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Research Article

Robust Adaptive Control and L_2 Disturbance Attenuation for Uncertain Hamiltonian Systems with Time Delay

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This paper deals with the robust stabilizability and L_2 disturbance attenuation for a class of time-delay Hamiltonian control systems with uncertainties and external disturbances. Firstly, the robust stability of the given systems is studied, and delay-dependent criteria are established based on the dissipative structural properties of the Hamiltonian systems and the Lyapunov-Krasovskii (L-K) functional approach. Secondly, the problem of L_2 disturbance attenuation is considered for the Hamiltonian systems subject to external disturbances. An adaptive control law is designed corresponding to the time-varying delay pattern involved in the systems. It is shown that the closed-loop systems under the feedback control law can guarantee the γ-dissipative inequalities be satisfied. Finally, two numerical examples are provided to illustrate the theoretical developments.

1. Introduction

Systems with unknown delayed states are often encountered in practice, such as communication systems, engineering systems, and process control systems. For this reason, robust stability analysis for uncertain time-delay control systems has attracted a considerable amount of interests in recent years [1–9]. The Lyapunov-Krasovskii (L-K) method is always employed, and the results are often obtained in the form of linear matrix inequalities (LMIs). However, robust stabilization of nonlinear systems with time delays has been a challenging problem. As is well known, the control design of nonlinear systems is a difficult process. The existence of time delay in nonlinear systems further degrades the control performance and sometimes makes the closed-loop stabilization difficult [10–12]. More recently, Mahmoud and El Ferik obtained some new results on dissipative analysis and state feedback synthesis for a class of nonlinear systems with time-varying delays and convex polytypic uncertainties [12]. This class consists of linear time-delay systems subject to nonlinear cone-bounded perturbations. Hu et al. in [10] integrated the sliding mode control method with the robust H_∞ technique and developed a discrete-time sliding mode controller for a class of time-delay uncertain systems with stochastic nonlinearities. The nonlinearities are described by statistical means.

On the other hand, for affine nonlinear systems with disturbances, the L_2-gain analysis and the L_2 disturbance attenuation are always important issues [13]. Almost all these studies deal with the existence of solutions to some partial differential inequality, which reflects the dissipative behavior of the system under consideration for a certain supply rate which is called passivity-based control design method. This kind of method is used to achieve a γ-dissipative inequality which not only guarantees asymptotic stability but also renders the L_2-gain from disturbance to the penalty signal less than or equal to a given level $\gamma > 0$. The key to solve the problem of L_2 disturbance attenuation is to find a proper storage function that ensures the γ-dissipative inequality holding.

As an important class of nonlinear systems, port-controlled Hamiltonian systems (PCH) proposed by [14, 15] have attracted increasing attentions in the field of nonlinear control theory [16–18]. The Hamilton function in a PCH system is considered as the sum of potential energy (excluding gravitational potential energy) and kinetic energy in physical systems, and it can be used as a good candidate of Lyapunov functions for many physical systems. Due to this and its nice structure with clear physical meaning, the PCH system has drawn a good deal of attention in practical control designs [19–23]. In [21], with a proper penalty signal, the γ-dissipativity was achieved by making a sufficiently large damping injection in the design stage. Wang et al. in [23] proposed an energy-based adaptive L_2 disturbance attenuation control scheme for the power systems with superconducting magnetic energy storage (SMES) units. Besides, Hamiltonian systems with time delay also have been studied [24–27]. Reference [25] addresses the stabilization problem of a class of Hamiltonian systems with state time delay and input saturation. The problem of L_2-disturbance attenuation for time-delay port-controlled Hamiltonian systems is studied in [26]. The case that there are time-invariant uncertainties belonging to some convex bounded polytypic domains is also considered in [26], and an L_2 disturbance attenuation control law is proposed. In practice, dynamic uncertainties often arise from many different control engineering applications. The inevitable uncertainties may enter a nonlinear system in a much more complex way. In addition to polytypic uncertainties, systems may encounter modeling error, parameter perturbations, and external disturbances. However, to the best of our knowledge, the analysis and synthesis for time-delay Hamiltonian systems with parametric perturbations have not been discussed yet. It is well worth pointing out that with the help of Hamiltonian realization [28, 29], the control problem of a large class of time-delay nonlinear systems with uncertainties can be solved via the Hamiltonian system framework. Thus, study of time-delay Hamiltonian control systems with uncertainties and disturbances is a meaningful topic.

Motivated by the above observations, in this paper we study a class of time-delay Hamiltonian systems model with uncertainties and external disturbances. We derive sufficient condition for which the uncertain time-delay Hamiltonian system along with the proposed feedback controller is robustly stable for all admissible uncertainties. The condition is given in terms of linear matrix inequalities. Furthermore, the problem of L_2 disturbance attenuation is examined using the parametric adaptive methodology for delay-dependent case. The L_2 feedback adaptive control law can guarantee that the closed-loop time-delay Hamiltonian system is asymptotically stable and the L_2 performance is achieved. The effectiveness of the proposed methods in this paper is illustrated by numerical examples.

The paper is organized as follows. Section 2 presents the problem formulation and some preliminaries. The main results are proposed in Section 3. Section 4 illustrates the obtained results by several numerical examples, which is followed by the conclusion in Section 5.

Notations. $\mathbb{R}^n$ denotes the n-dimension Euclidean space, and $\mathbb{R}^{n\times m}$ is the real matrices with dimension $n \times m$; $\|\cdot\|$ stands for either the Euclidean vector norm or the induced matrix 2-norm; $\|x\|_{\mathscr{C}} = \max_{t-h\leqslant\varphi\leqslant t}\|x(\varphi)\|$, where $\mathscr{C} = \mathscr{C}([-h,0],\mathbb{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h,0]$ into $\mathbb{R}^n$; $L_2^n[0,\infty)$ denotes the set of all measurable functions $x : [0,\infty) \rightarrow \mathbb{R}^n$ that satisfy $\int_0^\infty |x(t)|^2 dt < \infty$. $\mathbb{C}^i$ denotes the set of all functions with continuous ith partial derivatives. The notation $X \geqslant Y$ (resp., $X > Y$) where X and Y are symmetric matrices means that the matrix $X - Y$ is positive semidefinite (resp., positive definite); $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and the minimum of eigenvalue of a real symmetric matrix A. The notation $*$ represents the elements below the main diagonal of a symmetric matrix; A^{T} denotes the transposed matrix of A; $(\cdot)'$ and $[\cdot]'$ denote the derivative of the variable inside the brackets. What is more, for the sake of simplicity, throughout the paper, we denote $\partial H/\partial x$ by ∇H.

2. Problem Statement and Preliminaries

Consider the following class of time-delay Hamiltonian systems with parametric uncertainties and external disturbances:

$$\begin{aligned}\dot{x}(t) &= [J(x,p) - R(x,p)]\nabla H(x,p) \\ &\quad + [J^*(x_\tau) - R^*(x_\tau)]\nabla H(x_\tau) + g_1 u(t) + g_2\omega(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; $x_\tau := x(t-d(t)) \in \mathscr{C}$ stands for the delayed state; $u \in \mathbb{R}^s$ is the control input; $\omega \in L_2^m[0,\infty)$ is the disturbance input; $H(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is the Hamilton function which satisfies $H(x) \geqslant 0$, $H(0) = 0$; p is an unknown constant vector and denotes the disturbance parameter; $J(x,p), J^*(x_\tau) \in \mathbb{R}^{n\times n}$ are skew-symmetric structure matrices; $R(x,p), R^*(x_\tau) \in \mathbb{R}^{n\times n}$ are positive semidefinite symmetric matrices; g_1 and g_2 are gain matrices of appropriate dimensions; $g_1 g_1^{\mathrm{T}}$ is nonsingular.

The delay $d(t)$ is a time-varying continuous function which satisfies

$$\begin{aligned}0 \leqslant d(t) \leqslant h, \\ \dot{d}(t) \leqslant \mu < 1,\end{aligned} \tag{2}$$

where the bounds h and μ are known positive scalars.

The initial condition is $x(t) = \phi(t)$, $t \in [-h,0]$.

Throughout the paper, we suppose that the following assumptions are satisfied.

Assumption 1. The matrices $R(x,p)$ and $R^*(x_\tau)$ satisfy

$$R(x,p) \geqslant \overline{R}, \qquad R^*(x_\tau) \geqslant \overline{R}^*, \tag{3}$$

where $\overline{R}, \overline{R}^* \geqslant 0$ are known constant matrices.

Assumption 1 means that $R(x,p)$ and $R^*(x_\tau)$ are unknown, but they are bounded by known nonnegative constant matrices. To illustrate that this assumption is reasonable, an example is given below.

Example 2. Consider two functional matrices

$$R_1(x,p)=\begin{pmatrix}\left(1+p^2x_1^2\right)^2+x_2^2 & x_2\\ x_2 & 2\end{pmatrix},$$
$$R_2(x_\tau)=\begin{pmatrix}2+x_1^2(t-\tau) & 0 & 0\\ 0 & \sin^2(x_2(t-\tau)) & 0\\ 0 & 0 & 0\end{pmatrix}, \quad (4)$$

where p is unknown constant and τ is the time delay.

It is easy to find two corresponding matrices

$$\overline{R}=\begin{pmatrix}1&0\\0&1\end{pmatrix},$$
$$\overline{R}^*=\begin{pmatrix}2&0&0\\0&0&0\\0&0&0\end{pmatrix}, \quad (5)$$

which satisfy $R_1(x,p)\geqslant\overline{R}$ and $R_2(x_\tau)\geqslant\overline{R}^*$.

Assumption 3. The Hamilton function $H(x)$ and its gradient $\nabla H(x)$ satisfy

(A1) $H(x)\in\mathbb{C}^2$,

(A2) $\varepsilon_1(\|x\|)\leqslant H(x)\leqslant\varepsilon_2(\|x\|)$,

(A3) $\epsilon_1(\|x\|)\leqslant\nabla^{\mathrm{T}}H(x)\cdot\nabla H(x)\leqslant\epsilon_2(\|x\|)$,

(A4) $\pi_1(\|x\|)\leqslant[(\nabla H(x))']^{\mathrm{T}}\cdot[\nabla H(x)]'\leqslant\pi_2(\|x\|)$,

where $\varepsilon_1,\varepsilon_2,\epsilon_1,\epsilon_2,\pi_1,\pi_2$ all belong to $\mathscr{K}$-class functions.

Remark 4. Assumption 3 not only guarantees the existence of $\nabla H(x)$ and $[\nabla H(x)]'$ but also guarantees that $H(x)$, $\nabla H(x)$, and $[\nabla H(x)]'$ are bounded in terms of x. We shall note that the assumption is not very conservative to Hamilton functions and the majority of Hamilton functions in Hamiltonian systems can easily satisfy these conditions.

Assumption 5. There exists a function $\Phi(x)$ such that

$$[J(x,p)-R(x,p)]\,\Delta_H(x,p)=g_1\Phi(x)\theta \quad (6)$$

holds for all $x\in\mathbb{R}^n$, where $\theta\in\mathbb{R}^s$ denotes an unknown parametric vector, $\Delta_H(x,p)=\nabla H(x,p)-\nabla H(x,0)$.

In what follows, we shall address the problems of robust stability and the disturbance attenuation of system (1). Specifically, the objective of this paper can be summarized as follows.

(i) Robust Stability Problem. In the absence of disturbances ω, develop LMI-based conditions, and find an adaptive control law of the form

$$u=\alpha\left(x,\widehat{\theta}\right),\qquad \dot{\widehat{\theta}}=\varsigma(x) \quad (7)$$

so that the closed-loop system under the control law can be asymptotically stable.

(ii) L_2 Disturbance Attenuation Problem. Given a penalty signal $z=q(x)$ and a disturbance attenuation level $\gamma>0$, find an adaptive feedback control law

$$u=\beta\left(x,\widehat{\theta}\right),\qquad \dot{\widehat{\theta}}=\rho(x) \quad (8)$$

and a positive storage function $V(x,x_\tau,\widetilde{\theta})$ such that the γ-dissipation inequality

$$\dot{V}\left(x,x_\tau,\widetilde{\theta}\right)+Q(x,x_\tau)\leqslant\frac{1}{2}\left\{\gamma^2\|\omega\|^2-\|z\|^2\right\},\quad \forall\omega\in L_2^m[0,\infty) \quad (9)$$

holds along the closed-loop systems consisting of (1) and the feedback law, where $Q(x,x_\tau)$ is a nonnegative definite symmetric matrix.

We conclude this section by recalling an auxiliary result to be used in this paper.

Lemma 6 (see [30]). *For given matrices $Y=Y^{\mathrm{T}}$, D and E with appropriate dimensions,*

$$Y+DF(t)E+E^{\mathrm{T}}F^{\mathrm{T}}(t)D^{\mathrm{T}}<0 \quad (10)$$

holds for all $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t)\leqslant I$ if and only if there exists $c>0$ such that

$$Y+c^{-1}DD^{\mathrm{T}}+cE^{\mathrm{T}}E<0. \quad (11)$$

3. Main Results

3.1. Robust Stabilization. In the absence of external disturbances, namely, $\omega=0$, and under Assumption 5, system (1) can be transformed into

$$\dot{x}=[J(x,p)-R(x,p)]\nabla H(x)+[J^*(x_\tau)-R^*(x_\tau)]\nabla H(x_\tau)+g_1\Phi(x)\theta+g_1u. \quad (12)$$

In this subsection, we will put forward a robust stabilization result for system (12). Delay-dependent criteria are developed as follows.

Theorem 7. *Consider system* (12). *Suppose that Assumptions 1 and 3 hold. If there exist matrices*

$$0<P_1=P_1^{\mathrm{T}},\qquad 0<Z_1=Z_1^{\mathrm{T}},\qquad 0<M_1=M_1^{\mathrm{T}},$$
$$0\leqslant X=X^{\mathrm{T}}=\begin{pmatrix}X_{11}&X_{12}&X_{13}\\X_{21}&X_{22}&X_{23}\\X_{31}&X_{32}&X_{33}\end{pmatrix}, \quad (13)$$

any appropriately dimensioned matrices E, F, T, B_1, B_2, and a scalar ε > 0 such that the following conditions hold:

$$J^*(x_\tau) - R^*(x_\tau) = E\Delta(x_\tau)F + T, \quad (14)$$

$$\Xi_1 = \begin{pmatrix} -\overline{R} - \overline{R}^* & \frac{B_1^{\mathrm{T}}}{2} + hX_{12} & \frac{B_2^{\mathrm{T}}}{2} + hX_{13} \\ * & \Phi_{22} & -\frac{B_2^{\mathrm{T}}}{2} + hX_{23} \\ * & * & -M_1 + hZ_1 + hX_{33} \end{pmatrix} < 0, \quad (15)$$

$$\Theta = \begin{pmatrix} X_{11} + \varepsilon^{-1}EE^{\mathrm{T}} & X_{12} & X_{13} & \frac{1}{2}T \\ * & X_{22} & X_{23} & \frac{1}{2}B_1 \\ * & * & X_{33} & \frac{1}{2}B_2 \\ * & * & * & Z_1 + \varepsilon^{-1}F^{\mathrm{T}}F \end{pmatrix} \geqslant 0, \quad (16)$$

where

$$\Delta^{\mathrm{T}}(x_\tau)\Delta(x_\tau) \leqslant I,$$
$$\Phi_{22} = -(1-\mu)P_1 - \frac{B_1}{2} - \frac{B_1^{\mathrm{T}}}{2} + hX_{22}, \quad (17)$$

then the closed-loop systems under the feedback control law

$$\begin{aligned} u = &- g_1^{\mathrm{T}}\left(g_1 g_1^{\mathrm{T}}\right)^{-1}\Big\{(P_1 + hX_{11})\nabla H(x) - \Phi(x)\widehat{\theta} \\ &+ \left[\nabla^{\mathrm{T}}H(x)\cdot\nabla H(x)\right]^{-1}\nabla H(x) \\ &\times\left[(\nabla H(x(t)))'\right]^{\mathrm{T}} M_1[\nabla H(x(t))]'\Big\}, \\ \dot{\widehat{\theta}} = &\, K_1\Phi^{\mathrm{T}}(x)g_1^{\mathrm{T}}\nabla H(x) \end{aligned} \quad (18)$$

is asymptotically stable, where $K_1 > 0$ is an adaptive gain matrix with appropriate dimension.

Proof. Substituting (18) into (12) yields

$$\begin{aligned} \dot{x} = &\left[J(x,p) - R(x,p)\right]\nabla H(x) \\ &+ \left[J^*(x_\tau) - R^*(x_\tau)\right]\nabla H(x_\tau) \\ &+ g_1\Phi(x)\left(\theta - \widehat{\theta}\right) - (P_1 + hX_{11})\nabla H(x) \\ &- \left[\nabla^{\mathrm{T}}H(x)\nabla H(x)\right]^{-1}\nabla H(x)\left[(\nabla H(x(t)))'\right]^{\mathrm{T}} \\ &\times M_1[\nabla H(x(t))]', \\ \dot{\widehat{\theta}} = &\, K_1\Phi^{\mathrm{T}}(x)g_1^{\mathrm{T}}\nabla H(x). \end{aligned} \quad (19)$$

Choose a Lyapunov functional described as

$$\begin{aligned} V_1\left(x, x_\tau, \widetilde{\theta}\right) = &\, H(x) + \frac{1}{2}\widetilde{\theta}^{\mathrm{T}}K_1^{-1}\widetilde{\theta} \\ &+ \int_{t-d(t)}^{t}\nabla^{\mathrm{T}}H(x(\varphi))P_1\nabla H(x(\varphi))\,d\varphi \\ &+ \int_{-h}^{0}\int_{t+\beta}^{t}\left[(\nabla H(x(\alpha)))'\right]^{\mathrm{T}} \\ &\times Z_1[\nabla H(x(\alpha))]'\,d\alpha\,d\beta, \end{aligned} \quad (20)$$

where $\widetilde{\theta} = \theta - \widehat{\theta}$.

Since $H(x) \in \mathbb{C}^2$, $P_1 > 0$, $Z_1 > 0$, and (A3) in Assumption 3 holds, we have the following inequalities:

$$\begin{aligned} &\int_{t-d(t)}^{t}\nabla^{\mathrm{T}}H(x(\varphi))P_1\nabla H(x(\varphi))\,d\varphi \\ &\leqslant \int_{t-d(t)}^{t}\left\|\nabla^{\mathrm{T}}H(x(\varphi))P_1\nabla H(x(\varphi))\right\|d\varphi \\ &\leqslant \iota_p\int_{t-d(t)}^{t}\epsilon_2(\max\|x(\varphi)\|)\,d\varphi \\ &= h\iota_p\epsilon_2(\|x\|_{\mathscr{C}}), \end{aligned} \quad (21)$$

where $\iota_p = \lambda_{\max}(P_1) > 0$.

Moreover, according to (A4) in Assumption 3, we have

$$\begin{aligned} &\int_{-h}^{0}\int_{t+\beta}^{t}\left[(\nabla H(x(\alpha)))'\right]^{\mathrm{T}}Z_1[\nabla H(x(\alpha))]'\,d\alpha\,d\beta \\ &\leqslant \int_{-h}^{0}\int_{t+\beta}^{t}\iota_z\pi_2(\|x(\alpha)\|)\,d\alpha\,d\beta \\ &= \frac{1}{2}h^2\iota_z\pi_2(\|x\|_{\mathscr{C}}), \end{aligned} \quad (22)$$

where $\iota_z = \lambda_{\max}(Z_1) > 0$.

Combining (21) and (22), from (A2) in Assumption 3, we obtain

$$\begin{aligned} V_1\left(x, x_\tau, \widetilde{\theta}\right) \leqslant &\, \varepsilon_2(\|x\|) + \kappa\left\|\widetilde{\theta}\right\|^2 \\ &+ h\iota_p\epsilon_2(\|x\|_{\mathscr{C}}) + \frac{1}{2}h^2\iota_z\pi_2(\|x\|_{\mathscr{C}}), \end{aligned} \quad (23)$$

where $\kappa = \lambda_{\max}(K_1^{-1}) > 0$.

Let $\nu(\|\chi\|_{\mathscr{C}}) = \varepsilon_2(\|x\|) + \kappa\|\widetilde{\theta}\|^2 + h\iota_p\epsilon_2(\|x\|_{\mathscr{C}}) + (1/2)h^2\iota_z\pi_2(\|x\|_{\mathscr{C}})$, $\chi = \left[x^{\mathrm{T}}\ \ x_\tau^{\mathrm{T}}\ \ \widetilde{\theta}^{\mathrm{T}}\right]^{\mathrm{T}}$. Obviously, it belongs to $\mathscr{K}$-class function. So, we obtain

$$\varepsilon_1(\|\chi(0)\|) \leqslant V_1\left(x, x_\tau, \widetilde{\theta}\right) \leqslant \nu(\|\chi\|_{\mathscr{C}}). \quad (24)$$

According to the Newton-Leibnitz formula, it follows that

$$\nabla H(x) - \int_{t-d(t)}^{t}[\nabla H(x(\alpha))]'\,d\alpha - \nabla H(x_\tau) = 0; \quad (25)$$

then for any matrices B_1 and B_2 with appropriate dimensions, we have

$$\left\{\nabla^{\mathrm{T}} H(x)\left[J^*\left(x_\tau\right)-R^*\left(x_\tau\right)\right]+\nabla^{\mathrm{T}} H\left(x_\tau\right) B_1 +[\nabla H(x(t))]^{\prime} B_2\right\} \cdot\left[\nabla H(x)-\nabla H\left(x_\tau\right)-\int_{t-d(t)}^t[\nabla H(x(\alpha))]^{\prime} d \alpha\right] \equiv 0. \tag{26}$$

As is well known, for any positive definite matrix $X \geqslant 0$ and a vector function η, the following inequality holds:

$$h \eta^{\mathrm{T}}(t) X \eta(t)-\int_{t-d(t)}^t \eta^{\mathrm{T}}(t) X \eta(t) d \alpha \geqslant 0. \tag{27}$$

Noting that

$$\nabla^{\mathrm{T}} H(x) J(x, p) \nabla H(x) =\frac{1}{2} \nabla^{\mathrm{T}} H(x)\left[J(x, p)+J^{\mathrm{T}}(x, p)\right] \nabla H(x)=0 \tag{28}$$

and combining (26) and (27) and using Assumption 1, we can evaluate the derivative of $V_1(x, x_\tau, \widetilde{\theta})$ along the trajectory of the closed-loop system (19) as follows:

$$\begin{aligned}
\dot{V}_1&\left(x, x_\tau, \widetilde{\theta}\right) \\
&= \nabla^{\mathrm{T}} H(x)[J(x, p)-R(x, p)] \nabla H(x) \\
&\quad+\nabla^{\mathrm{T}} H(x)\left[J^*\left(x_\tau\right)-R^*\left(x_\tau\right)\right] \nabla H\left(x_\tau\right) \\
&\quad+\nabla^{\mathrm{T}} H(x) g_1 \Phi(x) \widetilde{\theta}+\nabla^{\mathrm{T}} H(x) P_1 \nabla H(x) \\
&\quad-\nabla^{\mathrm{T}} H(x)\left(P_1+h X_{11}\right) \nabla H(x) \\
&\quad-\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} M_1[\nabla H(x(t))]^{\prime} \\
&\quad-\left(1-\dot{d}(t)\right) \nabla^{\mathrm{T}} H\left(x_\tau\right) P_1 \nabla H\left(x_\tau\right) \\
&\quad-\widetilde{\theta}^{\mathrm{T}} \Phi^{\mathrm{T}}(x) g_1^{\mathrm{T}} \nabla H(x) \\
&\quad+h\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} Z_1[\nabla H(x(t))]^{\prime} \\
&\quad-\int_{t-h}^t\left[(\nabla H(x(\alpha)))^{\prime}\right]^{\mathrm{T}} Z_1[\nabla H(x(\alpha))]^{\prime} d \alpha \\
&\leqslant -\nabla^{\mathrm{T}} H(x) \overline{R} \nabla H(x)-h \nabla^{\mathrm{T}} H(x) X_{11} \nabla H(x) \\
&\quad-\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} M_1[\nabla H(x(t))]^{\prime} \\
&\quad-(1-\mu) \nabla^{\mathrm{T}} H\left(x_\tau\right) P_1 \nabla H\left(x_\tau\right) \\
&\quad-\int_{t-d(t)}^t\left[(\nabla H(x(\alpha)))^{\prime}\right]^{\mathrm{T}} Z_1[\nabla H(x(\alpha))]^{\prime} d \alpha \\
&\quad+h\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} Z_1[\nabla H(x(t))]^{\prime} \\
&\quad-\nabla^{\mathrm{T}} H(x) \overline{R}^* \nabla H(x) \\
&\quad+\nabla^{\mathrm{T}} H\left(x_\tau\right) B_1 \nabla H(x)-\nabla^{\mathrm{T}} H\left(x_\tau\right) B_1 \nabla H\left(x_\tau\right) \\
&\quad+\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} B_2 \nabla H(x) \\
&\quad-\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} B_2 \nabla H\left(x_\tau\right) \\
&\quad-\int_{t-d(t)}^t \nabla^{\mathrm{T}} H(x)\left[E \Delta\left(x_\tau\right) F+T\right][\nabla H(x(\alpha))]^{\prime} d \alpha \\
&\quad-\int_{t-d(t)}^t \nabla^{\mathrm{T}} H\left(x_\tau\right) B_1[\nabla H(x(\alpha))]^{\prime} d \alpha \\
&\quad-\int_{t-d(t)}^t\left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} \\
&\qquad\times B_2[\nabla H(x(\alpha))]^{\prime} d \alpha+h \eta_1^{\mathrm{T}} X \eta_1 \\
&\quad-\int_{t-d(t)}^t \eta_1^{\mathrm{T}}(t) X \eta_1(t) d \alpha,
\end{aligned} \tag{29}$$

where $\eta_1=\left[\nabla^{\mathrm{T}} H(x) \;\; \nabla^{\mathrm{T}} H(x_\tau) \;\; [(\nabla H(x(t)))^{\prime}]^{\mathrm{T}}\right]^{\mathrm{T}}$.

Let

$$\eta_2 =\left[\nabla^{\mathrm{T}} H(x) \;\; \nabla^{\mathrm{T}} H\left(x_\tau\right) \;\; \left[(\nabla H(x(t)))^{\prime}\right]^{\mathrm{T}} \;\; \left[(\nabla H(x(\alpha)))^{\prime}\right]^{\mathrm{T}}\right]^{\mathrm{T}}; \tag{30}$$

according to (15)-(16) and using Lemma 6, we get that

$$\dot{V}_1\left(x, x_\tau, \widetilde{\theta}\right) \leqslant \eta_1^{\mathrm{T}} \Xi_1 \eta_1-\int_{t-d(t)}^t \eta_2^{\mathrm{T}} \Theta \eta_2 d \alpha \leqslant \eta_1^{\mathrm{T}} \Xi_1 \eta_1. \tag{31}$$

Furthermore, since $\Xi_1<0$, according to Assumption 3, there exists a continuous nondecreasing function $\epsilon(\|\chi\|)$, $\chi=[x^{\mathrm{T}} \;\; x_\tau^{\mathrm{T}} \;\; \widetilde{\theta}^{\mathrm{T}}]^{\mathrm{T}}$ such that

$$\dot{V}_1\left(x, x_\tau, \widetilde{\theta}\right) \leqslant-\epsilon(\|\chi(0)\|). \tag{32}$$

According to the Lyapunov-Krasovskii stability theorem, we can conclude that the closed-loop system (19) consisting of system (12) and the control law (18) is asymptotically stable. This completes the proof. □

3.2. L_2 Disturbance Attenuation. In what follows, we consider the L_2 disturbance attenuation problem of systems (1). Given a disturbance attenuation level $\gamma > 0$, choose the following penalty function:

$$z = h(x) g_1^{\mathrm{T}} \nabla H(x), \tag{33}$$

where $h(x) \in \mathbb{R}^{q\times s}$ is weighing matrix.

For time delay $d(t)$ satisfying (2), we have the following result.

Theorem 8. *Consider system* (1). *Suppose that Assumptions 1–5 hold. If there exist matrices*

$$0 < P_2 = P_2^{\mathrm{T}}, \qquad 0 < Z_2 = Z_2^{\mathrm{T}}, \qquad 0 < M_2 = M_2^{\mathrm{T}}, \tag{34}$$

$$0 \leqslant \overline{X} = \begin{pmatrix} \overline{X}_{11} & \overline{X}_{12} & \overline{X}_{13} \\ \overline{X}_{21} & \overline{X}_{22} & \overline{X}_{23} \\ \overline{X}_{31} & \overline{X}_{32} & \overline{X}_{33} \end{pmatrix}, \tag{35}$$

any appropriately dimensioned matrices E, F, T, B_3, B_4 and a scalar $\varepsilon > 0$ such that (14) *and the following conditions hold*

$$\Xi_2 = \begin{pmatrix} \overline{\Phi}_{11} = -\overline{R} - \overline{R}^* - \dfrac{1}{2\gamma^2}\left(g_1 g_1^{\mathrm{T}} - g_2 g_2^{\mathrm{T}}\right) & \dfrac{B_3^{\mathrm{T}}}{2} + h\overline{X}_{12} & \dfrac{B_4^{\mathrm{T}}}{2} + h\overline{X}_{13} \\ * & -(1-\mu) P_2 - \dfrac{B_3}{2} - \dfrac{B_3^{\mathrm{T}}}{2} + h\overline{X}_{22} & -\dfrac{B_4^{\mathrm{T}}}{2} + h\overline{X}_{23} \\ * & * & -M_2 + hZ_2 + h\overline{X}_{33} \end{pmatrix} < 0,$$

$$\overline{\Theta} = \begin{pmatrix} \overline{X}_{11} + \varepsilon^{-1} E E^{\mathrm{T}} & \overline{X}_{12} & \overline{X}_{13} & \dfrac{1}{2} T \\ * & \overline{X}_{22} & \overline{X}_{23} & \dfrac{1}{2} B_3 \\ * & * & \overline{X}_{33} & \dfrac{1}{2} B_4 \\ * & * & * & Z_2 + \varepsilon F^{\mathrm{T}} F \end{pmatrix} \geqslant 0, \tag{36}$$

then the L_2 disturbance attenuation problem of system (1) *can be solved by the feedback control law:*

$$\begin{aligned} u = & -g_1^{\mathrm{T}}\left(g_1 g_1^{\mathrm{T}}\right)^{-1}\Big\{\left(P_2 + h\overline{X}_{11}\right)\nabla H(x) - \Phi(x)\hat{\theta} \\ & + \left[\nabla^{\mathrm{T}} H(x) \nabla H(x)\right]^{-1} \nabla H(x) \\ & \times \left[(\nabla H(x(t)))'\right]^{\mathrm{T}} M_2 [\nabla H(x(t))]'\Big\} \\ & - \left[\frac{1}{2} h^{\mathrm{T}}(x) h(x) + \frac{1}{2\gamma^2} I_m\right] g_1^{\mathrm{T}} \nabla H(x), \\ \dot{\hat{\theta}} = & K_2 \Phi^{\mathrm{T}}(x) g_1^{\mathrm{T}} \nabla H(x), \end{aligned} \tag{37}$$

where $K_2 > 0$ is an adaptive gain matrix with appropriate dimension.

Moreover, the γ-dissipation inequality

$$V_2\left(x, x_\tau, \tilde{\theta}\right) + Q\left(x, x_\tau\right) \leqslant \frac{1}{2}\left\{\gamma^2 \|\omega\|^2 - \|z\|^2\right\} \tag{38}$$

holds along the trajectories of the closed-loop systems consisting of (1) *and* (37), *where*

$$Q\left(x, x_\tau\right) = -\eta_1^{\mathrm{T}} \Xi_2 \eta_1 + \int_{t-d(t)}^{t} \eta_2^{\mathrm{T}} \overline{\Theta} \eta_2 \, d\alpha \tag{39}$$

with

$$\begin{aligned} \eta_1 &= \left[\nabla^{\mathrm{T}} H(x) \;\; \nabla^{\mathrm{T}} H(x_\tau) \;\; \left[(\nabla H(x(t)))'\right]^{\mathrm{T}}\right]^{\mathrm{T}}, \\ \eta_2 &= \left[\nabla^{\mathrm{T}} H(x) \;\; \nabla^{\mathrm{T}} H(x_\tau) \;\; \left[(\nabla H(x(t)))'\right]^{\mathrm{T}} \;\; \left[(\nabla H(x(\alpha)))'\right]^{\mathrm{T}}\right]^{\mathrm{T}}. \end{aligned} \tag{40}$$

The storage function is given as

$$\begin{aligned} V_2\left(x, x_\tau, \tilde{\theta}\right) &= H(x) + \int_{t-d(t)}^{t} \nabla^{\mathrm{T}} H(x(\varphi)) P_2 \nabla H(x(\varphi)) \, d\varphi \\ &+ \int_{-h}^{0} \int_{t-d(t)}^{t} \left[\nabla^{\mathrm{T}} H(x(\alpha))\right]' Z_2 [\nabla H(x(\alpha))]' \, d\alpha \, d\beta \\ &+ \frac{1}{2} \tilde{\theta}^{\mathrm{T}} K_2^{-1} \tilde{\theta}. \end{aligned} \tag{41}$$

Proof. Substituting (37) into (1) yields

$$\begin{aligned}\dot{x} = &\ [J(x,p) - R(x,p)]\nabla H(x)\\ &+ [J^*(x_\tau) - R^*(x_\tau)]\nabla H(x_\tau)\\ &- (P_2 + h\overline{X}_{11})\nabla H(x) + g_2\omega\\ &- [\nabla^{\mathrm{T}} H(x)\nabla H(x)]^{-1}\nabla H(x)\\ &\times [(\nabla H(x(t)))']^{\mathrm{T}} M_2 [\nabla H(x(t))]'\\ &+ g_1\Phi(x)(\theta - \widehat{\theta})\\ &- g_1\left[\frac{1}{2}h^{\mathrm{T}}(x)h(x) + \frac{1}{2\gamma^2}I_m\right] g_1^{\mathrm{T}}\nabla H(x),\\ &\dot{\widehat{\theta}} = K_2\Phi^{\mathrm{T}}(x)g_1^{\mathrm{T}}\nabla H(x).\end{aligned} \tag{42}$$

Evaluating the derivative of (41) along the trajectory of system (42) and using (26), (27), and Assumption 1, we get

$$\begin{aligned}&\dot{V}_2(x, x_\tau, \widetilde{\theta})\\ &= \nabla^{\mathrm{T}} H(x)[J(x,p) - R(x,p)]\nabla H(x)\\ &\quad + \nabla^{\mathrm{T}} H(x)[J^*(x_\tau) - R^*(x_\tau)]\nabla H(x_\tau)\\ &\quad - h\nabla^{\mathrm{T}} H(x)\overline{X}_{11}\nabla H(x) + \nabla^{\mathrm{T}} H(x)g_2\omega\\ &\quad - [(\nabla H(x(t)))']^{\mathrm{T}} M_2[\nabla H(x(t))]'\\ &\quad - \nabla^{\mathrm{T}} H(x)g_1\left[\frac{1}{2}h^{\mathrm{T}}(x)h(x) + \frac{1}{2\gamma^2}I_m\right] g_1^{\mathrm{T}}\nabla H(x)\\ &\quad - (1 - \dot{d}(t))\nabla^{\mathrm{T}} H(x_\tau)P_2\nabla H(x_\tau)\\ &\quad + h[(\nabla H(x(t)))']^{\mathrm{T}} Z_2[\nabla H(x(t))]'\\ &\quad - \int_{t-h}^{t}[(\nabla H(x(\alpha)))']^{\mathrm{T}} Z_2[\nabla H(x(\alpha))]'d\alpha\\ &\leqslant -\nabla^{\mathrm{T}} H(x)\overline{R}\nabla H(x) - h\nabla^{\mathrm{T}} H(x)\overline{X}_{11}\nabla H(x)\\ &\quad - [(\nabla H(x(t)))']^{\mathrm{T}} M_2[\nabla H(x(t))]'\\ &\quad - (1-\mu)\nabla^{\mathrm{T}} H(x_\tau)P_2\nabla H(x_\tau)\\ &\quad + h[(\nabla H(x(t)))']^{\mathrm{T}} Z_2[\nabla H(x(t))]'\\ &\quad - \int_{t-h}^{t}[(\nabla H(x(\alpha)))']^{\mathrm{T}} Z_2[\nabla H(x(\alpha))]'d\alpha\\ &\quad + \nabla^{\mathrm{T}} H(x)[J^*(x_\tau) - R^*(x_\tau)]\nabla H(x)\\ &\quad + \nabla^{\mathrm{T}} H(x_\tau)B_3\nabla H(x)\\ &\quad - \nabla^{\mathrm{T}} H(x_\tau)B_3\nabla H(x_\tau) + [(\nabla H(x(t)))']^{\mathrm{T}} B_4\nabla H(x)\\ &\quad - \int_{t-d(t)}^{t}\nabla^{\mathrm{T}} H(x)[J^*(x_\tau) - R^*(x_\tau)][\nabla H(x(\alpha))]'d\alpha\\ &\quad - \int_{t-d(t)}^{t}\nabla^{\mathrm{T}} H(x_\tau)B_3[\nabla H(x(\alpha))]'d\alpha\\ &\quad - [(\nabla H(x(t)))']^{\mathrm{T}} B_4\nabla H(x_\tau)\\ &\quad - \int_{t-d(t)}^{t}[(\nabla H(x(t)))']^{\mathrm{T}} B_4[\nabla H(x(\alpha))]'d\alpha\\ &\quad + h\eta_1^{\mathrm{T}}\overline{X}\eta_1\\ &\quad - \int_{t-d(t)}^{t}\eta_1^{\mathrm{T}}(t)\overline{X}\eta_1(t)d\alpha\\ &\quad - \frac{1}{2}\left\|\gamma\omega - \frac{1}{\gamma}\nabla^{\mathrm{T}} H(x)g_2\right\|^2\\ &\quad - \nabla^{\mathrm{T}} H(x)g_1\left[\frac{1}{2}h^{\mathrm{T}}(x)h(x) + \frac{1}{2\gamma^2}I_m\right] g_1^{\mathrm{T}}\nabla H(x)\\ &\quad + \frac{1}{2\gamma^2}\nabla^{\mathrm{T}} H(x)g_2g_2^{\mathrm{T}}\nabla H(x)\\ &\quad + \frac{1}{2}\{\gamma^2\|\omega\|^2 - \|z\|^2\}\\ &\quad + \frac{1}{2}\nabla^{\mathrm{T}} H(x)g_1h^{\mathrm{T}}(x)h(x)g_1^{\mathrm{T}}\nabla H(x).\end{aligned} \tag{43}$$

According to (35), (36), and Lemma 6, we have

$$\dot{V}_2(x, x_\tau, \widetilde{\theta}) - \eta_1^{\mathrm{T}}\Xi_2\eta_1 + \int_{t-d(t)}^{t}\eta_2^{\mathrm{T}}\overline{\Theta}\eta_2 d\alpha \leqslant \frac{1}{2}\{\gamma^2\|\omega\|^2 - \|z\|^2\}. \tag{44}$$

It is obvious that the γ-dissipation inequality (38) holds along the closed-loop system (42) which consist of (1) and (37). This completes the proof. □

4. Illustrative Examples

In this section, we give some examples to show how to apply the results proposed in this paper to investigate the robust stabilization and the L_2 disturbance attenuation for a class of time-delay nonlinear control systems with uncertainties and disturbances.

Let us consider the following 2-dimensional time-delay nonlinear control systems with parametric uncertainties and external disturbances:

$$\begin{aligned}\dot{x}_1(t) &= -4x_1^3(t) - 4x_1^3(t-d(t)) + 2u,\\ \dot{x}_2(t) &= -2x_1^3(t) - (2 + 3p + p^2)x_2(t) - 2x_1^3(t-d(t))\\ &\quad - 2x_2(t-d(t)) - x_2(t-d(t))\sin(x_2(t-d(t)))\\ &\quad + 3u + 0.5\omega,\\ x_1(t_0) &= \phi_1(t_0), \quad x_2(t_0) = \phi_2(t_0), \quad t_0 \in [-h, 0],\end{aligned} \tag{45}$$

where $d(t)$ is a time varying delay of the system (45); p is an unknown constant, $0 < p < 1$; ω is the disturbance input.

The system (45) can be realized into the following Hamiltonian system form:

$$\begin{aligned}\dot{x} = & [J(x,p) - R(x,p)]\nabla H(x,p) \\ & + [J^*(x_\tau) - R^*(x_\tau)]\nabla H(x_\tau) + g_1 u + g_2 \omega, \\ & x(t_0) = \phi(t_0), \quad t_0 \in [-h, 0]\end{aligned} \tag{46}$$

with

$$J(x,p) = \begin{pmatrix} 0 & 0.5 \\ -0.5 & 0 \end{pmatrix}, \qquad R(x,p) = \begin{pmatrix} 2 & 0.5 \\ 0.5 & 2+p \end{pmatrix},$$
$$J^*(x_\tau) = \begin{pmatrix} 0 & 0.5 \\ -0.5 & 0 \end{pmatrix},$$
$$R^*(x_\tau) = \begin{pmatrix} 2 & 0.5 \\ 0.5 & 2+\sin x_2(t-d(t)) \end{pmatrix}, \tag{47}$$
$$g_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \qquad g_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix},$$
$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

$$\begin{aligned} H(x,p) &= 0.5\left(x_1^4 + (1+p)x_2^2\right), \\ H(x_\tau) &= 0.5\left(x_1^4(t-d(t))\right) + x_2^2(t-d(t)). \end{aligned} \tag{48}$$

Let $E = F = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $T = \begin{pmatrix} -2 & 0 \\ -1 & -2 \end{pmatrix}$, $\Delta(x_\tau) = \begin{pmatrix} 0 & 0 \\ 0 & -\sin x_2(t-d(t)) \end{pmatrix}$, $\theta = (-1-0.5p)p$ and $\Phi(x) = x_2$. It is easy to verify that system (46) with the above values satisfies Assumptions 1–5 and the condition (14) of Theorem 7.

Firstly, we demonstrate the application of Theorem 7 by using LMI solver [31].

Set $\mu = 0.25$ and $h = 1$. Using the LMI control toolbox of MATLAB, the LMIs in Theorem 7 are solved to find the following matrices:

$$P_1 = \begin{pmatrix} 2.3387 & -0.0187 \\ -0.0187 & 2.3387 \end{pmatrix}, \qquad Z_1 = \begin{pmatrix} 0.3770 & 0.0447 \\ 0.0447 & 0.2949 \end{pmatrix},$$
$$M_1 = \begin{pmatrix} 1.0000 & 0.0000 \\ 0.0000 & 1.0000 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 0.7608 & 0.0232 \\ 0.0232 & 0.7653 \end{pmatrix},$$
$$X_{11} = \begin{pmatrix} 1.4363 & 0.0957 \\ 0.0957 & 1.4842 \end{pmatrix}, \qquad X_{12} = \begin{pmatrix} -0.1565 & -0.0155 \\ -0.0155 & -0.1583 \end{pmatrix},$$
$$X_{22} = \begin{pmatrix} 1.3787 & 0.0070 \\ 0.0070 & 1.3801 \end{pmatrix}, \qquad X_{33} = \begin{pmatrix} 0.5258 & -0.0203 \\ -0.0203 & 0.5359 \end{pmatrix},$$
$$B_2 = X_{13} = X_{23} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{49}$$

Thus a robust stabilizing controller is obtained as

$$u = -3.8229x_2 - x_2\left(\dot{x}_1^4 + \dot{x}_2^2\right)\left(x_1^4 + x_2^2\right)^{-1} - 0.0770x_1^3 - x_2\hat{\theta}. \tag{50}$$

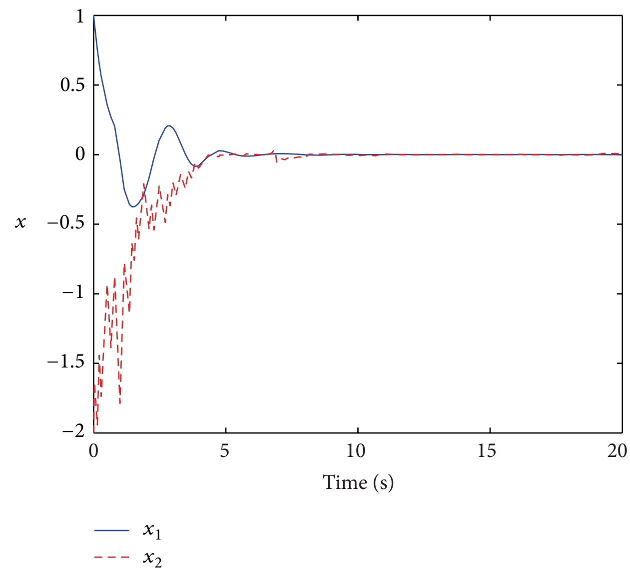

Figure 1: Responses of state x.

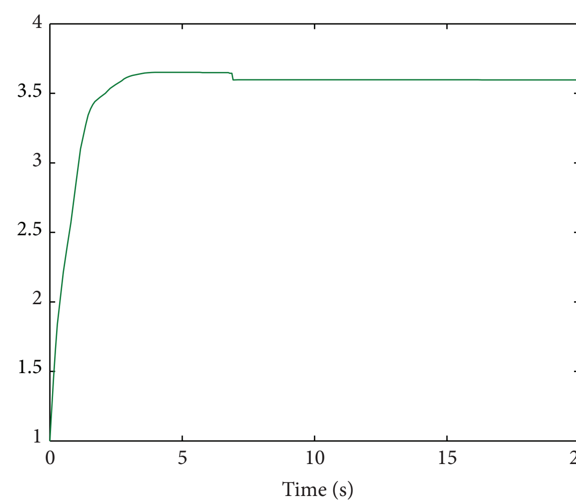

Figure 2: Parameter estimation $\hat{\theta}$.

The simulation with the initial condition $x(0) = \phi(0) = [1 \quad -2]^{\mathrm{T}}$ is given in Figures 1 and 2. It is clear that under the delay-dependent conditions, system (46) along with the controller (50) is asymptotically stable.

Next, we demonstrate the application of Theorem 8. We will check whether the designed L_2 disturbance attenuation controller according to Theorem 8 is effective in stabilizing the given time-delay Hamiltonian system (46) and has strong robustness against external disturbances.

Given a disturbance attenuation level γ, choose

$$z = h(x)\, g_1^{\mathrm{T}} \nabla H(x) \tag{51}$$

as the penalty function, where $h = [0.1 \quad 0.1]^{\mathrm{T}}$.

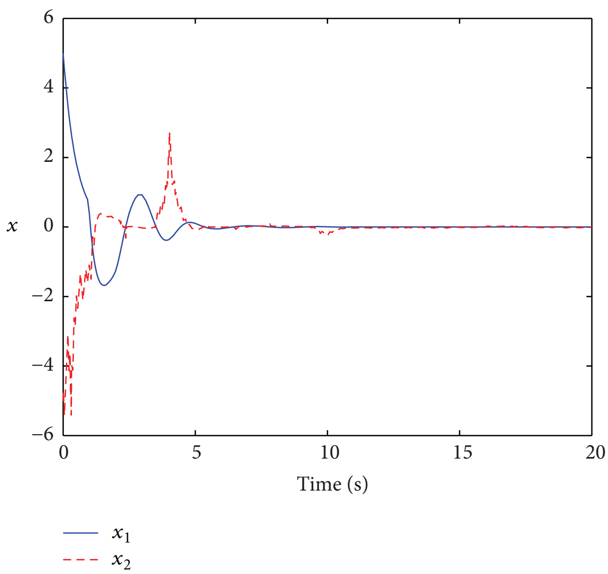

FIGURE 3: Responses of state x.

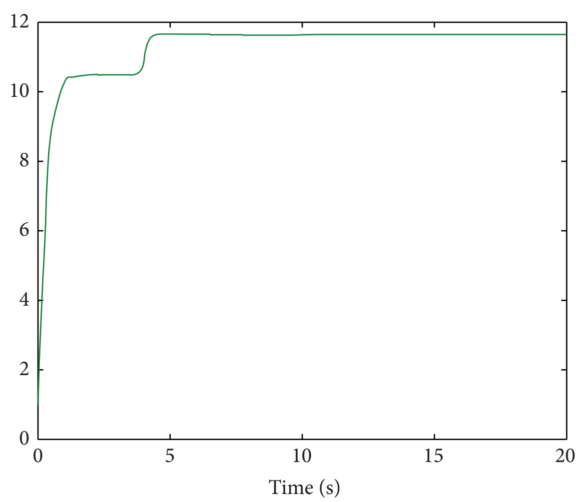

FIGURE 4: Parameter estimation $\hat{\theta}$.

Using the LMI control toolbox, the LMIs in Theorem 8 are solved to find the following matrices with $\mu = 0.25$, $h = 1$:

$$P_2 = \begin{pmatrix} 2.4230 & -0.0178 \\ -0.0178 & 2.4230 \end{pmatrix}, \qquad Z_2 = \begin{pmatrix} 0.3136 & 0.0426 \\ 0.0426 & 0.2926 \end{pmatrix},$$
$$M_2 = \begin{pmatrix} 1.0000 & 0.0000 \\ 0.0000 & 1.0000 \end{pmatrix}, \qquad B_3 = \begin{pmatrix} 0.7859 & 0.0223 \\ 0.0223 & 0.7939 \end{pmatrix},$$
$$\overline{X}_{11} = \begin{pmatrix} 1.4799 & 0.0894 \\ 0.0894 & 1.5238 \end{pmatrix}, \qquad \overline{X}_{12} = \begin{pmatrix} -0.1568 & -0.0160 \\ -0.0160 & -0.1551 \end{pmatrix},$$
$$\overline{X}_{22} = \begin{pmatrix} 1.4275 & 0.0069 \\ 0.0069 & 1.4304 \end{pmatrix}, \qquad \overline{X}_{33} = \begin{pmatrix} 0.5382 & -0.0194 \\ -0.0194 & 0.5477 \end{pmatrix},$$
$$B_4 = \overline{X}_{13} = \overline{X}_{23} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{52}$$

Then according to Theorem 8, a feedback adaptive controller can be obtained as

$$u = -4.4041x_2 - x_2\left(\dot{x}_1^4 + \dot{x}_2^2\right)\left(x_1^4 + x_2^2\right)^{-1} - 0.0716x_1^3 - x_2\hat{\theta}. \tag{53}$$

To illustrate the effectiveness of the adaptive control law (53), we carry simulation result with the following choices: the disturbance signal $\omega = \sin t$; the initial condition is $x(0) = \phi(0) = [5 \quad -5]^{\mathrm{T}}$; the disturbance attenuation level is chosen by $\gamma = 0.9$. The simulation results are shown in Figures 3 and 4, which are responses of the system's state and the parameter estimation, respectively. It can be seen from the simulation that the time-delay system converges to its equilibrium very quickly under the controller (53).

In general, from the simulations, we can conclude that the results presented in this paper are very practicable and effective in stabilization analysis and L_2 disturbance attenuation of time-delay Hamiltonian systems with parametric uncertainties and external disturbances. What is more, by using the result presented in this paper, we may solve the stability and control problem of some classes of time-delay nonlinear systems which can be realized into Hamiltonian systems form.

5. Conclusions

In this paper, the robust asymptotical stability and L_2 disturbance attenuation problem of a class of time-delay Hamiltonian control systems with parametric uncertainties and external disturbances have been investigated. Delay-dependent criteria are established. The proposed adaptive feedback control law, by which the asymptotic stability and the L_2 performance of the close-loop system is guaranteed, is determined by linear matrix inequalities constraints. Simulations show the effectiveness of the proposed method.

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