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Research Article

A Semi-Stochastic Propagation Model for the Study of Beam Tilting in Cellular Systems

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Base station antenna downtilt mitigates interference and improves the downlink performance of wireless systems. A semi-stochastic propagation model is presented and applied to the study of the impact of the base station beam tilting in cellular communications. The two-ray approximation of the proposed model is described analytically. Beam tilting is evaluated in relation to the base station antenna radiation pattern, the antennas height, the propagation environment, the bit error rate, and the signal-to-noise ratio at the receiver front end. Analytically derived expressions for the fading envelopes, the error probability, the optimum tilting angle, and the downlink capacity of a WCDMA system are derived. Theoretical analysis and simulation results are provided to show the characteristics of the model. Comparisons with data in the literature confirm its validity. Furthermore, the effect of beam tilting on system downlink performance in terms of bit error rate and capacity is investigated.

1. INTRODUCTION

The communication quality in the downlink of a mobile communication system is strongly affected by multipath distortion [1]. Among the proposed techniques, downtilting of the base station (BS) antenna gives good results providing a form of angle diversity [2, 3]. Maseng [4] analyzed the system bit rate in the mobile multipath channel for perfect reflection and omnidirectional BS antennas, showing that the maximum bit rate depends on the propagation environment, the communication range, and the complexity of the mobile station (MS) receiver. In [5], the effect of beam tilting was investigated in view of the multipath distortion under perfect and diffusely scattered reflection. The downlink bit rate as a function of the BS antenna radiation pattern, the antennas height, the sensitivity of the mobile receiver, the excess distance, and the characteristics of the propagation medium was examined in [6, 7]. Furthermore, in [8–10], optimum tilting was treated in view of the BS antenna characteristics, the cells sectorization, the site spacing, the number of signal paths, and the traffic volume showing that optimum tilting depends on various network parameters. Finally, in [11], an adaptive tilting method was proposed for interference mitigation and capacity enhancement in WCDMA systems, revealing the importance of beam tilting in wireless communications.

In this paper, the effect of beam tilting in the downlink performance of a cellular system is investigated. A semi-stochastic propagation model that describes the mobile radio channel is established. The two-ray line-of-sight (LOS) approximation of the model is described extensively. Analytically derived expressions for the fading envelopes, the bit error rate (BER), the optimum tilting angle, and the downlink capacity of a WCDMA system are obtained. The basic characteristics of the model are explained and its validity is confirmed. Furthermore, the issues of system performance and optimum tilting are addressed.

The rest of the paper is organized as follows. In Section 2, the system model is presented. In Section 3, the proposed propagation models are described and the mathematical formulation is given. Numerical examples, simulations, and discussions are provided in Section 4. Finally in Section 5, concluding remarks are drawn.

2. SYSTEM MODEL

In Figure 1 the model of a mobile system is illustrated. The horizontal distance between the BS and the MS is r, the

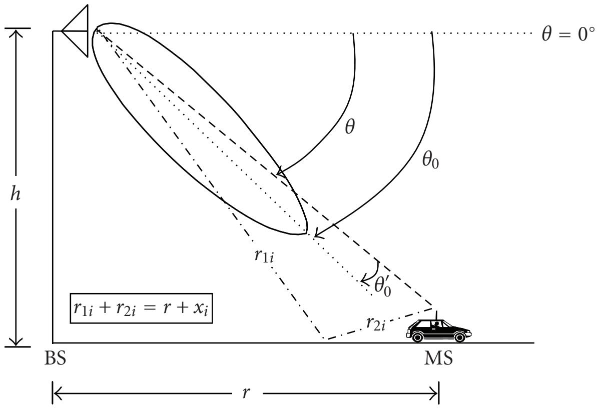

FIGURE 1: Two-dimensional model of a mobile system with tilted BS wave beam.

difference in height between their antennas (mentioned as BS antenna height for brevity) is h, and the additional path length (excess distance) of the ith reflected path relative to the direct path is x_i (it is $x_i = r_{1i} + r_{2i} - r$, where r_{1i} and r_{2i} are the horizontal distances of the ith path reflection point from the BS and the MS, resp.). The tilting angle of the BS antenna radiation pattern defined from an axis parallel to the azimuth plane θ_0; θ_0' is the tilting angle defined from the LOS axis, that is,

$$\theta_0' = \theta_0 - \tan^{-1}\left(\frac{h}{r}\right). \tag{1}$$

Without loss of generality, mechanical downtilting is assumed; further improvement can be achieved when BS antenna beam is electrically tilted due to improved interference suppression [12].

In the examples presented here, the elevation plane radiation pattern of the BS antenna contains $\csc\theta$ terms and is represented (in natural units) by

$$G(\theta) = \begin{cases} \exp\left[-k_1(\theta-\theta_0)^2\right], & -\dfrac{\pi}{2} \le \theta < \theta_0 + \dfrac{HP}{2}, \\ k_2 \csc(\theta - \theta_0), & \theta_0 + \dfrac{HP}{2} \le \theta < \dfrac{\pi}{2}, \end{cases} \tag{2}$$

where HP is the half-power beamwidth of the radiation pattern, $k_1 = 2\ln 2/HP^2$, and $k_2 = \sin(HP/2)/\sqrt{2}$.

3. MATHEMATICAL FORMULATION

In the wireless channel, the transmitted signal suffers multiple reflections. It is assumed that there exist $N-1$ echo signals with large amplitudes whose appearance influence the total signal level received by the MS antenna. The echo signals that are sufficiently far away have significantly smaller amplitudes and are left out. Assigning to each ray probability P_i, the effective BS radiation pattern (i.e., the pattern as seen at the receiver) is given [13] by

$$F(\theta) = \sum_{i=0}^{N-1}\left(1+\frac{x_i}{r}\right)^{-\alpha/2} P_i G(\theta^i) e^{j(2\pi x_i/\lambda + \phi_i)}, \tag{3}$$

where

$$\theta^i = \tan^{-1}\left(\frac{h}{r+x_i}\right). \tag{4}$$

The ϕ_i denotes the phase of the ith ray, and λ is the transmitted signal wavelength. Variables P_i and ϕ_i are independent and identically distributed (i.i.d.) random variables dependent on the propagation medium ($i = 0$ refers to the LOS ray). The path loss exponent α is a terrain-dependent factor; its values range from 3 to 4.3 in urban areas and are close to 3.9 and 4.4 in suburban and open (rural) space, respectively, [14]. The ratio

$$Y \equiv Y(\theta) \underset{\text{def}}{=} \frac{|F(\theta)|}{|G(\theta)|} = \left|\sum_{i=0}^{N-1} P_i X_i(\theta) e^{j(2\pi x_i/\lambda + \phi_i)}\right|, \tag{5}$$

where

$$X_i(\theta) = \frac{(1+x_i/r)^{-\alpha/2} G(\theta^i)}{G(\theta)}, \quad i = 0, 1, \dots, N-1, \tag{6}$$

is a measure of the fade depth. Using the identity

$$\left(\sum_i I_i\right)^2 + \left(\sum_j Q_j\right)^2 = \sum_i\sum_j \left(I_i I_j + Q_i Q_j\right) \tag{7}$$

after some manipulation, (5) becomes

$$Y = \frac{r^{\alpha/2}}{G(\theta)}\sqrt{A}. \tag{8}$$

where

$$A = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left\{P_i P_j \frac{G(\theta^i)G(\theta^j)}{\left[r^2 + (x_i + x_j)r + x_i x_j\right]^{a/2}} \times \cos\left(2\pi\frac{x_i - x_j}{\lambda} + \phi_i - \phi_j\right)\right\}. \tag{9}$$

The model's accuracy increases significantly when a substantial amount of data regarding terrain and antenna gain characteristics is known for every possible ray path. However, a major drawback of the model is its complexity. A common approximation used for the multipath fading channel is a two-ray approximation; for example, see [15–20].

A two-ray model which assumes a LOS and a ground reflected propagation path between the transmitter and the receiver will be presented now. Equal probabilities are assigned to both paths ($P_0 = P_1 = 1$). Let us consider $X_0(\theta) = 1$, $\phi_0 = 0$, $x_0 = 0$ and set $X \equiv X_1(\theta)$, $x \equiv x_1 - x_0 = x_1$, and $\phi \equiv \phi_1 - \phi_0 = \phi_1$. Then (8) is simplified into

$$Y = \sqrt{1 + X^2 + 2X\cos\left(\frac{2\pi x}{\lambda} + \phi\right)}, \tag{10}$$

where ϕ is uniformly distributed within $[-\pi, \pi]$ random variable [21, 22].

The pdf of Y, $f_Y(Y)$, is calculated (see Appendix A for a few details) by

$$f_Y(Y) = \begin{cases} k\dfrac{Y}{\sqrt{4X^2 - \left(1 + X^2 - Y^2\right)^2}}, & Y \in [Y_{\min}, Y_{\max}], \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

In this expression, k is a normalization factor intended to ensure unity cumulative distribution function. The terms $Y_{\min}$ and $Y_{\max}$ are

$$\begin{aligned} &Y_{\min} \\ &= \begin{cases} 1 - X, & \operatorname{mod}\left(\dfrac{x}{\lambda}\right) \in \left[0, \dfrac{1}{2}\right], \\ \sqrt{1 + X^2 + 2X\cos\left(\dfrac{2\pi x}{\lambda}\right)}, & \operatorname{mod}\left(\dfrac{x}{\lambda}\right) \in \left(\dfrac{1}{2}, \dfrac{3}{4}\right), \\ \sqrt{1 + X^2 - 2X\cos\left(\dfrac{2\pi x}{\lambda}\right)}, & \operatorname{mod}\left(\dfrac{x}{\lambda}\right) \in \left[\dfrac{3}{4}, 1\right), \end{cases} \\ &Y_{\max} \\ &= \begin{cases} \sqrt{1 + X^2 + 2X\cos\left(\dfrac{2\pi x}{\lambda}\right)}, & \operatorname{mod}\left(\dfrac{x}{\lambda}\right) \in \left[0, \dfrac{1}{4}\right], \\ \sqrt{1 + X^2 - 2X\cos\left(\dfrac{2\pi x}{\lambda}\right)}, & \operatorname{mod}\left(\dfrac{x}{\lambda}\right) \in \left(\dfrac{1}{4}, \dfrac{1}{2}\right), \\ 1 + X, & \operatorname{mod}\left(\dfrac{x}{\lambda}\right) \in \left[\dfrac{1}{2}, 1\right), \end{cases} \end{aligned} \tag{12}$$

where $\operatorname{mod}(x/\lambda)$ is the remainder on dividing x by λ. After some manipulation, we obtain that the expected value of Y is

$$\begin{aligned} \mathrm{E}\{Y\} = \frac{2}{\pi}(1 + X)\Biggl[E\left(\pi\left(\frac{x}{\lambda} + \frac{1}{2}\right) \Bigm| \frac{4X}{(1 + X)^2}\right) \\ - E\left(\frac{\pi x}{\lambda} \Bigm| \frac{4X}{(1 + X)^2}\right)\Biggr], \end{aligned} \tag{13}$$

where $E(\phi|m)$ is the incomplete elliptic integral of the 2nd kind.

Using (10)–(13), approximate expressions for the error probability and the optimum tilting angle are derived (see Appendix B). Without loss of generality, the coherent antipodal BPSK modulation scheme is assumed [23]. In the proposed two-ray channel model when the BS beam is tilted at an angle θ_0, the error probability is

$$P_e|_{\theta_0} \cong \frac{1}{2}\operatorname{erfc}\left(\mathrm{E}\{Y_{\theta_0}\}\mathrm{E}\left\{\frac{G_{\theta_0}(\theta)}{G_{\theta_0=0}(\theta)}\right\}\sqrt{\gamma}\right), \tag{14}$$

where γ is the bit energy-to-noise power spectral density at the MS receiver front end. The optimum tilting angle is found by expecting a maximum error probability $P_e^{\max}$ at an angle $\theta_{\text{edge}} = \tan^{-1}(h/d)$, where d is the sector dominance area (obviously the maximum of the main lobe points at the desired cell, i.e., $\theta_0 > \theta_{\text{edge}}$). It is defined by the BS antenna radiation pattern, the antennas height, the site spacing, the maximum acceptable bit error rate, the target signal-to-noise ratio at the MS receiver front end, and the propagation channel characteristics, and is given by the solution of

$$G_{\theta_0}(\theta_{\text{edge}}) = \frac{\operatorname{erfc}^{-1}(2P_e^{\max})}{\mathrm{E}\{Y\}\sqrt{\gamma}}G_{\theta_0=0}(\theta_{\text{edge}}), \tag{15}$$

where $\operatorname{erfc}^{-1}(x)$ is the inverse complementary error function. In the case of the antenna radiation pattern of (2), the optimum tilting angle is given by

$$\begin{aligned} \theta_{0,\,\text{opt}} \cong{}& \tan^{-1}\left(\frac{h}{r}\right) \\ &+ \sqrt{\frac{HP^2}{2\ln 2}\ln\left(\sqrt{\frac{2\gamma}{1 + (r/h)^2}}\cdot\frac{\csc(HP/2)}{\operatorname{erfc}^{-1}(2P_e^{\max})\mathrm{E}\{Y\}}\right)}. \end{aligned} \tag{16}$$

An approximate expression that describes the effect of BS antenna tilting on the downlink capacity of a WCDMA cellular network in terms of total deliverable bit rate has also been obtained (see Appendix C). The main assumptions are the equal downlink rate allocation to each of the K active users, the fixed channel bandwidth W, and the infinite BS transmission power. The total deliverable downlink bit rate satisfying a specific bit error rate target is inversely proportional to the target bit energy-to-noise power spectral density Γ at each user's receiver front end [24–26]. Users are uniformly distributed within the cells which are approximated as circles with radius d. Each user is allocated a single unique channelization code which is assumed orthogonal between users. To avoid any wastage of power, the bit error rate is fixed at the maximum that the quality-of-service (QoS) requirements allow. Without loss of generality, the target signal-to-noise ratio at each user's receiver front end and the BER QoS target are assumed equal for all users. Under these assumptions, the ratio between the maximum deliverable bit rate $R_{\max}^{\theta_0}$ when the BS antenna radiation pattern is tilted at a θ_0 angle and the corresponding rate with no tilting $R_{\max}^{0}$ is given by

$$\frac{R_{\max}^{\theta_0}}{R_{\max}^{0}} \cong \frac{4h^4}{d^4}\left(\int_{\cot^{-1}(d/h)}^{\pi/2}\frac{G_{\theta_0}(\theta)}{G_{\theta_0=0}(\theta)}\cdot\frac{\cos\theta}{\sin^3\theta}\,\mathrm{d}\theta\right)^2, \tag{17}$$

where [24–26]

$$R_{\max}^{0} = \frac{WK}{\Gamma\nu(K - 1)}, \tag{18}$$

where ν is the loss orthogonality factor due to fading, which depends on the propagation environment and the mobile's speed and position [9]. It has to be mentioned that BS antenna downtilt improves signal level within a cell due to better ability to aim the beam towards the intended dominance area, and simultaneously decreases interference towards other cells [9]. An optimum downtilt angle is a tradeoff between intercell interference mitigation and coverage threshold. However, under the assumption of unlimited BS transmission power, the cochannel interference

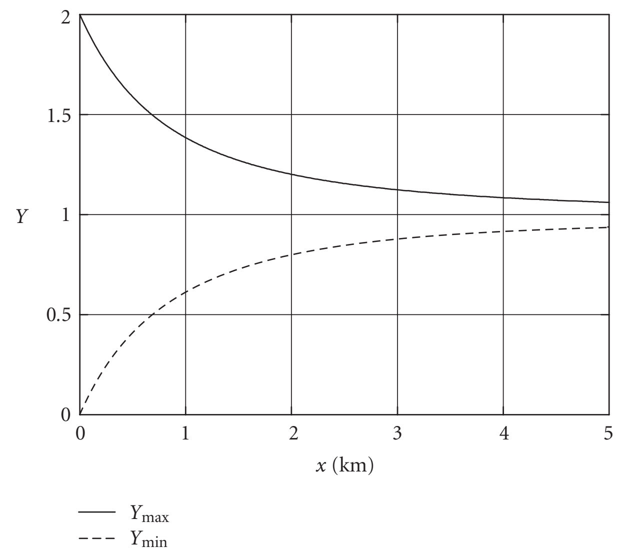

FIGURE 2: Fading envelopes versus excess distance (slow fading).

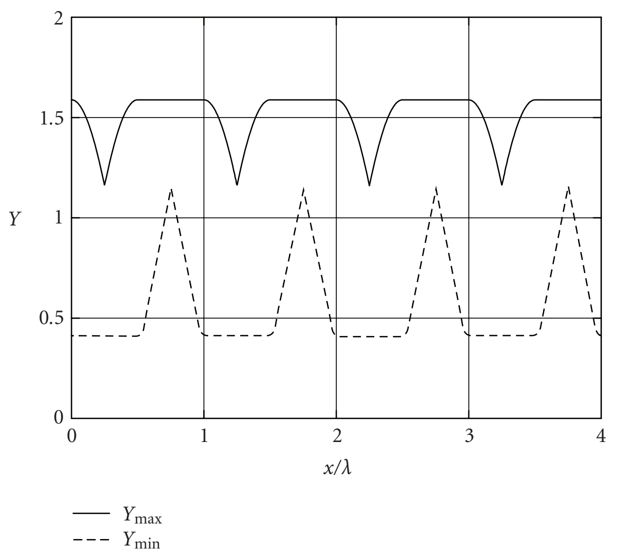

FIGURE 3: Fading envelopes versus normalized excess distance (fast fading).

from neighbouring cells is negligible [25]. In this case, capacity improvement is caused by the increase in the signal level only [27].

4. RESULTS AND DISCUSSIONS

In this section, the basic characteristics of the two-ray propagation channel model are examined. The impact of BS antenna downtilt on system performance and the choice of optimum tilting are investigated. Finally, the validity of the proposed model is verified by comparisons with data in the literature. In the following examples, the BS is located at the center of its cell, consisting of several sectors. The BS antenna radiation pattern is described by (2). If not otherwise specified, system parameters are $r = 2$ km, $h = 50$ m, $HP = 3°$, $\theta_0 = 3°$, $\alpha = 3.4$, and $\lambda = 0.33$ m.

Figures 2 and 3 depict the fading envelopes $Y_{\max}$ and $Y_{\min}$ as a function of the excess distance. In Figure 2, x ranges from zero to 5 km. As x increases, the range of the values of Y reduces. In Figure 3, it lies within a range of four signal wavelengths. In this case, the fading envelopes are periodic functions repeated every wavelength of the transmitted signal. In practice, the fading envelopes represent the envelopes of the received signal strength due to slow (Figure 2) and fast (Figure 3) fading [1, 23]. It has to be noticed that x depends on various system parameters, such as system geometry, path loss exponent, BS antenna radiation pattern, and bit rate (e.g., [6, 7, 12]) and is proportional to the excess delay of the channel. For the system under consideration, the typical values of x range from half to 2 km.

The dependence of $f_Y(Y)$ on the system parameters is analyzed in Figure 4. The f_Y^0 curve represents the example case previously mentioned (setting the excess distance equal to 1 km); in f_Y^1, f_Y^2, and f_Y^3, it is $r = 1$ km, $h = 25$ m, and $x = 500$ m, respectively. Finally in f_Y^4 and f_Y^5, θ_0 and HP are doubled. The pdfs are almost symmetrical with respect to $Y = 1$. The pdf spread reduces as r and θ_0 decrease, or as h, HP, and x increase, resulting in an improved system performance.

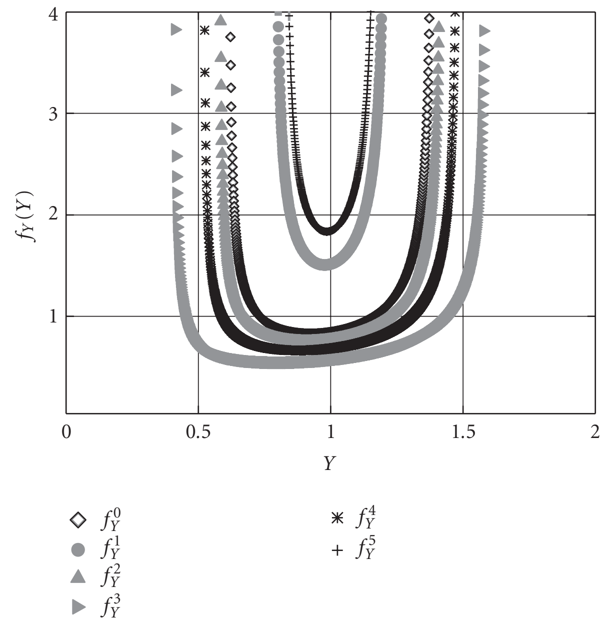

FIGURE 4: The pdf of Y for different system configurations.

Let us consider the MS at the edge of the cell. The error probability for BPSK modulation and for various BS antenna downtilting angles (measured from the LOS axis) is shown in Figure 5 ($P_e \equiv P_e|_{\theta_0'}$). Notice that beam downtilting improves system performance up to 3 dB. The optimum tilting angle, $\theta'_{0,\mathrm{opt}}$, ranges from one to two degrees (the same angle measured from an axis parallel to the azimuth plane, $\theta_{0,\mathrm{opt}}$, (see Figure 1) lies between two and three degrees). When a $P_e^{\max} = 10^{-6}$ is desired, a typical value for data packet transmission [23], and $\gamma_{\mathrm{dB}} = 10$ dB, (16) gives an optimum

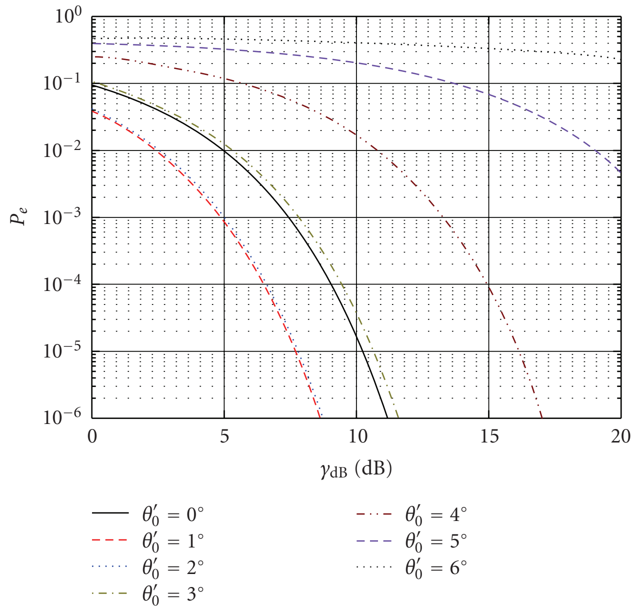

Figure 5: System performance dependence of the BS beam downtilting.

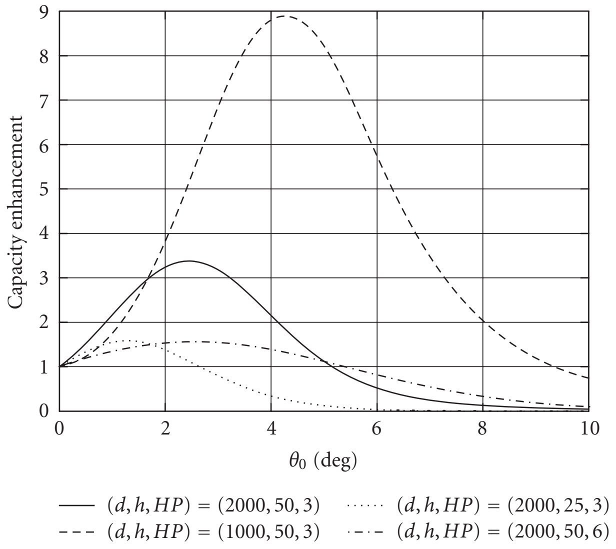

Figure 6: Impact of BS antenna downtilting on the capacity of a WCDMA cellular system (distances are measured in metres; angles are measured in degrees).

tilting angle $\theta_{0,\mathrm{opt}} = 2.68°$. Substituting this value in (1), r is the size of the cell dominance area [28], a $\theta'_{0,\mathrm{opt}} = 1.25°$ is found, in agreement with Figure 5. Notice that for tilting angles up to a point, a mobile station at the edge of the cell is within the main lobe of the BS radiation pattern. However, when the BS antenna beam is further tilted, its main lobe may not point to the desired user. As a result of great tilting angles system performance degrades.

Equation (16) shows that the optimum tilting angle increases with BS antenna height and decreases with site spacing, bit energy-to-noise power spectral density at the MS receiver front end, and maximum acceptable BER. However, its dependence of the BS antenna half-power beamwidth is more complicated. The optimum tilting angle increases with HP up to point. Differentiation of (16) with respect to HP shows that the optimum tilting angle is maximized when the half-power beamwidth of the BS antenna radiation pattern is the solution of

$$HP\cot\left(\frac{HP}{2}\right) - 4\ln\left(\sqrt{\frac{2\gamma}{1+(r/h)^2}}\cdot\frac{\csc(HP/2)}{\operatorname{erfc}^{-1}\left(2P_e^{\max}\right)\mathrm{E}\{Y\}}\right) = 0. \tag{19}$$

Further increase in half-power beamwidth reduces the optimum tilting angle.

The validity of the proposed model was confirmed by comparisons with data in the literature. In [9], Niemelä et al. investigated the impact of mechanical antenna downtilt on the performance of a macrocellular WCDMA network. In the simulations performed, a six-sectored configuration was assumed. The BS antenna heights were $\{25\,\mathrm{m}, 35\,\mathrm{m}, 45\,\mathrm{m}\}$. The BS antenna radiation pattern had a half-power beamwidth $HP = 6°$ and showed similarities to that of (2). The target downlink bit energy-to-noise ratio in the speech services was 8 dB. Their studies have shown that the optimum tilting angles were $\theta_{0,\mathrm{opt}}(\mathrm{deg}) = \{4.9, 5.9, 7.0\}$ and $\theta_{0,\mathrm{opt}}(\mathrm{deg}) = \{3.8, 4.8, 5.9\}$ for site spacing 1.5 km and 2 km, respectively. In order to compare these results with the ones derived from the proposed model, r was set as half the site spacing. An error probability of 10^{-3} is considered adequate for speech services [23]. In order to calculate $\mathrm{E}\{Y\}$, Monte Carlo simulations have been performed. The optimum tilting angles predicted by the two-ray model are $\theta_{0,\mathrm{opt}}(\mathrm{deg}) = \{4.5, 6.1, 7.4\}$ and $\theta_{0,\mathrm{opt}}(\mathrm{deg}) = \{3.3, 4.8, 6.0\}$ for $r = 750\,\mathrm{m}$ and $r = 1\,\mathrm{km}$, respectively, close to the ones reported in [9]. Similar conclusions are obtained from comparisons with results in [10]. For example, let us consider a three-sectored cell, $r = 500\,\mathrm{m}$, and BS antenna half-power beamwidth and height $HP = 10.2°$ and $h = 50\,\mathrm{m}$, respectively. In the case of speech services a $\theta_{0,\mathrm{opt}} = 12°$ was reported. Setting $\gamma_{\mathrm{dB}} = 8\,\mathrm{dB}$ and $P_e^{\max} = 10^{-3}$, the solution of (16) gives $\theta_{0,\mathrm{opt}} = 12.4°$.

In Figure 6, the capacity enhancement of a macrocellular WCDMA system in terms of the total downlink bit rate is presented. The users are uniformly distributed within the cell and are assigned equal bit rates (the rest of the assumptions are mentioned in the previous section). A significant improvement in the maximum deliverable downlink bit rate is observed when BS antenna radiation pattern is downtilted. Notice that capacity increases when the radiation pattern is tilted up to point. Further tilting degrades system performance (a similar behavior has been observed in the study of bit error rate). Significant capacity enhancement in sites with smaller dominance area is observed. In this case, the optimum tilting angle is higher. Similar results are derived as the BS antenna height increases. Finally, increase in half-power beamwidth degrades system performance. The derived conclusions are similar to the ones derived in [9, 28].

5. CONCLUSIONS

In this paper, the impact of BS beam tilting on the downlink performance of a cellular system has been investigated. A semi-stochastic propagation model has been proposed. The two-ray model's approximation has been extensively described. Approximate expressions for the fading envelopes, the error probability and the optimum tilting angle of a cellular system, and the downlink capacity enhancement of a WCDMA wireless network have been derived. Results have exhibited the characteristics of the model and treated the choice of the optimum tilting angle considering the base station to mobile user posistion, the antennas height, the site dominance area, the BS antenna radiation pattern, the excess distance, the signal energy, and the bit error rate. Simulations have shown the strong impact of tiling in the performance of a wireless system and the critical role of tilting angle. Comparisons with data in the literature exhibited the validity of the model. The proposed model is a low complexity and adequate tool for wireless networks performance study and optimization.

APPENDICES

A. THE PDF OF A FUNCTION OF A RANDOM VARIABLE

Suppose that $\mathbf{z}$ is a random variable and $g(z)$ is a function of a real variable z. The expression $\mathbf{y} = g(\mathbf{z})$ is a new random variable. In order to find the pdf $f_y(y)$ for a specific y, we solve the equation $y = g(z)$. Denoting its N real roots by z_n, that is, $y = g(z_1) = g(z_2) = \cdots = g(z_N)$, the pdf $f_y(y)$ is given [29] by

$$f_y(y) = \sum_{n=1}^{N} \frac{f_z(z_n)}{|g'(z_n)|}, \tag{A.1}$$

where $f_z(z)$ is the pdf of $\mathbf{z}$ and $g'(z)$ is the derivative of $g(z)$

B. ERROR PROBABILITY AND OPTIMUM TILTING ANGLE EXPRESSIONS

In this appendix, the approximate expressions for the error probability and the optimum tilting angle are derived. The error probability for coherent antipodal BPSK modulation over an AWGN channel with no fading is [23]

$$P_e^0 = \frac{1}{2}\mathrm{erfc}(\sqrt{\gamma}) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b^0}{N_0}}\right), \tag{B.1}$$

where E_b^0 is the bit energy and N_0 is the noise power spectral density at the MS receiver front end. In the proposed two-ray channel model, if the BS beam is not tilted, the bit energy at an angle θ is approximately (see (5))

$$E_b \cong E_b^0\cdot \mathrm{E}\left\{\frac{F_{\theta_0=0}(\theta)}{G_{\theta_0=0}(\theta)}\right\}^2 = E_b^0\cdot \mathrm{E}\{Y_{\theta_0=0}\}^2, \tag{B.2}$$

where $\mathrm{E}\{\cdot\}$ denotes the expected value. When the BS beam is tilted at an angle θ_0, using (5) the previous expression becomes

$$\begin{aligned} E_b &\cong E_b^0\cdot \mathrm{E}\left\{\frac{F_{\theta_0}(\theta)}{G_{\theta_0=0}(\theta)}\right\}^2 = E_b^0\cdot \mathrm{E}\left\{\frac{Y_{\theta_0}G_{\theta_0}(\theta)}{G_{\theta_0=0}(\theta)}\right\}^2 \\ \Longrightarrow E_b &\cong E_b^0\cdot \mathrm{E}\{Y_{\theta_0}\}^2\cdot \mathrm{E}\left\{\frac{G_{\theta_0}(\theta)}{G_{\theta_0=0}(\theta)}\right\}^2. \end{aligned} \tag{B.3}$$

Considering that

$$P_e|_{\theta_0} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{B.4}$$

and using (B.1) and (B.3), (14) is derived.

Estimation of the tilting angle that optimizes system performance is a tradeoff between other cell interference mitigation and coverage thresholds, and depends on each network demands. Its value is affected by various factors such as the system geometry, the BS antenna radiation pattern, the propagation medium, the link bit rate, and the cells sectorization. Intuitively, the required downtilt angle increases with the BS antenna height. Correspondingly, a cell with a small dominance area requires a large downtilt angle. However, the system geometry is not enough to define the optimum downtilt angle. As it has already been mentioned, in the present proposal the optimum tilting angle is found by expecting a maximum error probability at the edge of the cell. Notice also that the expectation of Y describes the propagation medium and does not depend on beam tilting. As a result, it holds that $\mathrm{E}\{Y_{\theta_0}\} = \mathrm{E}\{Y\}$. Based on the previous analysis and using (14), it easily comes that the optimum tilting angle is found from the solution of (15). In the case of the antenna radiation pattern of (2), it holds that

$$\begin{aligned} G_{\theta_0=0}(\theta_{\mathrm{edge}}) &= \frac{1}{\sqrt{2}}\sin\left(\frac{HP}{2}\right)\csc\theta_{\mathrm{edge}}, \\ G_{\theta_0}(\theta_{\mathrm{edge}}) &= \exp\left[-\frac{2\ln 2}{HP^2}(\theta_{\mathrm{edge}}-\theta_0)^2\right]. \end{aligned} \tag{B.5}$$

Substituting (B.5) in (15), (16) is easily derived.

C. DOWNLINK THROUGHPUT ENHANCEMENT BY BEAM TILTING

In this appendix, the formulation of the downlink capacity improvement due to beam tilting in a WCDMA cellular network in terms of total deliverable bit rate under the assumptions already mentioned in Section 3 is presented. For fixed W and N, the total deliverable bit rate is proportional to the bit energy at each user's receiver front end when the signal-to-noise ratio QoS target is also fixed [24, 25]. Therefore, (B.3) gives that the ratio between $R_{\max}^{\theta_0}$ and $R_{\max}^{0}$ is

$$\frac{R_{\max}^{\theta_0}}{R_{\max}^{0}} \cong \mathrm{E}\left\{\frac{G_{\theta_0}(\theta)}{G_{\theta_0=0}(\theta)}\right\}^2 \tag{C.1}$$

(it is reminded that $\mathrm{E}\{Y_{\theta_0}\} = \mathrm{E}\{Y\}$). As a result of the uniform distribution of the users within the circular cell, the pdf of their distance ξ from the BS is [30]

$$f_\xi(\xi) = \frac{2\xi}{d^2}, \quad \xi \in [0, d], \tag{C.2}$$

where $\xi = h\cot\theta, \theta \in [\cot^{-1}(d/h), \pi/2]$, ($d$ is the radius of the cell).

Using (A.1), the pdf $f_\theta(\theta)$ can be found. After some manipulation comes that

$$f_\theta(\theta) = \frac{2h^2\cos\theta}{d^2\sin^3\theta}. \tag{C.3}$$

From (C.1) and (C.3), (17) is easily derived.

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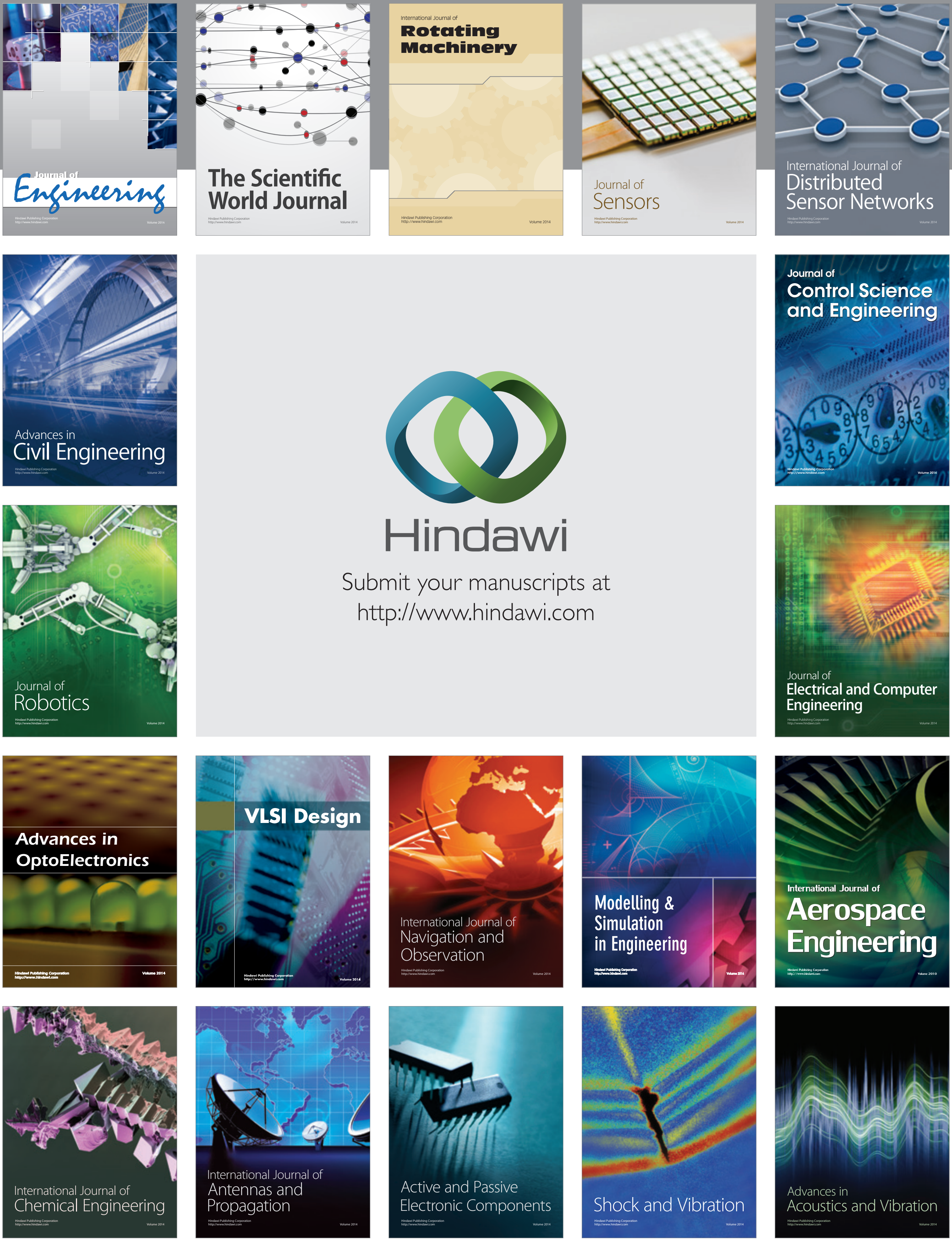
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