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"What is mathematics? It's neither physical nor mental, it's social. It's part of culture, it's part of history. It's like law, like religion, like money, like all those other things which are very real, but only as part of collective human consciousness....That's what math is."

For mathematician Reuben Hersh, mathematics has existence or reality only as part of human culture. Despite its seeming timelessness and infallibility, it is a social-cultural-historic phenomenon. He takes the long view. He thinks a lot about the ancient problems. What are numbers? What are triangles, squares and circles? What are infinite sets? What is the fourth dimension? What is the meaning and nature of mathematics?

In so doing he explains and criticizes current and past theories of the nature of mathematics. His main purpose is to confront philosophical problems: In what sense do mathematical objects exist? How can we have knowledge of them? Why do mathematicians think mathematical entities exist forever, independent of human action and knowledge?

Reuben Hersh is professor emeritus at the University of New Mexico, Albuquerque. He is the recipient (with Martin Davis) of the Chauvenet Prize and (with Edgar Lorch) the Ford Prize. Hersh is the author (with Philip J. Davis) of *The Mathematical Experience*, winner of the National Book Award in 1983. His new book, *What is Mathematics, Really?* is forthcoming (Oxford).

JOHN BROCKMAN: Reuben, got an interesting question?

REUBEN HERSH: What is a number? Like, what is two? Or even three? This is sort of a kindergarten question, and of course a kindergarten kid would answer like this: (raising three fingers). Or two (raising two fingers). That's a good answer and a bad answer. It's good enough for most purposes, actually. But if you get way beyond kindergarten, far enough to risk asking really deep questions, it becomes: what kind of a thing is a number?

Now, when you ask "What kind of a thing is a number?" you can think of two basic answers—either it's out there some place, like a rock or a ghost; or it's inside, a thought in somebody's mind. Philosophers have defended one or the other of those two answers. It's really pathetic, because anybody who pays any attention can see right away that they're both completely wrong.

A number isn't a thing out there; there isn't any place that it is, or any thing that it is. Neither is it just a thought, because after all, two and two is four, whether you know it or not.

Then you realize that the question is not so easy, so trivial as it sounds at first. One of the great philosophers of mathematics, Gottlob Frege, made quite an issue of the fact that mathematicians didn't know the meaning of One. What is One? Nobody could answer coherently. Of course Frege answered, but his answer was no better, or even worse, than the previous ones. And so it has continued to this very day, strange and incredible as it is. We know all about so much mathematics, but we don't know what it really is.

Of course when I say, "What is a number?" it applies just as well to a triangle, or a circle, or a differentiable function, or a self-adjoint operator. You know a lot about it, but what is it? What kind of a thing is it? Anyhow, that's my question. A long answer to your short question.

JB: And what's the answer to your question?

HERSH: Oh, you want the answer so quick? You have to work for the answer! I'll approach the answer by gradual degrees.

When you say that a mathematical thing, object, entity, is either completely external, independent of human thought or action, or else internal, a thought in your mind—you're not just saying something about numbers, but about existence—that there are only two kinds of existence. Everything is either internal or external. And given that choice, that polarity or dichotomy, numbers don't fit—that's why it's a puzzle. The question is made difficult by a false presupposition, that there are only two kinds of things around.

But if you pretend you 're not being philosophical, just being real, and ask what there is around, well for instance there's the traffic ticket you have to pay, there's the news on the TV, there's a wedding you have to go to, there's a bill you have to pay—none of these things are just thoughts in your mind, and none of them is external to human thought or activity. They are a different kind of reality, that's the trouble. This kind of reality has been excluded from metaphysics and ontology, even though it's well-known—the sciences of anthropology and sociology deal with it. But when you become philosophical, somehow this third answer is overlooked or rejected.

Now that I've set it up for you, you know what the answer is. Mathematics is neither physical nor mental, it's social. It's part of culture, it's part of history, it's like law, like religion, like money, like all those very real things which are real only as part of collective human consciousness. Being part of society and culture, it's both internal and external. Internal to society and culture as a whole, external to the individual, who has to learn it from books and in school. That's what math is.

But for some Platonic mathematicians, that proposition is so outrageous that it takes a lot of effort even to begin to consider it.

JB: Reuben, sounds like you're about to push some political agenda here, and it's not the Republican platform.

HERSH: You're saying my philosophy may be biased by my politics. Well, it's true! This is one of the many novel things in my book—looking into the correlation between political belief and belief about the nature of mathematics.

JB: Do you have a name for this solution?

HERSH: I call it humanistic philosophy of mathematics. It's not really a school; no one else has jumped on the bandwagon with that name, but there are other people who think in a similar way, who gave it different names. I'm not completely a lone wolf here, I'm one of the mavericks, as we call them. The wolves baying outside the corral of philosophy.

Anyhow, back to your other question. The second half of my book is about the history of the philosophy of mathematics. I found that this was best explained by separating philosophers of mathematics into two groups. One group I call *mainstream* and the other I call *humanists and mavericks*. The humanists and mavericks see mathematics as a human activity, and the mainstream see it as inhuman or superhuman. By the way, there have been humanists way back; Aristotle was one. I wondered whether there was any connection with politics. So I tried to classify each of these guys as either right-wing or left-wing, in relation to their own times. Plato was far right; Aristotle was somewhat liberal. Spinoza was a revolutionary; Descartes was a royalist, and so on. These are well known facts. There are some guys that you can't classify. It came out just as you are intimating: the humanists are predominantly left-wing and the mainstream predominantly right wing. Any explanation would be speculative, but intuitively it makes sense. For instance, one main version of mainstream philosophy of mathematics is Platonism. It says that all mathematical objects, entities, or whatever, including the ones we haven't discovered yet and the ones we never will discover—all of them have always existed. There's no change in the realm of mathematics. We discover things, our knowledge increases, but the actual mathematical universe is completely static. Always was, always will be. Well, that's kind of conservative, you know. Fits in with someone who thinks that social institutions mustn't change.

So this parallel exists. But there are exceptions. For instance, Bertrand Russell was a Platonist and a socialist. One of my favorite philosophers, Imre Lakatos, was a right-winger politically, but very radical philosophically. These correlations are loose and statistical, not binding. You can't tell somebody's philosophy from his politics, or vice versa.

I searched for a suitable label for my ideas. There were several others that had been used for similar points of view—social constructivism, fallibilism, quasi-empiricism, naturalism. I didn't want to take anybody else's label, because I was blazing my own trail, and I didn't want to label myself with someone else's school. The name that would have been most accurate was social conceptualism. Mathematics consists of concepts, but not individually held concepts; socially held concepts. Maybe I thought of humanism because I belong to a group called the Humanistic Mathematics Network. Humanism is appropriate, because it's saying that math is something human. There's no math without people. Many people think that ellipses and numbers and so on are there whether or not any people know about them; I think that's a confusion.

JB: Sounds like we're talking about an anthropic principle of mathematics here.

HERSH: Maybe so; I never thought of that. I had a serious argument with a friend of mine at the University of New Mexico, a philosopher of science. She said: "There are nine planets; there were nine planets before there were any people. That means there was the number nine, before we had any people."

There is a difficulty that has to be clarified. We do see mathematical things, like small numbers, in physical reality. And that seems to contradict the idea that numbers are social entities. The way to straighten this out has been pointed out by others also. We use number words in two different ways: as nouns and adjectives. This is an important observation. We say nine apples, nine is an adjective. If it's an objective fact that there are nine apples on the table, that's just as objective as the fact that the apples are red, or that they're ripe, or anything else about them, that's a fact. And there's really no special difficulty about that. Things become difficult when we switch unconsciously, and carelessly, between this real-world adjective interpretation of math words like nine, and the pure abstraction that we talk about in math class.

That's not really the same nine, although there's of course a correlation and a connection. But the number nine as an abstract object, as part of a number system, is a human possession, a human creation, it doesn't exist without us. The possible existence of collections of nine objects is a physical thing, which certainly exists without us. The two kinds of nine are different.

Like I can say a plate is round, an objective fact, but the conception of roundness, mathematical roundness, is something else.

Sad to say, philosophy is definitely an optional activity; most people, including mathematicians, don't even know if they have a philosophy, or what their philosophy is. Certainly what they do would not be affected by a philosophical controversy. This is true in many other fields. To be a practitioner is one thing; to be a philosopher is another. To justify philosophical activity one must go to a deeper level, for instance as in Socrates' remark about the unexamined life. It's pathetic to be a mathematician all your life and never worry, or think, or care, what that means. Many people do it. I compare this to a salmon swimming upstream. He knows how to swim upstream, but he doesn't know what he's doing or why.

JB: How does having a philosophy of mathematics affect its teaching?

HERSH: The philosophy of mathematics is very pertinent to the teaching of mathematics. What's wrong with mathematics teaching is not particular to this country. People are very critical about math teaching in the United States nowadays, as if it was just an American problem. But even though some other countries get higher test scores, the fundamental mis-teaching and bad teaching of mathematics is international, it's standard. In some ways we're not as bad as some other countries. But I don't want to get into that right now.

Let me state three possible philosophical attitudes towards mathematics:

Platonism says mathematics is about some abstract entities which are independent of humanity.

Formalism says mathematics is nothing but calculations. There's no meaning to it at all. You just come out with the right answer by following the rules.

Humanism sees mathematics as part of human culture and human history.

It's hard to come to rigorous conclusions about this kind of thing, but I feel it's almost obvious that Platonism and formalism are anti-educational, and interfere with understanding, and humanism at least doesn't hurt and could be beneficial.

Formalism is connected with rote, the traditional method which is still common in many parts of the world. Here's an algorithm; practice it for a while; now here's another one. That's certainly what makes a lot of people hate mathematics. (I don't mean that mathematicians who are formalists advocate teaching by rote. But the formalist conception of mathematics fits naturally with the rote method of instruction.)

There are various kinds of Platonists. Some are good teachers, some are bad. But the Platonist idea, that, as my friend Phil Davis puts it, pi is in the sky, helps to make mathematics intimidating and remote. It can be an excuse for a pupil's failure to learn, or for a teacher's saying "some people just don't get it."

The humanistic philosophy brings mathematics down to earth, makes it accessible psychologically, and increases the likelihood that someone can learn it, because it's just one of the things that people do. This is a matter of opinion; there's no data, no tests. But I'm convinced it is the case.

JB: How do you teach humanistic math?

HERSH: I'm going to sidestep that slightly, I'll tell you my conception of good math teaching. How this connects with the philosophy may be more tenuous.

The essential thing is interaction, communication. Only in math do you have this typical figure who was supposedly exemplified by Norbert Wiener. He walks into the classroom, doesn't look at the class, starts writing on the board, keeps writing until the hour is over and then departs, still without looking at the class.

A good math teacher starts with examples. He first asks the question and then gives the answer, instead of giving the answer without mentioning what the question was. He is alert to the body language and eye movements of the class. If they start rolling their eyes or leaning back, he will stop his proof or his calculation and force them somehow to respond, even to say "I don't get it." No math class is totally bad if the students are speaking up. And no math lecture is really good, no matter how beautiful, if it lets the audience become simply passive. Some of this applies to any kind of teaching, but math unfortunately is conducive to bad teaching.

It's so strange. Mathematical theorems may really be very useful. But nobody knows it. The teacher doen't mention it, the students don't know it. All they know is it's part of the course. That's inhuman, isn't it?

Here is an anecdote. I teach a class, which I invented myself, called Problem Solving for High School and Junior High School Teachers and Future Teachers. The idea is to get them into problem solving, having fun at it, feeling confident at it, in the hope that when they become teachers they will impart some of that to their class. The students had assignments; they were supposed to work on something and then come talk about it in class. One day I called for volunteers. No volunteers. I waited. Waited. Then, feeling very brave, I went to the back of the room and sat down and said nothing. For a while. And another while. Then a student went to the blackboard, and then another one.

It turned out to be a very good class. The key was that I was willing to shut up. The easy thing, which I had done hundreds of times, would have been to say, "Okay, I'll show it to you." That's perhaps the biggest difficulty for most, nearly all, teachers—not to talk so much. Be quiet. Don't think the world's coming to an end if there's silence for two or three minutes.

JB: Earlier you mentioned the word beauty. What's with beauty?

HERSH: Fortunately, I have an answer to that. My friend, Gian-Carlo Rota, dealt with that issue in his new book, *Indiscrete Thoughts*. He said the desire to say "How beautiful!" is associated with an insight. When something unclear or confusing suddenly fits together, that's beautiful. Maybe there are other situations that you would say are beautiful besides that, but I felt when I read that that he really had something. Because we talk about beauty all the time without being clear what we mean by it; it's purely subjective. But Rota came very close to it. Order out of confusion, simplicity out of complexity, understanding out of misunderstanding—that's mathematical beauty.