

## Research Article

# Analysis of an Impulsive One-Predator and Two-Prey System with Stage-Structure and Generalized Functional Response

Xiangmin Ma,<sup>1</sup> Yuanfu Shao,<sup>1</sup> Zhen Wang,<sup>1</sup> Xianjia Fang,<sup>1</sup> and Zhenguo Luo<sup>2</sup>

<sup>1</sup>School of Science, Guilin University of Technology, Guilin, Guangxi 541004, China

<sup>2</sup>Department of Mathematics, Hengyang Normal University, Hengyang, Hunan 421008, China

Correspondence should be addressed to Yuanfu Shao; shaoyuanfu@163.com and Zhenguo Luo; 981584207@qq.com

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An impulsive one-predator and two-prey system with stage-structure and generalized functional response is proposed and analyzed. By reasonable assumption and theoretical analysis, we obtain conditions for the existence and global attractivity of the predator-extinction periodic solution. Sufficient conditions for the permanence of this system are established via impulsive differential comparison theorem. Furthermore, abundant results of numerical simulations are given by choosing two different and concrete functional responses, which indicate that impulsive effects, stage-structure, and functional responses are vital to the dynamical properties of this system. Finally, the biological meanings of the main results and some control strategies are given.

## 1. Introduction and Model Formulation

In real world, the properties of one-predator and one-prey system have been studied widely and many valuable results have been obtained. If examining the cases that there are two preys for a predator, then the above system cannot reflect the real behaviors of individuals accurately, so scholars proposed three-species predator-prey system. The relationship between species in three-species system may take many forms, such as one prey and two predators [1], a food chain [2, 3], or two preys and one predator [4, 5]. On the other hand, for predator-prey model, in description of the relationship between predator and prey, a crucial element is the classic definition of a predator's functional response. Recently, the dynamics of predator-prey systems with different kinds of functional responses have been studied in relevant literature, such as Holling type [6], Crowley-Martin type [7–9], Beddington-DeAngelis type [10, 11], Watt type [12, 13], and Ivlev type [14]. For example, Gakkhar and Naji [15] investigated the dynamical behaviors of the following three-species system with nonlinear functional response:

$$x_1'(t) = x_1(t) \left( r_1 - a_{11}x_1(t) - \frac{\alpha_1 x_1(t) y(t)}{a_1 + b_1 x_1(t) + b_1 x_2(t)} \right),$$

$$\begin{aligned} x_2'(t) &= x_2(t) \left( r_2 - a_{22}x_2(t) - \frac{\alpha_2 x_2(t) y(t)}{a_1 + b_1 x_1(t) + b_1 x_2(t)} \right), \\ y'(t) &= \left( -d + \frac{m_1 \alpha_1 x_1(t) + m_2 \alpha_2 x_2(t)}{a_1 + b_1 x_1(t) + b_1 x_2(t)} \right) y(t), \end{aligned} \quad (1)$$

where  $x_1(t)$  and  $x_2(t)$  represent the two preys densities, respectively, and  $y(t)$  represents the density of predators depending on the two preys.

However, as Pei et al. [16] pointed out that system (1) could not provide an effective approach because there was no impulsive spraying pesticides or harvesting pest at different fixed moment. We know that pests may bring disastrous effects to their existing system when their amount reaches a certain level. For preventing large economic loss, chemical pesticides are often used in the process of pest management. As a matter of fact, the control on pests often makes pests reduce instantaneously in a short time. In the modeling process, these perturbations are often assumed to be in the form of impulses. Based on traditional models, impulsive differential equations are proposed and extensively used in some applied fields, especially in population dynamics; see

[17–19]. The theory of impulsive differential equation is now being recognized richer than the corresponding differential equation without impulses, which plays a key role in the development of biomathematics; see monographs [20, 21] and references cited therein.

On the other hand, the stage-structure for predator was also not considered in system (1). In real world, many species go through two or more life stages when they proceed from birth to death. For many animals, their babies are raised by their parents or are dependent on the nutrition from the eggs they stay in. The babies are too weak to produce babies or capture their prey; hence their competition with other individuals of the community can be ignored. Therefore, it is reasonable to introduce stage-structure into competitive or predator-prey models. Many researchers have incorporated it into biological models, where stage-structure is modeled by using a time delay [22–24]. Authors [5] pointed out that when the system contained time delay, it had more interesting behaviors. Their results showed that time delay could cause a stable equilibrium to become unstable and Hopf bifurcation could occur as the time delay crossed some critical values. These obtained results have shown that stage-structure plays a vital role in predator-prey models and stage-structured systems exhibit complicated properties. Moreover, Xu [25] showed that an important factor in modeling of predator-prey is the choice of functional response. Model with generalized functional response exhibited many universal properties, which could be applied to many fields because of its flexibility. Shao and Li [26] considered a predator-prey system with generalized functional response. Their results indicated that generalized functional response caused dynamical behaviors of the system to be very complex.

Based on these backgrounds, in this paper, develop- ing system (1) with stage-structure, generalized function response, and impulsive spraying pesticides, we will consider the following one-predator and two-prey system:

$$\begin{aligned}
 x_1'(t) &= x_1(t)(a_1 - b_1 x_1(t)) - \beta_1 f_1(x_1(t)) y_2(t), \\
 x_2'(t) &= x_2(t)(a_2 - b_2 x_2(t)) - \beta_2 f_2(x_2(t)) y_2(t), \\
 y_1'(t) &= (\lambda_1 \beta_1 f_1(x_1(t)) + \lambda_2 \beta_2 f_2(x_2(t))) y_2(t) \\
 &\quad - e^{-d_1 \tau} (\lambda_1 \beta_1 f_1(x_1(t-\tau)) + \lambda_2 \beta_2 f_2(x_2(t-\tau))) \\
 &\quad \cdot y_2(t-\tau) - d_1 y_1(t), \\
 y_2'(t) &= e^{-d_1 \tau} (\lambda_1 \beta_1 f_1(x_1(t-\tau)) + \lambda_2 \beta_2 f_2(x_2(t-\tau))) \\
 &\quad \cdot y_2(t-\tau) - d_2 y_2(t) - r y_2^2(t), \\
 &\quad t \neq nT, \\
 x_1(t^+) &= (1 - p_1) x_1(t), \\
 x_2(t^+) &= (1 - p_2) x_2(t), \\
 y_1(t^+) &= y_1(t), \\
 y_2(t^+) &= y_2(t), \\
 &\quad t = nT,
 \end{aligned} \tag{2}$$

where  $x_1(t)$  and  $x_2(t)$  represent the densities of two different preys, respectively, and we assume that there is no competition between the two preys.  $y_1(t)$  and  $y_2(t)$  denote the densities of immature predator and mature predator, respectively.  $a_i$  is the natural growth rate of  $x_i(t)$  ( $i = 1, 2$ ).  $b_i$  and  $r$  are coefficients of internal competition of prey  $x_i(t)$  ( $i = 1, 2$ ) and mature predator  $y_2(t)$ , respectively.  $\beta_i$  is capture rate of mature predator for  $x_i(t)$  ( $i = 1, 2$ ).  $\lambda_i$  represents the conversion rate of two preys into reproduction of mature predator.  $d_1$  and  $d_2$  are death rates of immature predator and mature predator, respectively.  $\tau$  is the mean length of juvenile period of predator. The term  $e^{-d_1 \tau}$  denotes the mature rate of immature predator. Function  $f_i(x_i(t))$  ( $i = 1, 2$ ) is adult predator's functional response.  $p_i$  ( $0 \leq p_i < 1$ ,  $i = 1, 2$ ) is partial impulsive harvesting of prey by catching or pesticides at moment  $t = nT$  ( $n = 1, 2, \dots$ ).

By use of impulsive differential equation theory and some analysis techniques, we aim to investigate the existence and global attractivity of predator-extinction periodic solution and the permanence of (2). Further, by numerical analysis, we try to find out the effects of impulsive and stage-structure on this system.

Since  $y_1(t)$  does not appear in the first, the second, and the fourth equation of system (2), we can simplify (2) and restrict our attention to the following system:

$$\begin{aligned}
 x_1'(t) &= x_1(t)(a_1 - b_1 x_1(t)) - \beta_1 f_1(x_1(t)) y_2(t), \\
 x_2'(t) &= x_2(t)(a_2 - b_2 x_2(t)) - \beta_2 f_2(x_2(t)) y_2(t), \\
 y_2'(t) &= e^{-d_1 \tau} (\lambda_1 \beta_1 f_1(x_1(t-\tau)) + \lambda_2 \beta_2 f_2(x_2(t-\tau))) \\
 &\quad \cdot y_2(t-\tau) - d_2 y_2(t) - r y_2^2(t), \\
 &\quad t \neq nT,
 \end{aligned} \tag{3}$$

$$x_1(t^+) = (1 - p_1) x_1(t),$$

$$x_2(t^+) = (1 - p_2) x_2(t),$$

$$y_2(t^+) = y_2(t),$$

$$t = nT,$$

with initial conditions:

$$\begin{aligned}
 (x_1(s), x_2(s), y_2(s)) &= (\varphi_1(s), \varphi_2(s), \varphi_3(s)) \\
 &\in C([-\tau, 0], R_+^3), \\
 \varphi_i(0) &> 0, \quad i = 1, 2, 3.
 \end{aligned} \tag{4}$$

From biological point of view, without loss of generality, in this paper, we assumed that  $f_i(x)$  ( $i = 1, 2$ ) is strictly increasing, differential with  $f_i(0) = 0$ , satisfying  $0 < f_i(x)/x < L_i$  (a constant) for all  $x > 0$ . Further, we only consider (3) in the following biological meaning region:

$$\begin{aligned}
 D = \{ &(x_1(t), x_2(t), y_1(t), y_2(t)) \mid x_1(t), x_2(t), y_1(t), \\
 &y_2(t) \geq 0 \}.
 \end{aligned} \tag{5}$$

The rest of this paper is organized as follows. In Section 2, we give some notations, definitions, and lemmas. By using lemmas and impulsive comparison theorem, we discuss the existence of predator-extinction solution and permanence of system (3) in Sections 3 and 4, respectively. In Section 5, numerical simulations are given to show the complicated dynamical behaviors of (3). Finally, we end this paper by a brief discussion in Section 6.

### 2. Preliminaries

In this section, some definitions and lemmas are introduced which are useful for our main results. Solution of (3), denoted by  $x(t) = (x_1(t), x_2(t), y_1(t), y_2(t))^T$ , is piecewise and continuous function:  $x : R_+ \rightarrow R_+^4$ ,  $x(t)$  is continuous on  $(nT, (n+1)T]$ ,  $n \in Z_+$ , and  $x(nT^+) = \lim_{t \rightarrow nT^+} x(t)$  exists. Obviously, the global existence and uniqueness of solution of (3) are guaranteed by the smoothness of  $f = (f_1, f_2, f_3, f_4)$ , where  $f$  denotes the mapping defined by right side of system (2). For more details refer to [20, 21].

**Lemma 1** (see [27]). *Consider the following differential equation:*

$$g'(t) = a_1 g(t - \tau) - a_2 g(t) - a_3 g^2(t), \quad (6)$$

where  $a_i$  ( $i = 1, 2, 3$ ) is a positive constant and  $g(t) > 0$ ,  $t \in [-\tau, 0]$ ; then we have the following:

- (i) if  $a_1 > a_2$ , then  $\lim_{t \rightarrow \infty} g(t) = (a_1 - a_2)/a_3$ ;
- (ii) if  $a_1 < a_2$ , then  $\lim_{t \rightarrow \infty} g(t) = 0$ .

**Lemma 2** (see [10]). *Consider the following impulsive system:*

$$\begin{aligned} x'(t) &= x(t)(a - bx(t)), \quad t \neq nT, \\ x(t^+) &= (1 - p)x(t), \quad t = nT, \end{aligned} \quad (7)$$

where  $a > 0, b > 0$ , and  $p > 0$ . If  $p < 1 - e^{-aT}$ , then system (7) has a positive periodic solution  $x^*(t)$  and for any solution  $x(t)$  of system (7), we have  $|x(t) - x^*(t)| \rightarrow 0$ , as  $t \rightarrow +\infty$ , where

$$\begin{aligned} x^*(t) &= \frac{a(1 - p - e^{-aT})}{b(1 - p - e^{-aT}) + bpe^{-a(t-nT)}}, \\ x^*(0^+) &= \frac{a(1 - p - e^{-aT})}{b(1 - e^{-aT})}, \end{aligned} \quad (8)$$

$$nT < t \leq (n+1)T.$$

By Lemma 2, we can easily know that if the following hypotheses (H1) and (H2) hold,

- (H1):  $p_1 < 1 - e^{-a_1T}$ ,
- (H2):  $p_2 < 1 - e^{-a_2T}$ .

Then (3) has a mature predator-extinction periodic solution  $(x_1^*(t), x_2^*(t), 0)$  for  $t \in (nT, (n+1)T]$ , and for any solution

$(x_1(t), x_2(t), y_2(t))$  of system (3), we have  $x_1(t) \rightarrow x_1^*(t)$ ,  $x_2(t) \rightarrow x_2^*(t)$ , and  $y_2(t) \rightarrow 0, t \rightarrow \infty$ , where

$$\begin{aligned} x_i^*(t) &= \frac{a_i(1 - p_i - e^{-a_iT})}{b_i(1 - p_i - e^{-a_iT}) + b_i p_i e^{-a_i(t-nT)}}, \\ i &= 1, 2, \quad t \in (nT, (n+1)T], \end{aligned} \quad (9)$$

with

$$x_i^*(0^+) = \frac{a_i(1 - p_i - e^{-a_iT})}{b_i(1 - e^{-a_iT})}, \quad i = 1, 2. \quad (10)$$

### 3. Global Attractivity of the Predator-Extinction Periodic Solution

In this section, we investigate the global attractivity of predator-extinction periodic solution  $(x_1^*(t), x_2^*(t), 0)$  of system (3).

**Theorem 3.** *Predator-extinction periodic solution  $(x_1^*(t), x_2^*(t), 0)$  of system (3) is globally attractive if (H1), (H2), and*

$$(H3): e^{-d_1\tau}(\lambda_1\beta_1 f_1(\eta_1) + \lambda_2\beta_2 f_2(\eta_2)) < d_2$$

hold, where  $\eta_1$  and  $\eta_2$  are defined in (15) and (16), respectively.

*Proof.* Since (H3) holds and  $f_1(x), f_2(x)$  are differential for all  $x > 0$ , we can choose two positive constants  $\varepsilon_1$  and  $\varepsilon_2$  to be sufficiently small such that

$$e^{-d_1\tau}(\lambda_1\beta_1 f_1(\eta_1 + \varepsilon_1) + \lambda_2\beta_2 f_2(\eta_2 + \varepsilon_2)) < d_2. \quad (11)$$

From the first equation of system (3), we have  $x_1'(t) \leq x_1(t)(a_1 - b_1 x_1(t))$ .

Consider the following impulsive comparison system:

$$\begin{aligned} u_1'(t) &= u_1(t)(a_1 - b_1 u_1(t)), \quad t \neq nT, \\ u_1(t^+) &= (1 - p_1)u_1(t), \quad t = nT. \end{aligned} \quad (12)$$

In view of Lemma 2, we obtain that

$$\begin{aligned} u_1^*(t) &= \frac{a_1(1 - p_1 - e^{-a_1T})}{b_1(1 - p_1 - e^{-a_1T}) + b_1 p_1 e^{-a_1(t-nT)}} = x_1^*(t), \\ t &\in (nT, (n+1)T], \end{aligned} \quad (13)$$

with

$$u_1^*(0^+) = \frac{a_1(1 - p_1 - e^{-a_1T})}{b_1(1 - e^{-a_1T})} = x_1^*(0^+), \quad (14)$$

which is unique and globally asymptotically stable positive periodic solution of (12). By use of comparison theorem of impulsive differential equation, there exists  $k_1 \in N$  such that, for the sufficiently small constant  $\varepsilon_1$  and all  $t \in (nT, (n+1)T]$  ( $n > k_1$ ), we have

$$\begin{aligned} x_1(t) &\leq x_1^*(t) + \varepsilon_1 \\ &\leq \frac{a_1(1 - p_1 - e^{-a_1T})}{b_1(1 - p_1 - e^{-a_1T}) + b_1 p_1 e^{-a_1T}} + \varepsilon_1 \triangleq \eta_1 \\ &+ \varepsilon_1. \end{aligned} \quad (15)$$

Similarly, there exists  $k_2 \in N$  such that, for the sufficiently small constant  $\varepsilon_2$  and all  $t \in (nT, (n+1)T]$  ( $n > k_2$ ), we have

$$\begin{aligned} x_2(t) &\leq x_2^*(t) + \varepsilon_2 \\ &\leq \frac{a_2(1-p_2-e^{-a_2T})}{b_2(1-p_2-e^{-a_2T})+b_2p_2e^{-a_2T}} + \varepsilon_2 \triangleq \eta_2 \\ &\quad + \varepsilon_2. \end{aligned} \quad (16)$$

Through observation of the third equation of (3), we have

$$\begin{aligned} y_2'(t) &\leq e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1+\varepsilon_1)+\lambda_2\beta_2f_2(\eta_2+\varepsilon_2)) \\ &\quad \cdot y_2(t-\tau) - d_2y_2(t) - ry_2^2(t). \end{aligned} \quad (17)$$

Consider the following differential comparison system:

$$\begin{aligned} u_2'(t) &= e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1+\varepsilon_1)+\lambda_2\beta_2f_2(\eta_2+\varepsilon_2)) \\ &\quad \cdot u_2(t-\tau) - d_2u_2(t) - ru_2^2(t). \end{aligned} \quad (18)$$

According to (11) and Lemma 1, we have  $\lim_{t \rightarrow \infty} u_2(t) = 0$ . Since  $y_2(\xi) = \varphi_3(\xi)$ ,  $\xi \in [-\tau, 0]$ ,  $u_2(t)$  is the solution of (18) with initial conditions  $u_2(\xi) = \varphi_3(\xi)$ ,  $\xi \in [-\tau, 0]$ ; by comparison theorem, we have  $\lim_{t \rightarrow \infty} y_2(t) \leq \lim_{t \rightarrow \infty} u_2(t) = 0$ . In view of the positivity of  $y_2(t)$ , we have  $\lim_{t \rightarrow \infty} y_2(t) = 0$ . It implies that for arbitrarily small positive constant  $\varepsilon_3$  and  $t$  large enough, we have

$$0 < y_2(t) < \varepsilon_3. \quad (19)$$

Further, from the first and the fourth equation of (3), we have

$$\begin{aligned} x_1'(t) &\geq x_1(t)(a_1 - \beta_1L_1\varepsilon_3 - b_1x_1(t)), \quad t \neq nT, \\ x_1(t^+) &= (1 - p_1)x_1(t), \quad t = nT. \end{aligned} \quad (20)$$

Considering the following comparison system of (20),

$$\begin{aligned} u_3'(t) &= u_3(t)(a_1 - \beta_1L_1\varepsilon_3 - b_1u_3(t)), \quad t \neq nT, \\ u_3(t^+) &= (1 - p_1)u_3(t), \quad t = nT, \end{aligned} \quad (21)$$

by Lemma 2, we get the positive periodic solution of system (21) as follows:

$$\begin{aligned} u_3^*(t) &= \frac{(a_1 - \beta_1L_1\varepsilon_3)(1 - p_1 - e^{-(a_1 - \beta_1L_1\varepsilon_3)T})}{b_1(1 - p_1 - e^{-(a_1 - \beta_1L_1\varepsilon_3)T}) + b_1p_1e^{-(a_1 - \beta_1L_1\varepsilon_3)(t-nT)}}, \quad (22) \\ &\quad t \in (nT, (n+1)T], \end{aligned}$$

with

$$\begin{aligned} u_3^*(0^+) &= \frac{(a_1 - \beta_1L_1\varepsilon_3)(1 - p_1 - e^{-(a_1 - \beta_1L_1\varepsilon_3)T})}{b_1(1 - e^{-(a_1 - \beta_1L_1\varepsilon_3)T})} \\ &= x_1^*(0^+). \end{aligned} \quad (23)$$

By comparison theorem, for given constant  $\varepsilon_1 > 0$  and  $t$  large enough, we have  $u_3^*(t) - \varepsilon_1 < x_1(t)$ . Let  $\varepsilon_3 \rightarrow 0$ , then  $u_3^*(t) \rightarrow x_1^*(t)$ , so we have  $x_1^*(t) - \varepsilon_3 < x_1(t)$ . It follows from (15) that  $x_1(t) < x_1^*(t) + \varepsilon_1$  for  $t$  sufficiently large, which implies that  $x_1(t) \rightarrow x_1^*(t)$  as  $t \rightarrow \infty$ . Similarly, we can obtain  $x_2(t) \rightarrow x_2^*(t)$  as  $t \rightarrow \infty$ . This is the end of the proof.  $\square$

## 4. Permanence of System (3)

Now we investigate the permanence of system (3). Before stating the theorem, we give the definition of permanence for system (3).

*Definition 4.* System (3) is said to be permanent, if there exist two positive constants  $m$  and  $M$ , such that, for any solution  $(x_1(t), x_2(t), y_2(t))$  of (3),  $m \leq x_1(t), x_2(t), y_2(t) \leq M$  holds for  $t$  sufficiently large.

**Theorem 5.** Suppose that conditions of (H1) and (H2) hold; moreover if the following conditions:

$$(H4): 1 - p_i - \exp(-(a_i - \beta_iL_i\eta_3)T) > 0, \quad i = 1, 2,$$

$$(H5): e^{-d_1\tau}(\lambda_1\beta_1f_1(\xi_1) + \lambda_2\beta_2f_2(\xi_2)) - d_2 - r\eta_3 > 0,$$

are satisfied, where  $\eta_3$ ,  $\xi_1$ , and  $\xi_2$  are defined in (27), (40), and (42), respectively, then system (3) is permanent.

*Proof.* Firstly, in view of (15) and (16), noticing that positive constants  $\varepsilon_1$  and  $\varepsilon_2$  are arbitrarily chosen and can be sufficiently small, we have

$$\begin{aligned} x_1(t) &\leq \eta_1, \\ x_2(t) &\leq \eta_2. \end{aligned} \quad (24)$$

Secondly, from the third equation of system (3), we have the following inequality:

$$\begin{aligned} y_2'(t) &\leq e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1) + \lambda_2\beta_2f_2(\eta_2))y_2(t-\tau) \\ &\quad - ry_2^2(t). \end{aligned} \quad (25)$$

Considering the following comparison equation,

$$\begin{aligned} u_4'(t) &= e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1) + \lambda_2\beta_2f_2(\eta_2))u_4(t-\tau) \\ &\quad - ru_4^2(t), \end{aligned} \quad (26)$$

by (H4) and Lemma 1, we have  $\lim_{t \rightarrow \infty} u_4(t) = e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1) + \lambda_2\beta_2f_2(\eta_2))/r$ . According to comparison theorem of differential equation, we get

$$\begin{aligned} y_2(t) &\leq \lim_{t \rightarrow \infty} u_4(t) \\ &\leq \frac{e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1) + \lambda_2\beta_2f_2(\eta_2))}{r} \triangleq \eta_3. \end{aligned} \quad (27)$$

Let  $M = \max\{\eta_1, \eta_2, \eta_3\}$ , by (24) and (27); then we have  $x_1(t), x_2(t), y_2(t) \leq M$ .

The following work is to find a constant  $m > 0$  with  $m < M$ , such that  $m \leq x_1(t), m \leq x_2(t)$ , and  $m \leq y_2(t)$ .

On one hand, from the first and the fourth equation of (3), combining inequality (27), we have

$$x_1'(t) \geq x_1(t)(a_1 - \beta_1L_1\eta_3 - b_1x_1(t)), \quad t \neq nT, \quad (28)$$

$$x_1(t^+) = (1 - p_1)x_1(t), \quad t = nT.$$

Consider the following comparison system:

$$u_5'(t) = u_5(t)(a_1 - \beta_1L_1\eta_3 - b_1u_5(t)), \quad t \neq nT, \quad (29)$$

$$u_5(t^+) = (1 - p_1)u_5(t), \quad t = nT.$$

According to Lemma 2 and (H5), by using comparison theorem, there exists an arbitrarily small constant  $\varepsilon_4 > 0$ , such

$$u_5^*(t) = \frac{(a_1 - \beta_1 L_1 \eta_3)(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 \eta_3)T))}{b_1(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 \eta_3)T)) + b_1 p_1 \exp(-(a_1 - \beta_1 L_1 \eta_3)(t - nT))}, \quad (30)$$

for  $t \in (nT, (n + 1)T]$ , and

$$\begin{aligned} u_5^*(0^+) &= \frac{(a_1 - \beta_1 L_1 \eta_3)(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 \eta_3)T))}{b_1(1 - \exp(-(a_1 - \beta_1 L_1 \eta_3)T))} \quad (31) \\ &= x_1^*(0^+). \end{aligned}$$

By using comparison theorem of impulsive differential equation, we can derive from (30) that

$$\begin{aligned} x_1(t) &\geq u_5^*(t) - \varepsilon_4 \\ &\geq \frac{(a_1 - \beta_1 L_1 \eta_3)(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 \eta_3)T))}{b_1(1 - \exp(-(a_1 - \beta_1 L_1 \eta_3)T))} \quad (32) \\ &\quad - \varepsilon_4 \hat{=} m_1 > 0 \end{aligned}$$

for  $t \in (nT, (n + 1)T]$ . Similarly, we have

$$\begin{aligned} x_2(t) &\geq \frac{(a_2 - \beta_2 L_2 \eta_3)(1 - p_2 - \exp(-(a_2 - \beta_2 L_2 \eta_3)T))}{b_2(1 - \exp(-(a_2 - \beta_2 L_2 \eta_3)T))} \quad (33) \\ &\quad - \varepsilon_4 \hat{=} m_2 > 0. \end{aligned}$$

that  $x_1(t) \geq u_5^*(t) - \varepsilon_4$  for  $t$  large enough, where  $u_5^*(t)$  is the unique and globally stable positive periodic solution of (29) with the following form:

On the other hand, in order to prove the stability of  $y_2(t)$ , we define a Lyapunov function as follows:

$$\begin{aligned} V(t) &= y_2(t) + \exp(-d_1\tau) \\ &\quad \cdot \int_{t-\tau}^t (\lambda_1 \beta_1 f_1(s) + \lambda_2 \beta_2 f_2(s)) y_2(s) ds. \end{aligned} \quad (34)$$

Calculating the derivative of  $V(t)$  along solution  $y_2(t)$  of system (3), we get

$$\begin{aligned} V'(t) &= (\exp(-d_1\tau) (\lambda_1 \beta_1 f_1(x_1(t)) + \lambda_2 \beta_2 f_2(x_2(t))) \\ &\quad - d_2 - r y_2(t)) y_2(t). \end{aligned} \quad (35)$$

According to (H4), we can choose a positive constant  $\varepsilon_5$  small enough such that

$$\begin{aligned} e^{-d_1\tau} (\lambda_1 \beta_1 f_1(\xi_1 - \varepsilon_5) + \lambda_2 \beta_2 f_2(\xi_2 - \varepsilon_5)) - d_2 \\ - r y_3 > 0. \end{aligned} \quad (36)$$

For some constant  $y_2^*$  ( $0 < y_2^* < \eta_3$ ), we claim that  $y_2(t) < y_2^*$  cannot be true for all  $t > t_0$ . Suppose that the claim is invalid, then there exists a positive constant  $t_0$  such that  $y_2(t) < y_2^*$  for all  $t > t_0$ . From system (3), we have

$$\begin{aligned} x_1'(t) &\geq x_1(t) (a_1 - \beta_1 L_1 y_2^* - b_1 x_1(t)), \quad t \neq nT, \\ x_1(t^+) &= (1 - p_1) x_1(t), \quad t = nT. \end{aligned} \quad (37)$$

From the unique solution  $u_6^*(t)$  of the comparison system of (37), we have  $x(t) \geq u_6^*(t) - \varepsilon_5$ , for  $t$  large enough, where

$$u_6^*(t) = \frac{(a_1 - \beta_1 L_1 y_2^*)(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 y_2^*)T))}{b_1(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 y_2^*)T)) + b_1 p_1 \exp(-(a_1 - \beta_1 L_1 y_2^*)(t - nT))} \quad (38)$$

is the unique solution of the following system:

$$\begin{aligned} u_6'(t) &= u_6(t) (a_1 - \beta_1 L_1 y_2^* - b_1 u_6(t)), \quad t \neq nT, \\ u_6(t^+) &= (1 - p_1) u_6(t), \quad t = nT, \end{aligned} \quad (39)$$

for  $t \in (nT, (n + 1)T]$ , with

$$u_6^*(0^+)$$

$$\begin{aligned} &= \frac{(a_1 - \beta_1 L_1 y_2^*)(1 - p_1 - \exp(-(a_1 - \beta_1 L_1 y_2^*)T))}{b_1(1 - \exp(-(a_1 - \beta_1 L_1 y_2^*)T))} \\ &\hat{=} \xi_1. \end{aligned} \quad (40)$$

Obviously  $u_6^*(t) \geq \xi_1$ ,  $t \in (nT, (n + 1)T]$ . Thus the inequality

$$x_1(t) \geq u_6^*(t) - \varepsilon_5 \geq \xi_1 - \varepsilon_5 \quad (41)$$

holds for  $t$  sufficiently large. Similarly we have

$$\begin{aligned} x_2(t) &\geq \frac{(a_2 - \beta_2 L_2 y_2^*)(1 - p_2 - \exp(-(a_2 - \beta_2 L_2 y_2^*)T))}{b_2(1 - \exp(-(a_2 - \beta_2 L_2 y_2^*)T))} \quad (42) \\ &- \varepsilon_5 \triangleq \xi_2 - \varepsilon_5. \end{aligned}$$

In view of (35), combining (41) and (42), we get

$$\begin{aligned} V'(t) &\geq (\exp(-d_1\tau) \\ &\cdot (\lambda_1 \beta_1 f_1(\xi_1 - \varepsilon_5) + \lambda_2 \beta_2 f_2(\xi_2 - \varepsilon_5)) - d_2 \quad (43) \\ &- r\eta_3) y_2(t). \end{aligned}$$

Let  $y_2^m \triangleq \min\{y_2(t) : t_1 \leq t \leq t_1 + \tau\}$ . We can prove  $y_2(t) \geq y_2^m$  for  $t > t_1$ . Otherwise, there exists  $t_2 > 0$  such that  $y_2(t) \geq y_2^m$  for  $t_1 \leq t \leq t_1 + \tau + t_2$ ,  $y_2(t_1 + \tau + t_2) = y_2^m$ , and  $y_2'(t_1 + \tau + t_2) \leq 0$ . However, from (43), we have

$$\begin{aligned} V'(t_1 + \tau + t_2) &\geq (\exp(-d_1\tau) \\ &\cdot (\lambda_1 \beta_1 f_1(\xi_1 - \varepsilon_5) + \lambda_2 \beta_2 f_2(\xi_2 - \varepsilon_5)) - d_2 \quad (44) \\ &- r\eta_3) y_2^m > 0. \end{aligned}$$

This is a contradiction. Hence, for all  $t > t_1$ , we have  $y_2(t) \geq y_2^m > 0$ .

In view of (36) and (43), we have  $V'(t) > 0$ , which leads to  $V(t) \rightarrow \infty$  as  $t \rightarrow \infty$ . It is a contradiction with  $V(t) \leq \eta_3 + \exp(-d_1\tau)(\lambda_1 \beta_1 f_1(\eta_1) + \lambda_2 \beta_2 f_2(\eta_2))\eta_3\tau$ . This implies that for any positive constant  $t_0$ ,  $y_2(t) < y_2^*$  cannot be true for all  $t > t_0$ ; then there are the following two cases.

*Case (a).*  $y_2(t) > y_2^*$  is true for all  $t$  large enough; then our aim is obtained.

*Case (b).*  $y_2(t)$  is oscillatory about  $y_2^*$ ; then we define

$$m_3 = \min \left\{ \frac{y_2^*}{2}, y_2^* \exp(-(d_2 + r y_2^*)\tau) \right\}. \quad (45)$$

Now we show  $y_2(t) \geq m_3$ . It is clear that there exist two positive constants  $\tilde{t}$  and  $\omega$  such that  $y_2(\tilde{t}) = y_2(\tilde{t} + \omega) = y_2^*$  and  $y_2(t) < y_2^*$ ,  $t \in (\tilde{t}, \tilde{t} + \omega)$ , with  $\tilde{t}$  being sufficiently large such that (37) holds for  $t \in (\tilde{t}, \tilde{t} + \omega)$ . By the continuous and bounded properties of  $y_2(t)$ , we know that  $y_2(t)$  is uniformly continuous. Therefore, there exists a constant  $t_3 > 0$  such that  $y_2(t) > y_2^*/2$  for all  $\tilde{t} \leq t \leq \tilde{t} + t_3$ . If  $\omega < t_3$ , then  $y_2(t) > y_2^*/2$ ; our aim is obtained. If  $t_3 < \omega < \tau$ , in view of (3), for  $\tilde{t} < t < \tilde{t} + \omega$ , combining assumption  $y_2(\tilde{t}) = y_2^*$  and  $y_2(t) < y_2^*$  ( $\tilde{t} < t < \tilde{t} + \omega$ ), we have  $y_2'(t) \geq -d_2 y_2(t) - r y_2^2(t) \geq (-d_2 - r y_2^*) y_2(t)$ . Easily we get  $y_2(t) \geq y_2^* \exp(-(d_2 + r y_2^*)\tau) \geq m_3$ . If  $\omega > \tau$ , analogously we derive that  $y_2(t) \geq m_3$  for  $\tilde{t} \leq t \leq \tilde{t} + \tau$ . Since the interval  $[\tilde{t}, \tilde{t} + \omega]$  is chosen at random and the choice of  $m_3$  is independent with the positive solution of (3), we conclude that  $y_2(t) \geq m_3$  holds for all  $t$  large enough.

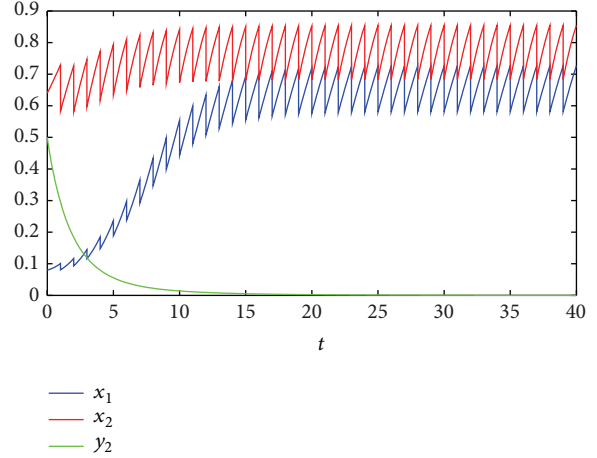


FIGURE 1: Dynamical behaviors of the predator-extinction solution of system (3) with initial value  $x_1(0) = 0.1$ ,  $x_2(0) = 0.8$ ,  $y_2(0) = 0.5$  and  $a_1 = 0.65$ ,  $a_2 = 1$ ,  $b_1 = 0.65$ ,  $b_2 = 1$ ,  $d_1 = 0.5$ ,  $d_2 = 0.7$ ,  $\beta_1 = 1$ ,  $\beta_2 = 1$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = 1$ ,  $p_1 = 0.2$ ,  $p_2 = 0.2$ ,  $c_1 = 1$ ,  $c_2 = 0.8$ ,  $c_3 = 1$ ,  $c_4 = 1$ ,  $c_5 = 1$ ,  $r = 0.25$ ,  $\tau = 1$ , and  $T = 1$ .

Based on the above analysis, letting  $m = \min\{m_1, m_2, m_3\}$ , we have  $m \leq x_1(t)$ ,  $m \leq x_2(t)$ , and  $m \leq y_2(t)$ . In addition,  $x_1(t), x_2(t), y_2(t) \leq M$  holds; hence we conclude that system (3) is permanent. The proof is complete.  $\square$

## 5. Numerical Simulation

For the generalized functional response of (3), there are many functional responses that meet the condition, such as Holling type I, Holling type II, Holling type III, Crowley-Martin type, Beddington-DeAngelis type, Watt type, and Ivlev type. In this section, we choose two concrete functional responses to illustrate the rationality of our results and try to find more dynamical behaviors of system (3). We choose such function response as Holling type II and Beddington-DeAngelis type as follows:

$$\begin{aligned} f_1(x_1(t)) &= \frac{x_1(t)}{c_1 + c_2 x_1(t)}, \\ f_2(x_2(t)) &= \frac{x_2(t)}{c_3 + c_4 x_2(t) + c_5 y_2(t)}. \end{aligned} \quad (46)$$

Firstly, let  $a_1 = 0.65$ ,  $a_2 = 1$ ,  $b_1 = 0.65$ ,  $b_2 = 1$ ,  $d_1 = 0.5$ ,  $d_2 = 0.7$ ,  $\beta_1 = 1$ ,  $\beta_2 = 1$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = 1$ ,  $p_1 = 0.2$ ,  $p_2 = 0.2$ ,  $c_1 = 1$ ,  $c_2 = 0.8$ ,  $c_3 = 1$ ,  $c_4 = 1$ ,  $c_5 = 1$ ,  $r = 0.25$ ,  $\tau = 1$ , and  $T = 1$ . By calculation, all parameters satisfy conditions of Theorem 3; then we obtain from Theorem 3 that a predator-extinction solution of system (3) exists, which is globally attractive. By numerical analysis with MATLAB, we get the following simulation figures of a predator-extinction solution and its global attractivity. Figure 1 shows the existence of a predator-extinction solution with only one initial value and Figure 2 shows the attractivity of the predator-extinction solution; that is, regardless of different initial values, species  $x_1$ ,  $x_2$ , and  $y_2$  converge to the predator-extinction solution.

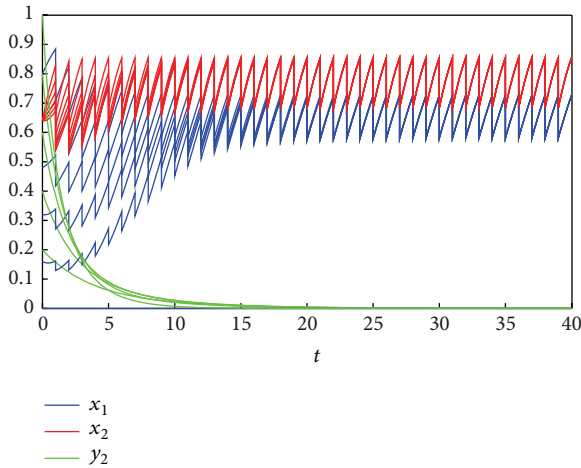


FIGURE 2: Dynamical behavior of system (3) with different initial values. These initial values are chosen randomly, and other parameters are the same as those in Figure 1. One can find that the solutions are globally attractive. The difference between Figures 1 and 2 is that more initial values are chosen in Figure 2 to show that the solutions are globally attractive.

Secondly, we choose another set of parameters to illustrate the permanence of system (3). Take  $a_1 = 0.65, a_2 = 1, b_1 = 0.65, b_2 = 1, d_1 = 0.5, d_2 = 0.2, \beta_1 = 1, \beta_2 = 1, \lambda_1 = 1, \lambda_2 = 1, p_1 = 0.2, p_2 = 0.2, c_1 = 1, c_2 = 0.8, c_3 = 1, c_4 = 1, c_5 = 1, r = 0.25, \tau = 1,$  and  $T = 1$ . One can verify that conditions of Theorem 5 are satisfied; then from Theorem 5, system (3) is permanent. By simulation, the results can be indicated clearly by Figure 3. Figure 3(a) shows the permanence of (3) and Figure 3(b) gives a positive periodic solution of this system.

Thirdly, in view of (H4), we know that pest population will die out if  $p_1$  and  $p_2$  are larger than the corresponding threshold. In order to investigate the influence of  $p_1, p_2$  and time delay  $\tau$ , we fix the same parameters in Figure 3 as follows. Consider that  $a_1 = 0.65, a_2 = 1, b_1 = 0.65, b_2 = 1, d_1 = 0.5, d_2 = 0.2, \beta_1 = 1, \beta_2 = 1, \lambda_1 = 1, \lambda_2 = 1, c_1 = 1, c_2 = 0.8, c_3 = 1, c_4 = 1, c_5 = 1, r = 0.25,$  and  $T = 1$ . If  $p_1 = 0.5$ , by simulation, pest  $x_1$  is driven to extinction (see Figure 4(a)), and if  $p_2 = 0.65$ , then, similarly, pest  $x_2$  becomes extinct (see Figure 4(b)). If  $p_1 = 0.5$  and  $p_2 = 0.65$  at the same time, then not only both pests are going to extinct but also their predator dies out due to lack of food (see Figure 4(c)), which is contrary to the conservation of biological diversity. From biological point of view, we only need to control these two pests at a rational level by adjusting the value of  $p_1$  and  $p_2$ , respectively. Furthermore, by simulation, if time delay  $\tau$  between immature predator and mature predator goes up to a threshold ( $\tau = 4$ ), the predator will die out (see Figure 4(d)), so we claim that the stage-structure also plays an important role in the permanence of system (3).

Finally, we consider the influence of impulsive period  $T$ . Take parameters in system (3) as  $a_1 = 0.65, a_2 = 1, b_1 = 0.65, b_2 = 1, d_1 = 0.5, d_2 = 0.1, \beta_1 = 1, \beta_2 = 1, \lambda_1 = 1, \lambda_2 = 1, p_1 = 0.1, p_2 = 0.1, c_1 = 10, c_2 = 0.8, c_3 = 10, c_4 = 1, c_5 = 1, r = 0.25, \tau = 0, x_1(0) = 0.7, x_2(0) = 0.8,$  and  $y_2(0) = 0.5$ . By simulation, we get the following bifurcation diagrams (see

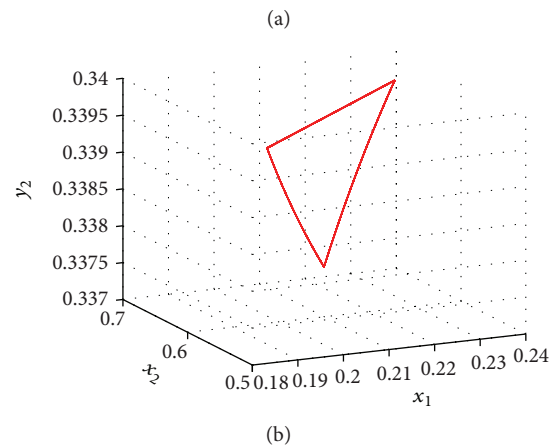
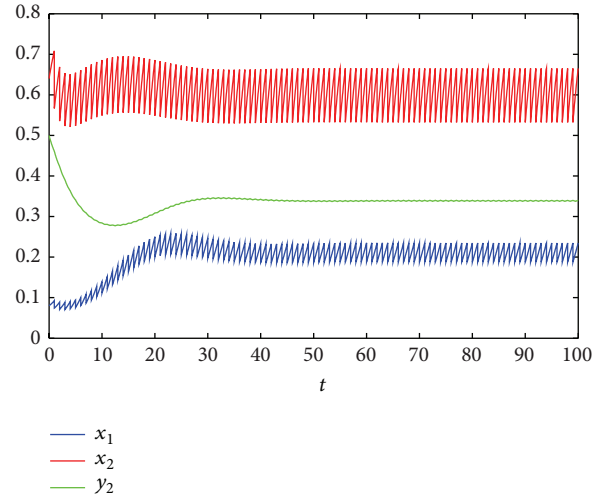


FIGURE 3: The permanence of system (3) with initial values of  $x_1(0) = 0.1, x_2(0) = 0.8, y_2(0) = 0.5,$  and  $d_2 = 0.2,$  and other parameters are the same as those in Figure 1. Obviously, all these species can coexist and their densities go into a bounded region. (a) Time series of  $x_1, x_2,$  and  $y_2,$  which indicate that the solution of (3) goes into a bounded region to be permanent. (b) Phase portrait of system (3), which implies a positive periodic solution.

Figure 5). Figure 5 indicates that bifurcation appears if  $T = 118.1, 143.9, 147.4,$  respectively, and if  $T \in [118.1, 147.4],$  more than one periodic solution appears. If a moderate pulse is given ( $T > 147$ ), then the system exhibits chaotic phenomena, including stable solutions, cycles, cascade, and chaos, which means the evolution of this system is unpredictable. In a word, the system analyzed here exhibits many complicated dynamical behaviors.

## 6. Discussion

In this paper, considering the complicated effects from the real world, we introduce impulsive spraying pesticides, stage-structure for predator, and generalized functional response into one-predator and two-prey system. Firstly, we investigate the existence and global attractivity of predator-extinction periodic solution under the condition that  $e^{-d_1\tau}(\lambda_1\beta_1f_1(\eta_1) +$

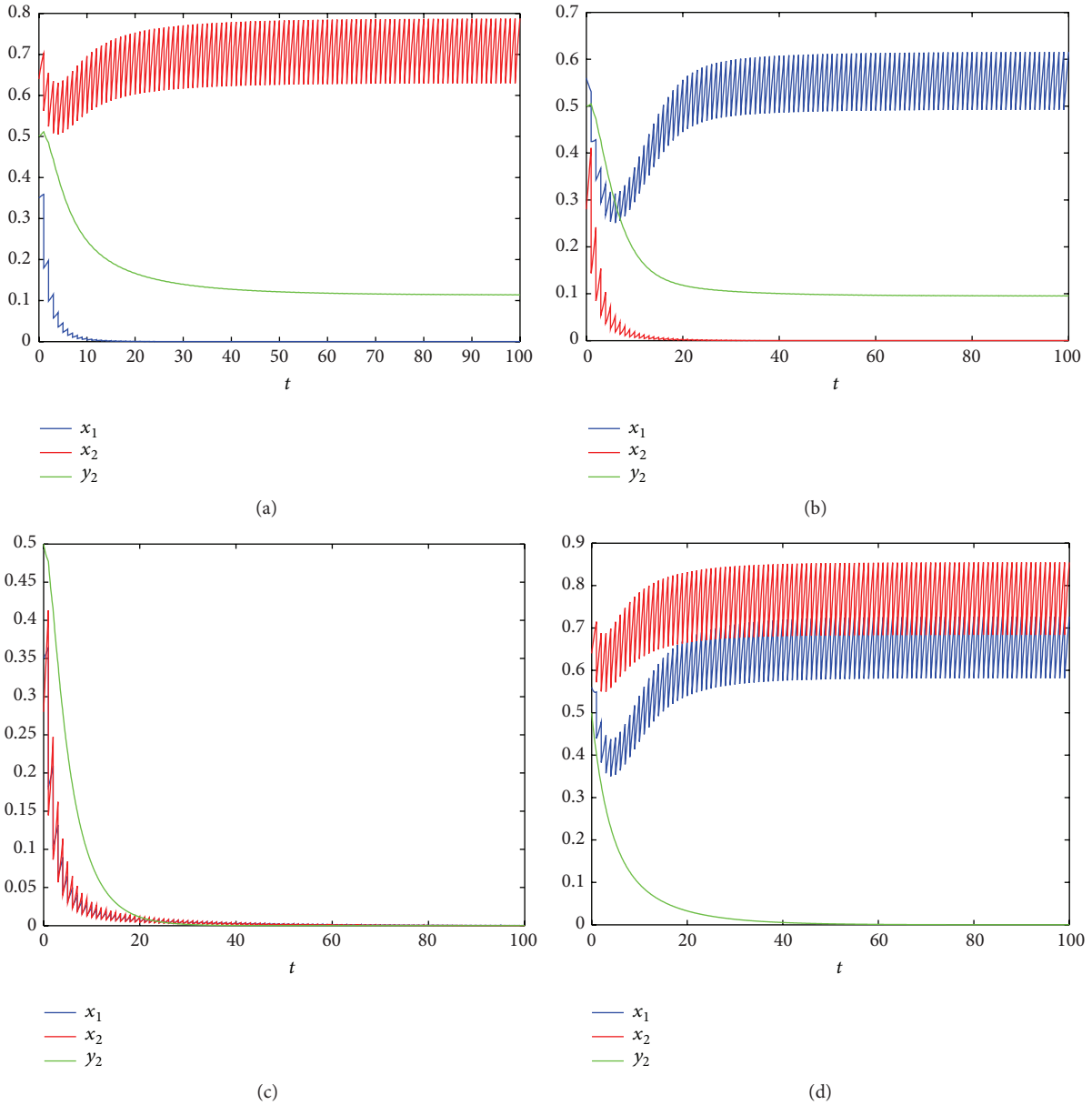


FIGURE 4: Dynamical behavior of system (3) with initial values  $x_1(0) = 0.7$ ,  $x_2(0) = 0.8$ , and  $y_2(0) = 0.5$ . (a) Time series of  $x_1$ ,  $x_2$ , and  $y_2$  with  $p_1 = 0.5$ ,  $p_2 = 0.2$ , which imply that overcatching of  $x_1$  will drive it to die out. (b) Time series of  $x_1$ ,  $x_2$ , and  $y_2$  with  $p_1 = 0.2$ ,  $p_2 = 0.65$ , which imply that overcatching of  $x_2$  will also drive it to die out. (c) Time series of  $x_1$ ,  $x_2$ , and  $y_2$  with  $p_1 = 0.5$ ,  $p_2 = 0.65$ . Figures show that the three species die out because of overcapturing of the two preys. (d) Time series of  $x_1$ ,  $x_2$ , and  $y_2$  with  $\tau = 4$ , the remaining parameters are as fixed as those in Figure 3. Comparing Figure 4 with Figure 3, one can find that the delay  $\tau$  large enough can lead  $y_2$  to die out.

$\lambda_2 \beta_2 f_2(\eta_2)) < d_2$ . Secondly, we obtain the sufficient conditions of the permanence. Finally, by numerical simulation with MATLAB, we further discuss some complicated dynamical behaviors of the system.

Our obtained results imply that if  $d_1$  or  $d_2$  is larger than a threshold (because of lack of food or catching the pest that died from insecticide), the predator will be extinct (see Figure 1), and if pesticides are used too much or harvesting is excessive on two pests, three species will all die out (see Figure 4(c)). In order to keep biological balance or biological diversity, some protective measures can be taken to ensure

$d_2$  is less than the threshold (such as disease prevention and releasing immature or mature predator); then the system will be permanent (see Figures 1–3). By comparing Figure 3 with Figures 4(a) and 4(b), if we change parameters  $p_1$  and  $p_2$ , respectively,  $x_1$  and  $x_2$  will die out effectively, but the rest of population will still survive, which can be used to provide a reliable control strategy: if impulsive period  $T$  is given, we can adjust  $p_1$ ,  $p_2$  to give a protection for the predator. It will not only reduce the economic loss but also protect environment from damage. Finally, impulsive period  $T$  affects the dynamical behaviors of the system heavily, which may



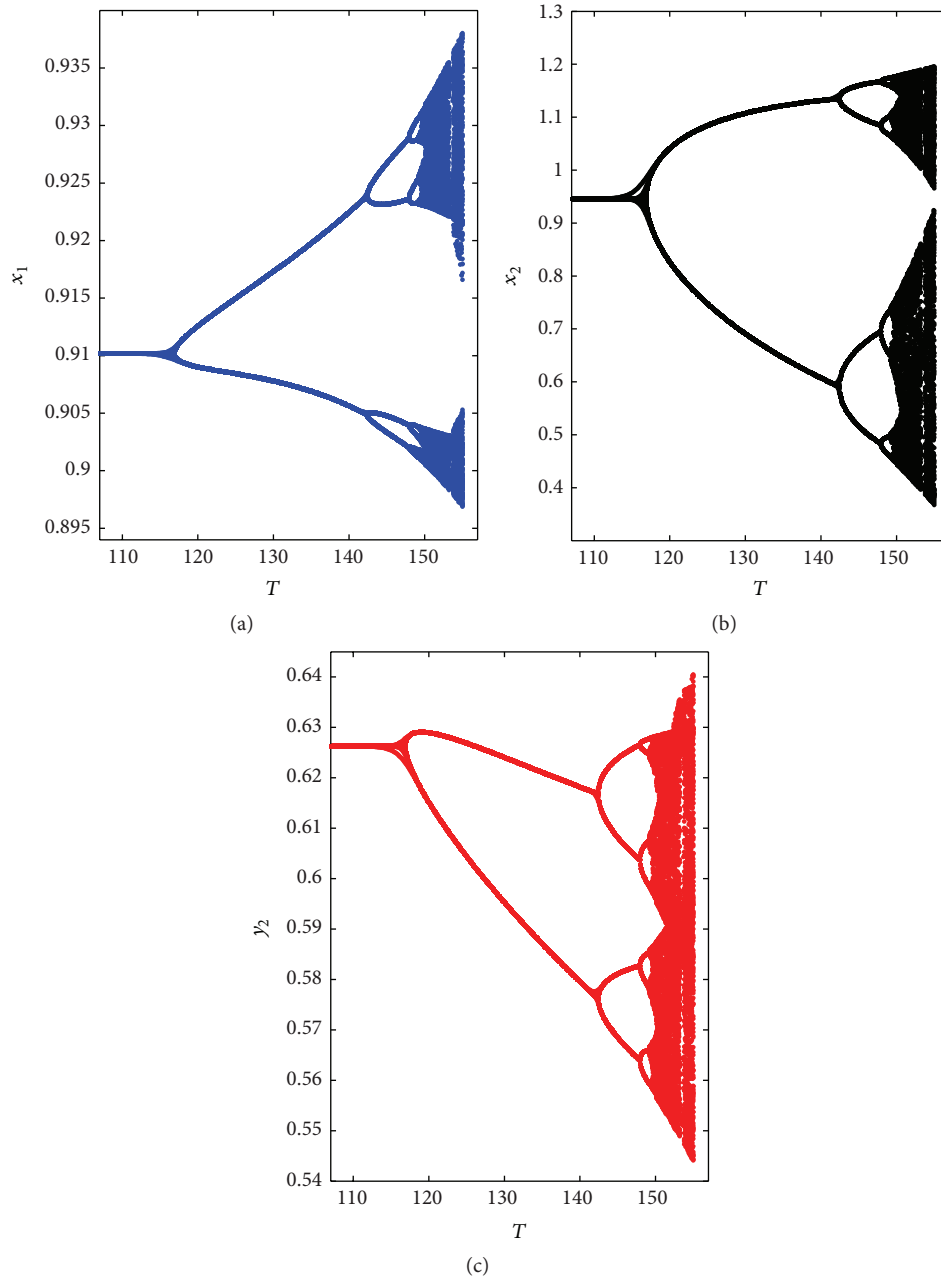


FIGURE 5: Bifurcation diagrams of system (3) with respect to impulsive period  $T$  on  $[107, 157]$ . (a) Bifurcation diagrams of  $x_1$ . (b) Bifurcation diagrams of  $x_2$ . (c) Bifurcation diagrams of  $y_2$ . Figure 5 indicates that bifurcation appears if  $T = 118.1, 143.9, 147.4$ , respectively, and if  $T \in [118.1, 147.4]$ , more than one periodic solution appears. If a moderate pulse is given ( $T > 147$ ), then the system shows chaotic phenomenon. The bifurcation diagrams include stable solutions, cycles, cascade, and chaos.

bring chaotic phenomena, including stable solutions, cycles, cascade, and chaos (see Figure 5).

In a word, our obtained results show that all parameters  $p_1, p_2, \tau$ , and  $T$  bring great effects on the properties of system (3), which can be applied to ecological resource management. The complicated dynamical behaviors imply that the influence from parameters  $p_1, p_2, \tau$ , and  $T$  is worthy of being studied and we will continue to study the potential dynamical properties in the near future.

### Conflict of Interests

None of the authors have any competing interests in the paper.

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