axis transient reactance of the ith generator (per unit),

u_{fi} is the voltage of the field circuit of the ith generator, the control input (per unit), M_i is the inertia coefficient of the ith generator (s), D_i is the damping constant (per unit), T_{d0i} is the field circuit time constant (s), P_{mi} is the mechanical power, assumed to be constant (per unit), P_{ei} is the active electrical power (per unit), ϵ_{i1} and ϵ_{i2} are bounded disturbances, I_{di} is the d-axis current (per unit), E_{qi} is the internal voltage (per unit), $Y_{ij} = G_{ij} + jB_{ij}$ is the admittance of line i-j (per unit), and $Y_{ii} = G_{ii} + jB_{ii}$ is the self-admittance of bus i (per unit).

There are signal transmission delays and random process in the modern power systems. The delays in the measuring data exist in such case that the exciter inputs are taken from remote buses. And assume that all the feedback wide-area signals have the time delay τ. Meanwhile, the generator torque can be regarded as a kind of random process because of random fluctuation in transient process, such as speed regulation, fluctuation of load, mechanical torsional vibration, and the changes of damping coefficients. Moreover, considering the imaginary control input is u_{fi} which feeds back both the local measurement information and the wide-area measurement signals, so the power system (1) should be modeled into differential-algebraic equations with time delay and stochasticity as follows:

$$
\begin{aligned}
d\delta_i &= (\omega_i - \omega_0)\, dt, \\
d\omega_i &= \left[\frac{\omega_0}{M_i} P_{mi} - \frac{D_i}{M_i}(\omega_i - \omega_0) - \frac{\omega_0}{M_i} P_{ei} + \epsilon_{i1}\right] dt \\
&\quad + \frac{\xi}{M_i}(\omega_i - \omega_0)\, dw(t), \\
dE'_{qi} &= \left[-\frac{1}{T_{d0i}} E_{qi} + \frac{1}{T_{d0i}} u_{fi}(t-\tau) + \epsilon_{i2}\right] dt,
\end{aligned} \tag{3}
$$

where ξ is random disturbance intensity and $w(t)$ is a zero-mean Wiener process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relative to an increasing family $(\mathcal{F}_t)_{t>0}$ of σ algebras $(\mathcal{F}_t)_{t>0} \subset \mathcal{F}$; here Ω is the samples space, $\mathcal{F}$ is σ algebra of subsets of the sample space, and P is the probability measure on $\mathcal{F}$. Moreover, we assume $E\{dw(t)\} = 0$, $E\{[dw(t)]^2\} = dt$, where E is the expectation operator.

Assume that $(\delta_i^{(0)}, \omega_0, E_{qi}'^{(0)})$, $i = 1, 2, \ldots, n$, are the preassigned operating points of system (3).

Setting $x_{i1} = \delta_i$, $x_{i2} = \omega_i - \omega_0$, $x_{i3} = E'_{qi}$, $(\omega_0/M_i)P_{mi} = a_i$, $D_i/M_i = b_i$, $(\omega_0/M_i)G_{ii} = c_i$, $\omega_0/M_i = d_i$, $1/T_{d0i} = e_i$, $(x_{di} - x'_{di})/T_{d0i} = h_i$, and $(1/T_{d0i})u_{fi}(t-\tau) = v_i(t-\tau)$, $i = 1, 2, \ldots, n$, then system (3) can be rewritten as follows:

$$
\begin{aligned}
dx_{i1} &= x_{i2} dt, \\
dx_{i2} &= \Bigg[a_i - b_i x_{i2} - c_i x_{i3}^2 + \epsilon_{i1} \\
&\quad - d_i x_{i3} \sum_{j=1, j\neq i}^{n} B_{ij} x_{j3} \sin\left(x_{i1} - x_{j1}\right) \Bigg] dt + \frac{\xi}{M_i} \\
&\quad \cdot x_{i2} dw(t), \\
dx_{i3} &= \Bigg[-\left(e_i + h_i B_{ii}\right) x_{i3} + v_i(t-\tau) + \epsilon_{i2} \\
&\quad + h_i \sum_{j=1, j\neq i}^{n} B_{ij} x_{j3} \cos\left(x_{i1} - x_{j1}\right) \Bigg] dt, \\
&\qquad i = 1, 2, \ldots, n.
\end{aligned} \tag{4}
$$

Inspired by [11], we introduce a prefeedback control law:

$$
\begin{aligned}
v_i(t-\tau) &= -\frac{2c_i h_i}{d_i} x_{i1}(t-\tau) x_{i3}(t-\tau) - k_i x_{i3}(t-\tau) \\
&\quad + \overline{u}_i + u_i(t-\tau), \quad i = 1, 2, \ldots, n,
\end{aligned} \tag{5}
$$

where the first term is to make system (4) have a Hamilton structure, the second and third terms are to guarantee the operating point of the system unchanged, $u_i(t-\tau)$ is the new reference input, and $\overline{u}_i$ and k_i are undetermined constants. To make the operating point of the system invariant, $\overline{u}_i$ and k_i have to satisfy

$$
\begin{aligned}
&-\left(e_i + h_i B_{ii}\right) E_{qi}'^{(0)} - \frac{2c_i h_i}{d_i} \delta_i^{(0)} E_{qi}'^{(0)} - k_i E_{qi}'^{(0)} + \overline{u}_i \\
&\quad + h_i \sum_{j=1, j\neq i}^{n} B_{ij} E_{qj}'^{(0)} \cos\left(\delta_i^{(0)} - \delta_j^{(0)}\right) = 0, \\
&\qquad i = 1, 2, \ldots, n,
\end{aligned} \tag{6}
$$

and $k_i = k_{i0}$ which is spelled out in [11]; what is more, this reference provides a kind of choice of $\overline{u}_i$ and k_i.

Furthermore, (5) can be rewritten as

$$
\begin{aligned}
&v_i(t-\tau) \\
&\quad = -\frac{2c_i h_i}{d_i} x_{i1}(t) x_{i3}(t) - k_i x_{i3}(t) + \overline{u}_i \\
&\quad - \frac{2c_i h_i}{d_i} \left[x_{i1}(t-\tau) x_{i3}(t-\tau) - x_{i1}(t) x_{i3}(t)\right] \\
&\quad - k_i \left[x_{i3}(t-\tau) - x_{i3}(t)\right] + u_i(t-\tau), \\
&\qquad i = 1, 2, \ldots, n.
\end{aligned} \tag{7}
$$

Let $x_i = [x_{i1}, x_{i2}, x_{i3}]^{\mathrm{T}}$, $\epsilon_i = [\epsilon_{i1}, \epsilon_{i2}]^{\mathrm{T}}$, then system (4) can be expressed as a dissipative Hamiltonian system as follows:

$$
\begin{aligned}
dx_i &= \Bigg\{ \left(J_i - R_i\right) \nabla H_i(x_i) + g_1 u_i(t-\tau) + \frac{2c_i h_i}{d_i} \\
&\quad \cdot g_1 \left[g_3^{\mathrm{T}} x_{i1}(t) g_1^{\mathrm{T}} x_{i3}(t) \right. \\
&\quad \left. - g_3^{\mathrm{T}} x_{i1}(t-\tau) g_1^{\mathrm{T}} x_{i3}(t-\tau)\right] + k_i g_1 \left[g_1^{\mathrm{T}} x_{i3}(t) \right. \\
&\quad \left. - g_1^{\mathrm{T}} x_{i3}(t-\tau)\right] + g_2 \epsilon_i \Bigg\} dt + g_4^{(i)}(x_i)\, dw(t), \\
&\qquad i = 1, 2, \ldots, n,
\end{aligned} \tag{8}
$$

where

$$\begin{aligned}
H_i(x_i) &= -\frac{a_i}{d_i}x_{i1} + \frac{c_i}{d_i}x_{i1}x_{i3}^2 + \frac{e_i + h_iB_{ii} + k_i}{2h_i}x_{i3}^2 \\
&\quad + \frac{1}{2d_i}x_{i2}^2 - \frac{\overline{u}_i}{h_i}x_{i3} \\
&\quad - x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos\left(x_{i1} - x_{j1}\right), \\
J_i &= \begin{pmatrix} 0 & d_i & 0 \\ -d_i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
R_i &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & b_id_i & 0 \\ 0 & 0 & h_i \end{pmatrix}, \\
g_1 &= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \\
g_2 &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
g_3 &= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \\
g_4^{(i)}(x_i) &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\xi}{M_i} & 0 \\ 0 & 0 & 0 \end{pmatrix} x_i.
\end{aligned} \tag{9}$$

$\nabla H_i(x_i)$ is the gradient of the Hamilton function $H_i(x_i)$, which satisfies $H_i(0) = 0$, $i = 1, 2, \ldots, n$.

Owing to each individual subsystem having the cross-variables, this structure does not provide the overall system a Hamilton structure. Thus, we need to find out a common Hamilton function for the n generators, which is regarded as the total energy of the whole system.

Let

$$\begin{aligned}
H(x) &= \sum_{i=1}^{n} H_i + \frac{1}{2}\sum_{i=1}^{n} x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos\left(x_{i1} - x_{j1}\right) \\
&= \sum_{i=1}^{n}\left[-\frac{a_i}{d_i}x_{i1} + \frac{c_i}{d_i}x_{i1}x_{i3}^2 + \frac{1}{2d_i}x_{i2}^2\right. \\
&\quad + \frac{e_i + h_iB_{ii} + k_i}{2h_i}x_{i3}^2 - \frac{\overline{u}_i}{h_i}x_{i3} \\
&\quad \left. - \frac{1}{2}x_{i3}\sum_{j=1,j\neq i}^{n} B_{ij}x_{j3}\cos\left(x_{i1} - x_{j1}\right)\right],
\end{aligned} \tag{10}$$

where $x = [x_1^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$. By using relation $B_{ij} = B_{ji}$, we can verify that

$$\frac{\partial H(x)}{\partial x_{ij}} = \frac{\partial H_i(x_i)}{\partial x_{ij}}, \quad i = 1, 2, \ldots, n, \; j = 1, 2, 3, \tag{11}$$

which imply that $H(x)$ is the common Hamilton function for the n generators. Furthermore, $H(x) \in C^2$ holds obviously.

Setting

$$\begin{aligned}
u &= \left[u_1, \ldots, u_n\right]^{\mathrm{T}}, \\
\epsilon &= \left[\epsilon_1^{\mathrm{T}}, \ldots, \epsilon_n^{\mathrm{T}}\right]^{\mathrm{T}}, \\
y &= \left[y_1^{\mathrm{T}}, \ldots, y_n^{\mathrm{T}}\right]^{\mathrm{T}},
\end{aligned} \tag{12}$$

then system (8) can be rewritten as follows:

$$\begin{aligned}
dx(t) &= \left\{(J - R)\nabla H(x) + G_1u(t-\tau)\right. \\
&\quad + 2G_1C\left[G_3^{\mathrm{T}}x(t)G_1^{\mathrm{T}}x(t) - G_3^{\mathrm{T}}x(t-\tau)G_1^{\mathrm{T}}(t-\tau)\right] \\
&\quad \left. + G_1KG_1^{\mathrm{T}}\left[x(t) - x(t-\tau)\right] + G_2\epsilon\right\}dt + G_4(x)\,dw(t), \\
x(t) &= \phi(t), \quad t \in [-\tau, 0],
\end{aligned} \tag{13}$$

where $J = \mathrm{Diag}\{J_1, \ldots, J_n\}$, $R = \mathrm{Diag}\{R_1, \ldots, R_n\}$, $C = \mathrm{Diag}\{c_1h_1/d_1, \ldots, c_nh_n/d_n\}$, $K = \mathrm{Diag}\{k_1, \ldots, k_n\}$,

$$\begin{aligned}
G_1 &= \begin{pmatrix} g_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_1 \end{pmatrix}_{3n\times n}, \\
G_2 &= \begin{pmatrix} g_2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_2 \end{pmatrix}_{3n\times 2n}, \\
G_3 &= \begin{pmatrix} g_3 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_3 \end{pmatrix}_{3n\times n}, \\
G_4(x) &= \begin{pmatrix} g_4^{(1)}(x_1) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & g_4^{(n)}(x_n) \end{pmatrix}_{3n\times n}.
\end{aligned} \tag{14}$$

Obviously, J is a skew-symmetric matrix, and R is a positive semidefinite matrix. In addition, we can choose $y = G_2^{\mathrm{T}}\nabla H(x)$ and $z = PG_1^{\mathrm{T}}\nabla H(x)$ as the output and the penalty signal, respectively, where P is a full column rank weighting matrix.

Definition 1. The stochastic time delay Hamiltonian system (13) is said to be robustly asymptotically stable in mean square, if there exists a controller $u(t-\tau)$ such that

$$\lim_{t\to\infty} E\left\{\left\|x(t) - x_0\right\|^2\right\} = 0, \tag{15}$$

where x_0 is the preassigned equilibrium and $x(t)$ is the solution of system (13) at time t under initial condition.

Consider the following cost function:

$$C(T_0) = E\left\{\int_0^{T_0} z^{\mathrm{T}}(t)\, z(t)\, dt\right\} - \gamma^2 E\left\{\int_0^{T_0} \epsilon^{\mathrm{T}}(t)\, \epsilon(t)\, dt\right\}, \quad \forall T_0 > 0. \tag{16}$$

Then H_∞ control objective of system (13) is to find a feedback controller:

$$u(t-\tau) = \alpha(t-\tau) \tag{17}$$

such that

$$C(\infty) < 0 \quad (T_0 \longrightarrow \infty), \tag{18}$$

for given $\gamma > 0$ and at the same time the closed-loop system is asymptotically stable when $\epsilon = 0$.

We conclude this section by recalling some auxiliary results to be used in this paper.

Lemma 2 (see [21]). *For system*

$$dx(t) = f(x(t), x(t-\tau))\, dt + g(x(t), x(t-\tau))\, dw(t), \quad \forall t \geq 0, \tag{19}$$

assume that $f(x,y)$ and $g(x,y)$ are locally Lipschitz in (x,y). If there exists a function $V(x,t) \in C^{2,1}(\mathscr{R}^n \times [-\tau,\infty); \mathscr{R}_+)$ such that for some constant $K > 0$ and any $t \geq 0$,

$$\mathscr{L}V \leq K(1 + V(x(t),t) + V(x(t-\tau), t-\tau)), \qquad \lim_{|x|\to\infty} \inf_{t\geq 0} V(x,t) = \infty, \tag{20}$$

where the differential operator $\mathscr{L}$ is defined as

$$\mathscr{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x(t), x(t-\tau)) + \frac{1}{2} \cdot tr\left\{g^{\mathrm{T}}(x(t), x(t-\tau)) \frac{\partial^2 V}{\partial x^2} g(x(t), x(t-\tau))\right\}, \tag{21}$$

then there exists a unique solution on $[-\tau,\infty)$ for any initial data $\{x(t) = \phi(t) : t \in [-\tau, 0]\} \in C^b_{\mathscr{F}_0}([-\tau,0]; \mathscr{R}^n)$.

Lemma 3. *For any given matrices $A \in \mathscr{R}^{n\times r}$ and $B \in \mathscr{R}^{n\times r}$, there holds*

$$tr\left(A^{\mathrm{T}}B\right) \leq \frac{1}{2}\left[tr\left(A^{\mathrm{T}}A\right) + tr\left(B^{\mathrm{T}}B\right)\right]. \tag{22}$$

Proof. This proof can be achieved by using the properties of matrix's trace. □

3. Main Results

3.1. Hamiltonian System. The H_∞ controller is given below for the stochastic Hamiltonian system (13) with input delay.

Theorem 4. *Consider system (13) and the following assumptions are satisfied:*

(A1) $\nabla H(x_0) = 0$;

(A2) $Hess(H(x_0)) > 0$;

(A3) $H(x) - H(x_0) \geq (\alpha_1/2)\|x - x_0\|^2$;

(A4) $\nabla^{\mathrm{T}} H(x) \cdot \nabla H(x) \geq \beta_1 \|x - x_0\|^2$.

If

$$2R + \frac{1}{\gamma^2} G_1 G_1^{\mathrm{T}} - \frac{1}{\gamma^2} G_2 G_2^{\mathrm{T}} \geq 0 \tag{23}$$

holds, then the H_∞ control problem of system (13) can be solved by the feedback control law:

$$\begin{aligned} u(t-\tau) = &-\frac{1}{2}\left(\frac{1}{\gamma^2} G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right) \nabla H(x(t-\tau)) \\ &- \frac{1}{4}\left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5 \nabla H(x(t-\tau)) \\ &+ 2CX(t-\tau) + KG_1^{\mathrm{T}} x(t-\tau) - M - 2N \\ &- \frac{1}{2}\tau\lambda_1\lambda_2 T - \frac{1}{4}\tau\lambda_1\lambda_2 \left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5, \end{aligned} \tag{24}$$

where x_0 is the preassigned equilibrium of system (13), $G_5 = \mathrm{Diag}\{g_5^{(1)}, \ldots, g_5^{(n)}\}$,

$$g_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\left(d_i^2 + 1\right)\epsilon^2}{M_i^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3\times 3}, \qquad X(t) = \begin{pmatrix} x_{11}(t)\, x_{13}(t) \\ x_{21}(t)\, x_{23}(t) \\ \ddots \\ x_{n1}(t)\, x_{n3}(t) \end{pmatrix}_{n\times 1}, \tag{25}$$

M, N, T are all positive constant matrices which satisfy $\|M\| \geq \|KG_1^{\mathrm{T}} x(t)\|$, $\|N\| \geq \|CX(t)\|$, $\|T\| \geq \|(1/\gamma^2) G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\|$, and λ_1 and λ_2 are constants which satisfy $\lambda_1 \geq \sup_{t\geq-\tau} \|Hess(H(x(t)))\|$, $\lambda_2 \geq \sup_{t\geq-\tau} \|\dot{x}(t)\|$.

Proof. Take a Lyapunov candidate function as follows:

$$V(x) = 2H(x) - 2H(x_0). \tag{26}$$

According to Itô differential formula, it follows that

$$dV(x) = \mathscr{L}V(x)\, dt + \nabla V(x)\, G_4(x)\, dw(t). \tag{27}$$

According to (21) in Lemma 2, one has

$$\begin{aligned}
&\mathscr{L}V(x) \\
&= \frac{1}{2}\operatorname{tr}\left\{g^{\mathrm{T}}(x(t), x(t-\tau))\frac{\partial^2 V}{\partial x^2} g(x(t), x(t-\tau))\right\} \\
&\quad + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x(t), x(t-\tau)) \\
&= \operatorname{tr}\left[G_4^{\mathrm{T}}(x)\operatorname{Hess}(H(x)) G_4(x)\right] \\
&\quad + 2\nabla^{\mathrm{T}} H(x)(J-R)\nabla H(x) \\
&\quad + 2\nabla^{\mathrm{T}} H(x) G_1 K G_1^{\mathrm{T}}[x(t) - x(t-\tau)] \\
&\quad + 4\nabla^{\mathrm{T}} H(x) G_1 C[X(t) - X(t-\tau)] \\
&\quad + 2\nabla^{\mathrm{T}} H(x) G_2 \epsilon \\
&\quad - \nabla^{\mathrm{T}} H(x) G_1\left(\frac{1}{\gamma^2}G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right)\nabla H(t-\tau) \\
&\quad - \frac{1}{2}\nabla^{\mathrm{T}} H(x) G_1\left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5 \nabla H(t-\tau) \\
&\quad + 4\nabla^{\mathrm{T}} H(x) G_1 C X(t-\tau) \\
&\quad + 2\nabla^{\mathrm{T}} H(x) G_1 K G_1^{\mathrm{T}} x(t-\tau) \\
&\quad - 2\nabla^{\mathrm{T}} H(x) G_1 (M + 2N) - \tau\lambda_1\lambda_2\nabla^{\mathrm{T}} H(x) G_1 T \\
&\quad - \frac{1}{2}\tau\lambda_1\lambda_2\nabla^{\mathrm{T}} H(x) G_1\left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5.
\end{aligned} \tag{28}$$

Based on the facts of Lemma 3 and Condition (22), we can achieve

$$\begin{aligned}
&\operatorname{tr}\left[G_4^{\mathrm{T}}(x)\operatorname{Hess}(H(x)) G_4(x)\right] \\
&\quad \le \frac{1}{2}\operatorname{tr}\left[G_4^{\mathrm{T}}(x)\operatorname{Hess}(H(x))\operatorname{Hess}^{\mathrm{T}}(H(x)) G_4(x)\right] \\
&\qquad + \frac{1}{2}\operatorname{tr}\left[G_4^{\mathrm{T}}(x) G_4(x)\right] = \nabla^{\mathrm{T}} H(x) G_5 \nabla H(x).
\end{aligned} \tag{29}$$

According to Newton-Leibniz formula, it follows that

$$\nabla H(x_\tau) = \nabla H(x) - \int_{t-\tau}^{t}\operatorname{Hess}(H(x(s)))\dot{x}(s)\,ds. \tag{30}$$

Therefore, the following equalities hold:

$$\begin{aligned}
&- \nabla^{\mathrm{T}} H(x) G_1\left(\frac{1}{\gamma^2}G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right)\nabla H(x(t-\tau)) \\
&\quad = -\nabla^{\mathrm{T}} H(x) G_1\left(\frac{1}{\gamma^2}G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right) \\
&\quad \cdot\left[\nabla H(x) - \int_{t-\tau}^{t}\operatorname{Hess}(H(x(s)))\dot{x}(s)\,ds\right], \\
&- \nabla^{\mathrm{T}} H(x) G_1\left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5 \nabla H(x(t-\tau)) \\
&\quad = -\nabla^{\mathrm{T}} H(x) G_1\left[G_1^{\mathrm{T}} G_1\right]^{-1} \\
&\quad \cdot G_1^{\mathrm{T}} G_5\left[\nabla H(x) - \int_{t-\tau}^{t}\operatorname{Hess}(H(x(s)))\dot{x}(s)\,ds\right].
\end{aligned} \tag{31}$$

According to the Mean Value Theorem of Integrals, there exists $\theta \in [t-\tau, t]$ that satisfies

$$\begin{aligned}
&\int_{t-\tau}^{t}\operatorname{Hess}(H(x(s)))\dot{x}(s)\,ds \\
&\quad = \tau\operatorname{Hess}(H(x(\theta)))\dot{x}(\theta).
\end{aligned} \tag{32}$$

Consequently, we have

$$\begin{aligned}
&\nabla^{\mathrm{T}} H(x) G_1\left(\frac{1}{\gamma^2}G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right) \\
&\quad \cdot\int_{t-\tau}^{t}\operatorname{Hess}(H(x(s)))\dot{x}(s)\,ds = \tau\nabla^{\mathrm{T}} H(x) \\
&\quad \cdot G_1\left(\frac{1}{\gamma^2}G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right)\operatorname{Hess}(H(x(\theta)))\dot{x}(\theta) \\
&\quad \le \tau\lambda_1\lambda_2\nabla^{\mathrm{T}} H(x) G_1 T.
\end{aligned} \tag{33}$$

Similarly, we further obtain

$$\begin{aligned}
&\frac{1}{2}\nabla^{\mathrm{T}} H(x) G_1\left(G_1^{\mathrm{T}} G_1\right)^{-1} \\
&\quad \cdot G_1^{\mathrm{T}} G_5\int_{t-\tau}^{t}\operatorname{Hess}(H(x(s)))\dot{x}(s)\,ds = \frac{1}{2}\tau\nabla^{\mathrm{T}} H(x) \\
&\quad \cdot G_1\left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5\operatorname{Hess}(H(x(\theta)))\dot{x}(\theta) \le \frac{1}{2} \\
&\quad \cdot \tau\lambda_1\lambda_2\nabla^{\mathrm{T}} H(x) G_1\left(G_1^{\mathrm{T}} G_1\right)^{-1} G_1^{\mathrm{T}} G_5.
\end{aligned} \tag{34}$$

Combining the above inequalities, we can conclude that

$$\begin{aligned}
&\mathscr{L}V(x) \\
&\quad \le -2\nabla^{\mathrm{T}} H(x) R\nabla H(x) + 2\nabla^{\mathrm{T}} H(x) G_2\epsilon \\
&\qquad - \nabla^{\mathrm{T}} H(x) G_1\left(\frac{1}{\gamma^2}G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}}\right)\nabla H(x) \\
&\quad = -2\nabla^{\mathrm{T}} H(x) R\nabla H(x) - \left\|\gamma\epsilon - \frac{1}{\gamma}G_2^{\mathrm{T}}\nabla H(x)\right\|^2 \\
&\qquad - \frac{1}{\gamma^2}\nabla^{\mathrm{T}} H(x) G_1 G_1^{\mathrm{T}}\nabla H(x) + \left(\gamma^2\epsilon^{\mathrm{T}}\epsilon - z^{\mathrm{T}} z\right) \\
&\qquad + \frac{1}{\gamma^2}\nabla^{\mathrm{T}} H(x) G_2 G_2^{\mathrm{T}}\nabla H(x) \\
&\quad \le -\nabla^{\mathrm{T}} H(x)\left(2R + \frac{1}{\gamma^2}G_1 G_1^{\mathrm{T}} - \frac{1}{\gamma^2}G_2 G_2^{\mathrm{T}}\right)\nabla H(x) \\
&\qquad + \left(\gamma^2\epsilon^{\mathrm{T}}\epsilon - z^{\mathrm{T}} z\right).
\end{aligned} \tag{35}$$

Taking (23) into account, it yields

$$\mathscr{L}V(x) \le \gamma^2 \epsilon^{\mathrm{T}} \epsilon - z^{\mathrm{T}} z. \tag{36}$$

Integrating (36) from 0 to T_0 leads to (18) which holds as $T_0 \to \infty$.

Next step we prove the closed-loop system where system (13) under the control law (24) is asymptotically stable in mean square when $\epsilon = 0$.

When $\epsilon = 0$, from (35), we can easily get that

$$\begin{aligned} &\mathscr{L}V(x) \\ &\le -2\nabla^{\mathrm{T}} H(x) R \nabla H(x) \\ &\quad - \nabla^{\mathrm{T}} H(x) G_1 \left(\frac{1}{\gamma^2} G_1^{\mathrm{T}} + P^{\mathrm{T}} P G_1^{\mathrm{T}} \right) \nabla H(x) \\ &= -\nabla^{\mathrm{T}} H(x) \left[2R + \frac{1}{\gamma^2} G_1 G_1^{\mathrm{T}} - \frac{1}{\gamma^2} G_2 G_2^{\mathrm{T}} \right] \nabla H(x) \\ &\quad - \frac{1}{\gamma^2} \nabla^{\mathrm{T}} H(x) G_2 G_2^{\mathrm{T}} \nabla H(x) \\ &\quad - \nabla^{\mathrm{T}} H(x) G_1 P^{\mathrm{T}} P G_1^{\mathrm{T}} \nabla H(x) \\ &\le -\nabla^{\mathrm{T}} H(x) \left(\frac{1}{\gamma^2} G_2 G_2^{\mathrm{T}} + G_1 P^{\mathrm{T}} P G_1^{\mathrm{T}} \right) \nabla H(x). \end{aligned} \tag{37}$$

Set

$$c_0 = \lambda_{\min} \left\{ \frac{1}{\gamma^2} G_2 G_2^{\mathrm{T}} + G_1 P^{\mathrm{T}} P G_1^{\mathrm{T}} \right\} > 0; \tag{38}$$

then we have

$$\mathscr{L}V(x) \le -c_0 \nabla^{\mathrm{T}} H(x) \nabla H(x). \tag{39}$$

Furthermore, owing to (A4) holding, there is

$$\mathscr{L}V(x) \le -c_0 \beta_1 \|x - x_0\|^2 \tag{40}$$

which implies

$$E\{\mathscr{L}V(x)\} \le -c_0 \beta_1 E\left\{\|x - x_0\|^2\right\}. \tag{41}$$

In addition, because of $E\{dw(t)\} = 0$, we further get

$$E\{dV(x)\} = E\{\mathscr{L}V(x)\}. \tag{42}$$

It is true that, for all $T > t_0$, $t_0 \in [-\tau, 0]$,

$$\begin{aligned} E\{V(T)\} - E\{V(t_0)\} &= \int_{t_0}^{T} E\{\mathscr{L}V(s)\}\, ds \\ &\le \int_{t_0}^{T} E\left\{-c_0 \beta_1 \|x(s) - x_0\|^2\right\} ds. \end{aligned} \tag{43}$$

Hence, one has

$$\frac{d}{dt} E\left\{\|x(T) - x_0\|^2\right\} \le -c_0 \beta_1 E\left\{\|x(T) - x_0\|^2\right\}. \tag{44}$$

From condition (A3), one has

$$\begin{aligned} &\alpha_1 \|x - x_0\|^2 \le V(x) = 2\left(H(X) - H(X_0)\right), \\ &E\left\{\alpha_1 \|x(T) - x_0\|^2\right\} \le E\{V(T)\}, \\ &\frac{d}{dt} E\left\{\alpha_1 \|x(T) - x_0\|^2\right\} \le \frac{d}{dt} E\{V(T)\} \\ &\qquad \le c_0 \beta_1 E\left\{\|x(T) - x_0\|^2\right\}. \end{aligned} \tag{45}$$

Set $c_1 = -c_0 \beta_1 / \alpha_1$; it follows that

$$\frac{d}{dt} E\left\{\|x(T) - x_0\|^2\right\} \le c_1 E\left\{\|x(T) - x_0\|^2\right\}. \tag{46}$$

Multiplying $e^{-c_1 T}$ to the two sides of inequality (44) yields

$$\begin{aligned} &e^{-c_1 T} \frac{d}{dt} E\left\{\|x(T) - x_0\|^2\right\} - e^{-c_1 T} c_1 E\left\{\|x(T) - x_0\|^2\right\} \\ &\le 0 \end{aligned} \tag{47}$$

which implies that

$$\frac{d}{dt}\left(e^{-c_1 T} E\left\{\|x(T) - x_0\|^2\right\}\right) \le 0. \tag{48}$$

Integrating inequality (48) from t_0 to T, we have

$$e^{-c_1 T} E\left\{\|x(T) - x_0\|^2\right\} - e^{-c_1 t_0} E\left\{\|x(t_0) - x_0\|^2\right\} \le 0; \tag{49}$$

that is,

$$\begin{aligned} E\left\{\|x(T) - x_0\|^2\right\} &\le e^{c_1 (T - t_0)} E\left\{\|x(t_0) - x_0\|^2\right\}, \\ &\qquad \forall T > t_0. \end{aligned} \tag{50}$$

Due to $c_1 < 0$, there is

$$\lim_{T \to \infty} E\left\{\|x(T) - x_0\|^2\right\} = 0. \tag{51}$$

According to Definition 1, we can conclude that system (13) under the control law (24) is robustly asymptotically stable in mean square with respect to x_0. This completes the proof. □

Remark 5. $\nabla H(x_0) = 0$ and $\mathrm{Hess}(H(x_0)) > 0$ guarantee that the equilibrium x_0 is the minimal point of $H(x)$. Moreover, in view of conditions (A1)–(A4), there hold $\nabla V(x_0) = 0$ and $\mathrm{Hess}(V(x_0)) > 0$, which together with $V(x_0) = 0$ lead to the fact that $V(x)$ is a positive definite function in some neighborhood of equilibrium x_0.

Remark 6. Owing to the fact of $H(x) \in C^2$, the solution of the closed-loop system (13) under the control law (24) is existent and unique on $[-\tau, \infty)$ for any initial data $\{x(t) = \phi(t) : t \in [-\tau, 0]\} \in C^b_{\mathscr{F}_0}([-\tau, 0]; \mathscr{R}^n)$ in some neighborhood of equilibrium x_0.

3.2. N-Machine Power System. In this subsection, we consider the n-machine power system (3).

First, we can verify that

$$H(x)=\sum_{i=1}^{n}\left\{-P_{mi}\delta_i+G_{ii}\delta_iE_{qi}'^2+\frac{M_i}{2\omega_0}(\omega_i-\omega_0)^2\right.$$
$$+\frac{1+(x_{di}-x_{di}')B_{ii}+k_iT_{d0i}}{2(x_{di}-x_{di}')}E_{qi}'^2-\frac{\bar{u}_iT_{d0i}}{x_{di}-x_{di}'}E_{qi}' \qquad (52)$$
$$\left.-\frac{1}{2}E_{qi}'\sum_{j=1,j\neq i}^{n}B_{ij}E_{qi}'\cos(\delta_i-\delta_j)\right\}\in C^2.$$

Choose the preassigned equilibrium

$$x_0=\left(\delta_i^{(0)},\omega_0,E_{qi}'^{(0)}\right),\quad i=1,2,\ldots,n \qquad (53)$$

satisfying

$$\mathrm{Hess}(H(x_0))=\mathrm{Hess}\left\{\sum_{i=1}^{n}\left\{-P_{mi}\delta_i^{(0)}\right.\right.$$
$$+G_{ii}\delta_i^{(0)}\left(E_{qi}'^{(0)}\right)^2$$
$$+\frac{1+(x_{di}-x_{di}')B_{ii}+k_iT_{d0i}}{2(x_{di}-x_{di}')}\left(E_{qi}'^{(0)}\right)^2 \qquad (54)$$
$$-\frac{1}{2}E_{qi}'^{(0)}\sum_{j=1,j\neq i}^{n}B_{ij}E_{qi}'^{(0)}\cos\left(\delta_i^{(0)}-\delta_j^{(0)}\right)$$
$$\left.\left.-\frac{\bar{u}_iT_{d0i}}{x_{di}-x_{di}'}E_{qi}'^{(0)}\right\}\right\}>0$$

and $\nabla^{\mathrm{T}}H(x)=0$; that is

$$P_{mi}+G_{ii}E_{qi}'^{(0)}$$
$$+E_{qi}'^{(0)}\sum_{j=1,j\neq i}^{n}B_{ij}E_{qi}'^{(0)}\sin\left(\delta_i^{(0)}-\delta_j^{(0)}\right)=0,$$
$$\frac{1+(x_{di}-x_{di}')B_{ii}+k_iT_{d0i}}{x_{di}-x_{di}'}E_{qi}'(0)-\frac{\bar{u}_iT_{d0i}}{x_{di}-x_{di}'} \qquad (55)$$
$$-\sum_{j=1,j\neq i}^{n}B_{ij}E_{qi}'(0)\cos(\delta_i(0)-\delta_j(0))$$
$$+2G_{ii}\delta_i(0)E_{qi}'(0)=0.$$

Meanwhile, we assume that there exist positive constants α_1,β_1 such that $H(x)-H(x_0)\geq(\alpha_1/2)\|x-x_0\|^2$ and $\nabla^{\mathrm{T}}H(x)\cdot\nabla H(x)\geq\beta_1\|x-x_0\|^2$ hold.

An H_∞ controller for system (3) is given in the following theorem.

Theorem 7. *Consider power system (3). If*

$$\begin{pmatrix}0 & 0 & 0\\ 0 & \dfrac{2D_i\omega_0}{M_i^2}-\dfrac{1}{\gamma'^2} & 0\\ 0 & 0 & \dfrac{2(x_{di}-x_{di}')}{T_{d0i}'}+\dfrac{1}{\gamma'^2}\end{pmatrix}\geq 0, \qquad (56)$$
$$i=1,2,\ldots,n$$

hold, then the H_∞ control problem of system (3) can be solved by the feedback control law

$$u_{fi}(t-\tau)=-2G_{ii}(x_{di}-x_{di}')\delta_i(t-\tau)E_{qi}'(t-\tau)$$
$$-k_iT_{d0i}E_{qi}'(t-\tau)+T_{d0i}\bar{u}_i-\frac{1}{2}T_{d0i}\left[\frac{1}{\gamma'^2}+p_i^2\right.$$
$$\left.+\frac{(\omega_0^2+M_i^2)\varepsilon^2}{M_i^4}\right]\left[2G_{ii}\delta_i(t-\tau)E_{qi}'(t-\tau)\right.$$
$$+\frac{1+(x_{di}-x_{di}')B_{ii}}{x_{di}-x_{di}'}E_{qi}'(t-\tau)$$
$$-\sum_{j=1,j\neq i}^{n}B_{ij}E_{qi}'(t-\tau)\cos(\delta_i(t-\tau)-\delta_j(t-\tau)) \qquad (57)$$
$$\left.+\frac{(k_iE_{qi}'(t-\tau)-\bar{u}_i)T_{d0i}}{x_{di}-x_{di}'}\right]+2G_{ii}(x_{di}-x_{di}')\delta_i(t$$
$$-\tau)E_{qj}'(t-\tau)+T_{d0i}k_iE_{qj}'(t-\tau)-(m_i+2n_i)$$
$$-\frac{1}{2}\tau\lambda_1\lambda_2t_i-\frac{1}{4}\tau\lambda_1\lambda_2\frac{(\omega_0^2+M_i^2)\varepsilon^2}{M_i^4},$$
$$i=1,2,\ldots,n,$$

where $\lambda_1,\lambda_2,m_i,n_i$, and t_i, are constants, which satisfy

$$\lambda_1\geq\sup_{t\geq-\tau}\left\|Hess\left[\frac{1+(x_{di}-x_{di}')B_{ii}+k_iT_{d0i}}{2(x_{di}-x_{di}')}E_{qi}'^2\right.\right.$$
$$+G_{ii}\delta_iE_{qi}'^2-\frac{\bar{u}_iT_{d0i}}{x_{di}-x_{di}'}E_{qi}'-P_{mi}\delta_i$$
$$\left.\left.-E_{qi}'\sum_{j=1,j\neq i}^{n}B_{ij}E_{qj}'\cos(\delta_i-\delta_j)\right]\right\|,$$
$$\lambda_2\geq\sup_{t\geq-\tau}\left\|\left(\dot{\delta}_i(t)\quad\dot{\omega}_i(t)\quad\dot{E}_{qi}'(t)\right)\right\|,$$
$$m_i\geq\left|k_iE_{qi}'\right|,$$

$$n_i \geq \left| \frac{2G_{ii}\left(x_{di} - x'_{di}\right)\delta_i(t-\tau)E'_{qi}(t-\tau)}{T_{d0i}} \right|,$$
$$t_i \geq \left| \frac{1}{\gamma'^2} + p_i^2 \right|, \quad i = 1, 2, \ldots, n. \tag{58}$$

$(\delta_i(t), \omega_i(t), E'_{qi}(t))$, $i = 1, 2, \ldots, n$, is the solution of the closed-loop system at time t under initial condition.

Proof. Taking

$$y_i = \begin{pmatrix} \frac{M_i}{\omega_0}\left(\omega_i(t) - \omega_0\right) \\ 2G_{ii}\delta_i(t)E'_{qi}(t) + \frac{1 + \left(x_{di} - x'_{di}\right)B_{ii}}{x_{di} - x'_{di}}E'_{qi}(t) \end{pmatrix} - \begin{pmatrix} 0 \\ \sum_{j=1, j\neq i}^{n} B_{ij}E'_{qj}(t)\cos\left(\delta_i(t) - \delta_j(t)\right) \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\left(k_iE'_{qi}(t) - \overline{u}_i\right)T_{d0i}}{x_{di} - x'_{di}} \end{pmatrix}, \tag{59}$$

$$z_i = p_i \left[2G_{ii}\delta_i(t)E'_{qi}(t) - \sum_{j=1, j\neq i}^{n} B_{ij}E'_{qj}(t)\cos\left(\delta_i(t) - \delta_j(t)\right) + \frac{1 + \left(x_{di} - x'_{di}\right)B_{ii}}{x_{di} - x'_{di}}E'_{qi}(t) + \frac{\left(k_iE'_{qi}(t) - \overline{u}_i\right)T_{d0i}}{x_{di} - x'_{di}} \right]$$

into consideration, then we can prove the result using the similar method in the proof of Theorem 4, where $p_i \geq 0$, $i = 1, 2, \ldots, n$ are the weighting constants. □

4. Illustrative Examples

To show the effectiveness of the proposed control strategy, we give a two-machine power system as shown in Figure 1. The generators G_1, G_2 are assumed to be connected to distant power systems and disturbed by random fluctuation. In simulating, a temporary short-circuit fault occurs at point K (see Figure 1) during the time 0.5 sec~1 sec. The system parameters used in this simulation are given in Table 1. Choose $\omega_0 = 1$, $\xi = 1$.

Taking the above parameters, system (3) can be expressed as

$$d\delta_1 = \left(\omega_1 - 1\right)dt,$$
$$d\omega_1 = \left(\frac{6}{8} - \frac{5}{8}\omega_1 - \frac{1}{8}P_{e1} + \epsilon_{11}\right)dt + 0.125\left(\omega_1 - 1\right)dw(t),$$

Table 1: Generators' data (all per unit except M_i, T_{d0i}, $i = 1, 2, \ldots, n$ in seconds).

M_1	P_{m1}	D_1	x_{d1}	x'_{d1}	T_{d01}
8	1	5	1	0.5	5
M_2	P_{m2}	D_2	x_{d2}	x'_{d2}	T_{d02}
9	1	6	1	0.4	6
B_{11}	B_{12}	B_{21}	B_{22}	G_{11}	G_{22}
4	1	1	10	1	1

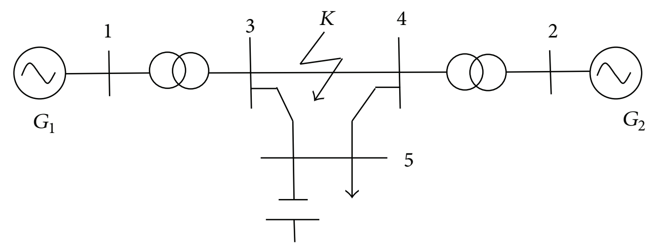

Figure 1: Two-machine power system.

$$dE'_{q1} = \left[-\frac{1}{5}E_{q1} + \frac{1}{5}u_{f1}(t-\tau) + \epsilon_{12}\right]dt,$$
$$d\delta_2 = \left(\omega_2 - 1\right)dt,$$
$$d\omega_2 = \left(\frac{7}{9} - \frac{6}{9}\omega_2 - \frac{1}{9}P_{e2} + \epsilon_{21}\right)dt + \frac{1}{9}\left(\omega_2 - 1\right)dw(t),$$
$$dE'_{q2} = \left[-\frac{1}{6}E_{q2} + \frac{1}{6}u_{f2}(t-\tau) + \epsilon_{22}\right]dt, \tag{60}$$

$$E_{q1} = E'_{q1} + 0.5I_{d1},$$
$$P_{e1} = E'^2_{q1} + E'_{q1}\sum_{j=1, j\neq i}^{2} E'_{qj}\sin\left(\delta_i - \delta_j\right),$$
$$I_{d1} = 4E'_{q1} - \sum_{j=1, j\neq i}^{2} E'_{qj}\cos\left(\delta_i - \delta_j\right),$$
$$E_{q2} = E'_{q2} + 0.6I_{d2}, \tag{61}$$
$$P_{e2} = E'^2_{q2} + E'_{q2}\sum_{j=1, j\neq i}^{2} E'_{qj}\sin\left(\delta_i - \delta_j\right),$$
$$I_{d2} = 10E'_{q2} - \sum_{j=1, j\neq i}^{2} E'_{qj}\cos\left(\delta_i - \delta_j\right).$$

Choosing the following preassigned operating point

$$\left(\delta_1^{(0)}, \omega_0, E'^{(0)}_{q1}, \delta_2^{(0)}, \omega_0, E'^{(0)}_{q2}\right) = \begin{bmatrix} 0.5 & 1 & 1 & 0.5 & 1 & 1 \end{bmatrix}, \tag{62}$$

then $\overline{u}_1 = 0.5$, $\overline{u}_2 = 16/15$, and $k_1 = k_2 = -0.1$.

It is easy to verify that system (60) with the above values satisfies conditions (A1)–(A4) of Theorem 4.

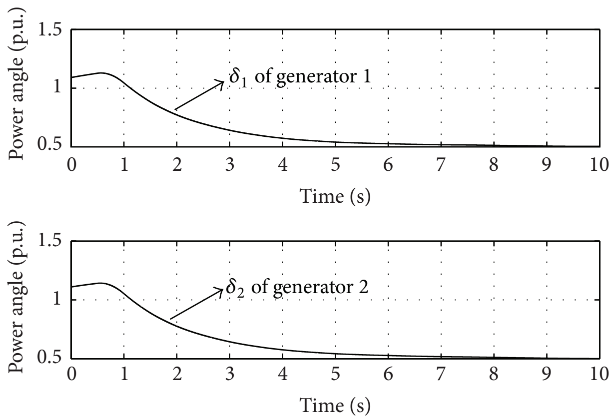

Figure 2: Power angle dynamic behavior while $\tau = 0.05$ s.

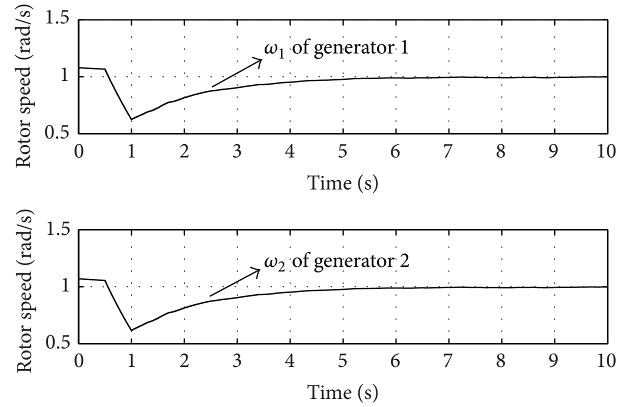

Figure 3: Rotor speed dynamic behavior while $\tau = 0.05$ s.

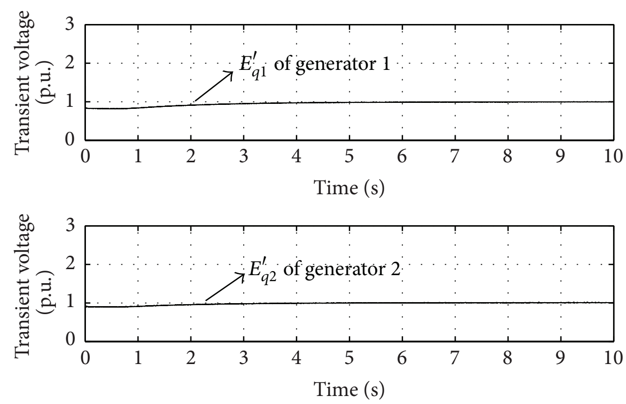

Figure 4: Transient voltage dynamic behavior while $\tau = 0.05$ s.

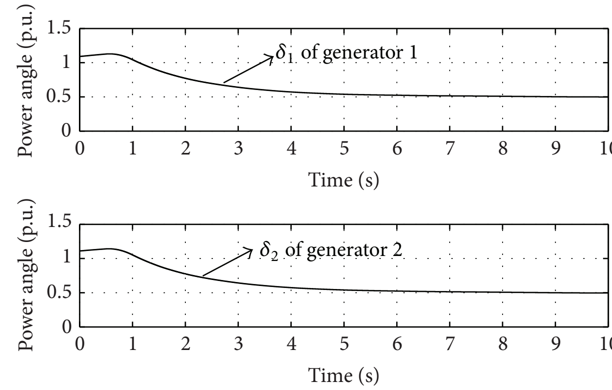

Figure 5: Power angle dynamic behavior while $\tau = 0.5$ s.

The fault indicates a unit step function; that is, $\epsilon_{11} = \epsilon_{12} = \epsilon_{21} = \epsilon_{22} = -1(t-0.5) + 1(t-1)$. For given $\gamma' = 4$, we can find $p_1 = p_2 = 1$ such that inequality (56) is satisfied.

We will test the effectiveness of the proposed control configuration at two different time delays $\tau = 0.5$ s and $\tau = 0.05$ s. The initial condition is $(\delta_1(0), \omega_1(0), E'_{q1}(0), \delta_2(0), \omega_2(0), E'_{q2}(0)) = [1.2 \ \ 1 \ \ 2 \ \ 1.2 \ \ 1 \ \ 2]$.

Take $\mu = 1/8^4$, $\lambda' = 40$, $m'_{11} = m'_{12} = 100$, $m'_{21} = m'_{22} = 100$, and $m'_3 = 40$. According to Theorem 7 proposed in this paper, system (60) is asymptotically stable in mean square for all $\tau \ge 0$ and $\epsilon = 0$ under the feedback control law

$$\begin{aligned}
u_{f1}(t-\tau) &= (4096)^{-1}\left[-29858.5\delta_1(t-\tau)E'_{q1}(t-\tau)\right.\\
&\quad - 62358.25E'_{q1}(t-\tau) + 12881.25E'_{q2}(t-\tau)\\
&\quad \cdot\cos(\delta_1(t-\tau)-\delta_2(t-\tau)) - 3202000\tau\\
&\quad \left.\cdot\operatorname{sgn}(5\omega_1-5) + 74646.25\right] - 4480\tau\operatorname{sgn}\left[5E'_{q1}(t)\right.\\
&\quad \left.+ 2\delta_1(t)E'_{q1}(t) - E'_{q2}(t)\cos(\delta_1(t)-\delta_2(t)) - 5\right],\\
u_{f2}(t-\tau) &= (4096)^{-1}\left[-35011\delta_2(t-\tau)E'_{q2}(t-\tau)\right.\\
&\quad - 162422.4E'_{q2}(t-\tau) + 17915.1E'_{q1}(t-\tau)\\
&\quad \cdot\cos(\delta_1(t-\tau)-\delta_2(t-\tau)) - 3842400\tau\\
&\quad \left.\cdot\operatorname{sgn}(6\omega_2-6) + 191094.4\right] - 5376\tau\\
&\quad \cdot\operatorname{sgn}\left[\left(\frac{2}{3}+10\right)E'_{q2}(t) + 2\delta_2(t)E'_{q2}(t)\right.\\
&\quad \left.- E'_{q1}(t)\cos(\delta_1(t)-\delta_2(t)) - \frac{64}{6}\right].
\end{aligned} \tag{63}$$

Simulations with the above initial condition and the delay $\tau = 0.5$ s and $\tau = 0.05$ s are given in Figures 2–4 and Figures 5–7, separately. Through Figures 2–7, we can see that the states of the system converge to the equilibrium $(\delta_1(0), \omega_1(0), E'_{q1}(0), \delta_2(0), \omega_2(0), E'_{q2}(0)) = [1.2 \ \ 1 \ \ 2 \ \ 1.2 \ \ 1 \ \ 2]$ eventually. Obviously, under the delayed feedback controller by using the proposed method, the robustness of the closed-loop system is guaranteed. It is also seen that the controller possesses insensitivity in regard to the types of time delay and stochastic disturbances.

5. Conclusion

This paper studied the H_∞ excitation controller design problem of a class of stochastic power systems with time-delay and

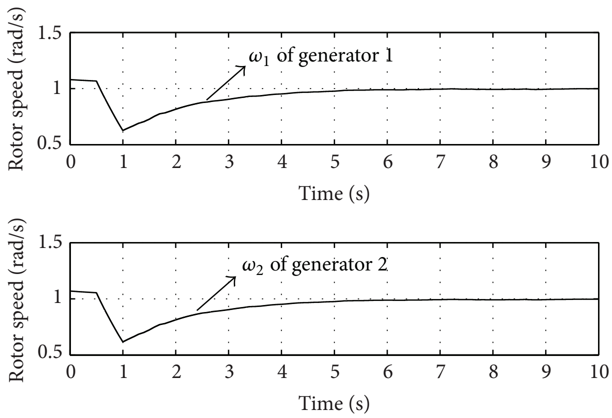

FIGURE 6: Rotor speed dynamic behavior while $\tau = 0.5$ s.

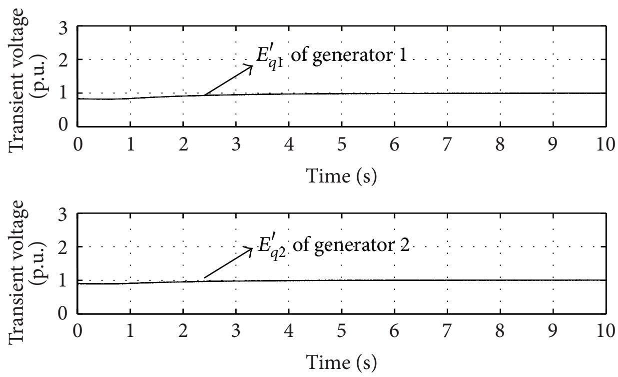

FIGURE 7: Transient voltage dynamic behavior while $\tau = 0.5$ s.

disturbances. In the design process, we used the prefeedback technique, Newton-Leibniz formula, and a few properties of norm. Besides, we obtain these results by nonlinear Hamilton function approach due to the special structural properties of the Hamiltonian systems. We also give a two-machine power system simulation and it shows that the results achieved in this paper are practicable in analyzing the H_∞ excitation control problem of stochastic power system in consideration of time-delay and disturbances.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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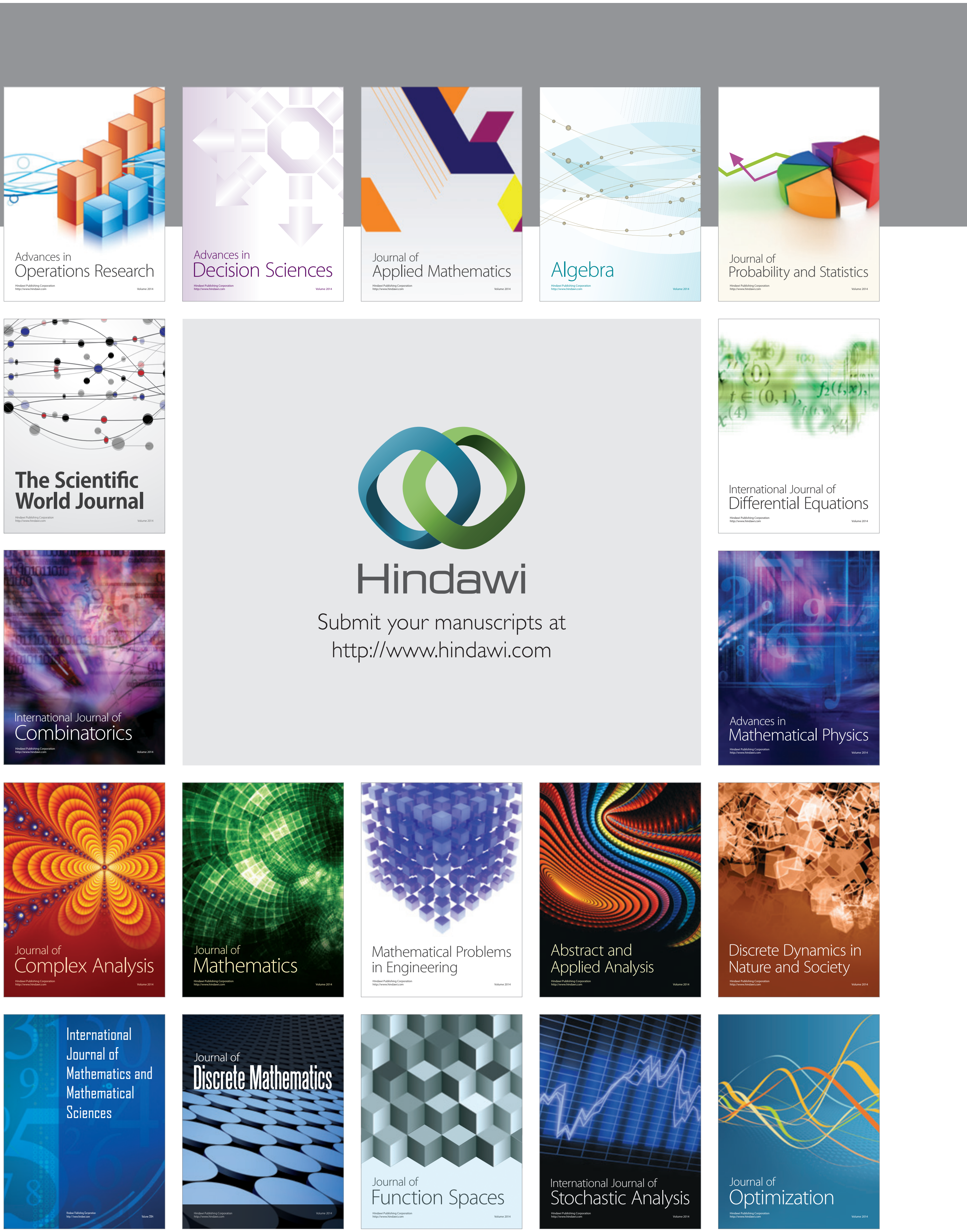
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