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*Research Article*

# Impulsive Synchronization of Multilinks Delayed Coupled Complex Networks with Perturb Effects

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This paper investigates impulsive synchronization of multilinks delayed coupled complex networks with perturb effects. Based on the comparison theory of impulsive differential system, a novel synchronization criterion is derived and an impulsive controller is designed simultaneously. Finally, numerical simulations demonstrate the effectiveness of the proposed synchronization criteria.

## 1. Introduction

In the past few decades, the problem of control and synchronization of complex dynamical networks has been extensively investigated in various fields of science and engineering due to its many potential practical applications [1–8]. One important consideration in practical networks is the existence of time delays because obstructions to the transmission of signals are inevitable in a biological neural network, in an epidemiological model, in a communications network, or in an electrical power grid. Since recently, there are many studies on dynamical networks with time delays [9–16]. Moreover, the multidelayed coupling consists of providing more information about the dynamics in nodes to the other nodes in the network, such as the transportation network; we all know that the transmission speed is different among highway network railway network, and airline network. So we can use timedelay to describe these networks [17]. In [17], the authors studied synchronization of a class of timedelayed complex dynamical networks with multilinks, and this model is suitable to investigate and simulate more realistic complex networks, so we should pay attention to this network with multilinks. Moreover, in [9–11, 17–20], the authors studied synchronization of complex network with time-varying coupling delay by designing a controller and adaptive updated laws. However, the controller designed for such an adaptive synchronization is usually quite complex, it will be useful to find a simple structure to solve this problem, the impulsive controller seems to have a simple structure, and impulsive control is an artificial control strategy which is cheaper to operate compared with other control strategy. Motivated by the above discussions, we investigate impulsive synchronization for such a complex networks model in this paper, and the novel synchronization criterion is derived.

The rest of this work is organized as follows. Section 2 gives the problem formulation. Section 3 gives synchronization scheme. Section 4 gives illustrative example. Section 5 gives the conclusion of the paper.

## 2. Problem Formulation

In [17], the authors achieve synchronization between two complex networks with multilinks by designing effective controller. For simplicity, the complex network model is written in the following form:

$$\begin{aligned}\dot{x}^i(t) &= f\left(x^i(t)\right) + \sum_{l=0}^{m-1}\sum_{j=1}^{N} a_{ij}^l x^j(t-\tau_l)\\ &= f\left(x^i(t)\right) + \sum_{j=1}^{N} a_{ij}^0 x^j(t) + \sum_{j=1}^{N} a_{ij}^1 x^j(t-\tau_1) + \cdots + \sum_{j=1}^{N} a_{ij}^{m-1} x^j(t-\tau_{m-1}),\end{aligned} \tag{2.1}$$

where $x^i = (x_1, x_2, \ldots, x_n)^T \in R^n$, $f : R^n \to R^n$ standing for the activity of an individual subsystem is a vector value function. $A_l = (a_{ij}^l)_{N\times N} \in R^{N\times N}(l = 0, 1, \ldots, m-1)$ is the $l$th subnetwork's topological structure. The definition of $a_{ij}^l$ is that in the $l$th sub-network, if there exists a link from node $i$ to $j$ $(i \neq j)$, then $a_{ij}^l \neq 0$. Otherwise, $a_{ij}^l = 0 \cdot \tau_l$ $(l = 0, 1, \ldots, m-1)$ is time-delay of the $l$th subnetwork compared to the zero subnetwork $(\tau_0 = 0)$ which is without time delayed.

*Remark 2.1.* In [17], $a_{ii}^l = -\sum_{j=1, i\neq j}^{N} a_{ij}^l$ is defined, we are not concerned whether the coupling matrix $A_l$ satisfies $a_{ii}^l = -\sum_{j=1, i\neq j}^{N} a_{ij}^l$ in this paper.

In the paper, we have the following mathematical preliminaries.

*Assumption 2.2.* We assume that $f(x^i(t))$ is Lipschitz continuous on $x^i(t)$, that is, there exists a positive constant $\eta > 0$ such that

$$\left| f\left(y^i(t)\right) - f\left(x^i(t)\right)\right| \leq \eta e^i(t), \quad \forall x^i(t), y^i(t) \in R^n. \tag{2.2}$$

*Assumption 2.3.* We also assume that $\sigma(x^i(t))$ is Lipschitz continuous on $x^i(t)$, and one can consider

$$\sigma\left(y^i(t),t\right)-\sigma\left(x^i(t),t\right)=M\left(x^i(t),y^i(t)\right)e^i(t), \tag{2.3}$$

where $\|M(x^i(t),y^i(t))\|\leq H,\ H>0$.

## 3. Synchronization Scheme

In this section, we will investigate impulsive synchronization of the complex networks with perturb functions. The multidelayed coupled complex network with perturb functions can be described by

$$\begin{aligned}\dot{x}^i&=f\left(x^i(t)\right)+\sum_{j=1}^{N}a_{ij}^0x^j(t)+\sum_{j=1}^{N}a_{ij}^1x^j(t-\tau_1)+\cdots\\&+\sum_{j=1}^{N}a_{ij}^{m-1}x^j(t-\tau_{m-1})+\sigma\left(x^i(t),t\right).\end{aligned} \tag{3.1}$$

We take the network given by (3.1) as the driving network and a response network with impulsive control scheme which is given by

$$\begin{aligned}\dot{y}^i&=f\left(y^i(t)\right)+\sum_{j=1}^{N}a_{ij}^0y^j(t)+\sum_{j=1}^{N}a_{ij}^1y^j(t-\tau_1)+\cdots\\&+\sum_{j=1}^{N}a_{ij}^{m-1}y^j(t-\tau_{m-1})+\sigma\left(y^i(t),t\right),\quad t\neq t_k,\end{aligned} \tag{3.2}$$

$$\Delta y^i=y^i(t_k^+)-y^i(t_k^-)=B^{ik}\left(y^i-x^i\right),\quad t=t_k, \tag{3.3}$$

where $y=(y_1,y_2,\ldots,y_n)^T\in R^n$.

Let $e^i(t)=y^i(t)-x^i(t)$, then we have the following error system:

$$\begin{aligned}\dot{e}^i&=f\left(y^i(t)\right)-f\left(x^i(t)\right)+\sum_{j=1}^{N}a_{ij}^0\left(y^j(t)-x^j(t)\right)+\sum_{j=1}^{N}a_{ij}^1\left(y^j(t-\tau_1)-x^j(t-\tau_1)\right)+\cdots\\&+\sum_{j=1}^{N}a_{ij}^{m-1}\left(y^j(t-\tau_{m-1})-x^j(t-\tau_{m-1})\right)+M\left(x^i(t),y^i(t)\right)e^i,\quad t\neq t_k,\end{aligned} \tag{3.4}$$

$$\Delta e^i=e^i(t_k^+)-e^i(t_k^-)=B^{ik}e^i,\quad t=t_k, \tag{3.5}$$

where $e(t_k^+)=\lim_{t\to t_k^+}e(t)$, $e(t_k)=\lim_{t\to t_k^-}e(t)=e(t_k^-)$.

**Theorem 3.1.** *Let Assumptions 2.2–2.3 hold,* $\alpha_{r-1} = \max(a_{ij}^{r-1})^2$, $r = 1, 2, \ldots, m$. *If there exists a constant* $\theta \geq 1$ *such that*

$$\ln \theta \rho_k + 2\left(\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t_{k+1} - t_k)\right) \leq 0, \tag{3.6}$$

*then the driving network* (3.1) *and the response network* (3.2) *can realize impulsive synchronization.*

*Proof.* We choose a nonnegative function as

$$\begin{aligned} V(t) = {} & \frac{1}{2} \sum_{i=1}^{N} \left(e^i(t)\right)^T e^i(t) + \sum_{i=1}^{N} \int_{t-\tau_1}^{t} (e_i(s))^T e_i(s) ds + \cdots \\ & + \sum_{i=1}^{N} \int_{t-\tau_{m-1}}^{t} (e_i(s))^T e_i(s) ds. \end{aligned} \tag{3.7}$$

Then the differentiation of $V$ along the trajectories of (3.4) is

$$\begin{aligned} \dot{V}(t) = {} & \sum_{i=1}^{N} \left(e^i(t)\right)^T \Bigg[ \left( f\left(y^i(t)\right) - f\left(x^i(t)\right)\right) + \sum_{j=1}^{N} a_{ij}^0 \left(y^j(t) - x^j(t)\right) \\ & + \sum_{j=1}^{N} a_{ij}^1 \left(y^j(t-\tau_1) - x^j(t-\tau_1)\right) + \cdots + \sum_{j=1}^{N} a_{ij}^{m-1} \left(y^j(t-\tau_{m-1}) - x^j(t-\tau_{m-1})\right) \Bigg] \\ & + \sum_{i=1}^{N} \left[ \left(e^i(t)\right)^T e^i(t) - \left(e^i(t-\tau_1)\right)^T e^i(t-\tau_1) \right] \\ & + \cdots + \sum_{i=1}^{N} \left[ \left(e^i(t)\right)^T e^i(t) - \left(e^i(t-\tau_{m-1})\right)^T e^i(t-\tau_{m-1}) \right] \\ & + \sum_{i=1}^{N} \left(e^i(t)\right)^T M\left(x^i(t), y^i(t)\right) e^i(t) \\ \leq {} & \sum_{i=1}^{N} \eta \left(e^i(t)\right)^T e^i(t) + \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}^0 \left(e^i(t)\right)^T e^j(t) + \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}^1 \left(e^i(t)\right)^T e^j(t-\tau_1) + \cdots \\ & + \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}^{m-1} \left(e^i(t)\right)^T e^j(t-\tau_{m-1}) \\ & + \sum_{i=1}^{N} \left[ \left(e^i(t)\right)^T e^i(t) - \left(e^i(t-\tau_1)\right)^T e^i(t-\tau_1) \right] + \cdots \\ & + \sum_{i=1}^{N} \left[ \left(e^i(t)\right)^T e^i(t) - \left(e^i(t-\tau_{m-1})\right)^T e^i(t-\tau_{m-1}) \right] + \sum_{i=1}^{N} H \left(e^i(t)\right)^T e^i(t) \end{aligned}$$

$$
\begin{aligned}
&\le \sum_{i=1}^{N}\eta\left(e^i(t)\right)^T e^i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^0\right)^2\left(e^i(t)\right)^T e^i(t)+\sum_{j=1}^{N}\left(e^j(t)\right)^T e^j(t)\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^1\right)^2\left(e^i(t)\right)^T e^i(t)\\
&\quad+\sum_{j=1}^{N}\left(e^j(t-\tau_1)\right)^T e^j(t-\tau_1)\ +\cdots+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^{m-1}\right)^2\left(e^i(t)\right)^T e^i(t)\\
&\quad+\sum_{j=1}^{N}\left(e^j(t-\tau_{m-1})\right)^T e^j(t-\tau_{m-1})+\sum_{i=1}^{N}\left[\left(e^i(t)\right)^T e^i(t)\ -\left(e^i(t-\tau_1)\right)^T e^i(t-\tau_1)\right]+\cdots\\
&\quad+\sum_{i=1}^{N}\left[\left(e^i(t)\right)^T e^i(t)-\left(e^i(t-\tau_{m-1})\right)^T e^i(t-\tau_{m-1})\right]+\sum_{i=1}^{N}H\left(e^i(t)\right)^T e^i(t)\\
&=\sum_{i=1}^{N}\eta\left(e^i(t)\right)^T e^i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^0\right)\left(e^i(t)\right)^T e^i(t)+\sum_{j=1}^{N}\left(e^j(t)\right)^T e^j(t)\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^1\right)^2\left(e^i(t)\right)^T e^i(t)+\cdots\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^{m-1}\right)^2\left(e^i(t)\right)^T e^i(t)+\sum_{i=1}^{N}\left(e^i(t)\right)^T e^i(t)+\cdots\\
&\quad+\sum_{i=1}^{N}\left(e^i(t)\right)^T e^i(t)+\sum_{i=1}^{N}H\left(e^i(t)\right)^T e^i(t)\\
&\le \sum_{i=1}^{N}\eta\left(e^i(t)\right)^T e^i(t)\ +\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^0\right)^2\left(e^i(t)\right)^T e^i(t)+\sum_{j=1}^{N}\left(e^j(t)\right)^T e^j(t)\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^1\right)^2\left(e^i(t)\right)^T e^i(t)+\cdots\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\left(a_{ij}^{m-1}\right)^2\left(e^i(t)\right)^T e^i(t)+\sum_{i=1}^{N}\left(e^i(t)\right)^T e^i(t)+\cdots+\sum_{i=1}^{N}\left(e^i(t)\right)^T e^i(t)\\
&\quad+\sum_{i=1}^{N}H\left(e^i(t)\right)^T e^i(t)=\left(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H\right)\sum_{i=1}^{N}\left(e^i(t)\right)^T e^i(t)\\
&\le 2\left(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H\right)\left\{\frac{1}{2}\sum_{i=1}^{N}\left(e^i(t)\right)^T e^i(t)+\sum_{i=1}^{N}\int_{t-\tau_1}^{t}\left(e_i(s)\right)^T e_i(s)ds\right.\\
&\qquad\qquad\qquad\qquad\qquad\qquad\left.+\cdots+\sum_{i=1}^{N}\int_{t-\tau_{m-1}}^{t}\left(e_i(s)\right)^T e_i(s)ds\right\}\\
&=2\left(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H\right)V(t).
\end{aligned}
\tag{3.8}
$$

This implies that

$$V(e(t)) \le V\left(e\left(t_{k-1}^{+}\right)\right)\exp\left(2\left(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H\right)(t-t_{k-1})\right), \quad t\in(t_{k-1},t_k],\ k=1,2,\dots \tag{3.9}$$

On the other hand, when $t=t_k$, we have

$$\begin{aligned}
V(e(t^{+})) &= \frac{1}{2}\sum_{i=1}^{N}\left(e^{i}(t_k)\right)^{T}\left[I+B^{ik}\right]^{T}\left[I+B^{ik}\right]e^{i}(t_k)+\sum_{i=1}^{N}\int_{t-\tau_j}^{t}(e_i(s))^{T}\left[I+B^{ik}\right]^{T}\left[I+B^{ik}\right]e_i(s)ds\\
&\quad+\sum_{i=1}^{N}\int_{t-d_j}^{t}(e_i(s))^{T}\left[I+B^{ik}\right]^{T}\left[I+B^{ik}\right]e_i(s)ds\\
&\le \lambda_{\max}\left[\left(I+B^{ik}\right)^{T}\left(I+B^{ik}\right)\right]\left\{\frac{1}{2}\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e^{i}(t)+\sum_{i=1}^{N}\int_{t-\tau_1}^{t}(e_i(s))^{T}e_i(s)ds\right.\\
&\qquad\left.+\cdots+\sum_{i=1}^{N}\int_{t-\tau_{m-1}}^{t}(e_i(s))^{T}e_i(s)ds\right\}\\
&\le \lambda_{\max}\left[\left(I+B^{ik}\right)^{T}(I+B)^{ik}\right]\left\{\frac{1}{2}\sum_{i=1}^{N}\left(e^{i}(t)\right)^{T}e(t)+\sum_{i=1}^{N}\int_{t-\tau_1}^{t}(e_i(s))^{T}e_i(s)ds\right.\\
&\qquad\left.+\cdots+\sum_{i=1}^{N}\int_{t-\tau_{m-1}}^{t}(e_i(s))^{T}e_i(s)ds\right\}\\
&= \rho_k V(e(t_k)),
\end{aligned} \tag{3.10}$$

where $\rho_k=\lambda_{\max}[(I+B^{ik})^{T}(I+B^{ik})]$.

When $t\in(t_0,t_1]$, $V(e(t))\le V(e(t_0^{+}))\exp(2(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H)(t-t_0))$, then

$$V(e(t_1)) \le V\left(e\left(t_0^{+}\right)\right)\exp\left(2\left(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H\right)(t_1-t_0)\right). \tag{3.11}$$

So,

$$V\left(e\left(t_1^{+}\right)\right) \le \rho_1 V(e(t_1)) \le \rho_1 V\left(e\left(t_0^{+}\right)\right)\exp\left(2\left(\eta+\sum_{r=1}^{m}\alpha_{r-1}+m+H\right)(t_1-t_0)\right). \tag{3.12}$$

In the same way, for $t \in (t_1, t_2]$, we have

$$\begin{aligned} V(e(t)) &\leq V(e(t_1^+)) \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t - t_1)\right) \\ &\leq \rho_1 V(e(t_0^+)) \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t - t_0)\right). \end{aligned} \tag{3.13}$$

In general for any $t \in (t_k, t_{k+1}]$, one finds that

$$V(e(t)) \leq V(e(t_0^+))\rho_1\rho_2 \cdots \rho_k \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t - t_0)\right) \tag{3.14}$$

Thus for all $t \in (t_k, t_{k+1}]$, $k = 1, 2, \ldots$, we have

$$\begin{aligned} V(e(t)) &\leq V(e(t_0^+))\rho_1\rho_2 \cdots \rho_k \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t - t_0)\right) \\ &\leq V(e(t_0^+))\rho_1\rho_2 \cdots \rho_k \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t_{k+1} - t_0)\right) \\ &= V(e(t_0^+))\rho_1 \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t_2 - t_1)\right)\rho_2 \\ &\quad \times \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t_3 - t_2)\right) \cdots \rho_k \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha^{r-1} + m + H\right)(t_{k+1} - t_k)\right) \\ &\quad \times \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha^{r-1} + m + H\right)(t - t_0)\right). \end{aligned} \tag{3.15}$$

From the assumptions given in the theorem

$$\rho_k \exp\left(2\left(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H\right)(t_{k+1} - t_k)\right) \leq \frac{1}{\theta}, \quad k = 1, 2, \ldots \tag{3.16}$$

we have $V(e(t)) \leq V(e(t_0^+))(1/\theta^k) \exp(2(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H)(t - t_0))$. That is $V(e(t)) \leq V(e(t_0^+))(1/\theta^k) \exp(2(\eta + \sum_{r=1}^{m} \alpha_{r-1} + m + H)(t - t_0)), t \geq t_0$.

When $\theta \geq 1$, from [21], this implies that the origin in system (3.4) is globally asymptotically stable or the driving network is synchronized with the response network asymptotically for any initial conditions. This completes the proof. □

*Remark 3.2.* Systems (3.1)-(3.2) are the time-invariant complex networks. As discussed in [22–24], systems (3.1)-(3.2) are the time-varying complex networks, which is a more complicated research issue.

*Remark 3.3.* Normally, it is difficult to control a complex networks by adding the controllers to all nodes, so it would be much better to use the pinning control method since the most complex networks have large number of nodes [25]. Regarding for the pinning control of the network systems (3.1)-(3.2), are the next research topic for us.

*Remark 3.4.* For the transportation network, we all know that the transmission speed is different among highway network, railway network and airline network. So we can use multilinks delayed to describe these networks [17]. Also impulsive control is an artificial control strategy which is cheaper to operate compared with other control strategy, so impulsive control method of the network systems (3.1)-(3.2) should have potential applications.

## 4. Illustrative Example

It is well known that the Lorenz system families are typical chaotic systems and the Lü chaotic system is a member of the families which is known as [26]

$$\dot{s} = \begin{pmatrix} -a & a & 0 \\ 0 & c & 0 \\ 0 & 0 & -b \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -s_1 s_3 \\ s_1 s_2 \end{pmatrix} \overset{\text{def}}{=} Ps + W(s), \tag{4.1}$$

when $a = 36, c = 20, b = 3$.

It is well known that the Lü attractor is bounded. Here we suppose that all nodes are running in the given bounded region. Our numerical analyses show that there exist constants $M_1 = 25, M_2 = 30, M_3 = 45$.

Satisfying $\|y_p\|, \|z_p\| \leq M_p$ for $1 \leq p \leq 3$. Therefore, one has

$$\begin{aligned} \|W(y) - W(z)\| &\leq \sqrt{\left(-y_3(y_1 - z_1) - z_1(y_3 - z_3)\right)^2 + \left(y_2(y_1 - z_1) + z_1(y_2 - z_2)\right)^2} \\ &\leq \sqrt{2M_1^2 + M_2^2 + M_3^2}\|y - z\| \approx 64.6142\|y - z\|. \end{aligned} \tag{4.2}$$

Obviously, $\|P\| \approx 52.9843$. Thus the Lü system satisfies Assumption 2.2, $\eta = 117.5985$. In the same way, it can be seen that the Chen system, the Lorenz system, the unified chaotic system and the Lorenz system families also satisfy Assumption 2.2. So, in the simulations, we select the Lü chaotic system as an example to show the effectiveness of the proposed method [27, 28].

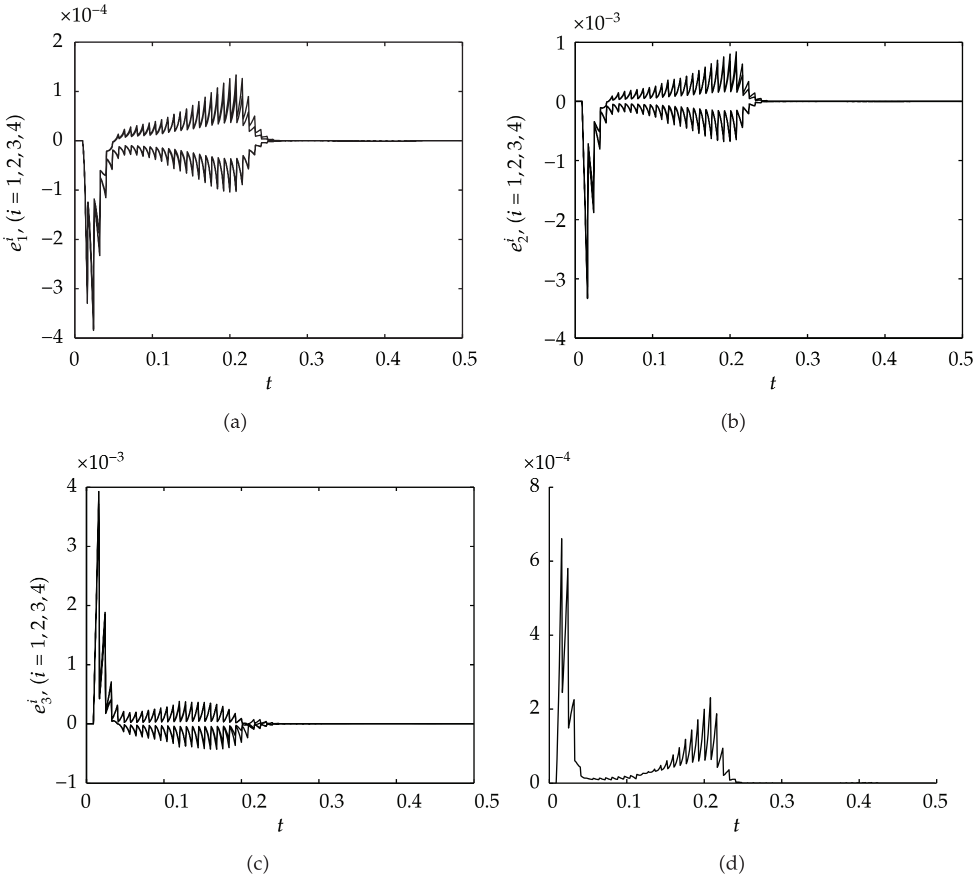


**Figure 1:** Synchronization errors with time $t$.

According to Section 3, we show that the network with 4 nodes described by

$$\begin{aligned}
\dot{x}^i(t) &= f\left(x^i(t)\right) + \sum_{j=1}^{4} a_{ij}^0 x^j(t) + \sum_{j=1}^{4} a_{ij}^1 x^j(t-\tau_1) + \sum_{j=1}^{4} a_{ij}^2 x^j(t-\tau_2) + 0.1x^i(t), \\
\dot{y}^i(t) &= f\left(y^i(t)\right) + \sum_{j=1}^{4} a_{ij}^0 y^j(t) + \sum_{j=1}^{4} a_{ij}^1 y^j(t-\tau_1) + \sum_{j=1}^{4} a_{ij}^2 y^j(t-\tau_2) + 0.1y^i(t), \quad t \neq t_k, \\
\Delta y^i &= y^i(t_k^+) - y^i(t_k^-) = B^{ik}\left(y^i - x^i\right), \quad t = t_k.
\end{aligned} \tag{4.3}$$

In numerical simulation, let

$$A_0 = \begin{pmatrix} 5 & -4 & -2 & 0 \\ 4 & -4 & 3 & -1 \\ 2 & 3 & -4 & 0 \\ 0 & -1 & 3 & -1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -3 & 3 & -1 & 0 \\ 1 & -4 & 5 & -1 \\ 2 & 1 & -2 & 0 \\ 0 & -3 & 0 & 2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 3 & -1 & 0 \\ 2 & -4 & 5 & -1 \\ 1 & 1 & -2 & 0 \\ 0 & -3 & 0 & 2 \end{pmatrix}, \tag{4.4}$$

we choose $\tau_1 = 0.1$, $\tau_2 = 0.2$, and the gain matrixes $B^{ik}(k = 1, 2, \ldots)$ as a constant matrix, $B^{ik} = B = \mathrm{diag}(-0.7, -0.8, -0.9)$, then $\rho_k = 0.09$. Let $\theta = 1.1$, from $\ln\theta\rho_k + 2(\eta + \sum_{r=1}^{m}\alpha_{r-1} + m + H)(t_{k+1} - t_k) \leq 0$, then $t_{k+1} - t_{\mathrm{k}} \leq 0.0085$, we let $t_{k+1} - t_k = 0.008$. All initial values are $x_1^i = 1 + 0.5\mathrm{i}$, $x_2^i = 2 + 0.7i$, $x_3^i = 2 + 0.8i$, $y_1^i = 1 - 0.6i$, $y_2^i = 2 - 0.8i$, $y_3^i = 3 - 0.8i$. Figure 1 shows the variance of the synchronization errors. We introduce the quantity $E(t) = \sqrt{\sum_{i=1}^{N}\|y^i(t) - x^i(t)\|^2/N}$ [29] which is used to measure the quality of the control process. It is obvious that when $E(t)$ no longer increases, two networks achieve synchronization.

## 5. Conclusion

This paper deals with the problem of impulsive synchronization of multilinks delayed coupled complex networks with perturb effects. On the basis of the comparison theory of impulsive differential system, the novel synchronization criteriion is derived and an impulsive controller is designed simultaneously. Finally, numerical simulations are presented to verify the effectiveness of the proposed synchronization criteria.

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