Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 903058, 13 pages
http://dx.doi.org/10.1155/2015/903058

Research Article

Robust Fault-Tolerant Tracking Control for Nonlinear Networked Control System: Asynchronous Switched Polytopic Approach

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Received 11 June 2015; Accepted 10 September 2015

Academic Editor: Xinggang Yan

This paper is concerned with the robust fault-tolerant tracking control problem for networked control system (NCS). Firstly, considering the locally overlapped switching law widely existed in engineering applications, the NCS is modeled as a locally overlapped switched polytopic system to reduce designing conservatism and solving complexity. Then, switched parameter dependent fault-tolerant tracking controllers are constructed to deal with the asynchronous switching phenomenon caused by the updating delays of the switching signals and weighted coefficients. Additionally, the global uniform asymptotic stability in the mean (GUAS-M) and desired weighted l_2 performance are guaranteed by combining the switched parameter dependent Lyapunov functional method with the average dwell time (ADT) method, and the feasible conditions for the fault-tolerant tracking controllers are obtained in the form of linear matrix inequalities (LMIs). Finally, the performance of the proposed approach is verified on a highly maneuverable technology (HiMAT) vehicle's tracking control problem. Simulation results show the effectiveness of the proposed method.

1. Introduction

In recent years, the networked control system (NCS) has received increasing interest due to the advantages of simple installation and maintenance, reduced weight, and power requirement (see [1–5]). However, the insertion of the communication network brings about some new drawbacks, such as the network-induced delays, data packet dropouts, and bandwidth limitation (see [6]), which may significantly deteriorate the performance of the system or even render the whole system unstable (see [3, 6]). For nonlinear NCS with both the nonlinear characteristics and the network transmission characteristics, the modeling, design, and analysis are more complex and challenging. Due to the increasing complexity, the probability of faults increases rapidly, which has motivated researchers to concentrate on fault-tolerant control for nonlinear NCS.

Several approaches on fault-tolerant control for nonlinear systems have been proposed, which include fuzzy approaches (see [7, 8]), neural network approaches (see [9, 10]), and switched system approaches (see [11–13]). Compared with other approaches, switched system approach combines the merits of less calculation amount with abundant engineering experience, which is of major interest in this paper.

Over last decades, switched system approaches for nonlinear systems have been studied intensively, and significant achievements have been obtained (see [12, 14–19]). In [12], a multiple Lyapunov function control method is presented for a broad class of switched nonlinear systems with input constraints. In [17], an observer-based fault-tolerant control method is proposed for a class of nonlinear switched systems that are output-input stable. A robust fault-tolerant control method is investigated for a class of uncertain switched nonlinear systems in lower triangular form in [18]. A safe-parking fault-tolerant scheme and a fault-tolerant scheme are designed for different switching schedules in [19].

Though these methods are effective switched system approaches, there are still some poorly developed but important theoretical issues in switched fault-tolerant control, such as the global stability certification and the slow variation requirement (see [20]). As a modified switched system analysis method, switched polytopic approach combines the advantages of higher precise modeling with lower computational complexity and has been applied widely. In [21], a switched polytopic system is established to describe the highly maneuverable technology vehicle within the full flight envelope and a robust dynamic output feedback control method is designed for the switched polytopic system. In [22], switched polytopic H_2 controllers are designed for linear parameter varying system to decrease design conservatism.

Before applying switched polytopic approach to engineering problems, significant researches remain to be done in constrained switching laws. "Locally overlapped" switching is a constrained switching law which exists in many practical engineering problems, where the order of the activated subsystems is predetermined. Meanwhile, the measurement and transmission of the switching signals, such as the time and system state, need external delays, which may result in the asynchronous phenomenon between the system modes and the switching signals (see [23, 24]). However, most switched polytopic methods are based on the assumption that the system modes and the corresponding controllers are switched synchronously, which is not always satisfied in industrial occasions.

Initiated by Zhang and his coworkers (see [25, 26]), many researchers have focused their attention on asynchronous switched system. In [27], state feedback stabilization controllers are designed for asynchronous switched system with time-varying state delays to guarantee the system's stability. Consider the existence of time delays and missing measurements simultaneously; [28] propose a state feedback controller in a more practical asynchronous switching case. To the best of the authors' knowledge, research about robust fault-tolerant tracking control for overlapped switched polytopic system under asynchronous switching is still challenging and has not been fully investigated yet.

Motivated by the above analysis, this paper investigates the robust fault-tolerant tracking control problem for nonlinear networked system under asynchronous switching. Firstly, the nonlinear NCS is modeled as a linear polytopic system by linearization on the equilibrium points. Secondly, considering the locally overlapped properties and asynchronous switching phenomenon, the polytopic system is augmented into a locally overlapped asynchronous switched polytopic system. Then, switched parameter dependent fault-tolerant tracking controllers are constructed to guarantee the tracking performance in the presence of external disturbances and faults. Moreover, the global uniform asymptotic stability in the mean (GUAS-M) of the system and the weighted l_2 performance are analyzed by combining the parameter dependent Lyapunov function method with average dwell time (ADT) method, and the feasible conditions and parameters for the controllers are obtained in the form of LMIs. Finally, a full envelope highly maneuverable technology (HiMAT) flight example is given to demonstrate the effectiveness of the proposed approach.

2. Model Description

The switched system builds a bridge between linear systems and complex uncertain systems, which make it possible to extend the abundant linear control theories to nonlinear systems effectively. The equilibrium points' linearization approach is a widely used nonlinear model describing method in engineering applications (see [3, 14, 15, 21]). Without loss of generality, suppose that the nonlinear model concerned in this paper can be linearized on equilibrium points; then a set of linear subsystems are obtained by linearization method and the dynamics of the original nonlinear system can be described by a linear switched system which consists of different equilibrium points. The model of ith equilibrium point can be presented as

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + B_{d,i} d(k) + B_{f,i} f(k) \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, $y \in R^c$ is the output, $d \in R^b$ is the external disturbance, and $f \in R^a$ is the unknown fault. Meanwhile, d and f are all belonging to $L_2[0,\infty)$; $i \in \Omega = \{1, 2, \ldots, M\}$ is mark number of the equilibrium points. For $\forall i \in \Omega$, the real system matrices A_i, B_i, $B_{d,i}$, $B_{f,i}$, and C are of appropriate dimensions.

The above linear system can only express the dynamics in the vicinity of the corresponding equilibrium point. To improve the precision of the system modeling, inspired by the switched linear parameter varying (LPV) approach, switched polytopic approach is adopted to describe the system dynamics.

The main idea of switched polytopic system modeling can be illustrated in Figure 1. Without loss of generality, suppose the dimension of the system state space is 2. All equilibrium points within the two-dimensional linear state space are divided into N regions according to the similar dynamics, and each region corresponds to one polytopic subsystem (PS). For each PS, referring to the gain scheduling approach (see [15]), the nonequilibrium point dynamics is described by dynamical weighting through several adjacent equilibrium points. For the whole state space, the system dynamics can be viewed as a linear switching between different PSs. Then the original dynamics of the nonlinear system can be described by a switched polytopic system.

After the manipulations above, the dynamics of the nonlinear system can be expressed by

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}(a_k)\, x(k) + B_{\sigma(k)}(a_k)\, u(k) \\ &\quad + B_{d,\sigma(k)}(a_k)\, d(k) + B_{f,\sigma(k)}(a_k)\, f(k) \\ y(k) &= Cx(k), \end{aligned} \tag{2}$$

where k is the sampling moment, switching law $\sigma(k) : N \rightarrow \Gamma$ represents the changing rule of the polytopic system along

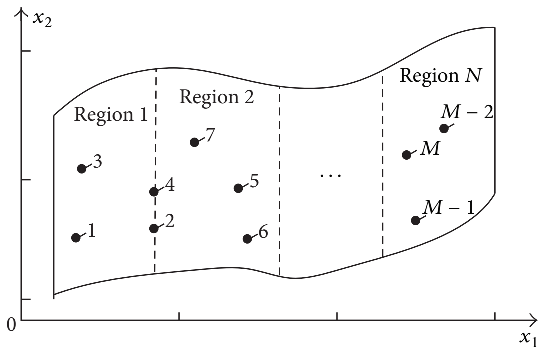

FIGURE 1: Linearization of equilibrium points for nonlinear system.

with time, $\Gamma = \{1, 2, \ldots, N\}$ is the mark number collection of the polytopic system. Moreover, the matrices in (2) satisfy the following equation:

$$\begin{bmatrix} A_m(a_k) & B_m(a_k) \\ B_{d,m}(a_k) & B_{f,m}(a_k) \end{bmatrix} = \sum_{i\in\Omega_m} a_{i,k} \begin{bmatrix} A_i & B_i \\ B_{d,i} & B_{f,i} \end{bmatrix}, \tag{3}$$

where $\sum_{i\in\Omega_m} a_{i,k} = 1,\ a_{i,k} \geq 0,\ \forall\sigma(k) = m \in \Gamma$.

Generally, the switching law is not always arbitrary in practical engineering. A constrained switching law with the property of "locally overlapped" (see [21]) exists extensively in many engineering problems, such as car shifting, temperature regulation, and assembly line work. As depicted in Figure 1, PS 1 contains equilibrium points 1, 2, 3, and 4 and PS 2 contains 2, 4, 5, 6, and 7. Equilibriums points 2 and 4 are their common equilibrium points. That means PS 1 and PS 2 are locally overlapped on points 2 and 4. Considering the locally overlapped characteristic for switching law $\sigma(k)$, the region division of the polytopic system must satisfy the following equation:

$$\begin{aligned} &\text{(a)} \bigcup_{m\in\Gamma} \Omega_m = \Omega \\ &\text{(b)}\ \Omega_{\sigma(k)} \cap \Omega_{\sigma(k+1)} \neq \emptyset, \quad \forall k \in N. \end{aligned} \tag{4}$$

Remark 1. Condition (a) ensures that the division results of the polytopic system can cover the whole nonlinear state space. Condition (b) ensures that switching occurs between two adjacent regions that share common equilibrium points. Under these conditions, only the common equilibrium points of the adjacent polytopic subsystems are used to calculate the corresponding polytopic weighted coefficient a_k. Therefore, the switching on the boundary of two adjacent regions will not result in a nonsmooth change of a_k. Then the system dynamics can vary smoothly.

3. Problem Formulation

In this section, parameter dependent robust fault-tolerant tracking controllers are proposed. The network's effects will be discussed firstly. The following assumptions are made to the network without loss of generality.

Assumption 1. The sampling period of the network is T. The sensors, the controllers, and actuators are all time-division-driven with the same time-driven period. The data packets that cannot be transmitted successfully on the sampling moment are discarded, which means the imperfect transmissions are all treated as dropouts (see [1]).

Assumption 2. Let $z(k)$ be the signal transmitted from the sensor to the controller; suppose the imperfect transmission can be represented by a Bernoulli distributed white sequence $\theta(k) \in \{0, 1\}$, where $\theta(k) = 1$ stands for a normal transmission and $\theta(k) = 0$ stands for a failed one (see [6]). Let $\Pr\{\theta(k) = 1\} = \rho$, and $\Pr\{\theta(k) = 0\} = 1 - \rho$.

Since there are zero-order holders in the control system, the real signal $z(k)$ can be written as

$$z(k) = \theta(k)\, y(k) + (1 - \theta(k))\, z(k-1). \tag{5}$$

The purpose of fault-tolerant tracking control is to make the output $y(k)$ track the command signal $r(k)$ and satisfy a desired H_∞ tracking performance even in fault case; define the tracking error $e(k)$ in the following equation:

$$e(k) = r(k) - y(k). \tag{6}$$

Since $y(k)$ cannot always be obtained on every sampling second, the actual available tracking error is $\bar{e}(k) = r(k) - z(k)$; the error integral action can be rewritten as follows:

$$\tilde{x}(k) = \sum_{i=0}^{k-1} \bar{e}(i) = \sum_{i=0}^{k-1} (r(i) - z(i)). \tag{7}$$

Then the tracking problem can be converted to find tracking controllers such that the following equation holds:

$$\lim_{k\to\infty} [\tilde{x}(k)] = 0. \tag{8}$$

Furthermore, one has

$$\begin{aligned} \tilde{x}(k+1) = {} & \tilde{x}(k) + r(k) - \theta(k)\, y(k) \\ & - (1 - \theta(k))\, z(k-1). \end{aligned} \tag{9}$$

To ensure the stability of the system and the tracking precision, the real output signal and the tracking error integral action are synthesized to construct the controllers in the following equation:

$$u(k) = K_{1,\sigma(k)} z(k) + K_{2,\sigma(k)} \tilde{x}(k), \tag{10}$$

where $K_{\sigma(k)} = [K_{1,\sigma(k)} \ \ K_{2,\sigma(k)}]$ are the controller parameters which need to be designed.

Then the local controller $K_m(a_k)$, $\forall\sigma(k) = m \in \Gamma$ of polytopic subsystem m can be written as

$$\begin{aligned} K_m(a_k) = \sum_{i\in\Omega_m} a_{i,k} K_i = \sum_{i\in\Omega_m} a_{i,k} [K_{1,i} \ \ K_{2,i}], \\ \sum_{i\in\Omega_m} a_{i,k} = 1,\ a_{i,k} \geq 0, \end{aligned} \tag{11}$$

where K_i is the controller's parameters of equilibrium point i, which is confirmed from (10).

Remark 2. The state parameters of the equilibrium points are all stored in internal storage and measured by sensors. The switching signal $\sigma(k)$ is confirmed by looking up the data tables online. The weighted coefficient a_k is calculated by graph functional method.

Remark 3. Due to the imperfect transmission of the system states transmitted by the network, the updating of $\sigma(k)$ and a_k may lag behind those of the system modes. The asynchronous polytopic switched system considered in this paper is more complex than that investigated in [25, 26], since both the delays of $\sigma(k)$ and a_k are considered.

Because of the unmatched interval between controllers and system modes, the Lyapunov function of the polytopic system will increase, but the increasing rate should be bounded. Define k_v and $k_{v+1}, v \in N$, as the active and over moments of subsystem $\sigma(k_v)$. The maximal updating delay of the state parameters is expressed by $\delta_m T$, where δ_m is a given constant. Under the influence of maximal asynchronous delay, the controllers in (10) can be rewritten as

$$\begin{aligned} u(k) = {} & K_{1,\sigma(k-\delta_m)}\left(a_{k-\delta_m}\right) z(k) \\ & + K_{2,\sigma(k)}\left(a_{k-\delta_m}\right) \tilde{x}(k). \end{aligned} \tag{12}$$

Define augmented state vector $\xi(k) = \left[x^T(k)\ \ z^T(k-1)\ \ \tilde{x}^T(k)\right]^T$ and augmented disturbance vector $w(k) = \left[d^T(k)\ \ f^T(k)\ \ r^T(k)\right]^T$. Combine (2), (5), and (9)~(12); the augmented locally overlapped polytopic system under asynchronous switching can be written as

$$\begin{aligned} \xi(k+1) &= \widehat{A}_n\left(a_{\bar{k}}\right)\xi(k) + \bar{\theta}(k)\widehat{A}_{1,n}\left(a_{\bar{k}}\right)\xi(k) \\ &\quad + \widehat{B}_n\left(a_{\bar{k}}\right) w(k) \\ e(k) &= \widehat{C}\xi(k) + \widehat{D}w(k) \\ &\qquad\qquad \forall k \in \left[k_v, k_v + \delta_m\right) \\ \xi(k+1) &= \overline{A}_m\left(a_{\bar{k}}\right)\xi(k) + \bar{\theta}(k)\overline{A}_{1,m}\left(a_{\bar{k}}\right)\xi(k) \\ &\quad + \overline{B}_m\left(a_{\bar{k}}\right) w(k) \\ e(k) &= \widehat{C}\xi(k) + \widehat{D}w(k) \\ &\qquad\qquad \forall k \in \left[k_v + \delta_m, k_{v+1}\right), \end{aligned} \tag{13}$$

where $\forall\{\sigma(k_v) = m, \sigma(k_v - \delta_m) = n\} \in \Gamma \times \Gamma$, $m \neq n$ and $\Omega_m \cap \Omega_n \neq \emptyset$; the system matrices are defined as

$$\begin{aligned} &\begin{bmatrix} \widehat{A}_n\left(a_{\bar{k}}\right) & \widehat{A}_{1,n}\left(a_{\bar{k}}\right) \\ \widehat{B}_n\left(a_{\bar{k}}\right) & 0 \end{bmatrix} = \sum_{l\in\Omega_n} a_{l,\bar{k}} \sum_{i\in\Omega_m} a_{i,k} \begin{bmatrix} \widehat{A}_{i,l} & \widehat{A}_{1,i,l} \\ \widehat{B}_i & 0 \end{bmatrix}, \\ &\begin{bmatrix} \overline{A}_m\left(a_{\bar{k}}\right) & \overline{A}_{1,m}\left(a_{\bar{k}}\right) \\ \overline{B}_m\left(a_{\bar{k}}\right) & 0 \end{bmatrix} = \sum_{l\in\Omega_m} a_{l,\bar{k}} \sum_{i\in\Omega_m} a_{i,k} \begin{bmatrix} \widehat{A}_{i,l} & \widehat{A}_{1,i,l} \\ \widehat{B}_i & 0 \end{bmatrix}, \\ &\widehat{A}_{i,l} = \begin{bmatrix} A_i + \rho B_i K_{1,l} C_i & (1-\rho) B_i K_{1,l} & B_i K_{2,l} \\ \rho C & (1-\rho) I & 0 \\ -\rho C & (\rho - 1) I & I \end{bmatrix}, \\ &\widehat{A}_{1,i,l} = \begin{bmatrix} B_i K_{1,l} C_i & -B_i K_{1,l} & 0 \\ C & -I & 0 \\ -C & I & 0 \end{bmatrix}, \\ &\widehat{B}_i = \begin{bmatrix} B_{d,i} & B_{f,i} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \\ &\widehat{C} = \begin{bmatrix} -C & 0 & 0 \end{bmatrix}, \\ &\widehat{D} = \begin{bmatrix} 0 & 0 & I \end{bmatrix}, \\ &\bar{k} = k - \delta_m, \\ &\bar{\theta}(k) = \theta(k) - \rho, \\ &E\left\{\bar{\theta}(k)\right\} = 0, \\ &E\left\{\bar{\theta}(k)\bar{\theta}(k)\right\} = \rho(1-\rho). \end{aligned} \tag{14}$$

According to the stability concept of the asynchronous polytopic switched system (see [29]), the augmented system (13) is said to be GUAS-M with a weighted l_2 performance $\gamma > 0$, if

(1) system (13) is GUAS-M with $w(k) \equiv 0$,

(2) under zero initial condition, the following inequality holds for all nonzero $w(k) \in L_2[0, \infty)$:

$$E\left\{\sum_{s=0}^{\infty}(1-\alpha)^s e^T(s) e(s)\right\} \leq \sum_{s=0}^{\infty}\gamma^2 w^T(s) w(s). \tag{15}$$

Remark 4. From the definition $\xi(k) = \left[x^T(k)\ \ z^T(k-1)\ \ \tilde{x}^T(k)\right]^T$, the augmented state vector $\xi(k)$ consists of system state, system output, and tracking error integral action, which means the GUAS-M of (13) can guarantee a sufficiently small tracking error. Furthermore, the performance index in (15) can guarantee the performance of the command tracking under the influences of the external disturbances and unknown faults.

4. Main Result

In this section, the global uniform asymptotic stability of (13) is analyzed. Firstly, the following definitions and lemmas that will be used in the derivation of the main results are introduced.

Definition 5 (see [30]). For a switching signal $\sigma(\cdot)$ and any $k > k_0$, let $N_\sigma[k_0, k)$ denote the number of switching $\sigma(\cdot)$ during the time interval $[k_0, k)$. If $N_\sigma[k_0, k) \le N_0 + (k - k_0)/\tau_a$ holds for $N_0 \ge 0$ and $\tau_a > 0$, then τ_a is called the average dwell time, and N_0 is the chatter bound.

Lemma 6. *Consider asynchronous switched polytopic system (13); let $0 < \alpha < 1$, $\beta > -1$, and $\mu \ge 1$ be given constants. Suppose that there exist C^1 functions $V_{\sigma(k)}$, $\sigma(k) = m \in \Gamma$, and two class κ_∞ functions κ_1 and κ_2 such that*

$$\kappa_1 (\|\xi(k)\|) \le V_m (a_k, \xi(k)) \le \kappa_2 (\|\xi(k)\|) \tag{16}$$

$$E(\Delta V_m(a_k, \xi(k))) \le \begin{cases} -\alpha V_m (a_k, \xi(k)) & \forall k \in [k_v + \delta_m, k_{v+1}) \\ \beta V_m (a_k, \xi(k)) & \forall k \in [k_v, k_v + \delta_m) \end{cases} \tag{17}$$

$$V_m (a_k, \xi(k)) \le \mu V_n (a_k, \xi(k)) \quad \forall (m, n) \in \Gamma \times \Gamma,\ m \ne n. \tag{18}$$

Then asynchronous switched polytopic system (13) is GUAS-M for any switching signal with ADT:

$$\tau_a > \tau_a^* = -\frac{\{\delta_m [\ln \tilde{\beta} - \ln \tilde{\alpha}] + \ln \mu\}}{\ln \tilde{\alpha}}, \tag{19}$$

where $\tilde{\alpha} = 1 - \alpha$, $\tilde{\beta} = 1 + \beta$, and $\delta_{\max} = \max \delta_l$, $\forall l \in N$.

Proof. Referring to the approach in [25], for $\forall k \in [k_v, k_{v+1})$, one has

$$\begin{aligned} &E(V_{\sigma(k)}(\xi(k))) \\ &\quad \le \tilde{\alpha}^{\mathcal{T}_\downarrow (k-k_v)} \tilde{\beta}^{\mathcal{T}_\uparrow (k-k_v)} E(V_{\sigma(k_v)}(\xi(k_v))) \\ &\quad \le \tilde{\alpha}^{(k-k_v)} \theta^{\delta_m} E(V_{\sigma(k_v)}(\xi(k_v))) \\ &\quad \le \tilde{\alpha}^{(k-k_v)} \theta^{\delta_m} \mu E(V_{\sigma(k_v - 1)}(\xi(k_v))) \le \cdots \\ &\quad \le \tilde{\alpha}^{(k-0)} (\theta^{\delta_m} \mu)^{N_\sigma[0,k)} V_{\sigma(0)}(\xi(0)) \\ &\quad \le \mu^{N_0} \theta^{N_0 \delta_m} (\tilde{\alpha} \theta^{\delta_m/\tau_a} \mu^{1/\tau_a})^k V_{\sigma(0)}(\xi(0)). \end{aligned} \tag{20}$$

If ADT τ_a satisfies (19), defining $\kappa \triangleq -\ln \tilde{\lambda} / (\delta_m \ln \theta + \ln \mu)$, one has

$$\begin{aligned} \tilde{\alpha} \theta^{\delta_m/\tau_a} \mu^{1/\tau_a} &< \tilde{\alpha} \theta^{-\delta_m \ln \tilde{\alpha}/(\delta_m \ln \theta + \ln \mu)} \mu^{-\ln \tilde{\alpha}/(\delta_m \ln \theta + \ln \mu)} \\ &= \tilde{\alpha} (\theta^{\delta_m} \mu)^\kappa = \tilde{\alpha} (e^{\delta_m \ln \theta + \ln \mu})^\kappa = \frac{\tilde{\alpha}}{\tilde{\alpha}} = 1. \end{aligned} \tag{21}$$

Therefore, one can obtain that $E(V_{\sigma(k)}(\xi(k))) \to 0$ when $k \to \infty$. Then, with the aid of (16), system (13) is GUAS-M by using the same approach in [31]. □

Lemma 7. *Consider asynchronous switched polytopic system (13); let $0 < \alpha < 1$, $\beta > -1$, $\mu > 1$, and $\gamma_m > 0$, $\forall m \in \Gamma$ be given constants. Suppose that there exist C^1 functions $V_{\sigma(k)}$, $\sigma(k) = m \in \Gamma$, such that (18) and (22) hold. Consider*

$$E(\Delta V_m(a_k, \xi(k))) \le \begin{cases} -\alpha V_m (a_k, \xi(k)) - \varphi(k) & \forall k \in [k_v + \delta_m, k_{v+1}) \\ \beta V_m (a_k, \xi(k)) - \varphi(k) & \forall k \in [k_v, k_v + \delta_m), \end{cases} \tag{22}$$

where $\varphi(k) = E\{e^T(k) e(k)\} - \gamma_m w^T(k) w(k)$. Then asynchronous switched polytopic system (13) is GUAS-M under any switching signal whose ADT τ_a satisfies (19) and has weighted l_2 performance of no more than $\gamma = \max_{m \in \Gamma} \{\sqrt{\mu^{N_0} \theta^{\delta_m N_0}} \gamma_m\}$.

Proof. Using the stochastic system analysis method in [32] for reference, replace $V_m(x(k))$ and $\Delta V_m(x(k))$ with $E(V_{\sigma(k)}(a_k, \xi(k)))$ and $E(\Delta V_{\sigma(k)}(a_k, \xi(k)))$. The stability analysis of Lemma 7 which is suitable for switched polytopic system (13) can be obtained directly. The proof is omitted here. □

Lemma 8 (see [33]). *For given matrix $C \in R^{n \times m}$, assume* $\operatorname{rank}(\mathbf{C}) = n$ *without loss of generality; then there always exists $C = U [\Sigma \ \ 0] E$ by performing the singular value decomposition to C, where $U \in R^{n \times n}$ and $E \in R^{m \times m}$ are two orthogonal matrices; $E = [E_1^T \ \ E_2^T]^T$, $E_1 \in R^{n \times m}$, $E_2 \in R^{(m-n) \times m}$, and $\Sigma =$* $\operatorname{diag}\{\sigma_1, \ldots, \sigma_n\}$ *with $\sigma_1, \ldots, \sigma_n$ denote the nonzero singular values of C.*

If $X \in R^{n \times n}$ is of the structure

$$X = U \Sigma X_1 \Sigma^{-1} U^T, \tag{23}$$

where $X_1 \in R^{n \times n} > 0$, then there exists a nonsingular matrix $\widehat{X} \in R^{m \times m}$ such that $XC = C\widehat{X}$.

4.1. Stability Analysis and Controller Design. Based on the above definition and lemmas, the following theorems provide the feasible conditions and solving methods of the fault-tolerant tracking controllers.

Theorem 9. *Consider system (13) and let $0 < \alpha < 1$, $\beta > -1$, and $\gamma_m > 0$, $\forall m \in \Gamma$ be given constants. Let $\vartheta = \sqrt{\rho(1 - \rho)}$; if there is existence of matrices $P_i > 0$, $i \in \Omega_m$, and $\forall m \in \Gamma$, such that the two inequalities,*

$$\Upsilon_{i,j,l} = \begin{bmatrix} -P_j & 0 & 0 & P_j \widehat{A}_{i,l} & P_j \widehat{B}_i \\ * & -P_j & 0 & \vartheta P_j \widehat{A}_{1,i,l} & 0 \\ * & * & -I & \widehat{C} & \widehat{D} \\ * & * & * & -(1-\alpha) P_i & 0 \\ * & * & * & * & -\gamma_m^2 I \end{bmatrix} < 0 \quad \forall i, j, l \in \Omega_m, \tag{24}$$

$$\overline{\Upsilon}_{i,j,l} = \begin{bmatrix} -P_j & 0 & 0 & P_j\widehat{A}_{i,l} & P_j\widehat{B}_i \\ * & -P_j & 0 & \vartheta P_j\widehat{A}_{1,i,l} & 0 \\ * & * & -I & \widehat{C} & \widehat{D} \\ * & * & * & -(1+\beta)P_i & 0 \\ * & * & * & * & -\gamma_m^2 I \end{bmatrix} < 0 \quad (25)$$

$$\forall i,j \in \Omega_m,\ l \in \Omega_n,\ \Omega_m \cap \Omega_n \neq \emptyset,\ m \neq n$$

hold, then system (13) is GUAS-M for any switching signal satisfying (26) and has a weighted l_2 performance $\gamma = \max\{\gamma_m\}$, $\forall m \in \Gamma$ defined in (15). Consider

$$\tau_a > \tau_a^* = -\frac{\{\delta_m [\ln\widetilde{\beta} - \ln\widetilde{\alpha}]\}}{\ln\widetilde{\alpha}}, \quad (26)$$

where $\theta = \widetilde{\beta}/\widetilde{\alpha}$, $\widetilde{\alpha} = 1-\alpha$, $\widetilde{\beta} = 1+\beta$, $\delta_{\max} = \max_{\forall l \in N}\delta_l$, and asterisk $(*)$ denotes a term that is induced by symmetry in symmetric block matrices.

Proof. First of all, choose (27) as a parameter dependent Lyapunov function:

$$V_m(\xi(k)) = V_m(a_k, \xi(k)) = \xi^T(k) P_m(a_k)\xi(k). \quad (27)$$

For $\forall\sigma(k) = m \in \Gamma$, the weighted matrices for PS ($\overline{A}_m(a_k)$, $\overline{A}_{1,m}(a_k)$, $\overline{B}_m(a_{\overline{k}})$) are $P_m(a_k) = \sum_{i\in\Omega_m} a_{i,k}P_i$. From Remark 1, one knows that the switching between two adjacent polytopic regions will not result in a step varying on weighted coefficient a_k. Therefore a_k is calculated by only the common equilibrium points within the adjacent region; one has

$$\begin{aligned} V_m(a_k, \xi(k_v)) &= \xi^T(k_v) P_m(a_{k_v})\xi(k_v) \\ &= \xi^T(k_v)\left(\sum_{s\in\{\Omega_m\cap\Omega_n\}} a_{s,k_v}P_s\right)\xi(k_v) \\ V_n(a_k, \xi(k_v)) &= \xi^T(k_v) P_n(a_{k_v})\xi(k_v) \\ &= \xi^T(k_v)\left(\sum_{s\in\{\Omega_m\cap\Omega_n\}} a_{s,k_v}P_s\right)\xi(k_v). \end{aligned} \quad (28)$$

Then one has $\mu = 1$ when switching occurs from (18). According to (19), the ADT of (13) only needs to satisfy the limitation in (26).

To make the following statement clear, the rest of the proof is divided into two parts.

Part I. System (13) is GUAS-M with $w(k) \equiv 0$.

When $w(k) = 0$ and $\forall k \in [k_v + \delta_m, k_{v+1})$, for any arbitrary $\xi(k) \neq 0$ and switching signal $\forall\sigma(k) = m \in \Gamma$, one has

$$\begin{aligned} &E\{V_m(a_{k+1}, \xi(k+1))\} - V_m(a_k, \xi(k)) \\ &\quad + \alpha V_m(a_k, \xi(k)) = E\{\xi^T(k+1) P_m(a_{k+1}) \\ &\quad \cdot \xi(k+1)\} - \xi^T(k)(1-\alpha)P_m(a_k)\xi(k) \\ &\quad = E\left\{\xi^T(k)\left(\overline{A}_m(a_{\overline{k}}) + \overline{\theta}(k)\overline{A}_{1,m}(a_{\overline{k}})\right)^T P_m(a_{k+1})\right. \\ &\quad \left.\cdot\left(\overline{A}_m(a_{\overline{k}}) + \overline{\theta}(k)\overline{A}_{1,m}(a_{\overline{k}})\right)\xi(k)\right\} - \xi^T(k)(1 \\ &\quad -\alpha)P_m(a_k)\xi(k) = \xi^T(k)\Lambda_{1,m}(a_k, a_{k+1})\xi(k), \end{aligned} \quad (29)$$

where $\Lambda_{1,m}(a_k, a_{k+1}) = \overline{A}_m^T(a_{\overline{k}})P_m(a_{k+1})\overline{A}_m(a_{\overline{k}}) + \vartheta^2\overline{A}_{1,m}^T(a_{\overline{k}})P_m(a_{k+1})\overline{A}_{1,m}(a_{\overline{k}}) - (1-\alpha)P_m(a_k)$.

In a similar way, when $w(k) = 0$ and $\forall k \in [k_v, k_v + \delta_m)$, for any arbitrary $\xi(k) \neq 0$ and switching signal $\forall\{\sigma(k) = m, \sigma(\overline{k}) = n\} \in \Gamma \times \Gamma$ and $\Omega_m \cap \Omega_n \neq \emptyset$, $m \neq n$, one has

$$\begin{aligned} &E\{V_m(a_{k+1}, \xi(k+1))\} - V_m(a_k, \xi(k)) \\ &\quad - \beta V_m(a_k, \xi(k)) \\ &\quad = E\{\xi^T(k+1)P_m(a_{k+1})\xi(k+1)\} \\ &\quad - \xi^T(k)(1+\beta)P_m(a_k)\xi(k) \\ &\quad = \xi^T(k)\Lambda_{2,m}(a_k, a_{k+1})\xi(k), \end{aligned} \quad (30)$$

where $\Lambda_{2,m}(a_k, a_{k+1}) = \widehat{A}_n^T(a_{\overline{k}})P_m(a_{k+1})\widehat{A}_n(a_{\overline{k}}) + \vartheta^2\widehat{A}_{1,n}^T(a_{\overline{k}})P_m(a_{k+1})\widehat{A}_{1,n}(a_{\overline{k}}) - (1+\beta)P_m(a_k)$.

Due to the fact that $P_i \geq 0$, $i \in \Omega_m$, $\forall m \in \Gamma$, by performing congruence transformation to (24) and (25) via $\operatorname{diag}\{P_j^{-T}, P_j^{-T}, I, I, I\}$ and further by Schur complement, (24) and (25) are equal to the following equation:

$$\begin{aligned} \Theta_{i,j,l} &= \begin{bmatrix} \widehat{A}_{i,l} & \widehat{B}_i \\ \vartheta\widehat{A}_{1,i,l} & 0 \\ \widehat{C}_i & \widehat{D}_i \end{bmatrix}^T \begin{bmatrix} P_j & 0 & 0 \\ 0 & P_j & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \widehat{A}_{i,l} & \widehat{B}_i \\ \vartheta\widehat{A}_{1,i,l} & 0 \\ \widehat{C} & \widehat{D} \end{bmatrix} \\ &\quad - \begin{bmatrix} (1-\alpha)P_i & 0 \\ 0 & \gamma_m^2 I \end{bmatrix} < 0 \quad \forall i,j,l \in \Omega_m \\ \overline{\Theta}_{i,j,l} &= \begin{bmatrix} \widehat{A}_{i,l} & \widehat{B}_i \\ \vartheta\widehat{A}_{1,i,l} & 0 \\ \widehat{C}_i & \widehat{D}_i \end{bmatrix}^T \begin{bmatrix} P_j & 0 & 0 \\ 0 & P_j & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \widehat{A}_{i,l} & \widehat{B}_i \\ \vartheta\widehat{A}_{1,i,l} & 0 \\ \widehat{C} & \widehat{D} \end{bmatrix} \\ &\quad - \begin{bmatrix} (1+\beta)P_i & 0 \\ 0 & \gamma_m^2 I \end{bmatrix} < 0 \\ &\quad \forall i,j \in \Omega_m,\ l \in \Omega_n,\ \Omega_m \cap \Omega_n \neq \emptyset,\ m \neq n. \end{aligned} \quad (31)$$

Noticing $(1,1)$ of matrices $\Theta_{i,j,l}$ and $\overline{\Theta}_{i,j,l}$, one has

$$\begin{aligned} \Lambda_{i,j,l} &= \widehat{A}_{i,l}^T P_j \widehat{A}_{i,l} + \vartheta^2 \widehat{A}_{1,i,l}^T P_j \widehat{A}_{1,i,l} - (1-\alpha) P_i < 0 \\ \widehat{\Lambda}_{i,j,l} &= \widehat{A}_{i,l}^T P_j \widehat{A}_{i,l} + \vartheta^2 \widehat{A}_{1,i,l}^T P_j \widehat{A}_{1,i,l} - (1+\beta) P_i < 0. \end{aligned} \tag{32}$$

Then

$$\begin{aligned} \Lambda_{1,m}(a_k, a_{k+1}) &= \sum_{l\in\Omega_m} a_{l,\bar{k}} \sum_{i\in\Omega_m} a_{i,k} \sum_{j\in\Omega_m} a_{j,k+1} \left(\Lambda_{i,j,l}\right) < 0 \\ \Lambda_{2,m}(a_k, a_{k+1}) &= \sum_{l\in\Omega_n} a_{l,\bar{k}} \sum_{i\in\Omega_m} a_{i,k} \sum_{j\in\Omega_m} a_{j,k+1} \left(\widehat{\Lambda}_{i,j,l}\right) \\ &< 0. \end{aligned} \tag{33}$$

From (33), one knows that every PS is GUAS-M. From (29) and (30), one has (34) holding:

$$E\left(\Delta V_m(\xi(k))\right) \le \begin{cases} -\alpha V_m(\xi(k)), & \forall k \in [k_v + \delta_m, k_{v+1}) \\ \beta V_m(\xi(k)), & \forall k \in [k_v, k_v + \delta_m). \end{cases} \tag{34}$$

Combining (26), (34), and Lemma 6, the asynchronous switched polytopic system (13) is GUAS-M under the ADT limitation of (26).

Part II. Under zero initial condition, the asynchronous switched polytopic system (13) has the weighted l_2 performance defined in (15) for all nonzero $w(k) \in L_2[0,\infty)$.

Defining the augmented vector $\zeta(k) = \left[\xi^T(k) \;\; w^T(k)\right]^T$, under zero initial condition, for any nonzero $w(k) \in L_2[0,\infty)$ and $\forall k \in [k_v + \delta_m, k_{v+1})$, one has

$$\begin{aligned} &E\left\{V_m(a_{k+1}, \xi(k+1))\right\} - V_m(a_k, \xi(k)) \\ &\quad + \alpha V_m(a_k, \xi(k)) + E\left\{e^T(k) e(k)\right\} - \gamma_m^2 w^T(k) \\ &\quad \cdot w(k) = E\left\{\xi^T(k+1) P_m(a_{k+1}) \xi(k+1)\right\} \\ &\quad - \xi^T(k)(1-\alpha) P_m(a_k)\xi(k) - \gamma_m^2 w^T(k) w(k) \\ &\quad + E\left\{\zeta^T \begin{bmatrix} \widehat{C}^T \\ \widehat{D}^T \end{bmatrix} \begin{bmatrix} \widehat{C}^T \\ \widehat{D}^T \end{bmatrix}^T \zeta\right\} \\ &\quad = \zeta^T \left\{ \begin{bmatrix} \overline{A}_m^T(a_{\bar{k}}) \\ \overline{B}_m^T(a_{\bar{k}}) \end{bmatrix} P_m(a_{k+1}) \begin{bmatrix} \overline{A}_m^T(a_{\bar{k}}) \\ \overline{B}_m^T(a_{\bar{k}}) \end{bmatrix}^T \right. \\ &\quad + \vartheta^2 \begin{bmatrix} \overline{A}_{1,m}^T(a_{\bar{k}}) \\ 0 \end{bmatrix} P_m(a_{k+1}) \begin{bmatrix} \overline{A}_{1,m}^T(a_{\bar{k}}) \\ 0 \end{bmatrix}^T \\ &\quad \left. + \begin{bmatrix} \widehat{C}^T \\ \widehat{D}^T \end{bmatrix} \begin{bmatrix} \widehat{C}^T \\ \widehat{D}^T \end{bmatrix}^T - \begin{bmatrix} (1-\alpha) P_m(a_k) & 0 \\ 0 & \gamma_m^2 I \end{bmatrix} \right\} \zeta \\ &\quad = \zeta^T(k) \Gamma_{1,m}(a_k, a_{k+1}, a_{\bar{k}}) \zeta(k), \end{aligned} \tag{35}$$

where

$$\begin{aligned} \Gamma_{1,m}(a_k, a_{k+1}, a_{\bar{k}}) &= \begin{bmatrix} \overline{A}_m(a_{\bar{k}}) & \overline{B}_m(a_{\bar{k}}) \\ \vartheta \overline{A}_{1,m}(a_{\bar{k}}) & 0 \\ \widehat{C}^T & \widehat{D}^T \end{bmatrix}^T \\ &\cdot \begin{bmatrix} P_m(a_{k+1}) & 0 & 0 \\ 0 & P_m(a_{k+1}) & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \overline{A}_m(a_{\bar{k}}) & \overline{B}_m(a_{\bar{k}}) \\ \vartheta \overline{A}_{1,m}(a_{\bar{k}}) & 0 \\ \widehat{C}^T & \widehat{D}^T \end{bmatrix} \\ &- \begin{bmatrix} (1-\alpha) P_m(a_k) & 0 \\ 0 & \gamma_m^2 I \end{bmatrix}. \end{aligned} \tag{36}$$

In a similar way, for any nonzero $w(k) \in L_2[0,\infty)$ and $\forall k \in [k_v + \delta_m, k_{v+1})$, under arbitrary switching signal $\{\sigma(k) = m, \sigma(\bar{k}) = n\} \in \Gamma \times \Gamma$, $\Omega_m \cap \Omega_n \neq \emptyset$, $m \neq n$, one has

$$\begin{aligned} &E\left\{V_m(a_{k+1}, \xi(k+1))\right\} - V_m(a_k, \xi(k)) \\ &\quad - \beta V_m(a_k, \xi(k)) + E\left\{e^T(k) e(k)\right\} \\ &\quad - \gamma_m^2 w^T(k) w(k) \\ &\quad = E\left\{\xi^T(k+1) P_m(a_{k+1}) \xi(k+1)\right\} \\ &\quad - \xi^T(k)(1+\beta) P_m(a_k)\xi(k) - \gamma_m^2 w^T(k) w(k) \\ &\quad + E\left\{\zeta^T \begin{bmatrix} \widehat{C}^T \\ \widehat{D}^T \end{bmatrix} \begin{bmatrix} \widehat{C}^T \\ \widehat{D}^T \end{bmatrix}^T \zeta\right\} \\ &\quad = \zeta^T(k) \Gamma_{2,mn}(a_k, a_{k+1}, a_{\bar{k}}) \zeta(k), \end{aligned} \tag{37}$$

where

$$\begin{aligned} \Gamma_{2,mn}(a_k, a_{k+1}, a_{\bar{k}}) &= \begin{bmatrix} \widehat{A}_n(a_{\bar{k}}) & \widehat{B}_n(a_{\bar{k}}) \\ \vartheta \widehat{A}_{1,n}(a_{\bar{k}}) & 0 \\ \widehat{C}^T & \widehat{D}^T \end{bmatrix}^T \\ &\cdot \begin{bmatrix} P_m(a_{k+1}) & 0 & 0 \\ 0 & P_m(a_{k+1}) & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \widehat{A}_n(a_{\bar{k}}) & \widehat{B}_n(a_{\bar{k}}) \\ \vartheta \widehat{A}_{1,n}(a_{\bar{k}}) & 0 \\ \widehat{C}^T & \widehat{D}^T \end{bmatrix} \\ &- \begin{bmatrix} (1+\beta) P_m(a_k) & 0 \\ 0 & \gamma_m^2 I \end{bmatrix}. \end{aligned} \tag{38}$$

It is easy to obtain (39) from (31):

$$\begin{aligned} \Gamma_{1,m}(a_k, a_{k+1}, a_{\bar{k}}) &= \sum_{l\in\Omega_m} a_{l,\bar{k}} \sum_{i\in\Omega_m} a_{i,k} \sum_{j\in\Omega_m} a_{j,k+1} \left(\Theta_{i,j,l}\right) \\ &< 0 \\ \Gamma_{2,mn}(a_k, a_{k+1}, a_{\bar{k}}) &= \sum_{l\in\Omega_n} a_{l,\bar{k}} \sum_{i\in\Omega_m} a_{i,k} \sum_{j\in\Omega_m} a_{j,k+1} \left(\overline{\Theta}_{i,j,l}\right) \\ &< 0. \end{aligned} \tag{39}$$

Thus one has

$$E\left(\Delta V_m(\xi(k))\right) \le \begin{cases} -\alpha V_m(\xi(k)) + E\left\{e^T(k)e(k)\right\} - \gamma_m^2 w^T(k)w(k), & \forall k \in [k_v+\delta_m, k_{v+1}) \\ \beta V_m(\xi(k)) + E\left\{e^T(k)e(k)\right\} - \gamma_m^2 w^T(k)w(k), & \forall k \in [k_v, k_v+\delta_m). \end{cases} \quad (40)$$

Combining (26), (40), and Lemma 7, one can obtain that system (13) is GUAS-M under the switching signal with the ADT τ_a satisfying (26). When $\mu = 1$ and $N_0 = 0$, the weighted l_2 performance in (15) is degenerated to $\gamma = \max\{\gamma_m\}$, $\forall m \in \Gamma$. This completes the proof. □

4.2. Controller Design. Based on Theorem 9, Theorem 10 provides the solving method for the tracking controllers in the form of (12).

Theorem 10. *Consider system (13) and let* $0 < \alpha < 1$, $\beta > -1$, *and* $\gamma_m > 0$, $\forall m \in \Gamma$ *be given constants. Let* $\vartheta = \sqrt{\rho(1-\rho)}$*; if there exist positive-definition matrices* S_{11i}, S_{3i}, $i \in \Omega_m$, $\forall m \in \Gamma$ *and matrices* $\overline{K}_{1,i}, \overline{K}_{2,i}$, $i \in \Omega_m$, $\forall m \in \Gamma$ $\forall i \in \Omega$, *such that for* $\forall (m,n) \in \Gamma \times \Gamma$ *(41) and (42) hold, then system (13) with controllers (12) is GUAS-M under the switching signal whose ADT* τ_a *is satisfying (26) and has the weighted* l_2 *performance defined in (15), where* $\gamma = \max\{\gamma_m\}$, $\forall m \in \Gamma$. *Consider*

$$\begin{bmatrix} -S_j & 0 & 0 & \overline{\varphi}_{14} & \widehat{B}_i \\ * & -S_j & 0 & \overline{\varphi}_{24} & 0 \\ * & * & -I & \overline{\varphi}_{34} & \widehat{D} \\ * & * & * & (1-\alpha)\left(S_i - S_l - S_l^T\right) & 0 \\ * & * & * & * & -\gamma_m^2 I \end{bmatrix} < 0 \quad (41)$$

$$\forall i, j, l \in \Omega_m$$

$$\begin{bmatrix} -S_j & 0 & 0 & \overline{\varphi}_{14} & \widehat{B}_i \\ * & -S_j & 0 & \overline{\varphi}_{24} & 0 \\ * & * & -I & \overline{\varphi}_{34} & \widehat{D} \\ * & * & * & (1+\beta)\left(S_i - S_l - S_l^T\right) & 0 \\ * & * & * & * & -\gamma_m^2 I \end{bmatrix} < 0 \quad (42)$$

$$\forall i, j \in \Omega_m,\ l \in \Omega_n,\ \Omega_m \cap \Omega_n \ne \emptyset,\ m \ne n,$$

where

$$\overline{\varphi}_{14} = \begin{bmatrix} A_i S_{1l} + \rho B_i \overline{K}_{1,l} C & (1-\rho) B_i \overline{K}_{1,l} & B_i \overline{K}_{2,l} \\ \rho C S_{1l} & (1-\rho) S_{2l} & 0 \\ -\rho C S_{1l} & (\rho - 1) S_{2l} & S_{3l} \end{bmatrix},$$

$$\overline{\varphi}_{24} = \vartheta \begin{bmatrix} B_i \overline{K}_{1,l} C & -B_i \overline{K}_{1,l} & 0 \\ C S_{1l} & -S_{2l} & 0 \\ -C S_{1l} & S_{2l} & 0 \end{bmatrix}, \quad (43)$$

$$\overline{\varphi}_{34} = \left[-C S_{1l} \ \ 0 \ \ 0\right].$$

Moreover, the parameters of the controllers can be determined by

$$K_i = \overline{K}_{1,i} S_{2i}^{-1},$$
$$K_{2,i} = \overline{K}_{2,i} S_{3i}^{-1}, \quad (44)$$
$$i \in \Omega_m,\ \forall m \in \Gamma,$$

where $S_i = \mathrm{diag}\{S_{1i}, S_{2i}, S_{3i}\} > 0$, $S_{1i} = E_{1i}^T S_{11i} E_{1i}$, $S_{2i} = U_i \Sigma_i S_{11i} \Sigma_i^{-1} U^T$, *and* $U, E = \left[E_1^T \ \ E_2^T\right]^T$ *are the orthogonal matrices in Lemma 8 which satisfy the singular value decomposition* $C = U\left[\Sigma \ \ 0\right] E$.

Proof. Let the matrix P_i in Theorem 9 have the form $P_i \triangleq \mathrm{diag}\{P_{1i}, P_{2i}, P_{3i}\}$ and let $S_i = P_i^{-1} = \mathrm{diag}\{S_{1i}, S_{2i}, S_{3i}\}$. By performing congruence transformation to (24) via $\mathrm{diag}\{S_j, S_j, I, S_l, I\}$, one has

$$\begin{bmatrix} -S_j & 0 & 0 & \widehat{A}_{i,l} S_l & \widehat{B}_i \\ * & -S_j & 0 & \vartheta \widehat{A}_{1,i,l} S_l & 0 \\ * & * & -I & \widehat{C} S_l & \widehat{D} \\ * & * & * & -(1-\alpha) S_l^T P_i S_l & 0 \\ * & * & * & * & -\gamma_m^2 I \end{bmatrix} < 0. \quad (45)$$

From $(S_i - S_l)^T S_i^{-1} (S_i - S_l) \ge 0$, one has $S_i - S_l - S_l^T \ge -S_l^T S_i^{-1} S_l$; then (45) can be formulated into

$$\begin{bmatrix} -S_j & 0 & 0 & \widehat{A}_{i,l} S_l & \widehat{B}_i \\ * & -S_j & 0 & \vartheta \widehat{A}_{1,i,l} S_l & 0 \\ * & * & -I & \widehat{C} S_l & \widehat{D} \\ * & * & * & (1-\alpha)\left(S_i - S_l - S_l^T\right) & 0 \\ * & * & * & * & -\gamma_m^2 I \end{bmatrix} < 0, \quad (46)$$

where

$$\widehat{A}_{i,l} S_l = \begin{bmatrix} A_i S_{1l} + \rho B_i K_{1,l} C S_{1l} & (1-\rho) B_i K_{1,l} S_{2l} & B_i K_{2,l} S_{3l} \\ \rho C S_{1l} & (1-\rho) S_{2l} & 0 \\ -\rho C S_{1l} & (\rho - 1) S_{2l} & S_{3l} \end{bmatrix}, \quad (47)$$

$$\widehat{A}_{1,i,l} S_l = \begin{bmatrix} B_i K_{1,l} C S_{1l} & -B_i B_i K_{1,l} S_{2l} & 0 \\ C S_{1l} & -S_{2l} & 0 \\ -C S_{1l} & S_{2l} & 0 \end{bmatrix}.$$

From Lemma 8, for any roll full rank matrix $C \in R^{c\times n}$, the singular value decomposition of C is $C = U[\Sigma \;\; 0]E$, where $E = \begin{bmatrix} E_1^T & E_2^T \end{bmatrix}^T$. Suppose that S_{1i} and S_{2i} can be written as

$$S_{1i} = E_1^T S_{11i} E_1, \qquad S_{2i} = U\Sigma S_{11i}\Sigma^{-1}U^T. \tag{48}$$

Then one has

$$CS_{1i} = S_{2i}C. \tag{49}$$

Further defining the matrix variables $\overline{K}_{1,l} \triangleq K_{1,l}S_{2l}$ and $\overline{K}_{2,l} \triangleq K_{2,l}S_{3l}$, $l \in \Omega_m$, $\forall m \in \Gamma$, one can readily obtain that (41) holds by substituting the system matrices, (48) and (49), into (46). In a similar way, (42) holds according to (25). If the solutions of (41) and (42) exist, then the parameters of admissible controllers are obtained by (44) according to the definitions of $\overline{K}_{1,i}$ and $\overline{K}_{2,i}$. The proof is completed. □

To minimize l_2 performance γ_i, set $\eta_i = \gamma_i^2$ and solve the following optimization problem:

$$\begin{aligned} &\min \quad \{\eta_i\} \\ &\text{s.t.} \quad \text{LMIs (41) and (42)}. \end{aligned} \tag{50}$$

5. Numerical Example

In this section, the effectiveness of the proposed method will be demonstrated. The longitudinal short-period dynamics of the HiMAT flight vehicle in [34] is used as the simulation model. The system states are angle of attack (AOA) α and pitch rate q. The control inputs are δ_e (elevator deflection), δ_v (elevon deflection), and δ_c (canard deflection). To make the statement clear, the simulation process is divided into three steps.

Step 1 (asynchronous switched polytopic model description). In this paper, 20 linear equations are obtained by using linearization techniques on the equilibrium points within the flight envelope as depicted in Figure 2. The trim conditions for every operating point are illustrated in Table 1 in [21].

According to the partition method for polytopic regions in (4), the flight envelope is divided into 3 PSs. Consider

$$\begin{aligned} \Gamma &= \{1,2,3\}, \\ \Omega_1 &= \{1,2,3,4,5,6,7\}, \\ \Omega_2 &= \{6,7,8,9,10,11,12,13\}, \\ \Omega_3 &= \{12,13,14,15,16,17,18,19,20\}. \end{aligned} \tag{51}$$

From the partition result, one knows that Ω_1 and Ω_2 have the common equilibrium points 6 and 7, while Ω_2 and Ω_3 have the common equilibrium points 12 and 13.

The sampling period of the network is chosen as $T = 0.02$ s (see [1]). The packet dropout rate is chosen as 0.05, and that means $E\{\theta(k)\} = \rho = 0.05$. The varying case of $\theta(k)$ is shown in Figure 3.

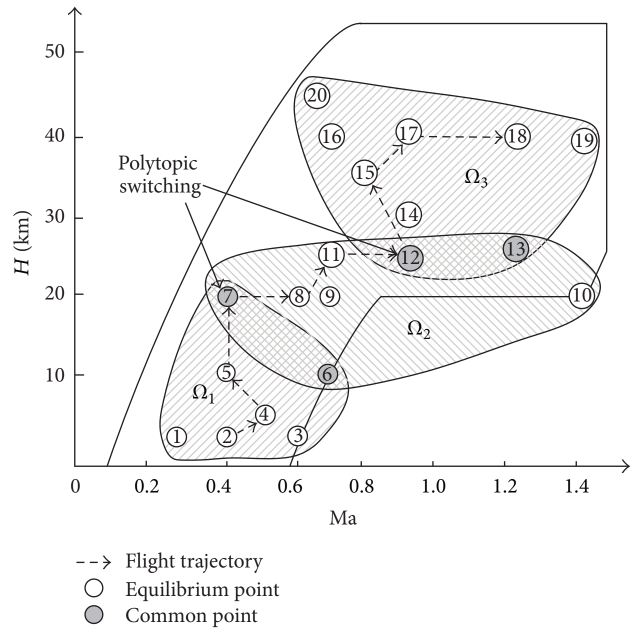

Figure 2: Flight envelope of the HiMAT vehicle.

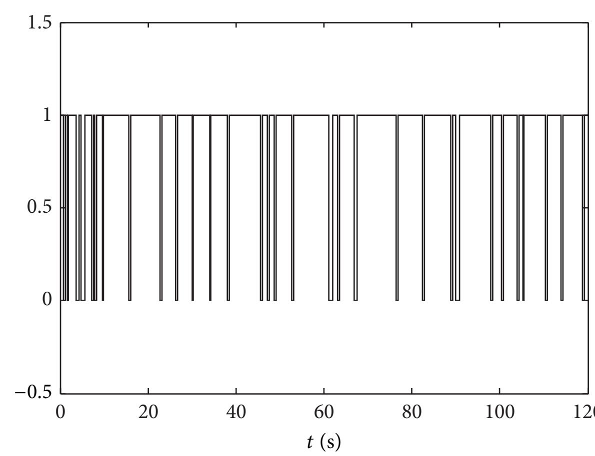

Figure 3: Network transmission characteristic $\theta(k)$.

The system matrices A_i and B_i in (1) can be obtained by the discretization of the equilibrium equations. The parameters of the system dynamics on every sampling moment are interpolated by (3). The disturbance $d(k)$ is supposed to be a disturbance of harmonics wind gust which is generated by an exogenous system described by

$$\begin{aligned} \mu(k+1) &= \begin{bmatrix} 0.9922 & 0.1247 \\ -0.1247 & 0.9922 \end{bmatrix}\mu(k) \\ d(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix}\mu(k), \end{aligned} \tag{52}$$

where the initial value of $\mu(k)$ is set to be $\begin{bmatrix} 0.01 & 0 \end{bmatrix}^T$. It has been shown that this model can be used to describe many kinds of disturbances in engineering (see [35]).

The fault $f(k)$ is set to be a bias fault of the elevator ξ_e in the simulation, while the elevon ξ_v and the canard ξ_c are set to be fault-free. Thus, the matrix $B_{f,i}$ should be equal to the first column of the system matrices B_i. Concretely, the elevator bias fault can be described by

$$f = \begin{cases} 0, & t < 40\text{ s} \\ -0.1\text{ rad}, & t \geq 40\text{ s}. \end{cases} \quad (53)$$

The remaining system matrices are given as $B_{d,i} = [0.01 \;\; 0.01]^T$, $i \in \Omega_m$, $\forall m \in \Gamma$, $C = [1 \;\; 0]$.

Step 2 (controller design). Given that $\alpha = 0.01$, $\beta = 0.01$, and the maximal updating delay of asynchronous switching caused by the network transmission is assumed to be $\delta_{\max} = 5$. Then, according to (26), it can be obtained that the allowed ADT $\tau_a^* = 10$ (0.2 s). The flight trajectory is chosen as 2-4-5-7-8-11-12-15-17-18 as depicted in Figure 2. The system dynamics of the HiMAT flight vehicle in the simulation can be described by a switching process between the chosen equilibrium points. The simulation time is set to be 120 s; then, according to Definition 5, it can be calculated that $\tau_a = 40\text{ s} > \tau_a^*$.

By setting γ_m as the optimization variable simultaneously, the YALMIP toolbox in MATLAB (see [36]) is adopted to solve the optimization problem of (50). The obtained minimal H_∞ inhibition performance is $\gamma^* = 1.9237$, and the parameters of the controllers on the equilibrium equations can be obtained as follows:

$$K_2 = \begin{bmatrix} 40.3090 & -19.5439 \\ 44.9207 & -21.9663 \\ 105.9873 & -52.5667 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 41.5828 & -20.5246 \\ 28.5197 & -14.0826 \\ 101.2166 & -50.9280 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 52.3953 & -24.9725 \\ 56.8270 & -27.2393 \\ 137.6653 & -66.6733 \end{bmatrix},$$

$$K_7 = \begin{bmatrix} 76.6366 & -35.9072 \\ 78.2575 & -36.7847 \\ 198.7008 & -94.0566 \end{bmatrix},$$

$$K_8 = \begin{bmatrix} 67.3576 & -32.4876 \\ 22.6869 & -10.8490 \\ 148.3147 & -72.1669 \end{bmatrix},$$

$$K_{11} = \begin{bmatrix} 62.4412 & -30.0254 \\ 36.3125 & -17.4411 \\ 162.6648 & -78.7089 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} 46.4942 & -23.0647 \\ 37.6284 & -18.6757 \\ 148.6060 & -73.9876 \end{bmatrix},$$

$$K_{15} = \begin{bmatrix} 68.7054 & -32.5613 \\ 65.4663 & -31.0557 \\ 218.1911 & -103.8117 \end{bmatrix},$$

$$K_{17} = \begin{bmatrix} 81.5338 & -38.6687 \\ 71.2526 & -33.7991 \\ 275.2488 & -130.6901 \end{bmatrix},$$

$$K_{18} = \begin{bmatrix} 135.3210 & -65.0831 \\ 99.7142 & -47.8251 \\ 339.1521 & -161.1666 \end{bmatrix}. \quad (54)$$

The controller gains for the equilibrium points are given above. According to (11), the parameters of controllers within each PS can be obtained by linear interpolation. In this paper, the gain-scheduled subcontrollers are interpolated in triangular regions. Three equilibrium points that have the smallest geometrical distance to the current one are chosen. For example, within the flight region $H \in [2.5, 10)$ and $Ma \in [0.5, 0.7)$, the gain-scheduled subcontrollers are interpolated by three gains K_3, K_4, and K_6. The weighted coefficient a_k satisfies

$$\begin{gathered} a_{3,k} + a_{4,k} + a_{6,k} = 1 \\ a_{3,k}h_3 + a_{4,k}h_4 + a_{6,k}h_6 = h_a \\ a_{3,k}Ma_3 + a_{4,k}Ma_4 + a_{6,k}Ma_6 = Ma, \end{gathered} \quad (55)$$

where h_i and Ma_i, $i \in \Omega$ are altitude and Mach number for the ith equilibrium point.

Step 3 (controller verification). To illustrate the effectiveness of the proposed method, the command signal $r(k)$ is chosen to be the angle of attack.

From the partition result in Step 2, there exist two polytopic switchings. One occurs on the common equilibrium point 7, and the other occurs on the common equilibrium point 12. The simulation results are depicted in Figures 4~7.

The response of the angle of attack is shown in Figure 4, and it can be concluded that the tracking performance of the angle of attack can be satisfied within the flight envelope, even when a constant bias fault occurs during the time interval [40 s, 120 s]. As depicted in the partial enlarged detail, when asynchronous switching occurs on 75 s (equilibrium point 12), the angle of attack jumps about 0.02° and converges to nearly zero within about 4 s. That shows the effectiveness of the smooth switching for weighted coefficient a_k.

The control surface deflections are shown in Figures 5~7 which are practical and acceptable, and the simulation results of fault case and no fault case have been given together. Obviously, comparing with the no fault case, the control inputs

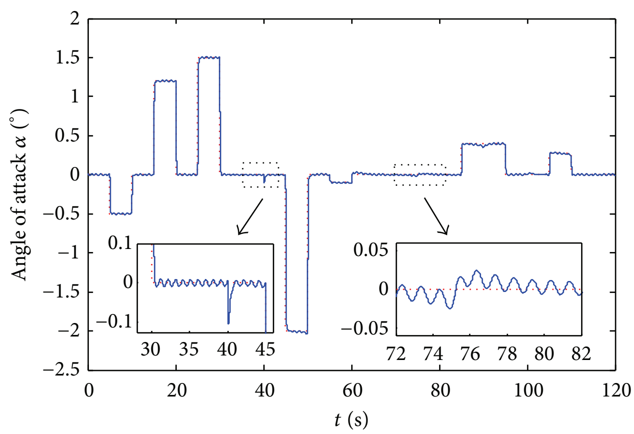

FIGURE 4: Angel of attack response.

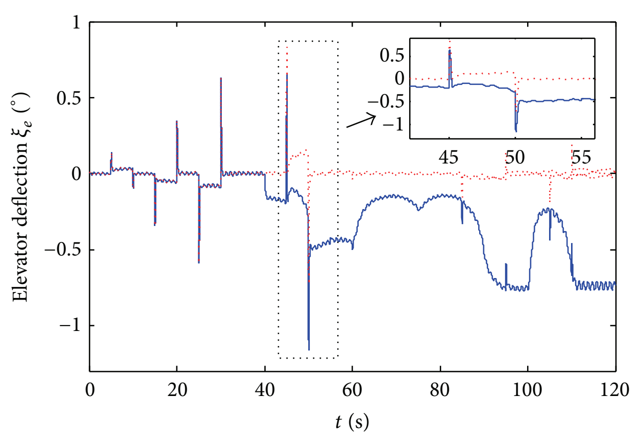

FIGURE 5: Elevator deflection.

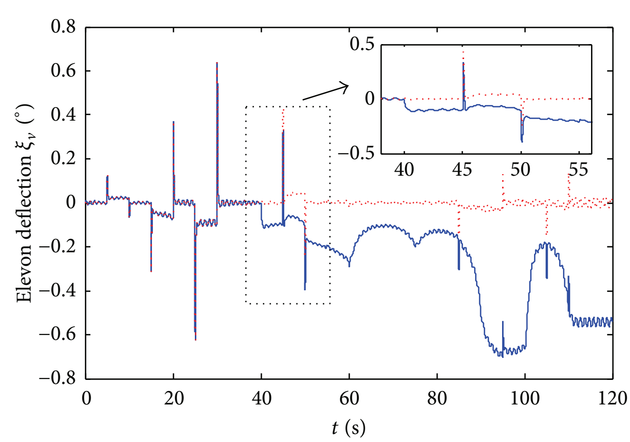

FIGURE 6: Elevon deflection.

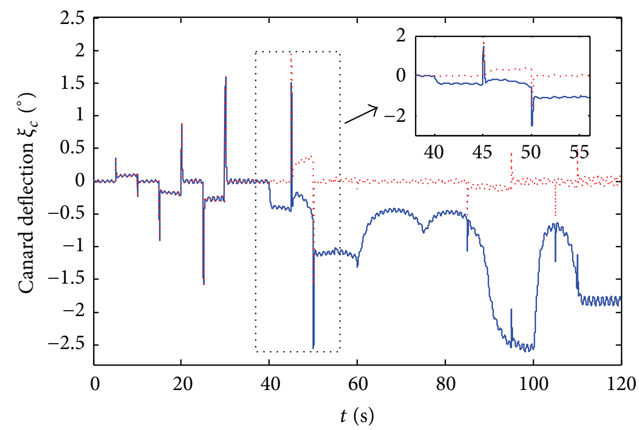

FIGURE 7: Canard deflection.

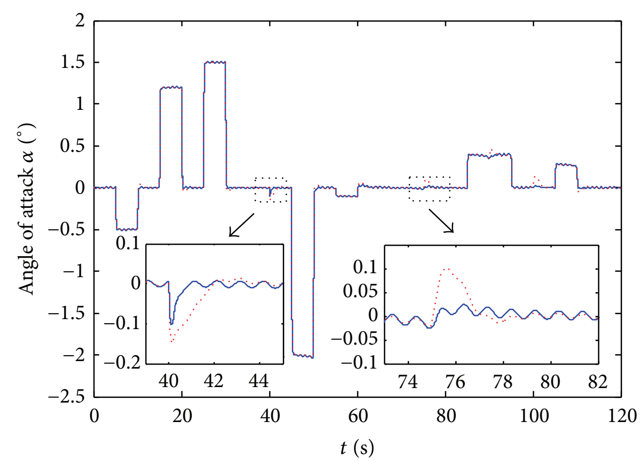

FIGURE 8: Contrastive simulation.

become larger when the fault occurs, and the influences of the fault on the system have been reduced.

To verify the advantage of the proposed controller for switched systems under asynchronous switching, contrastive simulation is made with a gain-scheduled switched controller in [37]. The simulation result of AOA response is shown in Figure 8. It can be seen that both of the two controllers achieve good tracking performance, and the tracking of the proposed controller is closer to the AOA command compared with the gain-scheduled controller. Moreover, the proposed controller shows better tracking performance when asynchronous switching occurs. It validates that the proposed method is more suitable for switched polytopic system with asynchronous phenomenon.

6. Conclusions

This paper proposed a robust fault-tolerant tracking control approach for nonlinear NCS under asynchronous switching.

Considering the imperfect transmission problems, a locally overlapped polytopic switched system with Bernoulli process is established to present the nonlinear NCS dynamics. The parameters of the tracking controllers are obtained by linear interpolation within polytopic subsystems and the locally overlapped regions. To overcome the asynchronous switching phenomenon caused by the updating delays of the switching signals and weighted coefficients, an asynchronous switching synthesis approach is used to obtain a prescribed tracking performance. A simulation of the HiMAT flight vehicle is given to show the effectiveness of the proposed approach. Compared with a gain-scheduled switched controller, the proposed controller has a better tracking performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (nos. 61374012 and 61273083).

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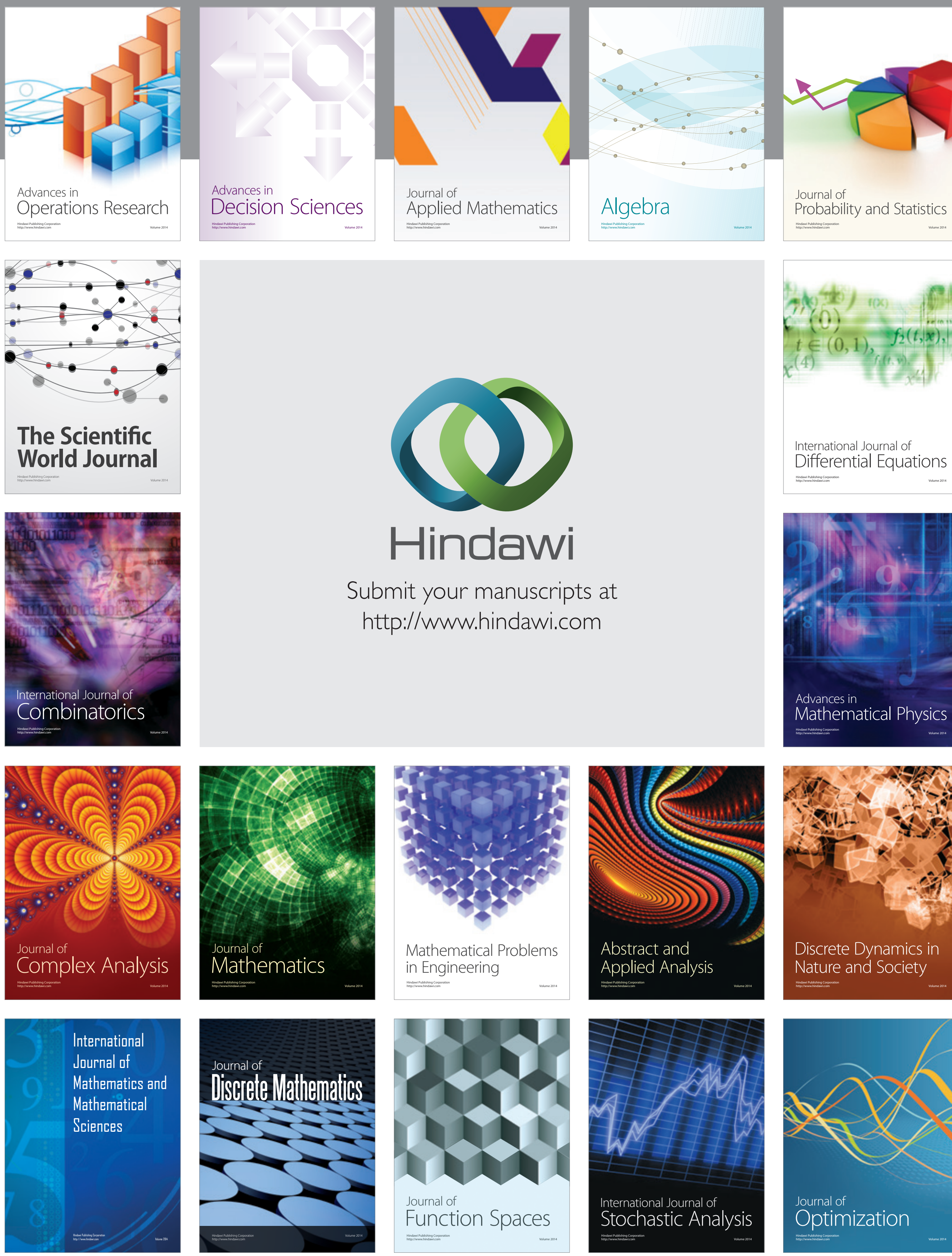
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