

Research Article

Robust Active MPC Synchronization for Two Discrete-Time Chaotic Systems with Bounded Disturbance

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This paper proposes a synchronization scheme for two discrete-time chaotic systems with bounded disturbance. By using active control method and imposing some restriction on the error state, the computation of controller's feedback matrix is converted to the min-max optimization problem. The theoretical results are derived with the aid of predictive model predictive paradigm and linear matrix inequality technique. Two example simulations are performed to show the effectiveness of the designed control method.

1. Introduction

Since the pioneering work of Pecora and Carroll [1], synchronization of chaotic systems has attracted more and more interest due to its significant applications in many fields such as chemical systems [2], ecological systems [3], physical systems [4], and secure communications [5]. From then, many methods have been proposed to study synchronization of chaotic systems. A vast variety of synchronization schemes have been proposed and applied in many research fields, such as adaptive control method [6], feedback control method [7], model predictive control method [8], impulse control method [9], and sliding mode method [10]. However, most of the designed methods have been applied only to investigate continuous-time chaotic systems' synchronization.

In practice, discrete-time chaotic dynamical systems are more important than continuous ones, and many models including neural networks, biological process, physical process, and chemical process are described by discrete-time chaotic dynamical models [11]. Therefore, the research of discrete-time chaotic systems' synchronization also plays an important role. Recently, many researches pay more and more attention to the synchronization of discrete-time chaotic systems for its applications in many fields such as secure communication and cryptology, and many synchronization methods are proposed like variable structure control [12],

H_∞ control [13], backstepping scheme [14, 15], digital filter method [16], nonlinear control [17], adaptive control [18, 19], active model predictive control [8], and so on [20–25].

However, most of the aforementioned researches consider the ideal condition without disturbance, and it is not real case in practice. The disturbance not only destroys the system's performance but also even makes the system not stable. So it is important to consider the discrete-time chaos synchronization with additive disturbance.

Based on our previous work [8], in this paper we propose a synchronization scheme for two discrete-time chaotic systems with bounded disturbance. The designed method is mainly based on the model predictive control (MPC), and the most difficulty is how to eliminate the influence of the additive disturbance on the stability of closed system. With the aid of quadratic boundedness, it is proved that the designed closed system is stable and realizes the two discrete-time chaotic systems' synchronization.

Notations. x' is the transpose vector x , and for (semi-)positive-definite matrix T , $\|x\|_T = x'Tx$. I is the identity matrix. $\varepsilon_P := \{\xi : \xi'P\xi \leq 1\}$ defines the ellipsoid. $\text{Co}\{\cdot\}$ denotes a convex combination of the elements in $\{\cdot\}$. The symbol $*$ induces a symmetric structure in linear matrix inequalities. $x(i | k)$ is the prediction value of x at time $k + i$.

2. Preliminary

In this section, the definition of quadratic boundedness is revisited [26]. Consider the following discrete-time system described by

$$x(k+1) = G(k)x(k) + H(k)w(k), \quad (1)$$

where $x(k) \in R^n$ is the state vector and $w(k) \in R^p$ is the noise vector. It is assumed that $w(k)$ belongs to an ellipsoidal compact set ε_T with $T > 0$. $G(k)$ and $H(k)$ are two matrices with appropriated dimension, and suppose that they are unknown but belong to a known bounded set.

Definition 1 (see [26]). A set φ is said to be strictly quadratically bounded with a common Lyapunov matrix of system (1) for all $w(k) \in \varepsilon_T$, $(G(k), H(k)) \in \varphi$, $k = 0, 1, \dots$, if $x'(k)Px(k) > 1$ implies $(Gx+Hw)'P(Gx+Hw) < x'(k)Px(k)$, for any $w(k) \in \varepsilon_T$ and $(G, H) \in \varphi$.

Definition 2 (see [26, 27]). A set φ is said to be a positively invariant set for system (1) for all $w(k) \in \varepsilon_T$, $(G(k), H(k)) \in \varphi$, $k = 0, 1, \dots$, if $x \in \varphi$ implies $Gx + Hw \in \varphi$, for any $w(k) \in \varepsilon_T$ and $(G, H) \in \varphi$.

3. Problem Formulation and Controller Design

Consider the following form of master chaotic system:

$$x(k+1) = A(k)x(k) + f(x(k), k) + w(k), \quad (2)$$

where $x \in R^n$ is the system's state vector and $A(k) \in \Omega = \text{Co}\{A_1, A_2, \dots, A_L\}$ is unknown constant matrix with appropriate dimension. $f(x(k), k)$ is the nonlinear part and $w(k) \in \varepsilon_{P_w} \subset R^n$ is the bounded disturbance vector. The controlled slave chaotic system is described by

$$y(k+1) = A(k)y(k) + f(y(k), k) + u(k), \quad (3)$$

where $y \in R^n$ is the slave system's state vector and $u(k)$ is the designed controller.

Define the system error

$$e(k) = y(k) - x(k). \quad (4)$$

Then we can get the error system

$$\begin{aligned} e(k+1) &= A(k)e(k) + f(y(k), k) - f(x(k), k) \\ &\quad - w(k) + u(k). \end{aligned} \quad (5)$$

With the aid of the active control technique [8, 28, 29], we select the following controller:

$$\begin{aligned} u(k) &= u_1(k) - f(y(k), k) + f(x(k), k), \\ u_1(k) &= F(k)x(k), \end{aligned} \quad (6)$$

where $F(k)$ will be computed in the following optimization problem. Substituting (6) into (5), we can get

$$e(k+1) = [A(k) + F(k)]e(k) - w(k); \quad (7)$$

that is,

$$e(k+1) = \sum_{l=1}^L w_l(k) [A_l + F(k)]e(k) - w(k). \quad (8)$$

Lemma 3 (see [26]). For system (8), the following facts are equivalent:

(a) For all allowable $w(k) \in \varepsilon_{P_w}$, $k \geq 0$, (8) is strictly quadratically bounded with a common Lyapunov matrix $M > 0$.

(b) For any $w(k) \in \varepsilon_{P_w}$ and any $l \in \{1, \dots, L\}$, $e^T M e > 1$ implies

$$[(A_l + F)e - w]^T M [(A_l + F)e - w] < e^T M e \quad (9)$$

and $e^T M e = 1$ implies

$$[(A_l + F)e - w]^T M [(A_l + F)e - w] \leq e^T M e. \quad (10)$$

(c) For any allowable $w(k) \in \varepsilon_{P_w}$, $k \geq 0$, the ellipsoid ε_M is a positively invariant set for (8).

(d) There exists $\alpha \in (0, 1)$, such that

$$\begin{bmatrix} (1-\alpha)M & * & * \\ 0 & \alpha P_w & * \\ M(A_l + F) & -M & M \end{bmatrix} \geq 0, \quad l \in \{1, \dots, L\}. \quad (11)$$

The target of the following part is to solve a dynamic feedback MPC where, at each time k , the feedback matrix is computed through the following optimization problem:

$$\min_{\beta < 1, \gamma, Q, F} \max_{[A] \in \Omega, w(k+i) \in \varepsilon_{P_w}, i=0, 1, \dots, N-1} J_N(k), \quad (12)$$

where

$$J_N(k) = \sum_{i=0}^N \|e(i|k)\|_R \quad (13)$$

and R is a symmetric positive-definite weighting matrix. Consider a quadratic Lyapunov function $V(e) = e^T P e$ ($P > 0$) of the error state e with $V(0) = 0$. At every sampling time k , suppose the error state e and V satisfy the following condition:

$$e(k) \in \varepsilon_{Q^{-1}} \quad (14)$$

$$\begin{aligned} e(i_1|k) &\in \varepsilon_{Q^{-1}} \setminus \varepsilon_{\beta^{-1}Q^{-1}}, \\ 0 &\leq i_1 < N \end{aligned} \quad (15)$$

$$e(i_2|k) \in \varepsilon_{\beta^{-1}Q^{-1}}, \quad \forall i_2 \geq N \quad (16)$$

$$\|e(i|k)\|_{Q^{-1}}^2 \geq 1 \implies$$

$$\|e(i|k)\|_{Q^{-1}}^2 - \|e(i+1|k)\|_{Q^{-1}}^2 \geq \frac{1}{\gamma} \|e(i|k)\|_R^2, \quad (17)$$

$$0 \leq i < N,$$

where $Q^{-1} = P$. It is obvious that condition (14) consists of the initial condition of the min-max problem, (15) implies $e(k)$ within $\varepsilon_{Q^{-1}}$, (15) guarantees that system (7) is quadratically bounded with a common Lyapunov matrix Q^{-1} , and (16) is used to guarantee convergence of $e(k)$ towards $\varepsilon_{\beta^{-1}Q^{-1}}$ when $e(k) \notin \varepsilon_{Q^{-1}}$. N need not to be known and $\beta < 1$ is to ensure that $e(k) \notin \varepsilon_{\beta^{-1}Q^{-1}}$. Note that, since $e(k)$ can only convergence to a neighborhood of $e(k) = 0$, there exist $k > 0$ when (16) becomes infeasible. In this case, we will stop computing the optimization problem.

Since $w(k) \in \varepsilon_{P_w}$, $e(k) \notin \varepsilon_{\beta^{-1}Q^{-1}}$ is equivalent to

$$\beta \|w(k+i)\|_{P_w}^2 \leq \|e(i|k)\|_{Q^{-1}}^2 \implies \|e(i|k)\|_{Q^{-1}}^2 - \|e(i+1|k)\|_{Q^{-1}}^2 \geq \frac{1}{\gamma} \|e(i|k)\|_R. \quad (18)$$

By applying (8) and using the S-procedure [30], we can get that the result of (18) is satisfied if and only if

$$\exists \alpha \geq 0 : \begin{bmatrix} \gamma Q^{-1} - \gamma [A+F]^T Q^{-1} [A+F] & * \\ \gamma Q^{-1} [A+F] & -\gamma Q^{-1} \end{bmatrix} \quad (19)$$

$$-\alpha \gamma \begin{bmatrix} Q^{-1} & 0 \\ 0 & -\beta P_w \end{bmatrix} \geq 0.$$

Using Schur complement it is shown that (19) is guaranteed by

$$\begin{bmatrix} (1-\alpha)Q^{-1} & * & * & * & * \\ 0 & \alpha\beta P_w & * & * & * \\ -Q^{-1}[A_l+F] & Q^{-1} & Q^{-1} & * & * \\ R^{1/2} & 0 & 0 & \gamma I & * \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \geq 0 \quad (20)$$

$$l = 1, \dots, L.$$

By pre- and postmultiplying both sides of (20) with $\text{diag}\{Q, I, Q, I, I\}$, then one can obtain

$$\begin{bmatrix} (1-\alpha)Q & * & * & * & * \\ 0 & \alpha\beta P_w & * & * & * \\ -[A_l+F] & I & Q & * & * \\ QR^{1/2} & 0 & 0 & \gamma I & * \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \geq 0 \quad l = 1, \dots, L. \quad (21)$$

Summing (17) from $i = 0$ to $i = N - 1$ and applying (14) yield

$$J_N(k) \leq \gamma \|e(k)\|_{Q^{-1}}^2 \leq \gamma. \quad (22)$$

Hence, (18) is imposed not only for quadratic boundedness but also for optimality (γ will be minimized). Then we can get the following theorem.

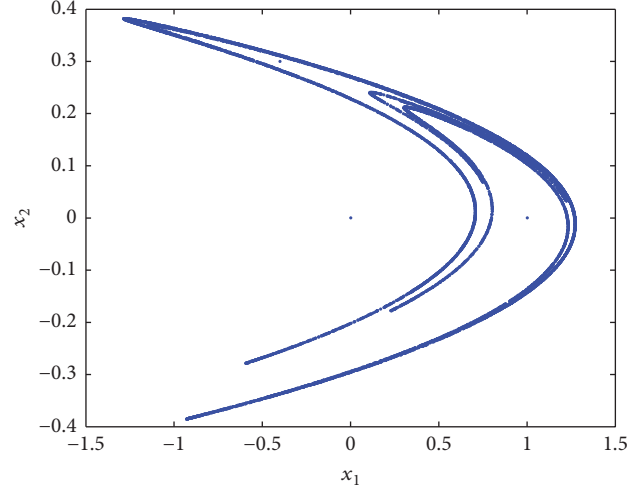


FIGURE 1: Attractor of Hénon map.

Theorem 4. The master system (2) and slave system (3) can realize the synchronization with controller (6), and the feedback matrix F is computed through the following optimization:

$$\min_{\beta < 1, \gamma, Q, F} \quad (23)$$

subject to

$$\begin{bmatrix} 1 & * \\ e(k) & Q \end{bmatrix} \geq 0$$

$$\begin{bmatrix} (1-\alpha)Q & * & * & * & * \\ 0 & \alpha\beta P_w & * & * & * \\ -[A_l+F] & I & Q & * & * \\ QR^{1/2} & 0 & 0 & \gamma I & * \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \geq 0 \quad l = 1, \dots, L. \quad (24)$$

4. Numerical Examples

Example 1. This example verifies the effectiveness of the proposed robust active MPC control method in solving the synchronization between two identical discrete chaotic systems. Consider the well-known discrete Hénon map [31]:

$$\begin{aligned} x_1(k+1) &= x_2(k) + 1 - ax_2(k) \\ x_2(k+1) &= bx_1(k). \end{aligned} \quad (25)$$

Corresponding to (2), $A = \begin{pmatrix} 0 & 1 \\ b & 0 \end{pmatrix}$, $f(x(k)) = \begin{pmatrix} 1-ax_2(k)^2 \\ 0 \end{pmatrix}$. The system has a strange attractor when $a = 1.4$, $b = 0.3$, and its attractor is shown in Figure 1 with the initial value $x(0) = (0, 0)^T$. The master system is selected as

$$\begin{aligned} x_1(k+1) &= x_2(k) + 1 - ax_2(k) + 0.25 \sin(k) \\ x_2(k+1) &= bx_1(k) + 0.25 \cos(k). \end{aligned} \quad (26)$$

When parameter b varied between 0.2 and 0.4, the matrix A belongs to the polytope $\Omega = \text{Co}\{A_1, A_2\}$, where $A_1 = \begin{pmatrix} 0 & 1 \\ 0.2 & 0 \end{pmatrix}$ and $A_2 = \begin{pmatrix} 0 & 1 \\ 0.4 & 0 \end{pmatrix}$. And the slave system is chosen as

$$y(k+1) = Ay(k) + f(y(k), k) + u(k), \quad (27)$$

where $y = (y_1, y_2)^T$ is the slave system's state and it is obvious that $P_w = 4I$. When we select $\alpha = 0.5$, $\beta = 0.9$, and $R = 2I$, and the initial values are selected as $x(0) = (16, -8)^T$, $y(0) = (2, 6)^T$, then the error of the master system and slave system is shown in Figure 2 with controller (6).

Example 2. In this example, the proposed robust active MPC controller is applied to stabilize the uncertain generalized hyperchaotic Hénon map [32]:

$$\begin{aligned} x_1(k+1) &= 1 + x_3(k) - ax_2(k)^2 \\ x_2(k+1) &= 1 + bx_2(k) - ax_1(k)^2 \\ x_3(k+1) &= bx_1(k). \end{aligned} \quad (28)$$

When $a = 1.4$, $b = 0.2$ the generalized Hénon map has an attractor which is shown in Figure 3 with the initial values $x(0) = (0.1, 0.05, 0.3)^T$. The master system is selected as

$$\begin{aligned} x_1(k+1) &= 1 + x_3(k) - ax_2(k)^2 + 0.25 \sin(k) \\ x_2(k+1) &= 1 + bx_2(k) - ax_1(k)^2 + 0.25 \cos(k) \\ x_3(k+1) &= bx_1(k) 0.15 \sin(2k). \end{aligned} \quad (29)$$

When parameter b varied between 0.1 and 0.3, the matrix A belongs to the polytope $\Omega = \text{Co}\{A_1, A_2\}$, where $A_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0.1 & 0 \\ 0.1 & 0 & 0 \end{pmatrix}$ and $A_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0.3 & 0 \\ 0.3 & 0 & 0 \end{pmatrix}$. The slave system is selected as

$$y(k+1) = Ay(k) + g(y(k), k) + u(k), \quad (30)$$

where $y(k) = (y_1(k), y_2(k), y_3(k))^T$, $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & b & 0 \\ b & 0 & 0 \end{pmatrix}$, $f(y(k)) = \begin{pmatrix} 1 - ay_2(k)^2 \\ 1 - ay_1(k)^2 \\ 0 \end{pmatrix}$.

When solving the optimization (23), we select the parameters as $P_w = 4I$, $\alpha = 0.5$, $\beta = 0.9$, and $R = I$ and the simulation error of the master system and the slave system is shown in Figure 4 with the initial value of $x(0) = (16, -8, 1)^T$ and $y(0) = (0, 4, -5)^T$.

5. Conclusions

In this paper, the robust active MPC synchronization for two discrete-time chaotic systems with bounded disturbance is studied. With the help of the active control technique, the synchronization problem is changed into a min-max optimization problem. On the basis of discrete Lyapunov stability theory, we prove the stability of the closed-loop system. Two simulation examples demonstrate that the designed control technique can synchronize the two uncertain discrete-time chaotic systems.

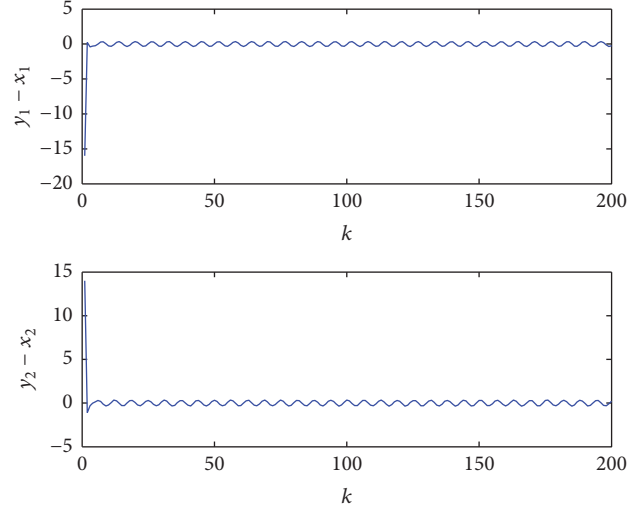


FIGURE 2: Synchronization errors between the uncertain disturbance Hénon maps.

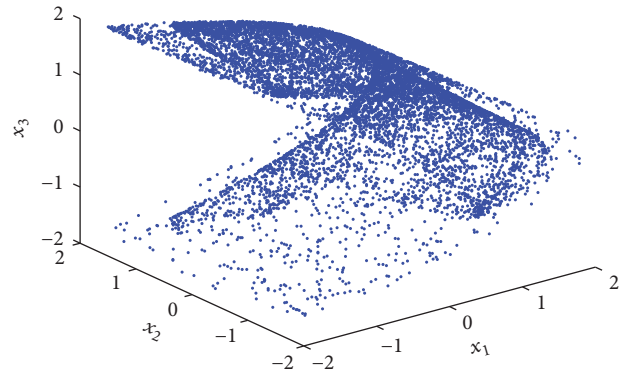


FIGURE 3: Attractor of the generalized Hénon map.

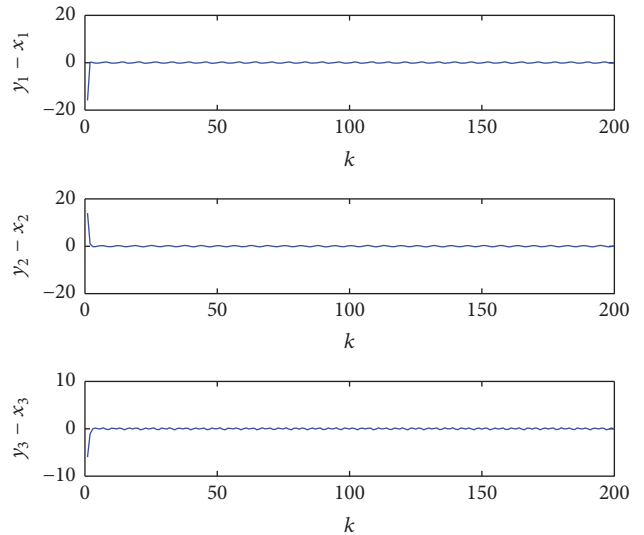


FIGURE 4: Synchronization errors between the generalized Hénon maps.

Conflicts of Interest

The author declares no conflicts of interest.

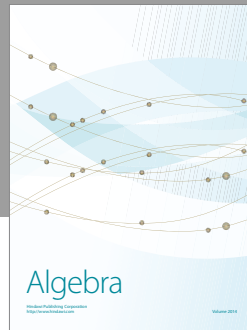
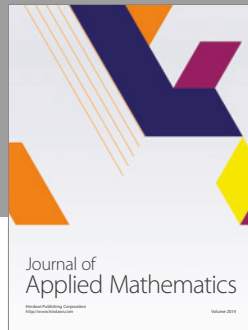
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