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Research Article

Solving Ratio-Dependent Predator-Prey System with Constant Effort Harvesting Using Homotopy Perturbation Method

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Due to wide range of interest in use of bioeconomic models to gain insight into the scientific management of renewable resources like fisheries and forestry, homotopy perturbation method is employed to approximate the solution of the ratio-dependent predator-prey system with constant effort prey harvesting. The results are compared with the results obtained by Adomian decomposition method. The results show that, in new model, there are less computations needed in comparison to Adomian decomposition method.

1. Introduction

Partial differential equations which arise in real-world physical problems are often too complicated to be solved exactly, and even if an exact solution is obtainable, the required calculations may be practically too complicated, or it might be difficult to interpret the outcome. Very recently, some promising approximate analytical solutions are proposed such as Exp-function method, Adomian decomposition method (ADM), variational iteration method (VIM), and homotopy perturbation method (HPM).

HPM is the most effective and convenient method for both linear and nonlinear equations. This method does not depend on a small parameter. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$, which is considered as a "small parameter." HPM has been shown to effectively, easily, and accurately solve a large class of linear and nonlinear problems with components converging to accurate solutions. HPM was first proposed by He [1–7] and was successfully applied to various engineering problems.

The motivation of this paper is to extend the homotopy perturbation method (HPM) [8–17] to solve the ratio-dependent predator-prey system. The results of HPM are compared with those obtained by the ADM [18]. Different from ADM, where specific algorithms are usually used to determine the Adomian polynomials, HPM handles linear and nonlinear problems in simple manner by deforming a difficult problem into a simple one. The HPM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations.

In this paper, we assume that the predator in model is not of commercial importance. The prey is subjected to constant effort harvesting with r, a parameter that measures the effort being spent by a harvesting agency. The harvesting activity does not affect the predator population directly. It is obvious that the harvesting activity does reduce the predator population indirectly by reducing the availability of the prey to the predator. Adopting a simple logistic growth for prey population with $e > 0$, $b > 0$, and $c > 0$ standing for the predator death rate, capturing rate, and conversion rate, respectively, we formulate the problem as

$$\begin{aligned} \frac{dx}{dt} &= x(1-x) - \frac{bxy}{y+x} - rx, \\ \frac{dy}{dt} &= \frac{cxy}{y+x} - ey, \end{aligned} \tag{1.1}$$

where $x(t)$ and $y(t)$ represent the fractions of population densities for prey and predator at time t, respectively. Equations (1.1) are to be solved according to biologically meaningful initial conditions $x(0) \geq 0$ and $y(0) \geq 0$ [18].

2. Applications

In this section, we will apply the HPM to nonlinear differential system of ratio-dependant predator-prey,

$$H(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0, \quad p \in [0,1], \ r\varepsilon\Omega, \tag{2.1}$$

where $A(v)$ is a general differential operator which can be divided into a linear part $L(v)$ and a nonlinear part $N(v)$ and $f(r)$ is a known analytical function. $p \in [0,\ 1]$ is an embedding parameter, while u_0 is an initial approximation of the equation which should be solved, and satisfies the boundary conditions.

According to the HPM (relation (2.1)), we can construct a homotopy of system as follows:

$$\begin{aligned} &(1-p)\left(v_2\dot{v}_1 + v_1\dot{v}_1 - \dot{x}_0 \mathrm{y}_0 - \dot{x}_0 x_0\right) + p\left(v_2\dot{v}_1 + v_1\dot{v}_1 - (1-b-r)v_1v_2 + v_2v_1^2 - (1-r)v_1^2 + v_1^3\right) = 0, \\ &(1-p) \times \left(v_2\dot{v}_2 + v_1\dot{v}_2 - \dot{\mathrm{y}}_0\mathrm{y}_0 - \mathrm{x}_0\dot{\mathrm{y}}_0\right) + p\left(v_2\dot{v}_2 + v_1\dot{v}_2 + (e-c)v_1v_2 + \mathrm{e}v_2^2\right) = 0, \end{aligned} \tag{2.2}$$

where dot denotes differentiation with respect to t, and the initial approximations are as follows:

$$\begin{aligned} v_{1,0}(t) &= x_0(t) = x(0), \\ v_{2,0}(t) &= y_0(t) = y(0). \end{aligned} \tag{2.3}$$

Assume that the solution of (2.2) can be written as a power series in p as follows:

$$\begin{aligned} \nu_1 &= \nu_{1,0} + p\nu_{1,1} + p^2\nu_{1,2} + p^3\nu_{1,3} + \cdots, \\ \nu_2 &= \nu_{2,0} + p\nu_{2,1} + p^2\nu_{2,2} + p^3\nu_{2,3} + \cdots, \end{aligned} \tag{2.4}$$

where $\nu_{i,j}$ $(i, j = 1, 2, 3, \ldots)$ are functions yet to be determined. Substituting (2.3) and (2.4) into (2.2), and arranging the coefficients of p powers, we have

$$\begin{aligned}
&(v_{2,0}\dot{v}_{1,0} + v_{1,0}\dot{v}_{1,0}) \\
&\quad + (v_{1,0}^3 - v_{1,0}^2 + v_{1,0}\,\dot{v}_{1,1} + v_{2,0}\dot{v}_{1,1} + r\,v_{1,0}v_{2,0} + bv_{1,0}v_{2,0} - v_{1,0}v_{2,0} + v_{2,0}v_{1,0}^2 + rv_{1,0}^2)p \\
&\quad + (v_{1,1}\,\dot{v}_{1,1} + v_{1,0}\,\dot{v}_{1,2} + v_{2,0}\,\dot{v}_{1,2} + v_{2,1}\,\dot{v}_{1,1} + 2r\,v_{1,0}v_{1,1} + b\,v_{1,0}v_{2,1} + 2\,v_{2,0}\,v_{1,0}\,v_{1,1} + r\,v_{1,1}\,v_{2,0} \\
&\qquad + r\,v_{1,0}\,v_{2,1} + b\,v_{1,1}\,v_{2,0} - v_{1,0}\,v_{2,1} - v_{1,1}v_{2,0} + v_{2,1}v_{1,0}^2 - 2\,v_{1,0}\,v_{1,1} + 3\,v_{1,0}^2v_{1,1})p^2 \\
&\quad + (v_{1,1}\dot{v}_{1,2} + v_{1,2}\dot{v}_{1,1} + v_{1,0}\dot{v}_{1,3} + v_{2,1}\dot{v}_{1,2} + v_{2,0}\dot{v}_{1,3} + v_{2,2}\dot{v}_{1,1} + v_{2,0}v_{1,1}^2 - v_{1,0}v_{2,2} - v_{1,2}v_{2,0} \\
&\qquad - v_{1,1}v_{2,1} + v_{2,2}v_{1,0}^2 + rv_{1,1}^2 + 3v_{1,0}v_{1,1}^2 - v_{1,1}^2 + bv_{1,1}v_{2,1} + bv_{1,0}v_{2,2} + bv_{1,2}v_{2,0} + rv_{1,0}v_{2,2} \\
&\qquad + rv_{1,1}v_{2,1} + rv_{1,2}v_{2,0} + 2v_{2,0}v_{1,0}v_{1,2} + 2rv_{1,0}v_{1,2} + 2v_{2,1}v_{1,0}v_{1,1} + 3v_{1,0}^2v_{1,2} \\
&\qquad - 2v_{1,0}v_{1,2})p^3 + \cdots = 0, \\
&(v_{2,0}\dot{v}_{2,0} + v_{1,0}\dot{v}_{2,0}) \\
&\quad + (ev_{1,0}v_{2,0} - cv_{1,0}v_{2,0} + v_{2,0}\dot{v}_{2,1} + v_{1,0}\dot{v}_{2,1} + ev_{2,0}^2)p \\
&\quad + (v_{2,1}\dot{v}_{2,1} + ev_{1,0}v_{2,1} - cv_{1,0}v_{2,1} + ev_{1,1}v_{2,0} - cv_{1,1}v_{2,0} + 2ev_{2,0}v_{2,1} + v_{2,0}\dot{v}_{2,2} + v_{1,1}\dot{v}_{2,1} + v_{1,0}\dot{v}_{2,2})p^2 \\
&\quad + (ev_{2,1}^2 + v_{2,1}\dot{v}_{2,2} + v_{2,2}\dot{v}_{2,1} + v_{2,0}\dot{v}_{2,3} + v_{1,1}\dot{v}_{2,2} + v_{1,2}\dot{v}_{2,1} + v_{1,0}\dot{v}_{2,3} + ev_{1,0}v_{2,2} + ev_{1,1}v_{2,1} \\
&\qquad - cv_{1,0}v_{2,2} - cv_{1,1}v_{2,1} + ev_{1,2}v_{2,0} - cv_{1,2}v_{2,0} + 2ev_{2,0}v_{2,2})p^3 + \cdots = 0.
\end{aligned} \tag{2.5}$$

In order to obtain the unknown of $\nu_{i,j}(x, t)$, $i, j = 1, 2, 3, \ldots$, we must construct and solve the following system which includes 6 equations, considering the initial conditions of $\nu_{i,j}(0) = 0$, $i, j = 1, 2, 3, \ldots$:

$$\begin{aligned}
&v_{2,0}\dot{v}_{1,0} + v_{1,0}\dot{v}_{1,0} = 0, \\
&v_{1,0}^3 - v_{1,0}^2 + v_{1,0}\,\dot{v}_{1,1} + v_{2,0}\dot{v}_{1,1} + v_{1,0}v_{2,0} + bv_{1,0}v_{2,0} - v_{1,0}v_{2,0} + v_{2,0}v_{1,0}^2 + rv_{1,0}^2 = 0, \\
&v_{1,1}\,\dot{v}_{1,1} + v_{1,0}\,\dot{v}_{1,2} + v_{2,0}\,\dot{v}_{1,2} + v_{2,1}\,\dot{v}_{1,1} + 2r\,v_{1,0}v_{1,1} + b\,v_{1,0}v_{2,1} + 2\,v_{2,0}\,v_{1,0}\,v_{1,1} + r\,v_{1,1}\,v_{2,0} \\
&\qquad + r\,v_{1,0}\,v_{2,1} + b\,v_{1,1}\,v_{2,0} - v_{1,0}\,v_{2,1} - v_{1,1}v_{2,0} + v_{2,1}v_{1,0}^2 - 2\,v_{1,0}\,v_{1,1} + 3\,v_{1,0}^2v_{1,1} = 0, \\
&v_{2,0}\dot{v}_{2,0} + v_{1,0}\dot{v}_{2,0} = 0, \\
&ev_{1,0}v_{2,0} - cv_{1,0}v_{2,0} + v_{2,0}\dot{v}_{2,1} + v_{1,0}\dot{v}_{2,1} + ev_{2,0}^2 = 0, \\
&v_{2,1}\dot{v}_{2,1} + ev_{1,0}v_{2,1} - cv_{1,0}v_{2,1} + ev_{1,1}v_{2,0} - cv_{1,1}v_{2,0} + 2ev_{2,0}v_{2,1} + v_{2,0}\dot{v}_{2,2} + v_{1,1}\dot{v}_{2,1} + v_{1,0}\dot{v}_{2,2} = 0.
\end{aligned} \tag{2.6}$$

From (2.4), if the first three approximations are sufficient, then setting $p = 1$ yields the approximate solution of (1.1) to

$$\begin{aligned} x(t) &= \lim_{p \to 1} v_1(t) = \sum_{k=0}^{k=3} v_{1,k}(t), \\ y(t) &= \lim_{p \to 1} v_2(t) = \sum_{k=0}^{k=3} v_{2,k}(t). \end{aligned} \tag{2.7}$$

Therefore,

$$\mathrm{v}_{1,0}(t) = x_0(t) = x(0), \tag{2.8}$$

$$v_{1,1}(t) = -\frac{\mathrm{x}_0\left(x_0^2 - x_0 - y_0 + x_0y_0 + ry_0 + by_0 + rx_0\right)t}{x_0 + y_0}, \tag{2.9}$$

$$\begin{aligned} v_{1,2}(t) = \frac{1}{2(x_0+y_0)^3}((x_0t^2(&3y_0x_0^2 - x_0^2by_0 + 2x_0^3by_0 + 3x_0^4r + 6x_0^3y_0^2 - 3y_0^3x_0 + x_0^3r^2 - 9x_0^3y_0 \\ &+ 6x_0^4y_0 - 9x_0^2y_0^2 + 2y_0^3x_0^2 - 2x_0^3r - 2ry_0^3 - 2by_0^3 + b^2y_0^3 + r^2y_0^3 \\ &+ x_0^2by_0r + 3x_0ry_0^2b + y_0^2x_0eb + bx_0^2y_0e - bx_0^2y_0c - 3x_0by_0^2 \\ &+ 3x_0y_0^2 + 3y_0^3x_0r + 3y_0^3x_0b - 6x_0^2ry_0 + 2x_0^5 - 3x_0^4 + y_0^3 + 2ry_0^3b \\ &+ 9x_0^3ry_0 - 6x_0ry_0^2 + 9x_0^2y_0^2r + 5x_0^2y_0^2b + x_0^3 + 3x_0r^2y_0^2 + 3x_0^2r^2y_0))), \end{aligned} \tag{2.10}$$

$$v_{2,0}(t) = y_0(t) = y(0), \tag{2.11}$$

$$v_{2,1}(t) = \frac{\mathrm{y}_0\left(-ex_0 + cx_0 - ey_0\right)t}{y_0 + x_0}, \tag{2.12}$$

$$\begin{aligned} v_{2,2}(t) = -\frac{1}{2(y_0+x_0)^3}((y_0t^2(&3y_0ex_0^2c + y_0^2cx_0e + 2ex_0^3c - cx_0^2y_0 - cx_0y_0^2 - c^2x_0^3 + cx_0^3y_0 \\ &+ cx_0^2y_0r + cx_0y_0^2b + cx_0^2y_0^2 + cx_0y_0^2r - e^2x_0^3 - 3y_0e^2x_0^2 \\ &- 3y_0^2e^2x_0 - y_0^3e^2))). \end{aligned} \tag{2.13}$$

We also obtained $v_{1,3}$ and $v_{2,3}$, but because they were too long to maintain, we skip them and only use them in the final numerical results. In this manner, the other components can be easily obtained by substituting (2.8) through (2.13) into (2.7) as follows:

$$\begin{aligned} x(t) = x(0) &- \left(\frac{\mathrm{x}_0\left(x_0^2 - x_0 - y_0 + x_0y_0 + ry_0 + by_0 + rx_0\right)t}{x_0 + y_0}\right) \\ &+ \frac{1}{2(x_0+y_0)^3}(x_0t^2(3y_0x_0^2 - x_0^2by_0 + 2x_0^3by_0 + 3x_0^4r + 6x_0^3y_0^2 - 3y_0^3x_0 + x_0^3r^2 - 9x_0^3y_0 \\ &\quad + 6x_0^4y_0 - 9x_0^2y_0^2 + 2y_0^3x_0^2 - 2x_0^3r - 2ry_0^3 - 2by_0^3 + b^2y_0^3 + r^2y_0^3 \\ &\quad + x_0^2by_0r + 3x_0ry_0^2b + y_0^2x_0eb + bx_0^2y_0e - bx_0^2y_0c - 3x_0by_0^2 + 3x_0y_0^2 \\ &\quad + 3y_0^3x_0r + 3y_0^3x_0b - 6x_0^2ry_0 + 2x_0^5 - 3x_0^4 + y_0^3 + 2ry_0^3b + 9x_0^3ry_0 - 6x_0ry_0^2 \\ &\quad + 9x_0^2y_0^2r + 5x_0^2y_0^2b + x_0^3 + 3x_0r^2y_0^2 + 3x_0^2r^2y_0)) + v_{1,3}\cdots, \\ y(t) = y(0) &+ \frac{\mathrm{y}_0\left(-ex_0 + cx_0 - ey_0\right)t}{y_0 + x_0} - \frac{1}{2(y_0+x_0)^3} \\ &\times (y_0t^2(3y_0ex_0^2c + y_0^2cx_0e + 2ex_0^3c - cx_0^2y_0 - cx_0y_0^2 - c^2x_0^3 + cx_0^3y_0 + cx_0^2y_0r \\ &\quad + cx_0y_0^2b + cx_0^2y_0^2 + cx_0y_0^2r - e^2x_0^3 - 3y_0e^2x_0^2 - 3y_0^2e^2x_0 - y_0^3e^2)) + v_{2,3}\cdots. \end{aligned} \tag{2.14}$$

3. Numerical results and comparison with ADM

For comparison with the results obtained by ADM [18], the parameter values in four cases are considered in Table 1.

Table 1: Parameter values used for illustration purposes.

Case	x_0	y_0	b	c	e	r
1	0.5	0.3	0.8	0.2	0.5	0.9
2	0.5	0.3	0.8	0.2	0.5	0.1
3	0.5	0.6	0.5	0.5	0.3	0.1
4	0.5	0.2	0.5	0.5	0.1	0.2

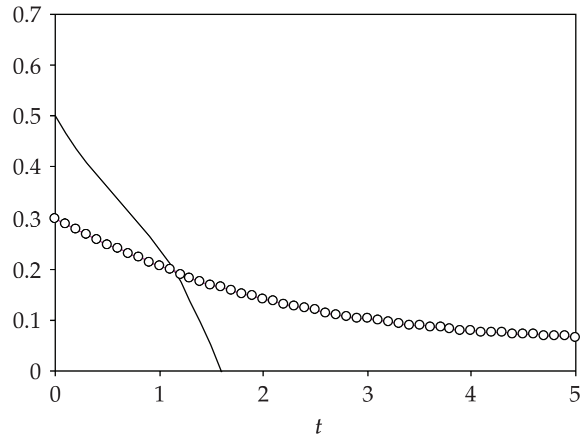

Figure 1: Population fraction versus time for Case 1: $r = 0.9$: (—) prey population fraction; (∘∘∘) predator population fraction.

Results of four terms approximation for $x(t), y(t)$ obtained by using HPM and ADM [18] are presented in (3.1), respectively:

$$\begin{aligned}
\text{Case 1}: x &\approx 0.5 - 0.35t + 0.19476t^2 - 0.107288t^3,\\
y &\approx 0.3 - 0.1125t + 0.018808t^2 - 0.0011284t^3,\\
\text{Case 2}: x &\approx 0.5 + 0.05t + 0.012265t^2 - 0.0016032t^3,\\
y &\approx 0.3 - 0.1125t + 0.024433t^2 - 0.00398199t^3,\\
\text{Case 3}: x &\approx 0.3 + 0.0799\text{t} + 0.00533\,\text{t}^2 - 0.00115\,t^3,\\
y &\approx 0.6 - 0.08t + 0.01866t^2 - 0.00231t^3,\\
\text{Case 4}: x &\approx 0.5 + 0.07857t - 0.016020\,t^2 - 0.00119873\,t^3,\\
y &\approx 0.2 + 0.051428t + 0.0055918t^2 + 0.00002245t^3,\\
\text{Case 1}: x &\approx 0.5 - 0.35000t + 0.19476t^2 - 0.10728t^3,\\
y &\approx 0.3 - 0.11250t + 0.018809t^2 - 0.0011286t^3,\\
\text{Case 2}: x &\approx 0.5 + 0.05000t + 0.012266t^2 - 0.0016034t^3,\\
y &\approx 0.3 - 0.11250t + 0.024434t^2 - 0.0039821t^3,\\
\text{Case 3}: x &\approx 0.3 + 0.08000\text{t} + 0.005333\,\text{t}^2 - 0.0011555\,t^3,\\
y &\approx 0.6 - 0.08000t + 0.018667t^2 - 0.0023112t^3,\\
\text{Case 4}: x &\approx 0.5 + 0.07857t - 0.016021\,t^2 - 0.0011984t^3,\\
y &\approx 0.2 + 0.051430t + 0.0055920t^2 + 0.00002246t^3.
\end{aligned} \tag{3.1}$$

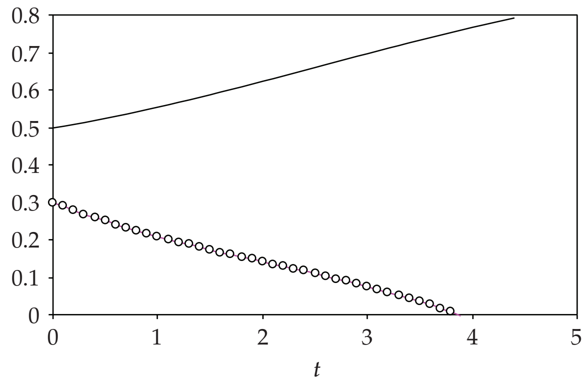

Figure 2: Population fraction versus time for Case 2: $r = 0.1$: (—) prey population fraction; (∘∘∘) predator population fraction.

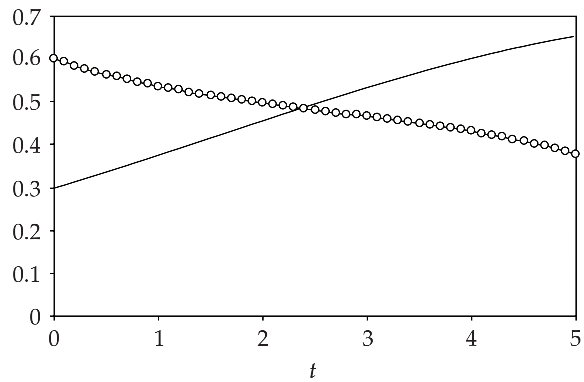

Figure 3: Population fraction versus time for Case 3: $r = 0.1$: (—) prey population fraction; (∘∘∘) predator population fraction.

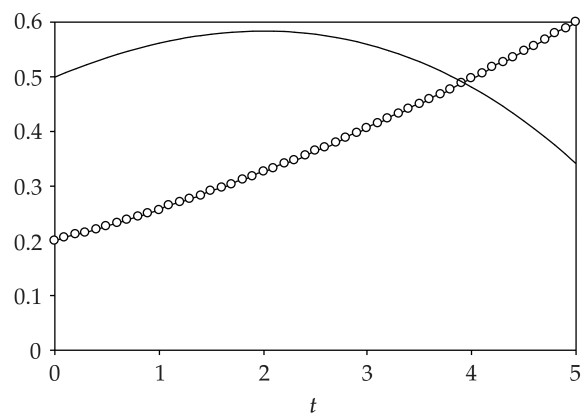

Figure 4: Population fraction versus time for Case 4: $r = 0.2$: (—) prey population fraction; (∘∘∘) predator population fraction.

Figures 1–4 show the relations between prey and predator populations versus time.

A noteworthy observation from Figure 1 is that prey and predator species can become extinct simultaneously for some values of parameters, regardless of the initial values. Thus, overexploitation of the prey population by constant effort harvesting process together with high predator capturing rate may lead to mutual extinction as a possible outcome of predator-pray interaction. In Figure 2, only the predator population gradually decreases and becomes extinct despite the availability of increasing prey population. This can be attributed to the effect of the predator death rate, being greater than the conversion rate and low constant prey harvesting as shown in Case 2 (see Table 1). Figures 3 and 4 illustrate the possibility of predator and prey long-term coexistence. Depending on the initial values, both prey and predator populations increase or reduce in order to allow long-term coexistence [18].

4. Conclusion

Homotopyperturbation method was employed to approximate the solution of the ratio-dependent predator-prey system with constant effort prey harvesting. The results obtained here were compared with results of Adomian decomposition method. The results show that there is less computations needed in comparison to ADM.

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