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Quantum Irreversibility And Noise In Mesoscopic Devices

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Abstract. Irreversibility in quantum processes in the Bunimovich stadium and rectangular billiard in the presence of noise is studied. For this purpose, a novel method based on Loschmidt echo and quantum trajectories, as defined in the de Broglie–Bohm formulation, is used. Our results indicate that the dynamics along the diagonal of the billiard is most sensitive to noise when the wave packet (or alternatively the quantum trajectories) collide with the corners of the billiard.

Keywords: Quantum irreversibility, quantum chaos, mesoscopic systems, electron transport
PACS: 05.45.Mt, 73.21.-b, 73.23.Ad

INTRODUCTION

In the last years there has been a growing interest in the study of chaotic phenomena in mesoscopic systems [1]. Technical developments have made possible the manufacturing of micro and nanostructures that allow charge transport without loss of electron phase coherence [2]. For example, the ballistic transport of electrons in semiconductor heterostructures has been recently studied experimentally [3, 4]. Moreover, when the system is confined in all dimensions and the sample is sufficiently clean, the correlations in the energy spectrum against variations of parameters, such as the sample geometry or the intensity of an external field, are very similar to those found in quantum billiards [5], thus making these simple systems excellent models for this kind of systems.

The classical dynamics in billiards is often chaotic [6], and "quantum chaos" is an active field of research [7]. Most problems in quantum chaos are of theoretical nature, falling within the category of semiclassical theories [8]. One important topic in quantum chaos that has received recently a great deal of attention is irreversibility [9, 10, 11]. This concept has been be related to the theory of chaos, which is classically interpreted as the result of exponential separation of trajectories. However, in quantum mechanics, this sensitivity is meaningless due to unitarity. For this reason, Peres proposed to consider the sensitivity to perturbations in quantum systems, as a mean to investigate the instability of quantum motion [12]. Such quantity, called Loschmidt echo (LE), in reference to the famous Loschmidt–Boltzmann correspondence, is defined as

$$M(t) = |\langle\psi|\exp(\mathrm{i}\hat{H}t)\exp(-\mathrm{i}\hat{H}_0 t)|\psi\rangle|^2, \tag{1}$$

($\hbar$ is set equal to unity throughout this paper). It measures the ability of a system to return to an initial state, $|\psi\rangle$, after a forward evolution with a Hamiltonian, $\hat{H}_0$, followed by a (imperfect) reverse evolution with a perturbed Hamiltonian $\hat{H} = \hat{H}_0 + \Sigma$. Alternatively, LE can be thought of as comparing the evolution of an initial state under different Hamiltonians (sensitivity to perturbations). For a given range of perturbation strengths, the LE decays exponentially at a rate given by the smallest quantity between the mean Lyapunov exponent and the level broadening following from the golden rule [9, 10].

Summarizing, two basic ingredients in the idea of LE are irreversibility and sensitivity to perturbations. However, the definition of LE's relies only on magnitudes evaluated at the end of the propagation process, an then is not able to provide any information about the involved history and the associated underlying physical mechanisms. (Actually, this can be a serious drawback as pointed out in [11].)

To avoid this problem, we propose here a new method to study irreversibility. It keeps the same basic philosophy of the original LE, but it is based in the use of quantum trajectories, as defined in the causal de Broglie–Bohm (BB) formulation of quantum mechanics [13]. This complementary quantum theory of motion [14] combines both the accuracy of the standard quantum description with intuitive explanations derived by a trajectory formalism, thus providing a powerful tool to understand the physics underlying microscopic phenomena (see for example Ref. [15]). This method has also an important additional advantage, since within its framework it is very easy to consider realistic perturbations, such as noise. This is particularly interesting in connection with billiards since, as stated before, they constitute ideal model for the transport of electrons in mesoscopic systems [7].

MODEL AND CALCULATIONS

The fundamental equations in the BB theory are derived by introducing the wave function in polar form, $\psi(\mathbf{r},t) = R(\mathbf{r},t)\, e^{iS(\mathbf{r},t)}$ into the time–dependent Schrödinger equation, obtaining the continuity and "quantum" Hamilton–Jacobi equations. The last one contains the so–called quantum potential which, together with V, determines the total forces acting on the system. Also, from it a quantum equation of motion can be defined: $m\dot{\mathbf{r}} = \nabla S$, from which quantum trajectories are obtained by numerical integration.

These orbits are used to define our measure of irreversibility. Starting from an ensemble of initial conditions reproducing the initial probability density distribution [16], we propagate them forward in time until a final value, t_f. We then propagate them backwards, introducing in the process a perturbation consisting of a kick given after every integration step. To avoid confusions, we will denote this "new" reversed time by τ. The effect of the kick consists of a displacement of the particle to a new position, randomly chosen within a given circle around the landing point. In this process we assume that the pilot wave function do not changes during the kick. Finally, the distance in configuration space, d, between both orbits is monitored as a function of τ. In this way, we compute a comparison, followed in time, of the unperturbed and perturbed dynamics of the system. From this, information about the mechanisms of irreversibility can be obtained. The idea behind this procedure is to mimic noise.

The systems that we have chosen to study are two degrees of freedom models consisting of a particle of mass 1/2 enclosed in a desymmetrized stadium billiard of radius $r = 1$ and area $1 + \pi/4$ with Dirichlet boundary conditions, and a rectangular billiard of side length $l = 1$ and the same area. The dynamics of the first system is known to be

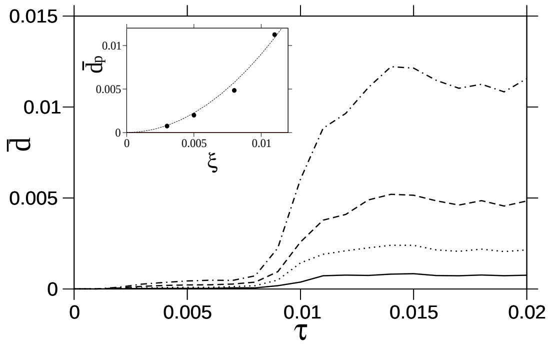

FIGURE 1. Averaged distance between forward and reversed quantum orbits as a function of the reverse time, τ, for different values of the perturbation strength parameter, ξ: 0.003 (full line), 0.005 (dotted line), 0.008 (dashed line), and 0.011 (dashed–dotted line). A value of $t_f = 0.02$ was used in the calculations. A fitting to a quadratic expression of the averaged distance at the plateaus (observed for $t > 0.015$) is shown in the inset.

classically ergodic and the second integrable.

RESULTS AND DISCUSSION

We first examine the dynamics of a wave packet running along the diagonal orbit going from the upper left (square) corner to the lower right (round) corner of the stadium at an energy value of E=2304, for which the period is $T = 0.0466$. As described in Ref. [17], the packet initially moves following the classical path with a slight dispersion. After the first rebound at the lower right corner ($t \simeq 0.009$), the packet spreads ($t \sim 0.019$), experiencing the well known defocalization effect described in [18]. Afterwards, the dispersed wave collides with the upper left corner, giving rise to a noticeable series of horizontal fringes ($t \simeq 0.028 - 0.047$). For subsequent times other rebounds take place, originating at $t \geq 0.066$ a complicated structure in the distribution of the quantum probability density.

In Fig. 1 we show the distance, $\overline{d}$, averaged over 20 quantum trajectories propagated, using a Gear stiff method with tolerance control, up to a final time of $t_f = 0.02$, for different values of the perturbation strength (parameterized by the kicking radius ξ). Notice that the origin of the reversed time, $\tau = 0$, corresponds to the final point of the forward propagation, t_f. As can be seen, the behavior of the four curves is the same. For the lowest times, $t \leq 0.008$, they grow very slowly and linearly. After that, the averaged distance increases dramatically in a cubic fashion for times up to the order of $t \sim 0.01$. And finally, for $0.015 < t < 0.020$, the values of $\overline{d}$ stabilize oscillating slightly around some sort of plateaus.

More interesting is the relationship existing between these results and the dynamics of the original packet [17]. The first part of the $\overline{d}$ vs. τ plots (showing a linear behavior) corresponds to a period of time in which the particle moves from the center of the stadium to just before the lower right corner. The packet here is in a semiclassical

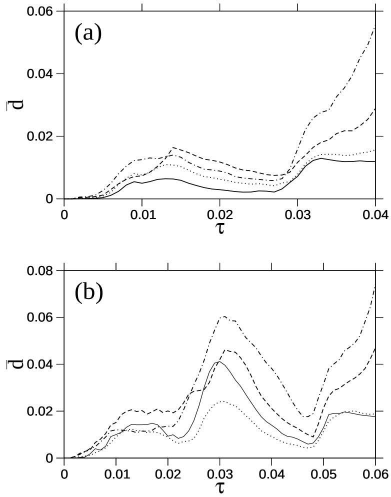

FIGURE 2. Averaged distance between forward and reversed quantum orbits as a function of the reverse time, τ, for the same values of the perturbation strength considered in Fig. 1 and values of $t_f = 0.04$ (a), and 0.06 (b).

regime, in which no much dispersion (irreversibility) is expected, in perfect agreement with our numerical findings. On the contrary, in the interval $\tau = 0.008 - 0.012$ the, now, perturbed dynamics include the bounce with the corner. Here a lot of interference of the packet with itself happens, and a great dispersion due to the perturbation takes place. This corresponds to the big, cubic growth observed in the computed values of $\overline{d}$. Also, for $\tau \sim 0.02$ we are at the echo, and then our results can be compared with those that would be obtained from the usual LE theory [9, 10]. This is done in the inset to the figure, where the functional form of $\overline{d}$ at the plateaus, $\overline{d}_p$, as a function of the magnitude of the perturbation, ξ, is shown. As it is seen, this dependence is quadratic with a very good accuracy, thus indicating that we are in a regime controlled by the Fermi golden rule [10]. Accordingly, we can conclude that noise–type perturbations like ours should be considered as generic from the point of view of the LE.

Let us discuss now what happens when longer final times, t_f, are considered, thus allowing a more complicated dynamics to enter into play. The results are shown in Fig. 2 for two values of this parameter. As can be seen similar results are obtained [17], i.e. big increases in $\overline{d}$ and then irreversibility, takes place only for those values of the time for which trajectories collide with the corners of the stadium, being there where the effect of the perturbation is stronger. Also, the plateaus in Fig. 2 are not completely flat, but rather they show a conspicuous decreasing behavior that, for example, in the case of Fig. 2(b) is quite important in the range $0.03 < \tau < 0.047$. The reason for this behavior can be understood if one considers that in these ranges of the reverse time, the packet is travelling from the upper left to the lower right corners, where it takes place a dynamics influenced by the self–focal point. This creates a quantum potential that forces the packet to return close to the original unperturbed path, which makes the separation

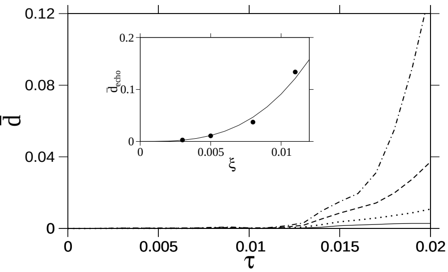

FIGURE 3. Same as Fig. 1 for a rectangular billiard of side length unity and area 1+π/4. The inset shows a cubic fit of the values of $\overline{d}$ at the echo time, $\tau = 0.02$, with the perturbation parameter, ξ.

$\overline{d}$ to go down.

To conclude this section, let us compare the results obtained so far with those corresponding to the rectangular stadium, for which the dynamics is completely integrable, and then regular and non–chaotic. The results are shown in Figs. 3 and 4. As can be seen the values for the trajectories separation is always greater than in the case of the (chaotic) stadium billiard considered before. This is in agreement with the results obtained by Prosen [19] for the standard LE. Moreover, the calculations shown in Figs. 3 and 4 do not show the presence of any plateau, as opposed to what happens in the case of the stadium. Finally, the values of $\overline{d}$ at the echo time increase cubically (see inset to Fig. 3) withe the parameter ξ controlling the perturbation strength, again behaving differently that in the case of the stadium billiard.

CONCLUDING REMARKS

In this paper we have presented a novel method to study irreversibility in quantum processes. This method is similar in spirit to the LE introduced by Peres [12], but recast in terms of quantum trajectories, as defined in BB theory. In this way, useful information about the history and mechanisms involved in the perturbation process are obtained.

This method has been applied to the stadium billiard with noise, a model which is adequate for electrons transport in mesoscopic cavities. Our principal results are

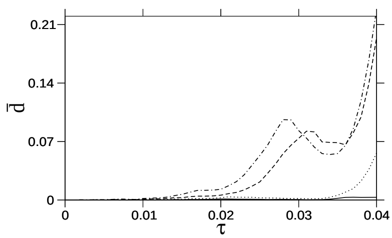

FIGURE 4. Same as Fig. 2(a) for a rectangular billiard of side length unity and area 1+π/4.

summarized as following. First, the dynamics is sensitive to the perturbation mainly when the particle is bouncing at the corners, points in which the trajectories separate from each other cubically in time, on average. Second, the noise–type perturbation that has been used in the present work behaves in a totally generic way, as it is indicated by the fact that the Fermi golden rule regime is found. Third, comparison with the results obtained for a rectangular billiard, in which the dynamics is regular, shows that in this later case the growth of $\overline{d}$ is greater than for the classically chaotic stadium billiard, in agreement with previous results obtained for the usual LE.

Finally, we should remark that in our calculation times beyond the Ehrenfest time have not been considered. For these longer times, large interference effects in the pilot wave guiding the quantum trajectories, and then the complexity of the associated quantum potential, is much higher and widespread over all configuration space. This point is very interesting and deserves further investigation, which will imply an enormous computational effort if a reasonable average over initial conditions is to be maintained.

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