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Research Article

Damage Detection Based on Cross-Term Extraction from Bilinear Time-Frequency Distributions

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Abundant damage information is implicated in the bilinear time-frequency distribution of structural dynamic signals, which could provide effective support for structural damage identification. Signal time-frequency analysis methods are reviewed, and the characters of linear time-frequency distribution and bilinear time-frequency distribution typically represented by the Wigner-Ville distribution are compared. The existence of the cross-term and its application in structural damage detection are demonstrated. A method of extracting the dominant term is proposed, which combines the short-time Fourier spectrum and Wigner-Ville distribution; then two-dimensional time-frequency transformation matrix is constructed and the complete cross-term is extracted finally. The distribution character of which could be applied to the structural damage identification. Through theoretical analysis, model experiment and numerical simulation of the girder structure, the change rate of cross-term amplitude is validated to identify the damage location and degree. The effectiveness of the cross-term of bilinear time-frequency distribution for damage detection is confirmed and the analytical method of damage identification used in structural engineering is available.

1. Introduction

In the whole life cycle, the components in the engineering structure will damage inevitably under the influence of the performance changes in materials, loads, and other uncertain factors. Detecting and finding the damage information exactly and in time is an important problem in the structural safety and life-cycle evaluation. In the structure detection and health monitoring, the signal can be divided into stationary ones and nonstationary ones by the variation in statistical properties in time domain or in frequency domain. Thus, the signals analysis methods include time-domain analysis and frequency-domain analysis and time-frequency domain analysis. The time-frequency analysis can combine the advantages of signals analysis in both time domain and frequency domain, especially for common nonstationary signals in engineering. The time-frequency analysis can accurately reflect the changes of frequency components with the time. The structural damage identification based on time-frequency analysis is an important research aspect of health monitoring.

The time-frequency analysis is a method for using time-frequency joint functions to describe the signal energy density or intensity in the time domain and frequency domain. The research in time-frequency analysis began in the 1940s, and it has become an important branch of signal analysis and processing, which is used in a wide range of applications such as astronomy, communications, physics, biology, medicine, and mathematics [1]. In the 21st century, the time-frequency analysis technology has been widely used in various fields of civil engineering. Especially, it provided an important analytical tool for structural dynamic analysis and damage identification. Time-frequency analysis technology promoted the industry's development and progress. Study on damage detection in engineering frequently focuses on linear time-frequency analysis such as short-time Fourier transform and wavelet transform; unfortunately the bilinear time-frequency distribution is rarely discussed in this field.

The linear time-frequency analysis evolved from the Fourier transform; the short-time Fourier transform provides more details than the Fourier transform and the time-varying spectrum with a sliding time window. Once the window function is selected, the time-frequency resolution will not be changed. This property makes the short-time Fourier transform to be limited in the analysis of mutation signals and nonstationary signals, especially for the mutation signal in damaged structures. The wavelet transform or the wavelet packet transform is a powerful tool for the nonstationary signal analysis. It uses the multiscale decomposition by selecting the wavelet function and flex-transition calculating. The wavelet transform can effectively obtain favorable localized characteristics with multiresolution analysis in the time-frequency domain from the signal. Therefore, it is widely used in the damage detection. However, the appropriate wavelet function for wavelet transform must be selected, and the results are not unique; once the wavelet function and multiscale are determined, its characteristics will be constant.

The bilinear time-frequency analysis was developed from energy spectrum or power spectrum, and Wigner-Ville distribution (WVD) is one of the most important distributions. WVD can be regarded as the distribution of signal energy in two-dimensional time-frequency space with many excellent properties such as symmetry, time shift, combinations, and complex conjugation relationship. It contains the information of the amplitude and the phase in the signal. The disadvantage of WVD is that it may produce cross-term interference in the multicomponent signals, which is the intrinsic property of bilinear time-frequency distribution [2]. The extended achievements have been obtained on time-frequency analysis based on WVD, but for the study on structural damage detection, further development of its application is needed.

The advantages and disadvantages of the linear time-frequency analysis method should be discussed dialectically, and as the typical method in bilinear time-frequency analysis, the improvement on WVD is a crucial problem and a breakthrough for damage detection.

2. Time-Frequency Analysis Methods Summary

The most classic traditional stationary signal analyzing and processing method is Fourier transform, which establishes a signal bridge from the time domain to the frequency domain, and it can be showed as follows:

$$\begin{aligned} s(f) &= \int_{-\infty}^{\infty} s(t)\, e^{-j2\pi f t}\, dt, \\ s(t) &= \int_{-\infty}^{\infty} s(f)\, e^{j2\pi t f}\, df. \end{aligned} \tag{1}$$

However, being an overall transformation (whole time domain or whole frequency domain), Fourier transform decomposed the entire signal into different frequency components. It cannot show signal's local time-frequency characteristics and some variations in the frequency components over time. Fourier transform analysis is only suitable for stationary signals, but signals are usually nonstationary ones in the engineering. The combination function of time and frequency is established to represent the signals and overcome the limitations of the Fourier transform. Time-frequency analysis is divided into linear time-frequency distribution and bilinear time-frequency distribution depending on the difference of the function construction.

2.1. Linear Time-Frequency Distribution. The Linear time-frequency distribution is the time-frequency analysis method based on the Fourier transform with the linear superposition. The linear time-frequency distribution is weighted according to the basic ingredients of decomposing the signal into the time domain and frequency domain.

The most common method for linear time-frequency representation includes the short-time Fourier transform and the wavelet transform [3].

The short-time Fourier transform achieves the localization signal in the time domain. The short-time window function is multiplied before the Fourier transform. The local spectrum at different times can be obtained by moving window in the timeline, which can be understood as the signal's Fourier transform near the time point. Short-time Fourier transform can be expressed as

$$\mathrm{STFT}_s(t, f) = \int s(\tau)\, h^*(\tau - t)\, e^{-j2\pi f \tau}\, d\tau, \tag{2}$$

where the window function $h^*(t)$ makes a great influence on the results of short-time Fourier transform, because the nonstationary signal is assumed stationary in the analysis window. The selection of window function should be related to local stationary length of the signal [4]. When a window function is selected, the short-time Fourier transform has only a single resolution in the time-frequency plane. For the time-varying nonstationary signals, it is difficult to find a perfect time window to adapt to different time segments.

The wavelet transform is a mathematical method proposed by French scholar in the 1980s, which was widely introduced to engineering application areas, especially in signal processing, linguistic analysis, pattern recognition and quantum physics areas, and so forth. The wavelet transform greatly boosted the signal analysis and processing method, because of its multiresolution characters, and each multiscale segment of the signal can be identified. Scholars have conducted the in-depth research in structural damage detection [5]. The wavelet transform can be defined as

$$\mathrm{WT}_s(a, t) = \frac{1}{\sqrt{a}} \int s(\tau)\, \varphi^*\left(\frac{\tau - t}{a}\right) d\tau, \quad a > 0, \tag{3}$$

where a is a scale factor and t is the translation factor. By selecting the appropriate retractable windows and the wavelet function, the wavelet transform can obtain the local characteristics of nonstationary signals in time domain and frequency domain. The wavelet analysis is essentially a time-scale analysis and it is suitable for analyzing self-similar structure of the signal. Meanwhile the wavelet transform has not the general applicability such as the signals in the language and images; therefore the wavelet transform has evolved into the discrete wavelet transform, the orthogonal wavelet transform, and the wavelet packet transform. Now, the wavelet transform is applied widely in damage detection, but choosing the wavelet function is still difficult and a key problem in the practical application. It is determined mainly through selecting by experience and experiment comparing [6–8].

2.2. Bilinear Time-Frequency Distribution. The bilinear time-frequency distribution is also called quadratic time-frequency distribution, which can visually and reasonably reveal the energy time-frequency distribution of the signal. The bilinear time-frequency distribution does not satisfy the linear superposition, at the same time there exist the inherent cross-terms. The bilinear time-frequency distribution is divided into Cohen bilinear time-frequency distribution and affine bilinear time-frequency distribution, which are all developed from WVD.

In 1932, Wigner proposed Wigner distribution, which applied to quantum mechanics initially. In 1948, Ville introduced it into the signal analysis field, and thus a new chapter in the field of signal analysis was opened. In 1970, Mark proposed the main problem of WVD that is the cross-interference term, which has also become a research focus in signal processing through half a century. WVD is defined as

$$\mathrm{WVD}_s(t,f)=\int s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi f\tau}d\tau. \qquad (4)$$

This equation does not contain window functions and it avoids the selection of the time-frequency resolution in the linear time-frequency representation. The time width and bandwidth attain the lower bound of uncertainty principle in Wigner-Ville distribution, so it has the characteristic of the highest resolution, energy concentration, and time-frequency edges. WVD becomes an important tool for the analysis of nonstationary time-varying signal. The problem of short-time Fourier transform is solved and the clear physical meaning is established.

In 1966, Cohen summarized and found that a series of time-frequency distribution are only WVD deformation, which can be expressed using a unified form by Kernel function. It is called Cohen time-frequency distribution [9]. This method can be expressed as

$$\begin{aligned}P_s(t,f)=\iiint & s\left(u+\frac{\tau}{2}\right)s^*\left(u-\frac{\tau}{2}\right)\\ &\times\phi(\tau,\theta)\,e^{-j2\pi(t\theta+\tau f-u\theta)}du\,d\tau\,d\theta,\end{aligned} \qquad (5)$$

where $A_s(\tau,\theta)=\int s(u+\tau/2)s^*(u-\tau/2)e^{j2\pi u\theta}du$, defined as fuzzy function. $\phi(\tau,\theta)$ is called Kernel function. Cohen-time class frequency distribution is two-dimensional Fourier transform weighted by kernel fuzzy function. So Cohen time-frequency distribution can be considered a two-dimensional Fourier transform of ambiguity function weighted by kernel function. Cohen time-frequency distribution is also called generalized bilinear time-frequency distribution. When the kernel function $\phi(\tau,\theta)=1$ the Cohen time-frequency distribution degenerates into WVD. The deficiency is that Cohen time-frequency distribution suppresses the cross-term using kernel function; meanwhile time-frequency resolution is decreased in the entire distribution [10].

Cohen time-frequency distribution is the collection of quadratic time-frequency energy distribution, which has the time-frequency shift invariance. Another type of energy distribution does not satisfy the time-frequency shift invariance, which is called the affine bilinear time-frequency distribution. It is achieved by time shifting and stretching transformation. This type of distribution can be expressed as

$$T_s(t,a)=\iint\Phi\left(\frac{\tau-t}{a},a\theta\right)\mathrm{WVD}_s(\tau,\theta)\,d\tau\,d\theta. \qquad (6)$$

The affine time-frequency distribution is also called scalogram [11], which is the smooth form of WVD essentially. Thus, WVD becomes a link of Cohen time-frequency distribution and affine time-frequency distribution. Scalogram is smoothed by WVD and is eliminated cross-term, but the disadvantage is the poor time-frequency resolution. One class of affine time-frequency distribution discretely uses the smoothing function of time domain and frequency domain. It is called affine smoothed pseudo-Wigner distribution [2]. This type of distribution can be expressed as

$$\begin{aligned}\mathrm{ASPW}_s(t,a)=\frac{1}{a}\iint & h\left(\frac{\tau}{a}\right)g\left(\frac{\theta-t}{a}\right)s\left(\theta+\frac{\tau}{2}\right)\\ &\times s^*\left(\theta-\frac{\tau}{2}\right)d\theta\,d\tau.\end{aligned} \qquad (7)$$

By selecting the window functions g and h and determining the time and scale resolution independently, the effect of affine smoothed pseudo-Wigner distribution is between scalogram and WVD. Given the excellent features of the affine bilinear time-frequency distribution, the application in damage detection needs to be researched. Especially choosing an adaptive window function can reflect the mutation point in the damage signals [12].

3. Existence and Extraction of the Cross-Term

The cross-term is an inherent property of bilinear time-frequency distribution. It is the cross-over effect between the different signal components of the multicomponent signal. For example, WVD can explain the cross-term of bilinear time-frequency distribution [13]. The signal $s(t)=s_1(t)+s_2(t)$, thus

$$\begin{aligned}\mathrm{WVD}_s(t,f)=\int & \left[s_1\left(t+\frac{\tau}{2}\right)+s_2\left(t+\frac{\tau}{2}\right)\right]\\ &\times\left[s_1^*\left(t-\frac{\tau}{2}\right)+s_2^*\left(t-\frac{\tau}{2}\right)\right]e^{-j2\pi f\tau}d\tau,\end{aligned} \qquad (8)$$

$$\begin{aligned}\mathrm{WVD}_s(t,f)=&\,\mathrm{WVD}_{s_1}(t,f)+\mathrm{WVD}_{s_2}(t,f)\\ &+2\,\mathrm{Re}\left\{\mathrm{WVD}_{s_1s_2}(t,f)\right\},\end{aligned} \qquad (9)$$

where $\mathrm{WVD}_{s_1s_2}(t,f)=\int s_1(t+\tau/2)s_2^*(t-\tau/2)e^{-j2\pi f\tau}d\tau$; the first two terms of the formula are auto-terms of signal components. The third is the cross-term of signal components. By (9), it is obvious that the WVD of the sum of the two signals is not equal to the sum of the respective WVD. If a multicomponent signal has n components, it will produce $n(n-1)/2$ cross-term. The cross-term exists between the auto-term elements, it is usually oscillatory, and its amplitude can achieve two times of the auto-term amplitude. It is traditionally considered that the cross-term provides the false spectrum distribution and makes the characteristics of time-frequency signals blurred while affecting the WVD physical explanation [14]. Therefore, inhibiting the cross-term has been the core problem around bilinear time-frequency distribution. Scholars made a lot of research work in this area. Designing the ideal kernel function can eliminate the effects of cross-term. The common distributions are the reduced cross-term distribution (RID), the pseudo-Wigner-Ville Distribution (PWD), the smoothed Wigner-Ville Distribution (SWD), the smoothed pseudo-Wigner-Ville Distribution (SPWD), and so on. Some researchers also use different methods or transform to inhibit cross-term. Mirela and Isar [15] combine the Gabor transform with WVD and get a new time-frequency distribution, which can effectively inhibit and weaken the cross-term interference of the signals. Wu et al. [16] combine the blind source separation (BSS) to suppress the cross-term of WVD. The above research works are eliminating the cross-term and reducing negative effects as springboard. On the other hand, the cross-term existence reflects the phase relationship and the degree of coherence between the multicomponent signal components. For the nonstationary signals in structural health monitoring or testing, it will be reflected in the transform of the amplitude, the phase, and the local time-frequency characteristics when the damage expands in a position of the structure from the damaged or undamaged monitoring signals. This change should also be reflected in the cross-term. Considering that the cross-term contains wealthy information of the location and the degree of structural damage, this intrinsic property of the cross-term can find the mutation points of the structural damage signals, which can accurately identify the location and the degree of structural damage.

Firstly, the time-frequency analysis of the structural damage signals should accurately extract and separate the cross-term by using bilinear time-frequency distribution. A method in this paper is that the difference value between the WVD and the bilinear time-frequency analysis by the cross-term suppression can obtain the cross-term [17]. This method has simple algorithm and definite physical meaning, but the cross-term will inevitably miss some important phase information. Meanwhile, some of the main items in signal components will be incorporated into the imprecise result, which cannot reflect the variation of the spectral characteristics through the cross-term in the damage signal. Haykin and Bhattacharya [18] proposed that using the cross-term of the WVD in the radar echo detects the icebergs floating in the sea, but his document only gives judgment through the existence of the cross-term and does not involve variation in the cross-term. Xiaofen et al. [19] proposed that the cross-term identification through the auto-term and the cross-term of the WVD have possibility in theoretical formula. Chaoshan et al. [20] established the function relationship between the time-frequency amplitude and the structural modal parameters. The curvature of the time-frequency amplitude in the signals of the measuring points by the WVD can identify the structural damage. The numerical simulation was established and lack of the practical application of engineering. Zhou et al. [21] proposed that the relative variation of statistics in the cross-term can identify the damage in the guyed mast. The numerical simulation and experimental research were established and proceed. This method identifies the location of the damage in the guyed mast. Researchers have also indicated that the cross-term can indeed reflect changes in spectral characteristics of the damage signal, but the extraction method of cross-term needs to be researched.

This paper proposed that the inner product is built up between the short-time Fourier transform and the bilinear time-frequency distribution represented by WVD. There is no cross-term in the short-time Fourier transform as the linear time-frequency distribution. The general results of the time-frequency analysis in the signals are obtained, but the resolution is worse. $\mathrm{STFT}_s(t, f)^2$ is called the short-time Fourier transform spectrogram, which is real-valued and nonnegative in the bilinear time-frequency distribution. It has the time-frequency shift invariance and better aggregation compared with SFFT. Generally, choosing a reasonable window function, it can significantly weaken the cross-term and strengthen the spectrum amplitude of the auto-terms in the damage signals [22]. In this paper, the inner product is built up with multiplication between short-time Fourier transform spectrogram and WVD, as shown in

$$T_s(t, f) = \left\langle \mathrm{STFT}_s(t, f)^2, \mathrm{WVD}_s(t, f) \right\rangle . \qquad (10)$$

In this equation, the multiplication of the spectrogram and the WVD is consistent with the direction of the inner product space in the amplitude of auto-terms in the damage signal spectrum. It strengthens the amplitude distribution of auto-terms. Oppositely, the inner product of the cross-term in the damage signal is very small or negligible. It can be seen that the amplitude magnitude of the auto-term and the cross-term are distinctly different. To some extent, it is the purification method that the highlighting auto-terms purify the time-frequency distribution. After getting the $T_s(t, f)$ distribution, comparing with the amplitude magnitude of the auto-term and the cross-term, then the reasonable amplitude threshold value Q_T is determined by experience and trial method; the two-dimensional time-frequency conversion matrix is constructed **E**. The time-frequency amplitude of the auto-terms distribution in the matrix is set as 0, which is greater than the threshold value Q_T. The time-frequency amplitude of the cross-term distribution in the matrix is set as 1, which is smaller than the threshold value Q_T. The inner product between the two-dimensional time-frequency conversion matrix **E** and the WVD is obtained. The complete pseudo-cross-term is extracted finally. The cross-term obtained by this method has the good aggregation and the higher time-frequency resolution. The introduction of the short-time Fourier transform spectrum highlights the auto-term distribution in the signals, and the complete cross-term is obtained which contains a wealth of structural damage information. It has the higher sensitivity and applicability for the damage identification. Flowchart of cross-term extraction is shown in Figure 1.

4. Numerical Example Analysis

4.1. Simple-Supported Beam Damage Simulated. In order to study the feasibility of application of the cross-term in the damage identification of structures and verify the effectiveness of above purification methods, this paper uses the finite element software ANSYS to build a simulation model of concrete simple-supported beams with the beam concrete grade of C30, beam section of 150 mm × 300 mm, beam length of 4000 mm, and effective span of 3600 mm and without considering the nonlinear behavior of steel and concrete as well as the isotropic materials. The Rayleigh damping is adopted to analyze the dynamic, so as to better simulate the free vibration and attenuation process of the concrete simple-supported beam. It is assumed that after the simple-supported beam bottom cracks at the mid-span, the damage of simple-supported beam mainly concentrates on the mid-span, and other sections are in the elastic working status. This paper simulated the cracks of the simple-supported beam bottom at the mid-span through reducing the cross-section dimensions in the finite element model and the simulated different degrees of the damage at the mid-span by setting the different sectional dimensions as shown in Figure 2. The degrees of the damage adopted by the finite element model are, respectively, 5%, 10%, 15%, and 20%, corresponding to the levels D1 to D4. Arrange the six measuring points of the acceleration along the span direction of the beam, as shown in Figure 3. The calculations of the model can be divided into the five working conditions, namely, nondestructive beam and beams with four different degrees of damage. The sudden unloading method is used in each working condition to simulate the load incentive of the simple-supported beam. The acceleration signals of each measuring point on the damped simple-supported beam in free vibration are collected.

After the time-frequency processing of the damage signals and nondestructive signals, the cross-term of the acceleration signals can be obtained on the basis of the purification method proposed in this paper, and the cross-term amplitude change rate curve of the damage signal and nondestructive signal is shown in Figure 4. In this figure, the change rate curve shows a peak value in the cross measuring points C3 and C4 with obvious discontinuity, where the damage part of the simple-supported beam is located. It is indicated that using the cross-term amplitude change rate as the damage indicator can accurately identify the damage part of the simple-supported beam. Judging from the relative position of damage curves at all levels, the cross-term amplitude change rate gradually increases along the growth of damage degree, so that the damage degree of the simple-supported beam can be accurately identified according to the progressive increase relationship, thus achieving the positioning and identification for the damage of simple-supported beam.

When the damage of the simple-supported beam is located in the 4/5 of the span, namely, the measuring point C5 in Figure 3, and other conditions remain the same, the cross-term amplitude change rate of the measuring points in different degrees of damage can be obtained, as shown in Figure 5. It can be seen that the discontinuity curve of the amplitude change rate declines significantly at the damage position, namely, the measuring point C5, especially in the case of 5% and 10% damage. And other measuring points remain the same. With the growth in the degree of damage, the cross-term amplitude change rates will be transmitted to both sides to some extent. Since the damage is very serious, and the dynamic characteristic of the structure changes significantly, the auto-term distribution changes greatly by reflecting in the data, so that the cross-term distribution far from the damage position also changes along with the auto-term. The structural damage degree can also be judged from Figure 5 according to the progressive increase relationship of change rate [23, 24].

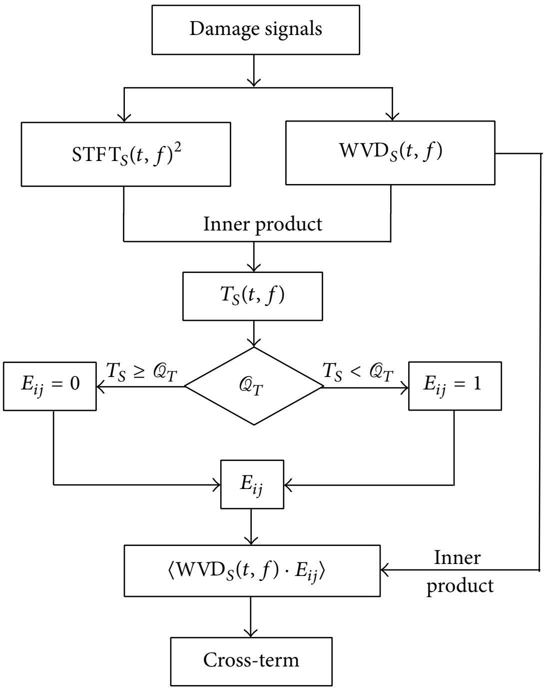

FIGURE 1: Flowchart of cross-term extraction.

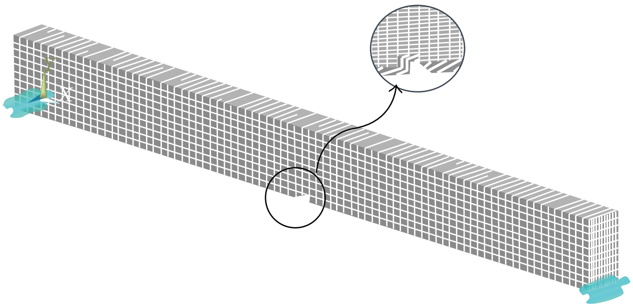

FIGURE 2: Simulation model of simple-supported beam.

Through the finite element modeling analysis on the simple-supported beam, it can be concluded that the cross-term obtained by the method described in this paper can accurately identify the damage position and the damage

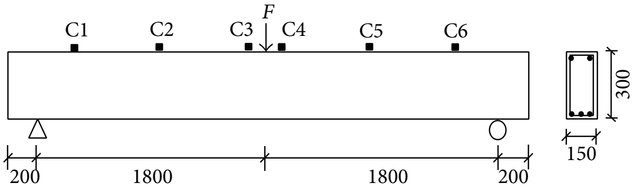

Figure 3: Design sketch of simple-supported beam.

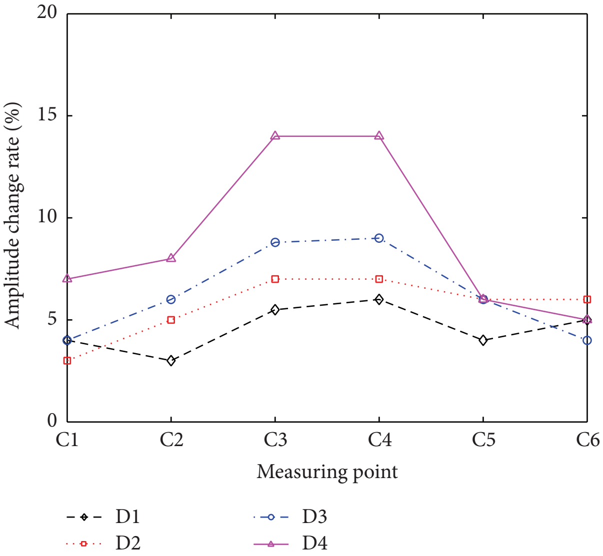

Figure 4: Damaged in the middle of C3 and C4.

degree of the simple-supported beam according to its amplitude change rate, which preliminarily verifies the effectiveness of bilinear cross-term of time-frequency distribution in the application of damage identification.

4.2. Cantilever Beam Damage Simulated. After getting the finite element analysis data of the simple-supported beam, this chapter further discusses the effects of cross-term on damage identification in the cantilever beam model. The calculation method for various parameters of the cantilever beam is the same as that of the simple-supported beam analyzed in the above chapter. The boundary conditions and the position of displacement loading point is modified at the end of the cantilever beam. The degree of damage and working conditions are the same as that of simple-supported beam. As one side of the cantilever beam is a free end with one measuring point, seven measuring points are divided into seven segments along the beam length; namely, the measuring points C1 to C7 are shown in Figure 6. For each working condition unit displacement to the cantilever end is applied and then released suddenly. After that, the acceleration signals of all measuring points on the cantilever beam in the damped cantilever beam free vibration are recorded. First when the damage is located in the middle of C1 and C2 on the cantilever beam, the cross-term amplitude change rate curve of damage signal and nondestructive signal can be obtained through methods mentioned above, as shown in Figure 7. The damage position of the cantilever beam is set as C6, as shown in Figure 8.

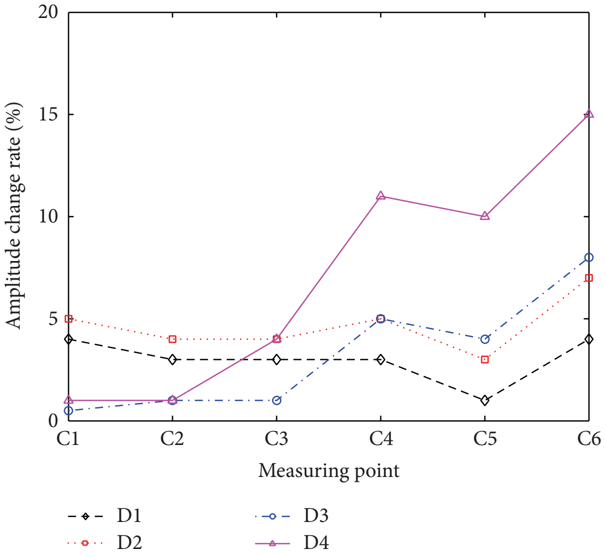

Figure 5: Damaged on C5.

It can be seen from Figures 7 and 8 that the mutation points of the cross-term amplitude change rate curve are the damage parts of the cantilever beam, possessing good identification ability in different degrees of damage, which once again verifies the effectiveness of the method described in this paper. The cross-term amplitude change rate curve has same rules for identifying the damage position and can also identify the damage degree of the structure according to the progressive increase relationship.

5. Damaged Experiment Research

5.1. Experiment. In this paper, the experimental testing is used to further verify the effectiveness of above method. With the reinforced concrete of simply supported beam in rectangle section as the object, the excitation test is conducted for the simply supported beam with the hammering method in the case that the mid-span has different degrees of damage [25]. The geometrical parameter of simply supported beam is the same as the finite element simulation model with upper main reinforcement of 2Φ10, lower of 3Φ10, and stirrup of Φ6@200. The acquisition system of the experiment consists of the IMC dynamic acquisition instrument and six ICP piezoelectricity acceleration sensors. In addition, the measuring points C1–C6 are arranged the same as the finite element analysis model, as shown in Figure 3. Figures 9 and 10 are the photos of the testing site.

The experiment uses the static force multistage loading way to simulate different degrees of damage of the simply supported beam in the cross. When gradually increasing vertical force to the cross, the crack on the lower part of the simply supported beam will spread continuously. And the residual deformation after loading also shows a progressive

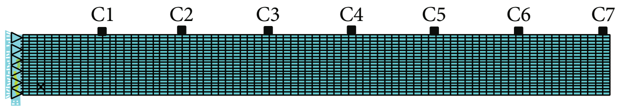

FIGURE 6: Design sketch of cantilever beam.

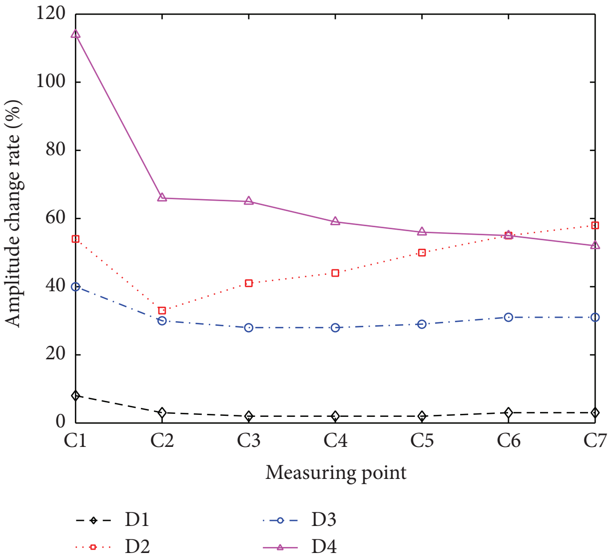

FIGURE 7: Damaged in the middle of C1 and C2.

increasing relationship. So the damage can be divided into four levels from D1 to D4. There are five working conditions in the experiment, namely, the nondestructive beam and the simply supported beam with four kinds of damage degree. After loading each working condition, the loading plate, force sensors, and other load weights are unloaded and removed. By measuring the increases in residual deformation of the simply supported beam in each stage, the rubber mallet hammers the mid-span of the beam and the acceleration signal in each damage degree from C1 to C6 is acquired. The time-history signal of acceleration at the measuring point C3 in different damage level is shown in Figures 11 and 12.

5.2. Data Analysis. Figures 13 to 15 are the results of the time-frequency analysis on the acceleration signals of the measuring point C3 at the damage level of D3 based on the purification processing method mentioned in (10). Through comparison, it can be seen that the time-frequency resolution of the short-time Fourier transform is lower than the resolution after the purification treatment. Since the test uses the hammering method for vibrating, coupled with the impact of environmental noise and the acquisition system, the clutter waves will inevitably appear in the signal. These factors are all presented in Figure 13, and two sides of the dominant term frequency are very obvious especially at the beginning of the hammering. But, in Figure 14, there are almost no other distributions except for the auto-term, but only the individual discrete points. When there is the environmental interference in the complicated structure or signal in the engineering project, the purification treatment to the signal can help accurately identify the dominant frequency distribution of the structure, so as to obtain clearer results of time-frequency analysis for signals.

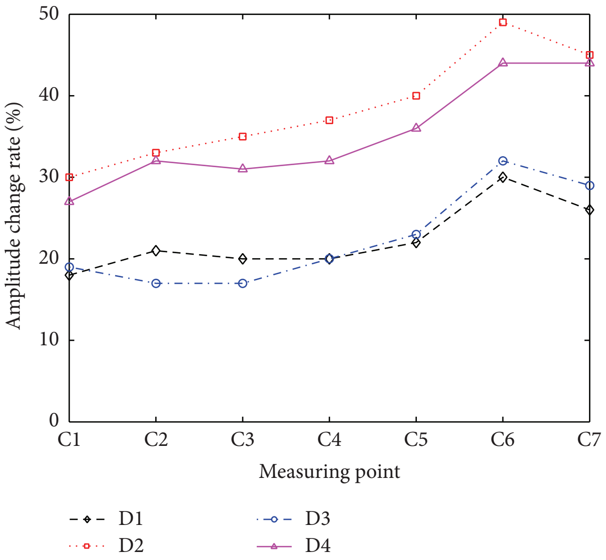

FIGURE 8: Damaged on C6.

FIGURE 9: Hammer load of simple-supported beam.

Figure 15 is the distribution of the cross-term obtained by the cross-term extraction method proposed in this paper. It can be seen from the figure that the cross-term has very wide frequency distribution and the quite low amplitude. Mainly it is concentrating on the high-frequency and low-frequency ends and satisfied the oscillation characteristics of the cross-term. Due to the simple frequency distribution of the simply supported beam, the cross-term mainly exists with the unimodal distribution at the same frequency.

The cross-term amplitude change rate of the damage signal and nondestructive signal of each measuring point at the different damage levels is shown in Figure 16. It can be intuitively seen from the figure that the change rate of the measuring points C3 and C4 in the cross of the simply supported beam is the largest. In addition, there are significant mutations on the cross-term amplitude change rate of the damage position, which can accurately identify the damage position of the cross. According to the force

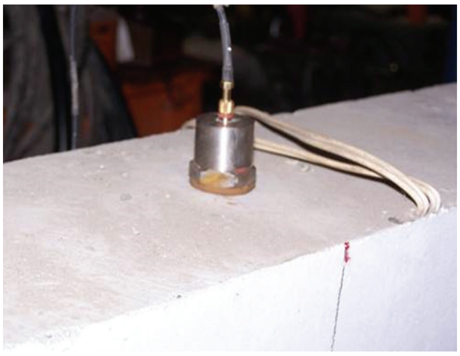

Figure 10: Signal acquisition from accelerometer.

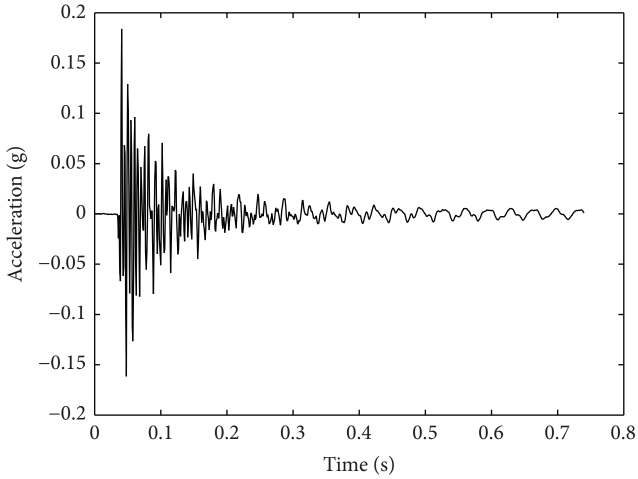

Figure 11: Acceleration signal on C3 for D3.

characteristics of the simply supported beam as well as the symmetry of the measuring points, the curve tendency is basically symmetric along the beam in the figure. And only the change rate of the measuring points C1 and C6 near the supporting point has some differences, whose reasons are related to the supporting conditions of the test beam. It also can be obtained from Figure 16 that, with the growth in the degree of damage, the cross-term amplitude change rate also increases accordingly. The cross-term amplitude change rate can not only accurately identify the damage position but also determine the damage levels of the structure on the basis of the progressive increasing relationship of the change rate.

It can be seen after summarizing above analysis results of experimental data and combing with Figure 4 that the curve obtained from the simulation model beam and test beam has similarity and consistent analysis results. It shows that the time-frequency analysis result of the signal purification method proposed by (10) in this paper possesses with higher time-frequency resolution; thus the dynamic characteristics of the structure can be accurately obtained. Through the cross-term amplitude change rate curve of damaged signal, the damage position of the structure can be accurately identified according to the position of mutations. Besides, the damage degree of the structure can also be confirmed on the

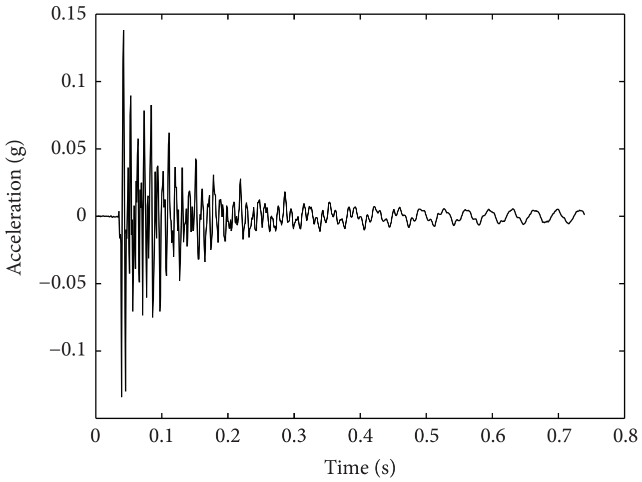

Figure 12: Acceleration signal on C3 for D4.

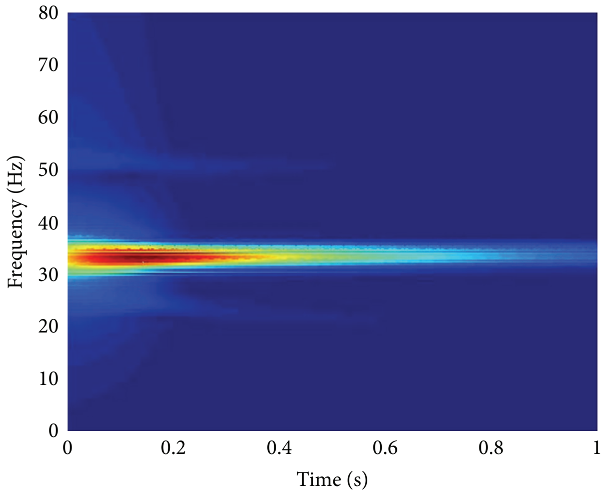

Figure 13: STFT of acceleration signals on C3 for D3.

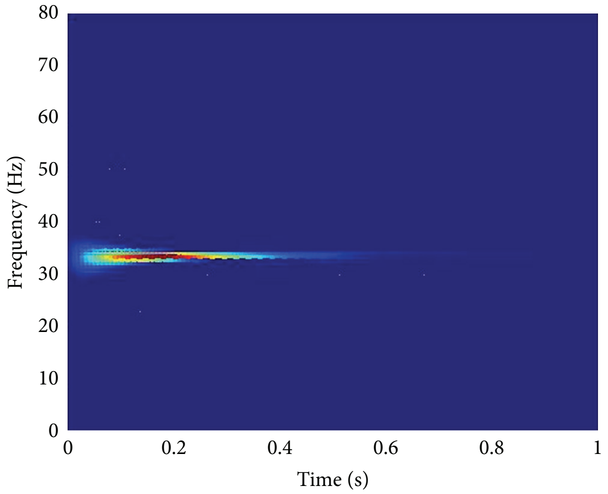

Figure 14: Refinement of acceleration signals on C3 for D3.

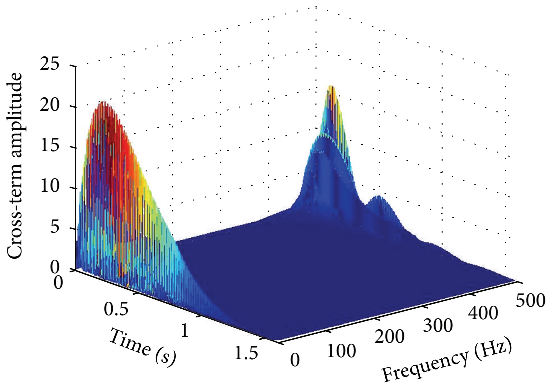

FIGURE 15: Cross-term distribution on C3 for D3.

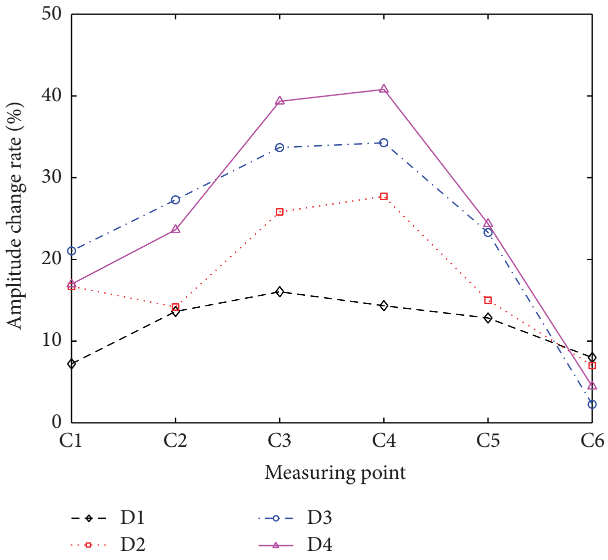

FIGURE 16: Damaged in the middle of C3 and C4.

basis of the progressive increasing relationship of the change rate.

6. Conclusion

In this paper, through the inner product between the short-time Fourier transform spectrogram and WVD, the cross-term of bilinear time-frequency distribution that is the Wigner-Ville distribution as the basis method of damage identification explores the purification of damaged signal. It has constructed two-dimensional time-frequency conversion matrix and extracted complete WVD cross-term distribution and selected the cross-term amplitude change rate as the damage indicator. Finally, the following conclusion can be obtained through the destructive testing of the simply supported beam and two simulation examples.

(1) The cross-term of the bilinear time-frequency distribution fully reflects the degree of coherence between the multicomponent signal components in the nonstationary signals. The damage position and the degree of the structure can be identified by using the variation of the cross-term in the damage signals.

(2) Using the inner product between the short-time Fourier conversion spectrum and WVD to purify the damaged signal of the structure can get the higher time-frequency resolution. Especially in the situation that there is environment interference in the complicated structures or signals, the purification of signal can accurately identify the dominant frequency distribution of the signal, so as to get clearer analysis results of signal time frequency.

(3) When taking the cross-term amplitude change rate as the indicator for damage identification of the structure, there are the significant mutation points in the cross-term amplitude change rate curve of damaged signal at the damage position of the structure, the damage position of the structure can be accurately judged. The analysis results of different damage levels show that the damage degree of the structure can be determined based on the progressive increasing relationship of the change rate curve. The change rate increases with the damage degrees of the structure.

(4) The example analysis and the experimental analysis show that the consistent results are obtained from the analogue simulation and destructive test of the simply supported beam, verifying the effectiveness of the cross-term using the bilinear time-frequency distribution on the damage identification and providing a reliable analysis method for the damage identification of engineering structures.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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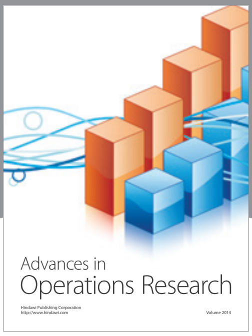
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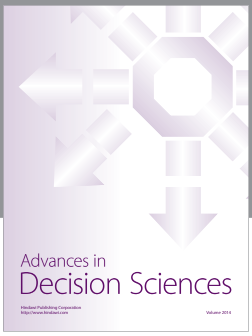
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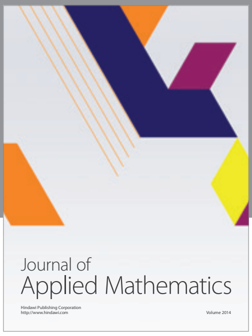
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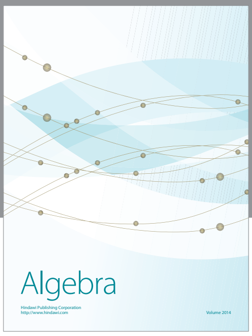
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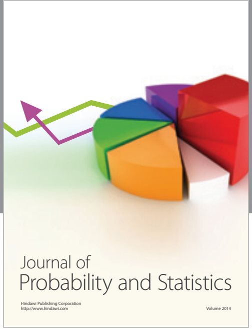
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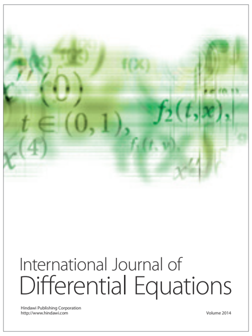
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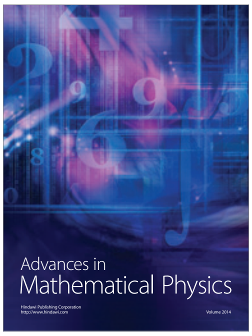
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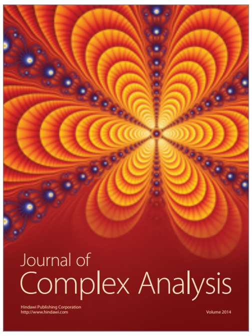
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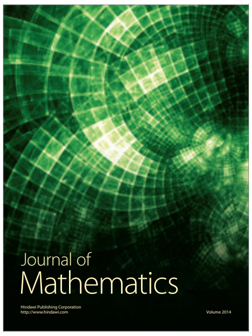
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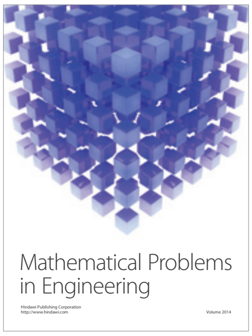
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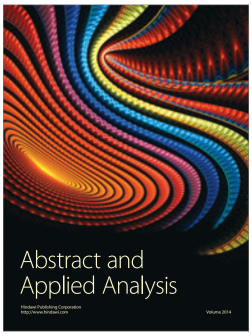
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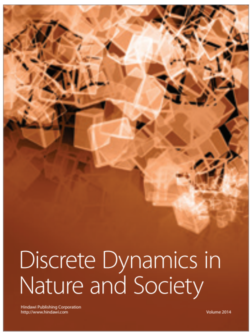
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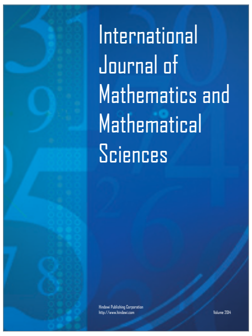
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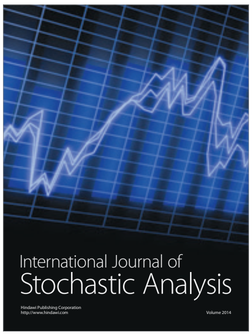
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