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OPEN CHANNEL HELIUM FLOW DURING RUPTURE EVENT

W. M. Soyars and J. L. Schiller

Fermi National Accelerator Laboratory
Batavia, IL, 60510, USA

ABSTRACT

Open channel fluid flow principles are applied to understand how helium might behave in the event of a release or spill in a long tunnel enclosure. An example would be helium flowing along the Tevatron accelerator tunnel as a result of some rupture of a header system. Buoyant forces would drive the helium along the ceiling until a vent to atmosphere is reached. An analogy is water flowing up to and over the crest of a dam. The momentum and continuity equations will be used to calculate the depth of helium from the ceiling as a function of length down the tunnel, for rectangular and semicircular cross sections. Testing was conducted on a scale model of the Tevatron tunnel, which generally indicated agreement between the simplified predictions and observations.

VARIABLE DEFINITIONS

R	radius of channel	m
h	thickness of helium from the ceiling	m
Fr	Froude Number = $V/\sqrt{(gh)}$	--
Re	Reynolds number = $\rho V R_h/\mu$	--
w	width of helium for semicircle	m
b	width of helium for rectangle	m
Pa	wetted perimeter of helium to air surface	m
Pw	wetted perimeter of helium to wall surface	m
τ_a	air interface viscous shear stress	N
τ_w	wall interface viscous shear stress	N
x	distance along tunnel floor from the leak	m
Q	volumetric flow rate	m^3/s
A	cross sectional area	m^2
g	acceleration due to gravity	m/s^2
ρ	density	kg/m^3
θ	the angle, starting at the origin of the tunnel radius, that defines the interface of the tunnel wall and the helium	rad
R_h	hydraulic radius = 4A/Pw	m
μ	viscosity of fluid	Pa-s
$c_{f,L}$	friction coefficient over length	--

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INTRODUCTION

The simultaneous flow of two stratified fluids is a rich and challenging subject. While the complexities of this topic are the subject of many detailed studies, the scope of this paper is to consider a simplified theoretical approach by use of first principles and approximations to gain a more basic understanding. To this end, the helium flow will be approximated as an open channel flow. This analysis should in the future be expanded to consider more precise models of different fluids of different densities flowing in strata, such as presented in some advanced fluids texts [1,2]. For example, such future work could account for instabilities in the stratified shear layer, which are typical in two-phase flow systems but have been ignored here. It should be stated that the authors' prime goals are to begin by making some simple theoretical predictions and to compare them to experimental results. The intent is to offer some insight into a practical engineering application, to promote safer use of helium in long enclosures.

Open channel flow is more complicated than fully flowing conduits. Applying conservation of mass and momentum to open channel flow with a free surface of variable depth results in equations similar in form to those describing compressible fluid flow in ducts of gradually varying area. An important dimensionless number that characterizes channel flow is the Froude number, the ratio of inertial forces to gravitational forces. Froude number defines the flow as either low velocity, tranquil, "subcritical" (when Fr<1) or high velocity, shooting, rapid, "supercritical" (when Fr>1). Subcritical channel flow allows disturbances to travel upstream, so conditions upstream are affected by downstream conditions, whereas supercritical channel flow does not allow disturbances to reach upstream since any elementary wave is swept downstream. When flow is such that its velocity is just equal to the velocity of an elementary wave, the flow is said to be critical (Fr = 1) [3].

For helium flow from a leak that builds up a layer of helium along the top of a channel, we are interested in the subcritical branch of the flow solution. When the helium flow reaches a vertical penetration, the flow is at its critical height, where energy is minimized and Fr=1. This is analogous to water flow over a broad weir or dam. For any given volumetric flow rate and channel geometry, the critical height of the flow can be determined.

DERIVATION

Momentum Balance

Assume the flow is steady state and 1-dimensional in the x direction. Velocity is constant over the flow cross-sectional area. Any velocity deviations in the x direction are slight, so accelerations normal to x are small compared to gravitation constant (valid when slope, depth, and cross sectional area changes are gradual). Density is constant. Channel floor is flat. Initial conditions are known at the vent end of the channel, where flow is at its critical depth.

Apply linear momentum equation to the control volume of FIGURE 1.

$$\sum \vec{F} = \sum (\dot{m}\vec{V})_{out} - \sum (\dot{m}\vec{V})_{in} = \vec{F}_{weight} + \vec{F}_{hydro.press.in} - \vec{F}_{hydro.press.out} - \vec{F}_{wallvisc.} - \vec{F}_{airvisc.}$$

$$\rho V^2 dA + 2\rho AVdV = 0 - dF_{hydro.press} - \tau_w P_w dx - \tau_a P_a dx \quad (1)$$

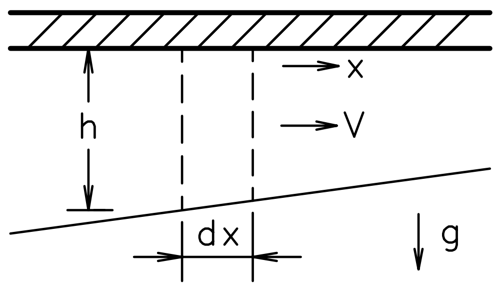

FIGURE 1. Sketch of He channel flow along top of ceiling

Consider the force of hydrostatic pressure, expressed as differential:

$$F_{hydro.press} = pA = \frac{1}{2}\rho ghA \rightarrow dF_{hydro.press.} = \frac{1}{2}\rho ghdA + \frac{1}{2}\rho gAdh \tag{2}$$

Consider mass continuity for a steady state control volume, expressed as differential:

$$(\rho VA)_{in} = (\rho VA)_{out} \rightarrow dV = \frac{-VdA}{A} \tag{3}$$

By substituting equations (2) and (3) into (1), the momentum equation becomes:

$$(1 - \frac{2V^2}{gh})dA + \frac{A}{h}dh = \frac{-2}{\rho gh}(\tau_w P_w + \tau_a P_a)dx \tag{4}$$

He-wall and He-air Boundary Viscous Shear Terms

In lieu of actual data for the channel in question, a useful approximation for the open channel flow's wall shear friction force can be obtained from pipe data by employing the hydraulic radius as the characteristic length [4]. For the Tevatron tunnel with flow rates of interest, a turbulent boundary layer is indicated. The average wall viscous shear stress can be approximated by [5]:

$$\tau_w = 0.079\,\mathrm{Re}_D^{-0.25}\frac{\rho V^2}{2} \tag{5}$$

The shear stress from the helium-air interface can be determined by modeling it as "Couette flow," a situation where the space between two parallel plates is separated by a gap filled with a constant viscosity fluid [6]. The upper plate moves steadily at a velocity V relative to the lower one. Pressure is everywhere constant. Apply "no-slip" conditions where the fluid meets a plate. Then by defining the height of the air in terms of tunnel height and helium depth, we can say:

$$\tau_a = \frac{\mu_{air}V}{R-h} \tag{6}$$

Rectangular Channel Mathematical Solution

For a rectangular channel with width b, equation (4) can be simplified:

$$\left(1 - \frac{V^2}{gh}\right)dh = -dx\left(\frac{\tau_w P_w}{gh\rho b} + \frac{\tau_a P_a}{gh\rho b}\right) \tag{7}$$

Separate variables, define some constants to simplify the expression, and analytically solve the differential equation by integration:

$$\int_{h_0}^{h(x)} \frac{h^3 + Z}{Bh^2 + Ch^3} dh = \int_0^x dx \qquad \text{where } Z = -\frac{Q^2}{b^2 g}; \;\; B = -\frac{\tau_w + \tau_a}{\rho g}; \;\; C = -\frac{2\tau_w}{\rho g b}$$

$$\frac{ZC^3 - B^3}{B^2C^2} \ln\left(\frac{B + Ch(x)}{B + Ch_0}\right) - \frac{ZC}{B^2} \ln\left(\frac{h(x)}{h_0}\right) - \frac{Z}{B}\left(\frac{1}{h(x)} - \frac{1}{h_0}\right) + \frac{1}{C}\left(h(x) - h_0\right) = x \qquad (8)$$

Semicircular Channel Mathematical Solution

In equation (4), apply the following geometry definitions:

$$A = \frac{\theta R^2}{2} - (R - h)\sqrt{2Rh - h^2}$$
$$P_w = \theta R$$
$$P_a = 2\sqrt{2Rh - h^2}$$
$$dA \approx w dh$$

Then the momentum equation (4) becomes:

$$\left[\frac{\left(1 - \frac{2V^2}{gh}\right) 2\sqrt{2Rh - h^2} + \frac{\theta R^2}{2h} - \frac{(R-h)\sqrt{2Rh - h^2}}{h}}{\frac{-2}{\rho g h}(\tau_w \theta R + 2\tau_a \sqrt{2Rh - h^2})}\right] dh = dx \qquad (9)$$

This formula cannot be analytically integrated. A spreadsheet is set up to numerically solve the differential equation.

ANALYTICAL RESULTS

Consider situations where the maximum flow length is taken to be 121.9 m, which approximates the longest distance in the Tevatron tunnel for helium to flow along the ceiling before reaching a vertical penetration. At this penetration, the helium flow is critical (Fr=1) and h can be calculated. Consider flow rates representing a range of rupture scenarios, and a variety of geometries, as shown in FIGURE 2. For the rectangular cross section case, the integrated differential equation, equation (8), was used to solve for helium depth h as a function of distance from the leak x. For the semicircular case, equation (9) is numerically solved. It should be recalled that this theory assumes perfect stratification of helium and air.

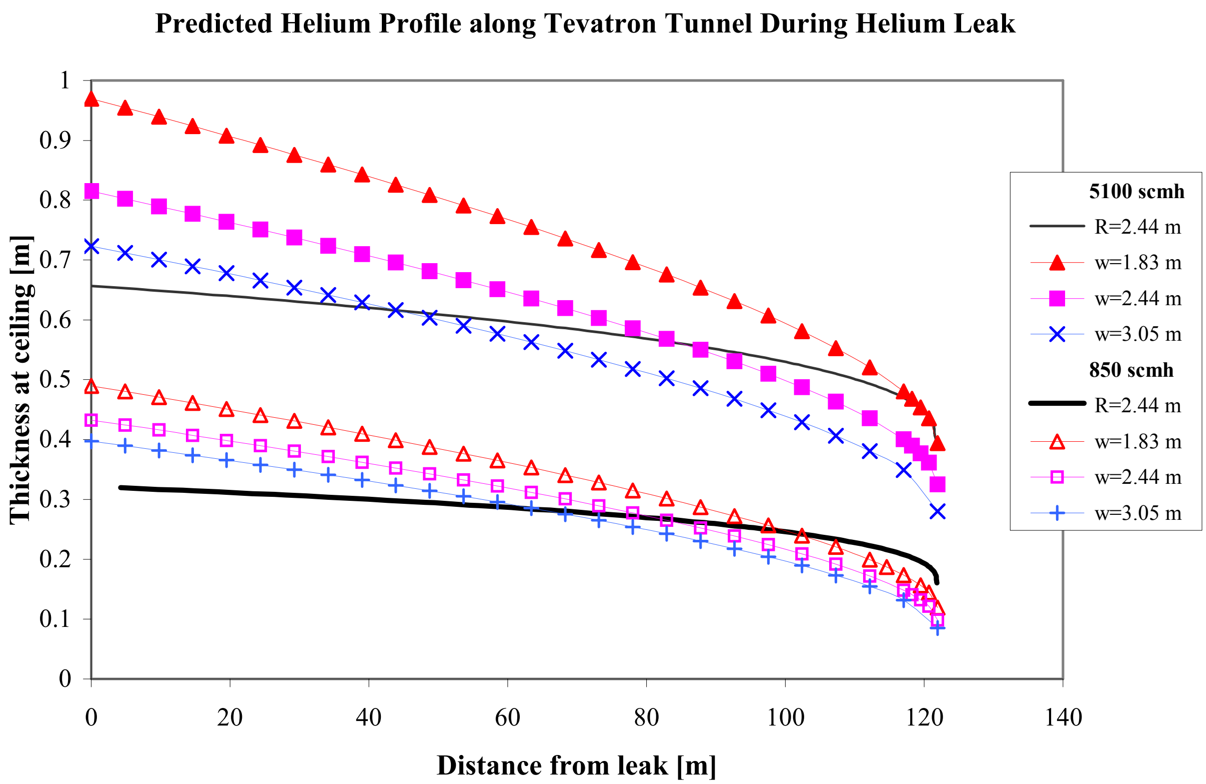

FIGURE 2. Predictions with Semi-Circular and Rectangular Geometries at Flow Rates of 5100 m^3/hr and 850 m^3/hr. Note, for rectangular cross-sections, the height of air is taken to be a constant 3.05 m.

EXPERIMENTAL DESIGN

To experimentally investigate the validity of the theoretical results, a 1/12 scale model of the tunnel was built with a radius of 0.20 m and length of 9.7 m. FIGURE 3 shows the model, a helium gas cylinder, and the author taking a helium concentration reading.

In the Tevatron, the helium leak rates of interest would produce a turbulent boundary layer at the wall-helium interface, where the shear stress is primarily dependent on Reynolds number, a function of ρ, μ, V, and R_h. Since the fluids used are fixed, the latter two variables are of concern in designing the model. The model radius was maximized, considering space and helium supply constraints. At this given size, turbulent flow could only be achieved by a flow rate that completely filled the model with helium for some portion of the length, which does not accurately portray the Tevatron open channel flow conditions. A larger scale model, which could produce a turbulent boundary layer at the wall, was prohibitively uneconomical. Therefore, laminar boundary conditions at the model's wall-helium interface were tolerated, and when predicting how the experiment would behave, the differential equation was modified so that the wall shear stress term, as given in equation (5), was based on laminar flow:

$$\tau_w = 16\,\mathrm{Re}_D^{-1}\frac{\rho V^2}{2} \qquad (10)$$

The model is capped at the helium supply end to force the flow in one direction. Oxygen concentration data points are taken at various distances from the helium source by inserting a 1.8 mm diameter probe and tube assembly up through the floor of the model at various heights. An Industrial Scientific TMX412 oxygen meter and SP400 pump (with rated speed of 1 l/min air) samples flow to record oxygen concentration. The probe has a 90-degree bend at the measuring end to minimize disruption to the air-helium interface layer.

DATA COLLECTION

Oxygen concentration measurements were taken from the bottom of the model floor towards the top, into areas of decreasing concentration. Whether the probe pointed into or away from the flow direction had no significant effect on the observations.

The supply tubing allowed helium to be injected either horizontally or vertically. There was a concern that vertical injection could have introduced swirling that tended to mix helium and air. However, with the exception of very near the injection point, the data taken during vertical and horizontal flow injection are very similar.

The raw oxygen meter output for this style sensor had to be corrected for carrier gas influence. This recognizes molecular weight affects that occur when helium is present, as opposed to nitrogen, upon which the standard meter response is based. In-house calibration of this particular sensor by the Beams Division Cryogenics Department allowed actual oxygen concentration to be determined.

FIGURE 3. Experimental model of Tevatron tunnel

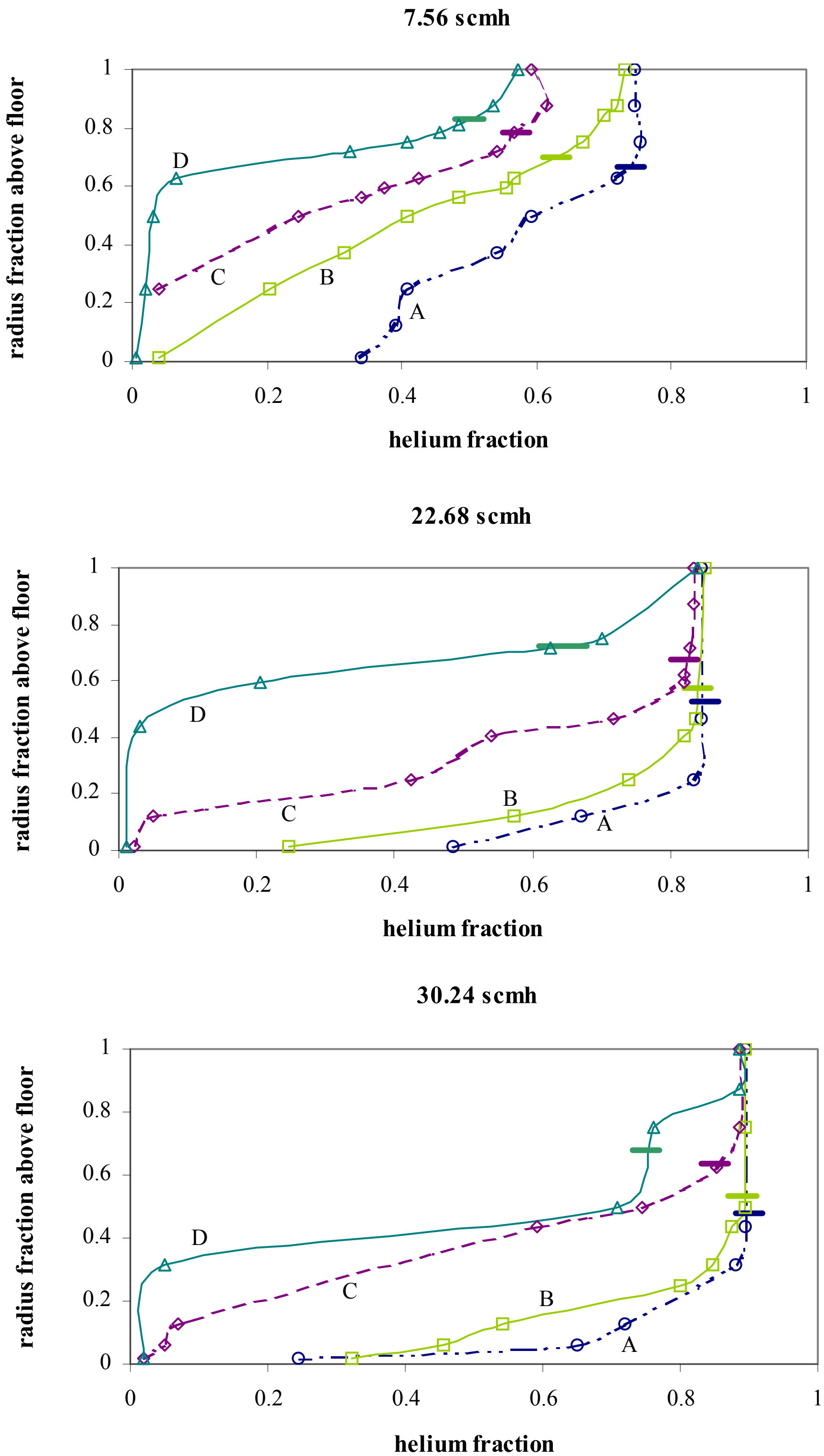

FIGURE 4. Helium concentration at various locations in the 20.3 cm radius model at distances of (A) 1.22m, (B) 5.18m, (C) 9.14m, and (D) 9.70m from source. Horizontal dash indicates theoretical interface prediction.

EXPERIMENTAL RESULTS

Oxygen concentrations were measured at different heights along the length of the mock-up tunnel, and for various flows. FIGURE 4 shows these observations. Each line represents a series of concentration measurements taken at a fixed distance from the flow inlet. The predicted He-air interface location from theory is indicated by a bold, horizontal dash crossing each line.

Since one of the theoretical assumptions was perfect stratification, the theory predicts helium fraction of 1 at heights above and helium fraction of 0 at heights below the theoretical interface line. Observations, however, deviated significantly from the stratified situation. Even at the top of the tunnel, pure helium was never achieved. Pure air was seen only near the bottom for the further distance cases. Toward the middle and especially the beginning, where helium is seen even very near the bottom, buoyant forces have not yet driven all the helium upward. Injection vortices, diffusion, turbulent eddies, and local instabilities in the interface are important mixing forces that are not considered in this theoretical analysis. Nevertheless, the simple theoretical predictions tend to indicate where the slope of the height versus helium fraction curve begins to characteristically change. This knee in the curve is a reasonable location to define as the approximate, experimentally determined, helium-air interface.

CONCLUSIONS

Models to predict the channel flow helium depth along a semicircular and rectangular length have been developed. This work is intended to more fully understand the behavior of helium system rupture flows in the Tevatron tunnel, some distance away from the local spill region. Theoretical results are based on a simple model which ignores factors such as wave interactions and instabilities between flow strata. Further improvements in the theoretical model are left for future work.

Predictions were made and observations taken on a small-scale mockup of the tunnel. In the analysis, an assumption was made that perfect stratification occurred, with pure helium in the upper portion and pure air in the lower. Experiments showed that buoyant forces were not completely dominant. Other forces, such as injection vortices, diffusion, turbulent eddies, and local instabilities tended to mix helium and air. Although mixing cannot be disregarded, the predicted interface between air and helium matched observations, especially at further distances for the helium source. Therefore, it may be reasonable to apply this theory to approximate flows resulting from unexpected, large helium releases in the Tevatron and similar tunnels.

REFERENCES

1. Yih, C. -S., *Stratified Flows*, Academic Press., New York, 1980.
2. Joseph, D. D., and Renardy, Y.Y., *Fundamentals of two-fluid dynamics, Part I: Mathematical Theory and applications*, Springer Verlag., New York, 1993.
3. Sabersky, R.H., Acosta, A.J.and Hauptmann, E.G., *Fluid Flow*, Macmillan Publishing Co., Inc., New York, 1971, pp. 357-360.
4. Sabersky, Acosta, and Hauptmann, pp 361-362.
5. Incropera, F. P. and Dewitt, D. P., *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, Inc., New York, 1985, p. 372.
6. Sabersky, Acosta, and Hauptmann, pp 10-11.