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Research Article

Evolving Center-Vortex Loops

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We consider coarse-graining applied to nonselfintersecting planar centervortex loops as they emerge in the confining phase of an SU(2) Yang-Mills theory. Well-established properties of planar curve-shrinking predict that a suitably defined, geometric effective action exhibits (mean-field) critical behavior when the conformal limit of circular points is reached. This suggests the existence of an asymptotic mass gap. We demonstrate that the initially sharp mean center-of-mass position in a given ensemble of curves develops a variance under the flow as is the case for a position eigenstate in free-particle quantum mechanics under unitary time evolution. A possible application of these concepts is an approach to high-T_c superconductivity based (a) on the nonlocal nature of the electron (1 fold selfintersecting center-vortex loop) and (b) on planar curve-shrinking flow representing the decrease in thermal noise in a cooling cuprate.

1. Introduction

The problem of how a confining ground state emerges in 4D nonabelian Yang-Mills theory was addressed by ’t Hooft in terms of the definition of an order parameter that is dual to the Wilson loop [1]. The lattice-gauge-theory measurement as well as analytical model approaches to the ’t Hooft loop expectation was performed by several authors starting shortly thereafter [2–7]. The according confinement scenario is a condensation of magnetic center vortices which induces narrow electric flux lines in between color-electric sources granting a potential which rises sufficiently rapid with an increasing distance of separation. Due to the absence of magnetic center monopoles the associated center vortices form closed flux loops.

Four-dimensional SU(2) Yang-Mills theory occurs in three phases: a deconfining, a preconfining, and a confining one. While the former two phases possess propagating gauge fields, a complete decoupling thereof takes place at a Hagedorn transition towards the confining phase [8–12]. Namely, by the decay of a preconfining ground state, consisting of collapsing magnetic (w.r.t. the gauge fields in the defining SU(2) Yang-Mills Lagrangian) flux lines of finite core-size d, see also [13], into nonselfintersecting or selfintersecting center-vortex loops [1], the mass m_D of the dual gauge field diverges. This, in turn, implies $d \to 0$. As a consequence, center-vortex loops (CVLs) with nonvanishing selfintersection number N become stable solitons in isolation. These solitons are classified according to their topology and center charge. That is, for $d \to 0$, the region of negative pressure P is confined to the vanishing vortex core, and the soliton becomes a particle-like ($P = 0$) excitation whose stability is in addition assured topologically by its selfintersection(s).

The purpose of our paper is to investigate the sector with $N = 0$ in some detail. Topologically, there is no reason for the stability of this sector's excitations, and we will argue that on average, and as a consequence of a noisy environment a planar CVL with $N = 0$ shrinks to nothingness within a finite "time." Here the role of "time" is played by a variable measuring the decrease of externally provided resolving power applied to the system. By "planar CVL" we mean an embedding of the $N = 0$ soliton into a 2D flat and spatial plane. For an isolated SU(2) Yang-Mills theory the role of the environment is played by the sectors with $N > 0$. If the SU(2) theory under consideration is part of a world subject to additional gauge symmetries, then a portion of such an environment arises from a mixing with these theories. In any case, a planar CVL at finite length L is acquiring mass by frequent interactions with the environment after it was generated by a process subject to an inherent, finite resolution Q_0. At the same time, the CVL starts shrinking towards a circular point. The latter becomes unresolvable starting at some finite resolution Q_*. That is, all properties that are related to the existence of extended lines of center flux, observable for $Q_0 \geq Q > Q_*$, do not occur for $Q \leq Q_*$, and the CVL vanishes from the spectrum of confining SU(2) Quantum Yang-Mills theory. Since CVLs with $N > 0$ have a finite mass (positive-integer multiples of the Yang-Mills scale Λ), we "observe" a gap in the mass spectrum of the theory when probing the system with resolution $Q \leq Q_*$.

The reader may object that the restriction of the shrinking dynamics to a 2D plane does not justify the conclusion about the aforementioned occurrence of a mass gap since CVLs evolve in 3D. However, according to the work of [14, 15] and for low-energy SU(2) Yang-Mills theory the assumption of planar, closed, and smooth curves accurately describing the late-"time" behavior of the evolution of any given center-vortex loop with $N = 0$ is valid because the CVL becomes planar after a finite "time" under the curve shrinking equation. Thus, to discuss the approach of the round-point limit (mass-gap property) we may pick planar initial conditions and consider the subsequent evolution in the plane. The formation of a singularity (a loop pinching off to form a cusp) is a planar phenomenon [15] which, however, is excluded to occur in low-energy, that is, weakly resolved SU(2) Yang-Mills theory because the energy required to form and resolve such a singularity is not available.

Notice that by embedding an $N = 0$ CVL of an isolated SU(2) Yang-Mills theory into a flat 2D surface at $m_D < \infty$, $d > 0$, a hypothetical observer measuring a positive (negative) curvature along a given sector of the vortex line experiences more (less) negative pressure in the intermediate vicinity of this curve sector leading to this sector's motion towards (away from) the observer, see Figure 1. The speed of this motion is a monotonic function of curvature. On average, this shrinks the CVL. Alternatively, one may globally consider the limit $m_D \to \infty$, $d \to 0$, that is, the confining phase of an SU(2) Yang-Mills theory, but now take into account the effects of an environment which locally relaxes this limit (by collisions) and thus also induces curve-shrinking. One

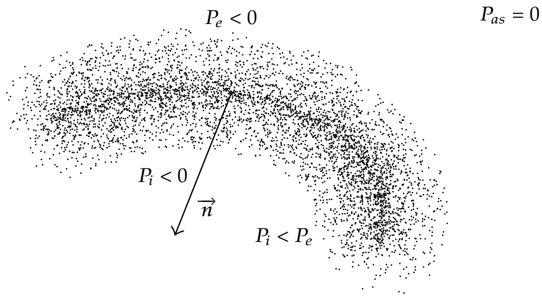


Figure 1: Highly space-resolved snapshot of an $N = 0$ CVL curve-sector. The pressure P_i in the region pointed to by the normal vector $\mathbf{n}$ is more negative than the pressure P_e, thus leading to a motion of the sector along $\mathbf{n}$.

possibility to describe this situation is by the following flow equation in the (dimensionless) parameter τ

$$\partial_\tau \mathbf{x} = \frac{1}{\sigma}\,\partial_s^2 \mathbf{x}, \tag{1.1}$$

where s is arc length, $\mathbf{x}$ is a point on the planar CVL, and σ is a string tension effectively expressing the distortions induced by the environment. After rescaling, $\widehat{x} \equiv \sqrt{\sigma}\mathbf{x}$, $\xi = \sqrt{\sigma}s$, (1.1) assumes the following form:

$$\partial_\tau \widehat{x}(u,\tau) = \partial_\xi^2 \widehat{x} = k(u,\tau)\mathbf{n}(u,\tau), \tag{1.2}$$

where u is a (dimensionless) curve parameter ($d\xi = |\partial_u \widehat{x}| du$), $\mathbf{n}$ the (inward-pointing) Euclidean unit normal, and k the scalar curvature, defined as

$$k \equiv \left|\partial_\xi^2 \widehat{x}\right| = \left|\frac{1}{|\partial_u \widehat{x}|}\partial_u\left(\frac{1}{|\partial_u \widehat{x}|}\partial_u \widehat{x}\right)\right|, \tag{1.3}$$

$|\mathbf{v}| \equiv \sqrt{\mathbf{v}\cdot\mathbf{v}}$, and $\mathbf{v}\cdot\mathbf{w}$ denotes the Euclidean scalar product of the vectors $\mathbf{v}$ and $\mathbf{w}$ lying in the plane. In the following, we resort to a slight abuse of notation by using the same symbol $\widehat{x}$ for the functional dependence on u or ξ.

It is worth mentioning that (1.2) expresses a special case of the local condition that the rate of decrease of the (dimensionless) curve length $L(\tau) = \int_0^{L(\tau)} d\xi = \int_0^{2\pi} du\, |\partial_u \widehat{x}(u,\tau)|$ is maximal w.r.t. a variation of the direction of the velocity $\partial_\tau \widehat{x}$ of a given point on the curve at fixed $|\partial_\tau \widehat{x}|$ [16]:

$$\frac{dL(\tau)}{d\tau} = -\int_0^{L(\tau)} d\xi\, k\mathbf{n}\cdot\partial_\tau \widehat{x}. \tag{1.4}$$

The 1D heat equation (1.2) is well understood mathematically [17, 18]. The evolution of a particular initial configuration of a planar CVL ($\tau = 0$) into the round-point limit ($\tau = T$) is depicted in Figure 2.

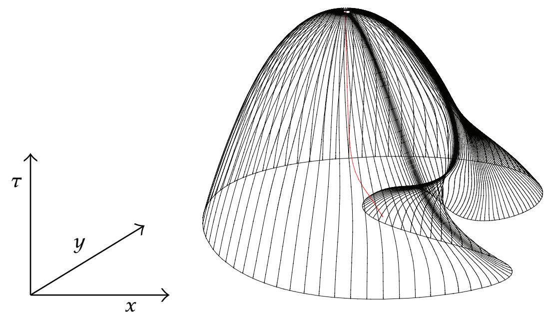


Figure 2: Plot of the evolution of a planar CVL under (1.2). The thick central line depicts the graph of the CVL's "center of mass." The flow is started at $\tau = 0$ and ends at $\tau = T$.

The present work interprets the shrinking of closed and 2D flat-embedded (planar) curves as a Wilsonian renormalization group evolution governed by an effective action, which is defined purely in geometric terms. In the presence of an environment represented by the parameter σ, this action possesses a natural decomposition into a conformal and a nonconformal factor. One of our goals is to show that the transition to the conformal limit of vanishing mean curve length really is a critical phenomenon characterized by a mean-field exponent if a suitable parameterization of the effective action is used. To see this, various initial conditions are chosen to generate an ensemble whose partition function is invariant under curve shrinking. A (second-order) phase transition is characterized by the critical behavior of the coefficient associated with the nonconformal factor in the effective action. That is, in the presence of an environment the (nearly massless) $N = 0$ sector in confining SU(2) Yang-Mills dynamics, generated during the Hagedorn transition, practically disappears after a finite "time" leading to an asymptotic mass gap. We also believe that $N = 0$ CVLs play the role of Majorana neutrinos in physics: their disappearance after a finite time and the absence of antiparticles would be manifestations of lepton-number violation [19–21] forbidden in the present standard model of particle physics, see [22] for a discussion of experimental signatures in this context.

In Section 2, we provide information on proven properties of curve-shrinking evolution. The philosophy underlying the statistics of geometric fluctuations in the $N = 0$ sector is elucidated in Section 3. In Section 4, we present our results for the renormalization-group flow of the effective action, give an interpretation, and compute the evolution of a local quantity. Finally, in Section 5 we summarize our work and give an outlook to the $N = 1$ case which we suspect to be relevant to high T_c superconductivity.

2. Prerequisites on Mathematical Results

In this section, we provide knowledge on the properties of the shrinking of embedded (nonselfintersecting) curves in a plane [17, 18]. It is important to stress that only for curve-shrinking in a plane are the following results valid. To restrict the motion of a CVL to a plane, however, is justified when physics close to the round-point limit is considered [14, 15], see Section 1.

The properties of the τ-evolution of smooth, embedded, and closed curves $\hat{x}(u,\tau)$ subject to (1.2) were investigated in [17] for the purely convex case and in [18] for the general case. The main result of [18] is that an embedded curve with finitely many points of inflection remains embedded and smooth when evolving under (1.2) and that such a curve flows to a circular point for $\tau \nearrow T$, where $0 < T < \infty$. That is, asymptotically the curve converges (w.r.t. the C^∞-norm) to a shrinking circle: $\lim_{\tau\to T} L(\tau) = 0$ and $\lim_{\tau\to T} A(\tau) = 0$, A being the (dimensionless) area enclosed by the curve, such that the isoperimetric ratio $L^2(\tau)/A(\tau)$ approaches the value 4π from above. For later use, we present the following two identities, see Lemmas 3.1.2 and 3.1.7 in [17]:

$$\partial_\tau L = -\int_0^L d\xi\ k^2 = -\int_0^{2\pi} du\ |\partial_u \hat{x}| k^2, \tag{2.1}$$

$$\partial_\tau A = -2\pi. \tag{2.2}$$

Setting $A(\tau = 0) \equiv A_0$, the solution to (2.2) is

$$\frac{A(\tau)}{A_0} = 1 - \frac{2\pi\tau}{A_0}. \tag{2.3}$$

By virtue of (2.3) the critical value T is related to A_0 as

$$T = \frac{A_0}{2\pi}. \tag{2.4}$$

3. Geometric Partition Function

We now wish to interpret curve-shrinking as a Wilsonian renormalization-group flow taking place in the $N = 0$ planar CVL sector. A partition function, defined as a statistical average (according to a suitably defined weight) over $N = 0$ CVLs, is to be left invariant under a decrease of the resolution determined by the flow parameter τ. Notice that, physically, τ is interpreted as a strictly monotonic decreasing (dimensionless) function of a ratio Q/Q_0 where Q (Q_0) are mass scales associated with an actual (initial) resolution applied to the system.

To devise a geometric ansatz for the effective action $S = S[\hat{x}(\tau)]$, which is a functional of the curve $\hat{x}$ representable in terms of integrals over local densities in ξ (reparametrization invariance), the following reflection on symmetries is in order. (i) scaling symmetry $\hat{x} \to \lambda\hat{x}$, $\lambda \in \mathbf{R}_+$: For both, $\lambda \to \infty$ and $\lambda \to 0$, implying $\lambda L \to \infty$ and $\lambda L \to 0$ at fixed L, the action S should be invariant under further finite rescalings (decoupling of the fixed length scale $\sigma^{-1/2}$), (ii) Euclidean point symmetry of the plane (rotations, translations and reflections about a given axis): Sufficient but not necessary for this is a representation of S in terms of integrals over scalar densities w.r.t. these symmetries. That is, the action density should be expressible as a series involving products of Euclidean scalar products of $(\partial^n/\partial\xi^n)\hat{x}$, $n \in \mathbf{N}_+$, or constancy. However, an exceptional scalar integral over a nonscalar density can be devised. Consider the area A, calculated as

$$A = \left| \frac{1}{2} \int_0^{2\pi} d\xi\ \hat{x} \cdot \mathbf{n} \right|. \tag{3.1}$$

The density $\hat{x} \cdot \mathbf{n}$ in (3.1) is not a scalar under translations.

We now resort to a factorization ansatz as

$$S = F_c \times F_{\mathrm{nc}}, \tag{3.2}$$

where in addition to Euclidean point symmetry F_c (F_{nc}) is (is not) invariant under $\hat{x} \to \lambda\hat{x}$. In principle, infinitely many operators can be defined to contribute to F_c. Since the evolution generates circles for $\tau \nearrow T$ higher derivatives of k w.r.t. ξ rapidly converge to zero [17]. We expect this to be true also for Euclidean scalar products involving higher derivatives $(\partial^n/\partial\xi^n)\hat{x}$. To yield conformally invariant expressions, such integrals need to be multiplied by powers of $\sqrt{A}$ and/or L or the inverse of integrals involving lower derivatives. At this stage, we are not capable of constraining the expansion in derivatives by additional physical or mathematical arguments. To be pragmatic, we simply set F_c equal to the isoperimetric ratio:

$$F_c(\tau) \equiv \frac{L(\tau)^2}{A(\tau)}. \tag{3.3}$$

We conceive the nonconformal factor F_{nc} in S as a formal expansion in inverse powers of L. Since we regard the renormalization-group evolution of the effective action as induced by the flow of an ensemble of curves, where the evolution of each member is dictated by (1.2), we allow for an explicit τ dependence of the coefficient c of the lowest nontrivial power $1/L$. In principle, this sums up the contribution to F_{nc} of certain higher-power operators which do not exhibit an explicit τ dependence. Hence, we make the following ansatz

$$F_{\mathrm{nc}}(\tau) = 1 + \frac{c(\tau)}{L(\tau)}. \tag{3.4}$$

The initial value $c(\tau = 0)$ is determined from a physical boundary condition such as the mean length $\overline{L}$ at $\tau = 0$ which determines the mean mass $\overline{m}$ of a CVL as $\overline{m} = \sigma\overline{L}$.

For latter use, we investigate the behavior of $F_{\mathrm{nc}}(\tau)$ as $\tau \nearrow T$ for an ensemble consisting of a single curve only and require the independence of the "partition function" under changes in τ. Integrating (2.1) in the vicinity of $\tau = T$ under the boundary condition that $L(\tau = T) = 0$, we have

$$L(\tau) = \sqrt{8\pi}\,\sqrt{T-\tau}. \tag{3.5}$$

Since $F_c(\tau \nearrow T) = 4\pi$ independence of the "partition function" under the flow in τ implies that

$$c(\tau) \propto \sqrt{T-\tau}. \tag{3.6}$$

That is, F_{nc} approaches constancy for $\tau \nearrow T$ which brings us back to the conformal limit by a finite renormalization of the conformal part $\int F_c$ of the action. In this parameterization of S, $c(\tau)$ can thus be regarded as an order parameter for conformal symmetry with mean-field critical exponent.

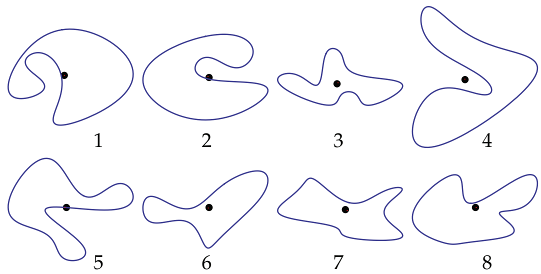


Figure 3: Initial curves contributing to the ensembles E_M, see text. Points locate the respective positions of the centers of mass.

4. Renormalization-Group Flow

4.1. Effective Action

Let us now numerically investigate the effective action $S[\hat{x}(\tau)]$ resulting from a partition function Z w.r.t. a nontrivial ensemble E. The latter is defined as the average

$$Z = \sum_i \exp(-S[\hat{x}_i(\tau)]) \tag{4.1}$$

over the ensemble $E = \{\hat{x}_1, \ldots\}$. Let us denote by E_M an ensemble consisting of M curves where E_M is obtained from E_{M-1} by adding a new curve $\hat{x}_M(u,\tau)$. In Figure 3, eight initial curves are depicted which in this way generate the ensembles E_M for $M = 1, \ldots, 8$.

We are interested in a situation where all curves in E_M shrink to a point at the same value $\tau = T$. Because of (2.3) and (2.4), we thus demand that at $\tau = 0$ all curves in E_M initially have the same area A_0. The effective action S in (3.2) (when associated with the ensemble E_M we will denote it as S_M) is determined by the function $c_M(\tau)$, compared with (3.4), whose flow follows from the requirement of τ-independence of Z_M:

$$\frac{d}{d\tau} Z_M = 0. \tag{4.2}$$

This is an implicit, first-order ordinary differential equation for $c(\tau)$ which needs to be supplemented with an initial condition $c_{0,M} = c_M(\tau = 0)$. A natural initial condition is to demand that the quantity:

$$\overline{L}_M(\tau = 0) \equiv \frac{1}{Z_M(\tau = 0)} \sum_{i=1}^{M} L[\hat{x}_i(\tau = 0)] \exp(-S_M[\hat{x}_i(\tau = 0)]) \tag{4.3}$$

coincides with the arithmetic mean $\widetilde{L}_M(\tau = 0)$ defined as

$$\widetilde{L}_M(\tau = 0) \equiv \frac{1}{M} \sum_{i=1}^{M} L[\hat{x}_i(\tau = 0)]. \tag{4.4}$$

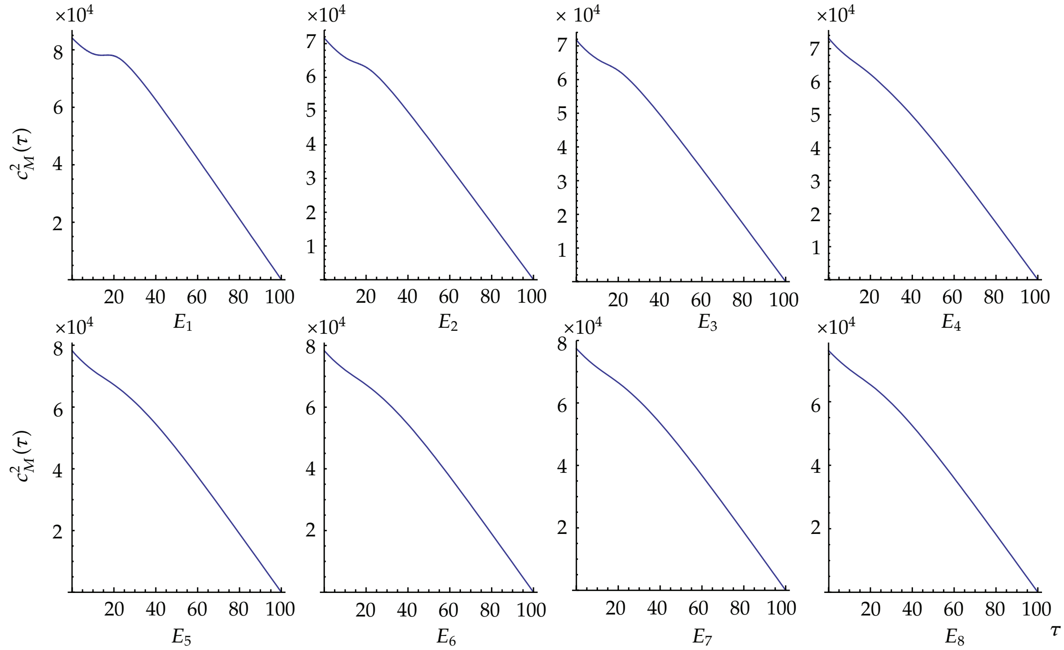


Figure 4: The square of the coefficient $c_M(\tau)$ entering the effective action of (3.2) by virtue of (3.4) for various ensemble sizes. Notice the early onset of the linear drop of $c_M^2(\tau)$ and the saturation in M for $M \geq 5$. The slope of $c_M^2(\tau)$ near $\tau = T$ does not depend on $c_{0,M}^2 \equiv c_M^2(\tau = 0)$ and thus not on the initial choice of $\overline{L}$.

From $\overline{L}_M(\tau = 0) = \widetilde{L}_M(\tau = 0)$ a value for $c_{0,M}$ follows. We also have considered a modified factor $F_{\mathrm{nc}}(\tau) = 1 + (c(\tau)/A(\tau))$ in (3.2). In this case the choice of initial condition $\overline{L}_M(\tau = 0) = \widetilde{L}_M(\tau = 0)$ leads to $F_{\mathrm{nc}}(\tau) \equiv 0$. While the geometric effective action thus is profoundly different for such a modification of $F_{\mathrm{nc}}(\tau)$ physical results such as the evolution of the variance of center-of-mass position agree remarkably well, see Section 4.2. That is, the geometric effective action itself is not a physical object. Rather, going from one ansatz for S_M to another describes a particular way of redistributing the weight in the ensemble which seems to have no significant impact on the physics. This is in contrast to quantum field theory and conventional statistical mechanics, where the action in principle is related to the physical properties of a given member of the ensemble.

For the curves depicted in Figure 3, we make the convention that $A_0 \equiv 2\pi \times 100$. It then follows that $T = 100$ by (2.4). The dependence $c_M^2(\tau)$ is plotted in Figure 4. According to Figure 4, it seems that the larger the ensemble the closer $c_M^2(\tau)$ to the evolution of a single circle of initial radius $R = \sqrt{A_0/\pi}$. That is, for growing M the function $c_M^2(\tau)$ approaches the form:

$$c_{\mathrm{as},M}^2(\tau) = k_M(T - \tau), \tag{4.5}$$

where the slope k_M depends on the strength of deviation from circles of the representatives in the ensemble E_M at $\tau = 0$, that is, on the variance ΔL_M at a given value A_0. Physically speaking, the value $\tau = 0$ is associated with a certain initial resolution of the measuring devise (the strictly monotonic function $\tau(Q)$, Q being a physical scale such as energy or momentum transfer, expresses the characteristics of the measuring device and the measuring process), the value of A_0 describes the strength of noise associated with the environment (A_0 determines how fast the conformal limit of circular points is reached), and the values of $c_{0,M}$ and k_M, see (4.5), are associated with the conditions at which the to-be-coarse-grained system is prepared. Notice that this interpretation is valid for the action $S_M = (L(\tau)^2/A(\tau))(1+c_M(\tau)/L(\tau))$ only.

4.2. Statistical Uncertainty of Center of Mass Position

We are now in a position to compute the flow of a more local "observable," namely, the mean "center-of-mass" (COM) position in a given ensemble and the statistical variance of the COM position. The COM position $\hat{x}_{\mathrm{COM}}$ of a given curve $\hat{x}(\xi,\tau)$ is defined as

$$\hat{x}_{\mathrm{COM}}(\tau) = \frac{1}{L(\tau)} \int_0^{L(\tau)} d\xi \hat{x}(\xi,\tau). \tag{4.6}$$

We will below present only results on the statistical variance of the COM position.

Let as assume that at $\tau = 0$, the ensembles E_M are modified such that a translation is applied to each representative letting its COM position coincide with the origin. Recall that such a modification $E_M \rightarrow E'_M$ does not alter the (effective) action (Euclidean point symmetry). That is, at $\tau = 0$ the statistical variance in the position of the COM is prepared to be nil, physically corresponding to an infinite resolution applied to the system by the measuring device.

The mean COM position $\overline{\hat{x}}_{\mathrm{COM}}$ over ensemble E'_M is defined as

$$\overline{\hat{x}}_{\mathrm{COM}}(\tau) \equiv \frac{1}{Z_M} \sum_{i=1}^{M} \hat{x}_{\mathrm{COM},i}(\tau) \exp(-S_M[\hat{x}_i(\tau)]). \tag{4.7}$$

The scalar statistical deviation $\Delta_{M,\mathrm{COM}}$ of $\overline{\hat{x}}_{\mathrm{COM}}$ over the ensemble E'_M is defined as

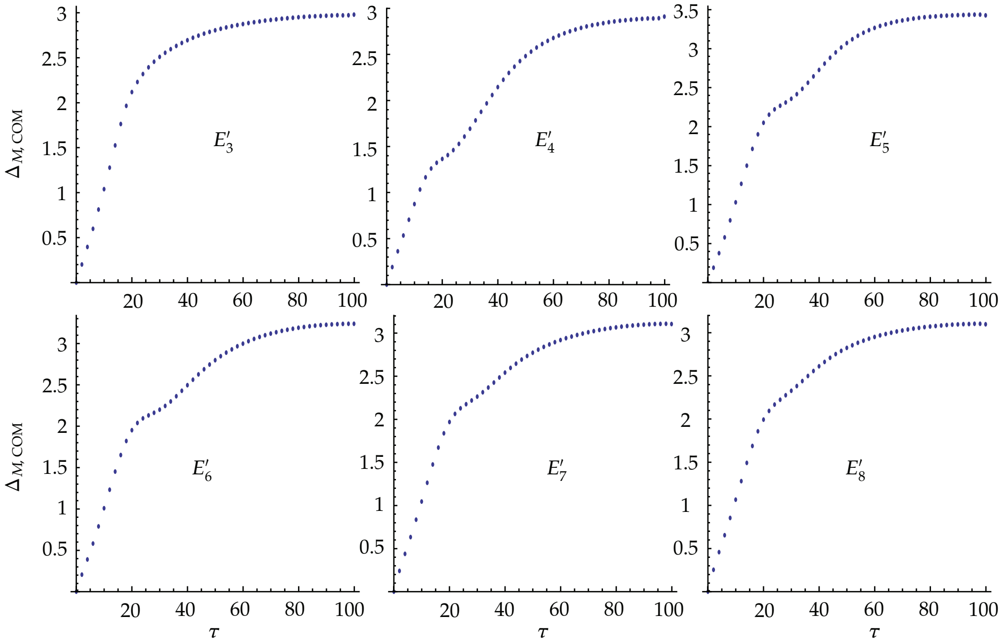
$$\Delta_{M,\mathrm{COM}}(\tau) \equiv \sqrt{\mathrm{var}_{M,\mathrm{COM};x}(\tau) + \mathrm{var}_{M,\mathrm{COM};y}(\tau)}, \tag{4.8}$$

where

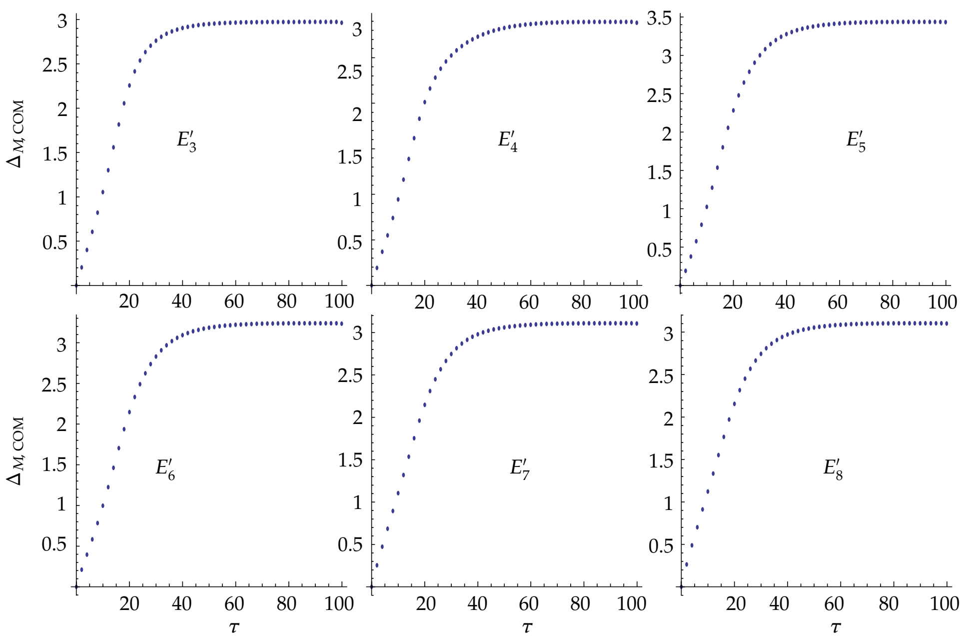
$$\begin{aligned} \mathrm{var}_{M,\mathrm{COM};x} &\equiv \frac{1}{Z_M} \sum_{i=1}^{M} \left(x_{\mathrm{COM},i}(\tau) - \overline{x}_{\mathrm{COM}}(\tau)\right)^2 \exp(-S_M[\hat{x}_i(\tau)]) \\ &= -\overline{x}^2_{\mathrm{COM}}(\tau) + \frac{1}{Z_M} \sum_{i=1}^{M} x^2_{\mathrm{COM},i}(\tau) \exp(-S_M[\hat{x}_i(\tau)]), \end{aligned} \tag{4.9}$$

and similarly for the coordinate y. In Figure 5, plots of $\Delta_{M,\mathrm{COM}}(\tau)$ are shown when $\Delta_{M,\mathrm{COM}}(\tau)$ is evaluated over the ensembles $E'_3, \ldots, E'_8$ with the action:

$$S_M = \frac{L(\tau)^2}{A(\tau)} \left(1 + \frac{c_M(\tau)}{L(\tau)}\right), \tag{4.10}$$



(a)



(b)

Figure 5: (a) plots of $\Delta_{M,\mathrm{COM}}(\tau)$ for $M = 3,\ldots,8$ when evaluated with the action $S_M = (L(\tau)^2/A(\tau))(1 + (c_M(\tau)/L(\tau)))$. Notice the rapid generation of an uncertainty in the COM position under the flow and its saturation when approaching the conformal limit $\tau \nearrow T$. There is also a saturation of this limiting value with a growing ensemble size. (b) plots of $\Delta_{M,\mathrm{COM}}(\tau)$ for $M = 3,\ldots,8$ when evaluated with the action $S_M = (L(\tau)^2/A(\tau))(1 + c_M(\tau)/A(\tau))$. Notice the qualitative agreement with the results displayed in (a).

and subject to the initial condition $\overline{L}_M(\tau = 0) = \widetilde{L}_M(\tau = 0)$. In Figure 5(b) the according plots of $\Delta_{M,\mathrm{COM}}(\tau)$ are depicted as obtained with the action

$$S_M = \frac{L(\tau)^2}{A(\tau)}\left(1 + \frac{c_M(\tau)}{A(\tau)}\right), \tag{4.11}$$

and subject to the initial condition $\overline{L}_M(\tau = 0) = \widetilde{L}_M(\tau = 0)$. In this case, one has $c_M(\tau) = -A(\tau)$ leading to equal weights for each curve in E'_M.

4.3. Quantum Mechanical versus Statistical Uncertainty

In view of the results obtained in Section 4.2, we would say that an ensemble of evolving planar CVLs in the $N = 0$ sector qualitatively resembles the quantum mechanics of a free-point particle (it is of no relevance at this point whether this particle carries spin or not) of mass m in 1D. Namely, an initially localized square of the wave function ψ with $|\psi(\tau = 0, x)|^2 \propto \exp[-(x^2/a_0^2)]$, where $\Delta x(\tau = 0) = a_0$, according to unitary time evolution in quantum mechanics evolves as $|\psi(\tau, x)|^2 = |\exp[-i(H\tau/\hbar)]\psi(\tau = 0, x)|^2 \propto \exp[-((x - (p/m)\tau)^2/a^2(\tau))]$, where $H = p^2/2m$ is the free-particle Hamiltonian, p is the spatial momentum, and $a(\tau) \equiv a_0\sqrt{1 + (\hbar\tau/ma_0^2)^2}$. In agreement with Heisenberg's uncertainty relation one has during the process that $\Delta x \Delta p = (\hbar/2)\sqrt{1 + (\tau\hbar/ma_0^2)^2} \geq \hbar/2$. Time evolution in quantum mechanics and the process of coarse-graining in a statistical system describing planar CVLs share the property that in both systems the evolution generates out of a small initial position uncertainty (corresponding to a large initial resolution Δp) a larger position uncertainty in the course of the evolution. Possibly, future development will show that interference effects in quantum mechanics can be traced back to the nonlocal nature of the degrees of freedom (CVLs) entering a statistical partition function.

5. Summary, Conclusions, and Outlook

5.1. Summary of Present Work

In this exploratory paper an attempt has been undertaken to interpret the effects of an environment on 2D planar center-vortex loops, as they emerge in the confining phase of an SU(2) Yang-Mills theory, in terms of a Wilsonian renormalization-group flow carried by purely geometric entities. Our (mainly numerical) analysis uses established mathematics on the shrinking of embedded curves in the plane. In the case of nonintersecting CVLs ($N = 0$) the role of the environment is played by the entirety of all sectors with $N > 0$ and, possibly, an explicit environment. In a particular parametrization of the effective action we observe critical behavior as the limit of circular points is approached during the evolution. That is, planar $N = 0$ CVLs on average disappear from the spectrum for resolving powers smaller than a critical, finite value. Using this formalism to compute the evolution of the mean values of local observables, such as the center-of-mass position, a behavior is generated that qualitatively resembles the associated unitary evolution in quantum mechanics. We also have found evidence that this situation is practically not altered when changing the ansatz for the effective action.

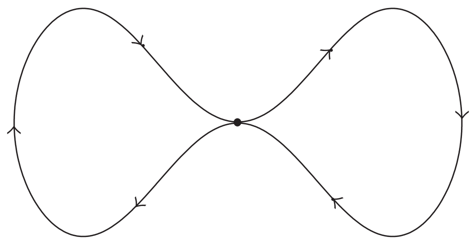


Figure 6: An isolated CVL with $N = 1$, which when associated with the confining phase of an SU(2) Yang-Mills theory of scale $\Lambda \sim m_e = 511$ keV, is interpreted as an electron or positron. The arrows depict one out of two possible directions of center flux responsible for the magnetic moment (spin), the intersection point is the location of mass and electric charge.

5.2. Outlook on Strongly Correlated Electrons in a Plane

Let us conclude this article with a somewhat speculative outlook on planar $N = 1$ CVLs. Setting the Yang-Mills scale (mass of the intersection in the CVL) of the associated SU(2) theory equal to the electron mass, this soliton is interpreted as an electron or positron, see [8, 9, 23, 24] for a more detailed discussion on the viability of such an assignment and Figure 6 for a display. Important for our purpose are the facts that the two-fold directional degeneracy of the center flux represents the two-fold degeneracy of the spin projection and that a large class of curve deformations (shifts of the intersection point) leaves the mass of the soliton invariant in the free case.

It is a remarkable fact that a high level of mathematical understanding exists for the behavior of curve-shrinking in a 2D plane [17, 18] even in the case of one selfintersection [25]. Incidentally, 2D quantum systems exist in nature which exhibit unconventional behavior. Specifically, the phenomenon of high T_c superconductivity appears to be strongly related to the two-dimensionality of electron dynamics as it is enabled by rare-earth doping of cuprate materials [26]. Apparently, the Coulomb repulsion between the electrons moving in the would-be valence band within the cuprate planes of high T_c superconductors (Mott insulators) is effectively screened by the interplane environment also providing for the very existence of these electrons by doping. The question then is how the long-range order of electronic spins, which at given (optimal) doping and at a sufficiently low temperature leads to superconductivity, emerges within the cuprate planes. As it seems, quantum Monte Carlo simulations of a transformed Hubbard model (t-J model) subject to Gutzwiller projection yield quantitative explanations of a number of experimental results related to the existence of the pseudogap phase (Nernst effect, nonlinear diamagnetic susceptibility), see [27, 28] and references therein.

We would here like to offer a sketch of an alternative approach to high T_c superconductivity being well aware of our ignorance on the details of present-day research in this field. The key idea is already encoded in Figure 6. Namely, according to confining SU(2) Yang-Mills theory the electron is a nonlocal object with the physics of its charge localization being only loosely related to the physics of its magnetic moment (spin): the magnetic moment, carried by the core of the flux line, microscopically manifested by (oppositely) moving (opposite) electric charges, receives contributions from vortex sectors that are spatially far separated (on the scale of the diameter of the intersection point) from the location of the isolated electric charge. This suggests that in certain physical circumstances,

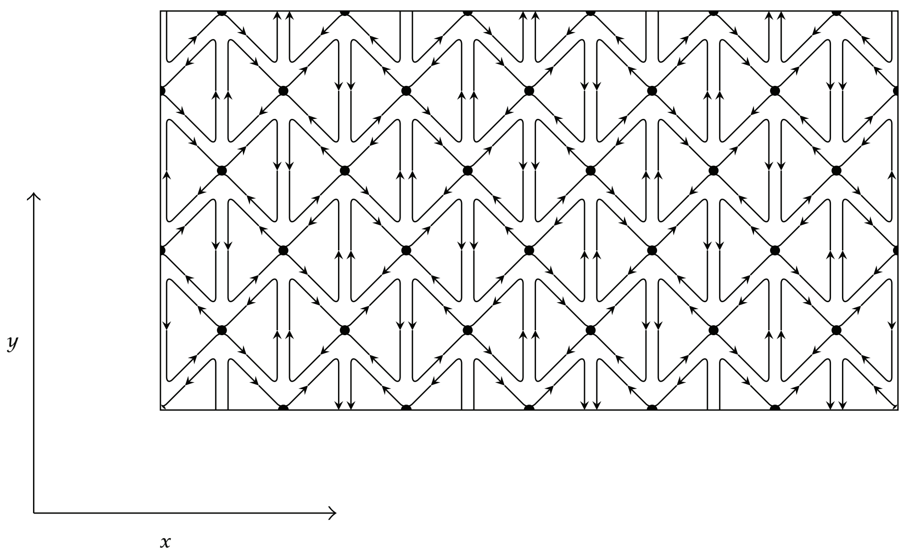


Figure 7: An array of strongly correlated electrons in the plane possibly representing the superconducting state in a cuprate. The equally directed center flux in adjacent vortex sections provides for an attractive force (Ampere's law) at intermediate distances. For a given electron, out of six neighboring vortex sectors there are four sectors with attraction and two sectors with repulsion. At short distance, there is repulsion since an overlap of CVL sections, leading to new intersection points each of mass $\sim m_e$, is topologically forbidden. Thus there is a typical equilibrium configuration contributing to long-range order in the 2D system. If the externally provided resolution (temperature) falls below a critical value, then statistical fluctuations of the position of an intersection point relative to another one (the location of the electronic charge) will vanish. That is, electrons no longer can disperse energy provided by the heat bath (phonons; spin fluctuations) and thus provide a 2D material free of electric resistivity.

where the ordering effect of interacting vortex lines becomes important, the postulate of a spinning point particle fails to describe reality.

Concerning the strong correlations in 2D electron dynamics responsible for high T_c superconductivity, we imagine a situation as depicted in Figure 7. Each electron's spin in the plane interacts with the spin of its neighbors as follows. Equally directed electric fluxes (dually interpreted center fluxes of $SU(2)_e$) attract one another, and there is attraction for four out of six vortex sectors defined by the neighboring electrons (Ampere's law), while two vortex sectors experience repulsion (that the electric charges of confining $SU(2)_e$ are seen by the photon is a consequence of the mixing between the corresponding two gauge groups: $SU(2)_e$ and $SU(2)_{CMB}$ [8, 9, 24]). Notice that for a given electron two of the adjacent electrons exhibit equal spin projection, while four of the adjacent electrons have opposite spins. This is in agreement with the observation that high T_c superconductivity is an effect not related to s-wave Cooper-pair condensation.

An overlap of vortex sectors, hypothetically leading to the creation of extra intersection points, is topologically forbidden. That is, the fluctuations in the energy density of the system are far too weak to create an intersection point of mass $m_e = 511$ keV. Therefore, at very small spatial separation repulsion must occur between adjacent vortex sectors. At a sufficiently low temperature and an optimal screening of Coulomb repulsion by the interplane environment (doping), this would lead to a typical equilibrium configuration as depicted in Figure 7 where the intersection points (electronic charge) do not move relative to one another. A local demolition of this highly ordered state would cost a finite amount of energy manifested in terms of the (gigantic) gaps measured experimentally in the cuprate systems. Applying an electric field vector with a component parallel to the plane would set into resistivity-free motion the thus locked electrons. For a macroscopic analogue, imagine a stiff table cloth being pulled over the table's surface in a friction-free fashion. The occurrence of the pseudogap phase would possibly be explained by local defects in the fabric of Figure 7 due to insufficient Coulomb screening and/or too large thermal noise (macroscopic vorticity; liquid of point-like defects in 2D).

Acknowledgment

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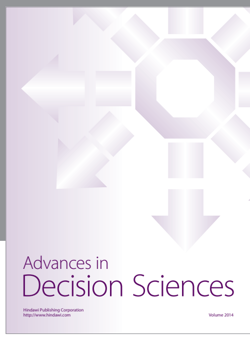

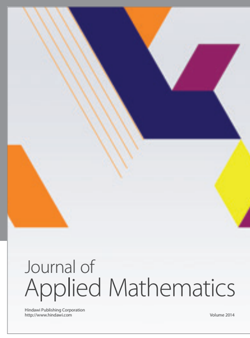

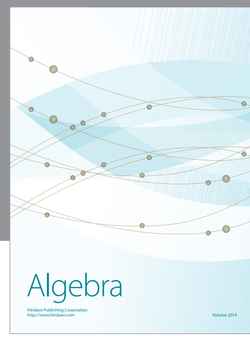

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