

Flavor Mixing, Neutrino Masses and Neutrino Oscillations

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December 8, 2018

Abstract

We study a model for the mass matrices of the leptons. We are able to relate the mass eigenvalues of the charged leptons and of the neutrinos to the mixing angles and can predict the masses of the neutrinos. We find a normal hierarchy - the masses are 0.004 eV, 0.01 eV and 0.05 eV. The atmospheric mixing angle is given by the mass ratios of the charged leptons and the neutrinos. We find 38 degrees, consistent with the experiments. The mixing element, connecting the first neutrino with the electron, is found to be 0.05.

We observe three lepton-quark families in nature. The first family consists of the electron, its neutrino and the u- and d-quarks. The members of the second family are the muon, its neutrino and the c- and s-quarks. The third family consists of the tauon, its neutrino and the t- and b-quarks. It is unknown, whether there is a connection between the number of families and the number of colors in quantum chromodynamics.

In the Standard Model 28 fundamental constants have to be introduced. Their values cannot be calculated - they have to be measured in the experiments. These constants are:

- the constant of gravity G ,
- the fine structure constant,
- the coupling constant of the weak interactions,
- the coupling constant of the strong interactions or the scale of QCD,
- the mass of the W-boson,
- the mass of the "Higgs"-boson,
- the masses of the three charged leptons,
- the masses of the three neutrinos,

- the masses of the six quarks,
- the four parameters, describing the flavour mixing of the quarks and
- the six parameters, describing the flavour mixing of the leptons.

For the masses of the quarks we assume the following values:

$$\begin{aligned}
u &: 5.3 \text{ MeV}, \\
d &: 7.8 \text{ MeV}, \\
s &: 146 \text{ MeV}, \\
c &: 1050 \text{ MeV}, \\
b &: 4600 \text{ MeV}, \\
t &: 174000 \text{ MeV}.
\end{aligned}$$

The quark masses are scale dependent. The values given above, except the one for the t-mass, are normalized at an energy scale of 1 GeV. The experimental errors are not given, but are at least 10 %, except for the t-mass, which is known to about 1 %.

The quarks of the same charge do mix. If the u-quark interacts with a W-boson, a mixture of d, s and b appears. These mixtures are described by the CKM matrix (ref.1). I prefer a description, which we introduced some time ago (ref. 2):

$$V = \begin{bmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \begin{bmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

In the case of three families there are three mixing angles and one phase parameter. The latter describes the CP-violation. The angles with the index u or d describe the mixing in the u-c sector or the d-s sector, the angle with no index describes the mixing between the (t,c)-system and the (b,s)-system.

I proposed years ago a simple texture 0 mass matrix for the quarks, given by the following matrix, valid both for the (u,c,t)-system and the (d,s,b)-system (ref.3):

$$M = \begin{bmatrix} 0 & A & 0 \\ A^* & C & B \\ 0 & B^* & D \end{bmatrix} \quad (2)$$

Such mass matrices are obtained, if in the electroweak theory special symmetries are present, either discrete reflection symmetries or continuous symmetries (ref.3). After diagonalization one finds the following relations for the mixing angles (ref. 4):

$$\theta_d = \tan^{-1} \sqrt{m_d/m_s} \quad , \quad \theta_u = \tan^{-1} \sqrt{m_u/m_c} \quad (3)$$

Taking as input the quark masses given above, we obtain for the mixing angle in the d-s- sector 13 degrees, and in the u-c-sector 4 degrees. The first angle is identical to the measured Cabibbo angle. These angles agree very well with the experimental data on the flavor mixing, if the phase angle is assumed to be close to 90 degrees.

Analogously we can describe the flavour mixing in the lepton sector, which can be studied in the neutrino oscillations. The experiments, carried out in

Japan (Kamiokande detector, ref. 5) and in Canada (SNO detector, ref. 6), give the following results for the mass-squared differences of the three neutrino mass eigenstates:

$$\Delta m_{21}^2 \simeq 8 \times 10^{-5} eV^2 \quad , \quad \Delta m_{32}^2 \simeq 2.5 \times 10^{-3} eV^2 \quad (4)$$

In neutrino oscillations only mass squared differences can be measured. No information can be obtained for the absolute magnitude of the neutrino masses. There is the possibility that the neutrino masses are nearly degenerate, e.g. masses like $m_1 = 0.94eV$, $m_2 = 0.95eV$, $m_3 = 1eV$ are possible. If one neutrino remains massless, one would have the masses $m_1 = 0eV$, $m_2 = 0.009eV$, $m_3 = 0.05eV$. In the latter case a hierarchy of the masses is present, but this hierarchy is much weaker than the mass hierarchy for the charged leptons. Below we shall calculate the neutrino masses.

Neutrino oscillations arise, since the neutrinos, produced by the weak interactions, are not mass eigenstates, but mixtures of mass eigenstates. Like for the quarks one has a 3x3 mixing matrix, which can be written as a product of three simple matrices, and a phase matrix, which is present only, if neutrinos are Majorana particles:

$$V = U \cdot P$$

$$U = \begin{bmatrix} c_l & s_l & 0 \\ -s_l & c_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \begin{bmatrix} c_\nu & -s_\nu & 0 \\ s_\nu & c_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$P = \begin{bmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here s_ν stands for $\sin \theta_\nu$ (θ_ν : solar mixing angle), s stands for $\sin \theta$ (θ : atmospheric mixing angle), and s_l stands for $\sin \theta_l$ (θ_l : reactor mixing angle). The latter has not been measured. The experimental results for the solar and the atmospheric mixing angles are (ref.5,6):

$$30^\circ \leq \theta_\nu \leq 39^\circ \quad , \quad 37^\circ \leq \theta \leq 53^\circ \quad (6)$$

We assume for the lepton mass matrices the same texture 0 pattern as for the quarks (ref.7). Thus we obtain the same relations between the mixing angles and the mass eigenvalues:

$$\tan \theta_l = \sqrt{m_e/m_\mu} \simeq 0.07 \quad , \quad \tan \theta_\nu = \sqrt{m_1/m_2} \quad (7)$$

Since the solar angle has been measured (we take 33 degrees), we find for the mass ratio of the first two neutrinos:

$$m_1/m_2 \approx 0.42 \quad (8)$$

The oscillation experiments provide us with the mass squared differences. The new relation (7) allows us to determine the neutrino masses. We find (in eV):

$$m_1 \approx 0.004 \quad , \quad m_2 \approx 0.01 \quad , \quad m_3 \approx 0.05$$

These neutrino masses are very small. We observe that the masses show a rather weak hierarchy, but the mass spectrum is not inverted. The first neutrino has the smallest mass.

If the neutrino masses are Majorana masses, one expects a neutrinoless double beta decay. The present limit on the Majorana mass is about 0.23 eV (ref. 8). This limit needs to be improved by about a factor 5. At this level the decay should be seen.

The atmospheric mixing angle is consistent with 45 degrees. The parameter C in eq. (2) might be zero, as originally assumed (see ref. 3). But the high mass of the t-quark does not allow this possibility for the quarks. It might work for the leptons. In this case the atmospheric mixing angle is related to the two angles, which are given by the corresponding mass ratios:

$$\begin{aligned} \tan \theta_1 &= \sqrt{m_\mu/m_\tau} \quad , \quad \tan \theta_2 = \sqrt{m_2/m_3} & (9) \\ \theta_1 &\approx 14^\circ \quad , \quad \theta_2 \approx 24^\circ \end{aligned}$$

The atmospheric mixing angle is given by the absolute value of the sum of the two angles, including a relative phase between the two terms. In order to get the direct sum, this phase must be 180 degrees. In this case we have for the atmospheric mixing angle:

$$\theta = \theta_1 + \theta_2 \simeq 38^\circ \quad (10)$$

We can not obtain a maximal mixing (45 degrees), but our result is consistent with the experiment.

We can predict the matrix element V_{3e} of the mixing matrix V :

$$V_{3e} = \sin \theta \sin \theta_l \approx 0.707 \sqrt{m_e/m_\mu} \approx 0.05 \quad (11)$$

A matrix element of this magnitude could be observed in the upcoming reactor neutrino experiments.

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