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Research Article

Optimal Transfer-Ordering Strategy for a Deteriorating Inventory in Declining Market

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The retailer's optimal procurement quantity and the number of transfers from the warehouse to the display area are determined when demand is decreasing due to recession and items in inventory are subject to deterioration at a constant rate. The objective is to maximize the retailer's total profit per unit time. The algorithms are derived to find the optimal strategy by retailer. Numerical examples are given to illustrate the proposed model. It is observed that during recession when demand is decreasing, retailer should keep a check on transportation cost and ordering cost. The display units in the show room may attract the customer.

1. Introduction

The management of inventory is a critical concern of the managers, particularly, during recession when demand is decreasing with time. The second most worrying issue is of transfer batching, the integration of production and inventory model, as well as the purchase and shipment of items. Goyal [1], for the first time, formulated single supplier-single retailer-integrated inventory model. Banerjee [2] derived a joint economic lot size model under the assumption that the supplier follows lot-for-lot shipment policy for the retailer. Goyal [3] extended Banerjee's [2] model. It is assumed that numbers of shipments are equally sized and the production of the batch had to be finished before the start of the shipment. Lu [4] allowed shipments to occur during the production period. Goyal [5] derived a shipment policy in which, during production, a shipment is made as soon as the buyer is about to face stock out and all the produced stock manufactured up to that point is shipped out. Hill [6] developed an optimal two-stage lot sizing and inventory batching policies. Yang and Wee [7] developed an integrated multilot-size production inventory model for deteriorating items. Law and Wee [8] derived an integrated production-inventory model for ameliorating and deteriorating items using DCE approach. Yao et al. [9] argued the importance of supply chain parameters when vendor-buyer adopts joint policy. The interesting papers in this areas are by Wee [10], Hill [11, 12], Vishwanathan [13], Goyal and Nebebe [14], Chiang [15], Kim and Ha [16], Nieuwenhuyse and Vandaele [17], Siajadi et al. [18], and their cited references. The aforesaid articles are dealing with integrated Vendor-buyer inventory model when demand is deterministic and known constant.

The aim of this paper is to determine the ordering and transfer policy which maximizes the retailer's profit per unit time when demand is decreasing with time. It is assumed that on the receipt of the delivery of the items, retailer stocks some items in the showroom and rest of the items is kept in warehouse. The floor area of the showroom is limited and wellfurnished with the modern techniques. Hence, the inventory holding cost inside the showroom is higher as compared to that in warehouse. The problem is how often and how many items are to be transferred from the warehouse to the showroom which maximizes the retailer's total profit per unit time. Here, demand is decreasing with time. This paper is organized as follows. Section 2 deals with the assumptions and notations for the proposed model. In Section 3, a mathematical model is formulated to determine the ordering-transfer policy which maximizes the retailer's profit per unit time. Section 4 deals with the establishment of the necessary conditions for an optimal solution. Using these conditions, the algorithms are developed. In Section 5, numerical examples are given. The sensitivity analysis of the optimal solution with respect to system parameter is carried out. The research article ends with conclusion in Section 5.

2. Mathematical Model

2.1. The Total Cost per Cycle in the Warehouse

The retailer orders Q-units per order from a supplier and stocks these items in the warehouse. The q-units are transferred from the warehouse to the showroom until the inventory level in the warehouse reaches to zero. Hence $Q = nq$. The total cost per cycle during the cycle time T in the warehouse is the sum of (1), the ordering cost A, and (2) the inventory holding cost, $h_w[(n(n-1)/2)q]t_1$.

2.2. The Total Cost per Unit Cycle in the Showroom

Initially, the inventory level is $L_0 \leq L$ due to the unit's transfer from the warehouse to the display area. The inventory level then depletes to R due to time-dependent demand and deterioration of units at the end of the retailer's cycle time, "t_1." A graphical representation of the inventory system is exhibited in Figure 1.

The differential equation representing inventory status at any instant of time t is given by

$$\frac{dI(t)}{dt} = -D(t) - \theta I(t), \quad 0 \leq t \leq t_1 \tag{2.1}$$

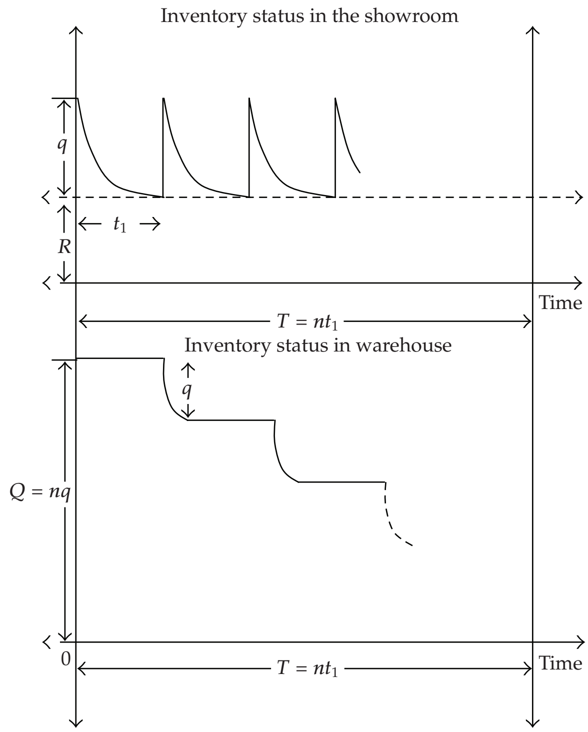

Figure 1: Combined inventory status for items in the warehouse and showroom.

with boundary condition $I(t_1) = R$. The solution of (2.1) is

$$I(t) = Re^{\theta(t_1-t)} + a\left(\frac{(e^{\theta(t_1-t)} - 1)(\theta + b)}{\theta^2} - \frac{b(t_1 e^{\theta(t_1-t)} - t)}{\theta}\right); \quad 0 \le t \le t_1. \tag{2.2}$$

The total cost incurred during the cycle time t_1 is the sum of the ordering cost, G and the inventory holding cost, where

inventory holding cost

$$\begin{aligned} &= h_d \int_0^{t_1} I(t)dt \\ &= h_d\left(-\frac{R}{\theta} + a\left(\frac{b\theta^2 t_1^2 - 2\theta - 2b - 2\theta^2 t_1}{2\theta^3}\right)\right) - h_d e^{\theta t_1}\left(a\left(\frac{\theta b t_1 - \theta - b}{\theta^3}\right) - \frac{R}{\theta}\right) \end{aligned} \tag{2.3}$$

Using (2.2) and $I(0) = q + R$, we get

$$q = \frac{Re^{\theta t_1}\theta^2 + ae^{\theta t_1}\theta + ae^{\theta t_1}b - a\theta - ab - abt_1 e^{\theta t_1}\theta - R\theta^2}{\theta^2}. \tag{2.4}$$

The revenue per cycle is

$$(P-C)q=\frac{(P-C)\left(Re^{\theta t_1}\theta^2+ae^{\theta t_1}\theta+ae^{\theta t_1}b-a\theta-ab-abt_1e^{\theta t_1}\theta-R\theta^2\right)}{\theta^2}. \tag{2.5}$$

Then inventory holding cost in the warehouse is

$$\frac{h_w n(n-1)t_1\left(Re^{\theta t_1}\theta^2+ae^{\theta t_1}\theta+ae^{\theta t_1}b-a\theta-ab-abt_1e^{\theta t_1}\theta-R\theta^2\right)}{2\theta^2}. \tag{2.6}$$

Hence, the total profit, ZP per cycle during the period $[0,T]$ is

$$\begin{aligned}ZP &= \text{Revenue} - [\text{total cost in the warehouse}] - [\text{total cost in the showroom}]\\ &= \left(\begin{array}{c}\dfrac{n(P-C)\left(Re^{\theta t_1}\theta^2+ae^{\theta t_1}\theta+ae^{\theta t_1}b-a\theta-ab-abt_1e^{\theta t_1}\theta-R\theta^2\right)}{\theta^2}-A\\ -\dfrac{h_w n(n-1)t_1\left(Re^{\theta t_1}\theta^2+ae^{\theta t_1}\theta+ae^{\theta t_1}b-a\theta-ab-abt_1e^{\theta t_1}\theta-R\theta^2\right)}{2\theta^2}-nG\\ -nh_d\left(-\dfrac{R}{\theta}+a\left(\dfrac{b\theta^2t_1^2-2\theta-2b-2\theta^2t_1}{2\theta^3}\right)\right)+nh_de^{\theta t_1}\left(a\left(\dfrac{\theta bt_1-\theta-b}{\theta^3}\right)-\dfrac{R}{\theta}\right)\end{array}\right).\end{aligned} \tag{2.7}$$

During period $[0,T]$, there are n-transfers at every t_1-time units. Hence, $T=nt_1$. Therefore, the total profit per time unit is

$$Z(n,R,t_1)=\frac{ZP}{T}=\frac{\left(\begin{array}{c}n(P-C)\left(Re^{\theta t_1}\theta^2+ae^{\theta t_1}\theta+ae^{\theta t_1}b-a\theta-ab-abt_1e^{\theta t_1}\theta-R\theta^2\right)/\theta^2-A-nG\\ +h_w n(n-1)t_1\left(Re^{\theta t_1}\theta^2+ae^{\theta t_1}\theta+ae^{\theta t_1}b-a\theta-ab-abt_1e^{\theta t_1}\theta-R\theta^2\right)/2\theta^2\\ -nh_d\left(-R/\theta+a\left(\left(b\theta^2t_1^2-2\theta-2b-2\theta^2t_1\right)/2\theta^3\right)\right)+nh_de^{\theta t_1}\left(a\left((\theta bt_1-\theta-b)/\theta^3\right)-R/\theta\right)\end{array}\right)}{nt_1}. \tag{2.8}$$

3. Necessary and Sufficient Condition for an Optimal Solution

The total profit per unit time of a retailer is a function of three variables, namely, n, R and t_1:

$$\frac{\partial^2 Z(n,R,t_1)}{\partial n^2}=-\frac{2A}{n^3t_1}<0. \tag{3.1}$$

Thus, the retailer's total profit per unit time is a concave function of n for fixed R and t_1.

Next, to determine the optimum cycle time for showroom, for given n, we first differentiate $Z(n, R, t_1)$ with respect to R. We get

$$\frac{\partial Z(n, R, t_1)}{\partial R} = \left(\frac{1 - e^{\theta t_1}}{t_1}\right)\left(-(P - C) + \frac{h_w(n - 1)t_1}{2} + \frac{h_d}{\theta}\right). \tag{3.2}$$

Depending on the sign of $(P - C)\theta - h_d$ three cases arise: Define $\Delta = (P - C)\theta - h_d$.

Case 1 ($\Delta < 0$). If $\Delta < 0$, then $Z(n, R, t_1)$ is a decreasing function of R for fixed R. It suggests that no transfer of units should be made from the warehouse to the showroom; so put $R = 0$ in $Z(n, R, t_1)$ and differentiate resultant expression with respect to t_1. We have

$$\begin{aligned}
&\left.\frac{\partial Z}{\partial t_1}\right|_{R=0} = 0 \\
&\frac{\begin{pmatrix} a(P-C)(1-bt_1)e^{\theta t_1} - (1/2)h_w(n-1)a\theta^2 t_1(1-bt_1)e^{\theta t_1} \\ +(1/2)\left(h_w(n-1)a\left(\left(1-e^{\theta t_1}\right)(\theta+b)+b\theta t_1 e^{\theta t_1}\right)\right)/\theta^2 - \left(\left(h_d a/\theta^2\right)(bt_1-1)\left(1-e^{\theta t_1}\right)\right) \end{pmatrix}}{t_1} \\
&- \frac{\begin{pmatrix} a(P-C)\left(\left(1-e^{\theta t_1}\right)(\theta+b)+bt_1 e^{\theta t_1}\theta\right)/\theta^2 + h_w(n-1)a\left(\left(1-e^{\theta t_1}\right)(\theta+b)+bt_1 e^{\theta t_1}\theta\right)t_1/2\theta^2 \\ -A/n - G - \left(h_d a\left(bt_1(2+\theta t_1)/2\theta^2 - (\theta+b)(1+\theta t_1)/\theta^3\right) - h_d a\left((b\theta t_1 - \theta - b)e^{\theta t_1}/\theta^3\right)\right) \end{pmatrix}}{t_1^2} = 0.
\end{aligned} \tag{3.3}$$

The sufficiency condition is $\partial^2 Z(n, R, t_1)/\partial t_1^2 < 0$, that is,

$$\frac{1}{2\theta^3 n t_1^3}\begin{pmatrix}
-4na\theta^3 t_1 Pe^{\theta t_1} + 4na\theta^3 t_1 Ce^{\theta t_1} + 4n\theta^2 Pae^{\theta t_1} - 4n\theta^2 Cae^{\theta t_1} \\
+4n\theta Pabe^{\theta t_1} - 4n\theta^2 Pa + 4n\theta^2 Ca - 4nG\theta^3 - 4A\theta^3 \\
-4n\theta Pab + 4n\theta Cab + 4nh_d a\theta + 4nh_d ab - 4n\theta^2 Pabt_1 e^{\theta t_1} \\
-4n\theta Cabe^{\theta t_1} + 4n\theta^2 Cabt_1 e^{\theta t_1} - 4nh_d a\theta e^{\theta t_1} - 4nh_d abe^{\theta t_1} \\
+4nh_d abt_1\theta e^{\theta t_1} + 2na\theta^4 t_1^2 Pe^{\theta t_1} + 2na\theta^3 t_1^2 Pbe^{\theta t_1} \\
-2na\theta^4 t_1^3 Pbe^{\theta t_1} - 2na\theta^4 t_1^2 Ce^{\theta t_1} - 2na\theta^3 t_1^2 Cbe^{\theta t_1} \\
+2na\theta^4 t_1^3 Cbe^{\theta t_1} - n^2 a\theta^4 t_1^3 h_w e^{\theta t_1} + n^2 a\theta^3 t_1^3 h_w be^{\theta t_1} \\
+n^2 a\theta^4 t_1^4 h_w be^{\theta t_1} + na\theta^4 t_1^3 h_w e^{\theta t_1} - na\theta^3 t_1^3 h_w be^{\theta t_1} \\
-na\theta^4 t_1^4 h_w be^{\theta t_1} - 2na\theta^3 t_1^2 h_d e^{\theta t_1} - 2na\theta^2 t_1^2 h_d be^{\theta t_1} \\
+2na\theta^3 t_1^3 h_d be^{\theta t_1} + 4na\theta^2 t_1 h_d e^{\theta t_1}
\end{pmatrix} < 0. \tag{3.4}$$

Thus, $Z(n, t_1)$, the total profit per unit time, is a concave function of t_1 for fixed n. There exists a unique t_1, denoted by t_1^{*1} such that $Z(n, t_1^{*1})$ is maximum. Substituting t_1^{*1} and $R^* = 0$ into (2.5) are obtain number of units to be transferred (say) q^{*1} for fixed n.

Note. Since $q^{*1} \leq L$ for all q, $q^{*1} = L$. If $q^{*1} > L$, then obtain t_1^{*1} using

$$t_1^{*1} = \frac{1}{\theta} \ln\left[1 + \frac{L\theta^2}{a(\theta + b)}\right]. \tag{3.5}$$

Case 2 ($\Delta = 0$). In this case, we made (2.8) as

$$Z(n,R,t_1) = \begin{pmatrix} \dfrac{h_w R e^{\theta t_1}}{2} + \dfrac{h_w a e^{\theta t_1}}{2\theta} + \dfrac{h_w ab e^{\theta t_1}}{2\theta^2} - \dfrac{h_w a}{2\theta} - \dfrac{h_w ab}{2\theta^2} - \dfrac{t_1 h_w ab e^{\theta t_1}}{2\theta} \\ -\dfrac{h_w R}{2} - \dfrac{G}{t_1} - \dfrac{A}{n t_1} - \dfrac{n h_w R e^{\theta t_1}}{2} - \dfrac{n h_w a e^{\theta t_1}}{2\theta} - \dfrac{n h_w ab e^{\theta t_1}}{2\theta^2} \\ +\dfrac{n h_w a}{2\theta} + \dfrac{n h_w ab}{2\theta^2} + \dfrac{n t_1 h_w ab e^{\theta t_1}}{2\theta} + \dfrac{n h_w R}{2} + \dfrac{h_d a}{\theta} - \dfrac{t_1 h_d ab}{2\theta} \end{pmatrix}. \tag{3.6}$$

Here,

$$\frac{\partial Z(n,R,t_1)}{\partial R} = -\frac{h_w}{2}(n-1)\left(e^{\theta t_1} - 1\right) < 0. \tag{3.7}$$

that is, $Z(n,R,t_1)$ is decreasing function of R for given n. So no transfer should be made from the warehouse to the showroom, that is, $R = 0$. So (3.6) becomes

$$Z(n,t_1) = \begin{pmatrix} \dfrac{h_w a e^{\theta t_1}}{2\theta} + \dfrac{h_w ab e^{\theta t_1}}{2\theta^2} - \dfrac{h_w a}{2\theta} - \dfrac{h_w ab}{2\theta^2} - \dfrac{t_1 h_w ab e^{\theta t_1}}{2\theta} \\ -\dfrac{G}{t_1} - \dfrac{A}{n t_1} - \dfrac{n h_w a e^{\theta t_1}}{2\theta} - \dfrac{n h_w ab e^{\theta t_1}}{2\theta^2} + \dfrac{n h_w a}{2\theta} \\ +\dfrac{n h_w ab}{2\theta^2} + \dfrac{n t_1 h_w ab e^{\theta t_1}}{2\theta} + \dfrac{h_d a}{\theta} - \dfrac{t_1 h_d ab}{2\theta} \end{pmatrix}. \tag{3.8}$$

The optimal value of t_1^{*2} can be obtained by solving

$$\frac{\partial Z(n,t_1)}{\partial t_1} = \begin{pmatrix} \dfrac{h_w a e^{\theta t_1}}{2} - \dfrac{t_1 h_w ab e^{\theta t_1}}{2} + \dfrac{G}{t_1^2} + \dfrac{A}{n t_1^2} \\ -\dfrac{n h_w a e^{\theta t_1}}{2} + \dfrac{h_w t_1 n ab e^{\theta t_1}}{2} - \dfrac{h_d ab}{2\theta} \end{pmatrix} = 0. \tag{3.9}$$

The sufficiency condition is

$$\frac{\partial^2 Z(n,t_1)}{\partial t_1^2} = -\left(\begin{array}{c} \dfrac{nh_w a\theta e^{\theta t_1}}{2} - \dfrac{nabh_w e^{\theta t_1}}{2} - \dfrac{nabt_1\theta h_w e^{\theta t_1}}{2} \\ -\dfrac{a\theta h_w e^{\theta t_1}}{2} + \dfrac{abh_w e^{\theta t_1}}{2} + \dfrac{t_1 h_w ab\theta e^{\theta t_1}}{2} + \dfrac{2G}{t_1^3} + \dfrac{2A}{nt_1^3} \end{array} \right) < 0, \quad \text{for } t_1 = t_1^{*2}. \tag{3.10}$$

Then, $Z(n,t_1^{*2})$ is a concave function of t_1^{*2} and hence $Z(n,t_1^{*2})$ is the maximum profit of the retailer. q^{*2} can be obtained by substituting value of t_1^{*2} in (2.5).

Note. Since $q^{*2} \leq L$ for all q, then $q^{*2} = L$. If $q^{*2} > L$, then obtain t_1^{*2} using,

$$t_1^{*2} = \frac{1}{\theta}\ln\left[1 + \frac{L\theta^2}{a(\theta+b)}\right]. \tag{3.11}$$

Case 3 ($\Delta > 0$). There are three subcases.

Subcase 3.1. $((P-C)\theta - h_d)/\theta t_1 < h_w(n-1)/2$ and then $\partial Z(n,R,t_1)/\partial R < 0$. It is same as Case 1.

The optimal transfer level of units in showroom is zero and there exists a unique t_1 (say) $t_1^{*3.1}$ such that $Z(n,t_1^{*3.1})$ is maximum.

Note. (1) $t_1^{*3.1} \leq 2((P-C)\theta - h_d)/\theta t_1 h_w(n-1)$ and then $t_1^{*3.1}$ is infeasible. (2) Because $q \leq L$ for all q, $q^{*3.1} = L$. If $q > L$, then obtain $t_1^{*3.1}$ using (2.5). (3) The number of transfers from the warehouse to the showroom must be at least 2.

Subcase 3.2. $((P-C)\theta - h_d)/\theta t_1 > h_w(n-1)/2$. Here, $\partial Z(n,R,t_1)/\partial R > 0$. Therefore, raise the inventory level to the maximum allowable quantity. So from $L = q + R$ and (2.5), we get

$$R = \frac{L\theta^2 - a\theta e^{\theta t_1} - abe^{\theta t_1} + a\theta + ab + abt_1\theta e^{\theta t_1}}{\theta^2 e^{\theta t_1}}. \tag{3.12}$$

Then R is a function of t_1. Substitute (3.12) into (2.8). The resultant expression for the total profit per unit time is function of n and t_1. The necessary condition for finding the optimal time $t_1^{*3.2}$ in showroom is

$$\frac{\partial Z(n,t_1)}{\partial t_1} = \left(\begin{array}{c} \frac{Pab}{\theta t_1 e^{\theta t_1}} - \frac{h_d ab}{2\theta} + \frac{G}{t_1^2} + \frac{A}{nt_1^2} - \frac{(P-C)L}{t_1^2} - \frac{(P-C)a}{\theta t_1^2} + \frac{h_d a}{\theta^2 t_1^2} + \frac{h_d L}{\theta t_1^2} + \frac{nh_w ab}{2\theta} - \frac{(P-C)ab}{\theta^2 t_1^2} \\ + \frac{h_d ab}{\theta^3 t_1^2} - \frac{h_w ab}{2\theta} - \frac{nh_w L\theta}{2e^{\theta t_1}} - \frac{CL}{t_1^2 e^{\theta t_1}} - \frac{CL\theta}{t_1 e^{\theta t_1}} + \frac{h_w a}{2e^{\theta t_1}} - \frac{h_d L}{\theta t_1^2 e^{\theta t_1}} - \frac{h_d L}{t_1 e^{\theta t_1}} + \frac{PL}{t_1^2 e^{\theta t_1}} + \frac{PL\theta}{t_1 e^{\theta t_1}} \\ + \frac{Pa}{\theta t_1^2 e^{\theta t_1}} + \frac{Pa}{t_1 e^{\theta t_1}} + \frac{Pab}{\theta^2 t_1^2 e^{\theta t_1}} - \frac{Ca}{\theta t_1^2 e^{\theta t_1}} - \frac{Ca}{t_1 e^{\theta t_1}} - \frac{Cab}{\theta^2 t_1^2 e^{\theta t_1}} - \frac{Cab}{\theta t_1 e^{\theta t_1}} - \frac{nh_w a}{2e^{\theta t_1}} - \frac{nh_w ab}{2\theta e^{\theta t_1}} \\ + \frac{h_w L\theta}{2e^{\theta t_1}} + \frac{h_w ab}{2\theta e^{\theta t_1}} - \frac{h_d a}{\theta^2 t_1^2 e^{\theta t_1}} - \frac{h_d a}{\theta t_1 e^{\theta t_1}} - \frac{h_d ab}{\theta^3 t_1^2 e^{\theta t_1}} - \frac{h_d ab}{\theta^2 t_1 e^{\theta t_1}} \end{array}\right). \tag{3.13}$$

The obtained $t_1 = t_1^{*3.2}$ maximizes the total profit, $Z(n, t_1^{*3.2})$, per unit time because

$$\frac{\partial^2 Z(n,t_1)}{\partial t_1^2} = \left(\begin{array}{c} -\frac{2CL}{t_1^3} + \frac{h_d L\theta}{t_1 e^{\theta t_1}} - \frac{2PL}{t_1^3 e^{\theta t_1}} - \frac{2PL\theta}{t_1^2 e^{\theta t_1}} - \frac{PL\theta^2}{t_1 e^{\theta t_1}} - \frac{2Pa}{\theta t_1^3 e^{\theta t_1}} \\ -\frac{2Pa}{t_1^2 e^{\theta t_1}} - \frac{Pa\theta}{t_1 e^{\theta t_1}} - \frac{2Pab}{\theta^2 t_1^3 e^{\theta t_1}} + \frac{2Ca}{\theta t_1^3 e^{\theta t_1}} + \frac{2Ca}{t_1^2 e^{\theta t_1}} + \frac{Ca\theta}{t_1 e^{\theta t_1}} \\ -\frac{2G}{t_1^3} + \frac{2h_d a}{\theta t_1^2 e^{\theta t_1}} + \frac{h_d ab}{\theta t_1 e^{\theta t_1}} + \frac{2Cab}{\theta^2 t_1^3 e^{\theta t_1}} + \frac{2Cab}{\theta t_1^2 e^{\theta t_1}} + \frac{Cab}{t_1 e^{\theta t_1}} \\ + \frac{nh_w a\theta}{2e^{\theta t_1}} + \frac{nh_w ab}{2e^{\theta t_1}} - \frac{h_w L\theta^2}{2e^{\theta t_1}} - \frac{2Cab}{\theta^2 t_1^3} - \frac{2A}{nt_1^3} - \frac{h_w ab}{2e^{\theta t_1}} \\ + \frac{2h_d a}{\theta^2 t_1^3 e^{\theta t_1}} + \frac{h_d a}{t_1 e^{\theta t_1}} + \frac{2h_d ab}{\theta^3 t_1^3 e^{\theta t_1}} + \frac{2h_d ab}{\theta^2 t_1^3 e^{\theta t_1}} - \frac{2Ca}{\theta t_1^3} + \frac{2Pa}{\theta t_1^3} \\ -\frac{2h_d a}{\theta^2 t_1^3} - \frac{2h_d L}{\theta t_1^3} + \frac{2h_d L}{t_1^2 e^{\theta t_1}} - \frac{Pab}{t_1 e^{\theta t_1}} - \frac{2h_d ab}{\theta^3 t_1^3} + \frac{nh_w L\theta^2}{2e^{\theta t_1}} + \frac{2CL}{t_1^3 e^{\theta t_1}} \\ + \frac{2CL\theta}{t_1^2 e^{\theta t_1}} + \frac{CL\theta^2}{t_1 e^{\theta t_1}} - \frac{h_w a\theta}{2e^{\theta t_1}} + \frac{2h_d L}{\theta t_1^3 e^{\theta t_1}} - \frac{2Pab}{\theta t_1^2 e^{\theta t_1}} + \frac{2Pab}{\theta^2 t_1^3} + \frac{2PL}{t_1^3} \end{array}\right) < 0. \tag{3.14}$$

Subcase 3.3. $((P-C)\theta - h_d)/\theta t_1 = h_w(n-1)/2$ and then

$$t_1^{*3.3} = \frac{2((P-C)\theta - h_d)}{\theta h_w (n-1)}. \tag{3.15}$$

Hence, one can obtain retransfer level of items in the showroom $R^{*3.3}$ and optimal units $q^{*3.3}$ transferred.

Algorithm

Step 1. Assign parametric values to A, G, h_d, h_w, P, C, a, b, θ, L.

Step 2. If $\Delta < 0$, then go to Algorithm 3.1.

Step 3. If $\Delta = 0$, then go to Algorithm 3.2.

Step 4. If $\Delta > 0$, then go to Algorithm 3.3.

Algorithm 3.1.

Step 1. Set $R = 0$ and $n = 1$.

Step 2. Obtain t_1^{*1} by solving (3.3) with Maple 11 (mathematical software) and q^{*1} from (2.5).

Step 3. If $q^{*1} < L$, then t_1^{*1} obtained in Step 2 is optimal; otherwise,

$$t_1^{*1} = \frac{1}{\theta} \ln\left[1 + \frac{L\theta^2}{a(\theta + b)}\right]. \tag{3.16}$$

Step 4. Compute $Z(n, t_1^{*1})$.

Step 5. Increment n by 1.

Step 6. Continue Steps 2 to 5 until $Z(n, t_1^{*1}) < Z((n-1), t_1^{*1})$.

Algorithm 3.2.

Step 1. Set $R = 0$ and $n = 2$.

Step 2. Obtain t_1^{*2} from (3.8) and q^{*2} from (2.5).

Step 3. If $q^{*2} < L$, then t_1^{*2} obtained in Step 2 is optimal; otherwise,

$$t_1^{*2} = \frac{1}{\theta} \ln\left[1 + \frac{L\theta^2}{a(\theta + b)}\right]. \tag{3.17}$$

Step 4. Compute $Z(n, t_1^{*2})$.

Step 5. Increment n by 1.

Step 6. Continue Steps 2 to 5 until $Z(n, t_1^{*2}) < Z((n-1), t_1^{*2})$.

Algorithm 3.3.

Step 1. Set $n = 2$.

Step 2. Solve (3.3) to compute $t_1^{*3.1}$ and determine $q^{*3.1}$ from (2.5) and $R = 0$.

Table 1

[Variations for b] [Fixed values $L = 150$, $A = 90$, $G = 10$, $b = 0.4$]						
b	n	t_1^{*1}	T^*	q^{*1}	Q^*	Z^*
0.40	6	0.138	0.830	135.48	812.94	1635.60
0.45	6	0.136	0.817	132.85	797.11	1629.22
0.50	6	0.133	0.804	130.34	782.04	1622.94

Table 2

[Variations for G] [Fixed values $L = 150$, $A = 90$, $b = 0.4$]						
G	n	t_1^{*1}	T^*	q^{*1}	Q^*	Z^*
10	9	0.152	1.368	148.4932	1336.439	1600.113
20	7	0.151	1.057	147.5394	1032.776	1560.089
30	6	0.138	0.828	135.1126	810.6756	1490.671

Step 3. If $q^{*3.1} \leq L$, then $t_1^{*3.1}$ obtained in Step 2 is optimal; otherwise,

$$t_1^{*3.1} = \frac{1}{\theta} \ln\left[1 + \frac{L\theta^2}{a(\theta + b)}\right] \tag{3.18}$$

is optimal.

Step 4. If $((P-C)\theta - h_d)/\theta t_1 < h_w(n-1)/2$ then Compute $Z(n, t_1^{*3.1})$, otherwise set $Z(n, t_1^{*3.1}) = 0$.

Step 5. Solve (3.13) to compute $t_1^{*3.2}$.

Step 6. If $((P-C)\theta - h_d)/\theta t_1 > h_w(n-1)/2$, then Substitute $t_1^{*3.2}$ into (3.12) to find R and Calculate $Z(n, t_1{}^{*3.2})$; otherwise set $Z(n, t_1^{*3.2}) = 0$.

Step 7. $Z(n, t_1^{*3}) = \max\{Z(n, t_1^{*3.1}), Z(n, t_1^{*3.2})\}$.

Step 8. Increment n by 1.

Step 9. Continue Steps 2 to 8 until $Z(n, t_1^{*3}) < Z((n-1), t_1^{*3})$.

4. Numerical Examples

Example 4.1. Consider the following parametric values in proper units: $[a, \theta, h_d, h_w, C, P] = [1000, 0.10, 0.6, 0.3, 1, 3]$. Here, $(P - C)\theta - h_d < 0$.

We apply Algorithm 3.1. The variations in demand rate b, transfer cost G, ordering cost A, and maximum allowable units L are studied (see Tables 1, 2, 3, and 4).

Example 4.2. Consider the following parametric values in proper units: $[a, \theta, h_d, h_w, C, P] = [1000, 0.20, 0.40, 0.10, 1, 3]$. Here, $(P - C)\theta - h_d = 0$. Using Algorithm 3.2, variations in

Table 3

[Variations for A] [Fixed values $L = 150$, $G = 10$, $b = 0.4$]						
A	n	t_1^{*1}	T^*	q^{*1}	Q^*	Z^*
50	6	0.149	0.894	145.631	873.7861	1679.377
60	6	0.146	0.876	142.7661	856.5966	1669.339
70	5	0.144	0.72	140.8545	704.2727	1663.394

Table 4

[Variations for L] [Fixed values $A = 90$, $G = 10$, $b = 0.4$]						
L	n	t_1^{*1}	T^*	q^{*1}	Q^*	Z^*
150	6	0.138	0.830	135.48	812.94	1635.60
250	5	0.156	0.778	151.90	759.50	1636.67
350	5	0.156	0.778	151.90	759.50	1636.67

Table 5

[Variations for b] [Fixed values $L = 150$, $A = 90$, $G = 10$, $P = 3$, $C = 1$, $\theta = 0.2$]						
b	n	t_1^{*2}	T^*	q^{*2}	Q^*	Z^*
0.4	10	0.151	1.508	148.43	1484.305	1746.88
0.425	10	0.149	1.487	146.14	1461.393	1743.27
0.45	10	0.147	1.467	143.94	1439.398	1739.70

Table 6

[Variations for G] [Fixed values $L = 150$, $A = 90$, $b = 0.4$, $P = 3$, $C = 1$, $\theta = 0.2$]						
G	n	t_1^{*2}	T^*	q^{*2}	Q^*	Z^*
10	10	0.1508	1.508	148.43	1484.305	1746.88
12	9	0.1493	1.3437	147.0036	1323.032	1734.124
14	8	0.1479	1.1832	145.6471	1165.176	1719.14

Table 7

[Variations for A] [Fixed values $L = 150$, $G = 10$, $b = 0.4$, $P = 3$, $C = 1$, $\theta = 0.2$]						
A	n	t_1^{*2}	T^*	q^{*2}	Q^*	Z^*
80	10	0.1548	1.548	152.3285	1523.285	1753.253
85	10	0.1528	1.528	150.393	1503.93	1750.13
90	10	0.1508	1.508	148.43	1484.31	1746.88

demand rate b, transferring cost G, ordering cost A, and maximum allowable number L on the decision variables and objective function are studied in Tables 5, 6, 7, and 8.

Example 4.3. Consider the following parametric values in proper units: $[a, \theta, h_d, h_w, C, P] = [1000, 0.40, 3, 1, 4, 12]$. Here, $(P-C)\theta - h_d > 0$. Using Algorithm 3.3, variations in demand rate;

Table 8

[Variations for L] [Fixed values $A = 90$, $G = 10$, $b = 0.4$, $P = 3$, $C = 1$, $\theta = 0.2$]						
L	n	t_1^{*2}	T^*	q^{*2}	Q^*	Z^*
100	22	0.099	2.185	98.31	2162.86	1715.17
150	10	0.151	1.508	148.43	1484.31	1746.88
175	8	0.170	1.358	166.76	1334.12	1748.55
200	8	0.170	1.358	166.76	1334.12	1748.55

Table 9

[Variations for b] [Fixed values $L = 150$, $A = 90$, $G = 30$, $P = 12$, $C = 4$, $\theta = 0.40$]							
b	n	t_1^{*3}	T^*	q^{*3}	Q^*	Z^*	R
0.40	3	0.151	0.452	150.74	452.22	7224.91	0
0.45	3	0.145	0.436	145.16	435.47	7195.76	4.845
0.50	3	0.141	0.422	140.16	420.48	7167.68	9.840

Table 10

[Variations for G] [Fixed values $L = 150$, $A = 90$, $b = 0.4$, $P = 12$, $C = 4$, $\theta = 0.4$]							
G	n	t_1^{*3}	T^*	q^{*3}	Q^*	Z^*	R
30	3	0.151	0.452	150.74	452.22	7224.91	0
20	3	0.137	0.412	138.01	414.02	7294.20	11.993
10	4	0.101	0.405	103.20	412.78	7381.82	46.804

Table 11

[Variations for A] [Fixed values $L = 150$, $b = 0.4$, $G = 30$, $P = 12$, $C = 4$, $\theta = 0.4$]							
A	n	t_1^{*3}	T^*	q^{*3}	Q^*	Z^*	R
90	3	0.151	0.452	150.74	452.22	7224.91	0
95	3	0.153	0.459	152.87	458.60	7214.22	0
100	3	0.155	0.465	154.97	464.90	7203.68	0

Table 12

[Variations for L] [Fixed values $A = 90$, $b = 0.4$, $G = 30$, $P = 12$, $C = 4$, $\theta = 0.4$]							
A	n	t_1^{*3}	T^*	q^{*3}	Q^*	Z^*	R
150	3	0.1508	0.452	150.74	452.22	7224.91	0
200	3	0.1502	0.451	153.04	459.13	7231.60	46.96
250	3	0.1496	0.449	155.38	466.13	7238.39	94.62

b, transferring cost G, ordering cost A, and maximum allowable number L on the decision variables and total profit per unit time are studied in Tables 9, 10, 11, and 12.

The following managerial issues are observed from Tables 1–12.

(1) Increase in demand rate b decreases t_1^*, q^*, and Z^*. It is obvious that retailer's total profit per unit time, cycle time in the warehouse, and procurement quantity from the supplier decrease as the demand decreases.

(2) Increase in transferring cost from the warehouse to the showroom increases t_1^*, q^* and decreases Z^*. Z^* decreases because the number of transfer increases.

(3) Increase in ordering cost decreases cycle time in showroom and units transferred from warehouse to the showroom and retailer's total profit per unit time. The cycle time in warehouse increases significantly.

(4) Increase in maximum allowable number in display area increases t_1^* and q^* but no significant change is observed in the total profit per unit time of the retailer. The cycle time in warehouse and procurement quantity from the supplier decreases significantly.

5. Conclusions

In this article, an ordering transfer inventory model for deteriorating items is analyzed when the retailer owns showroom having finite floor space and the demand is decreasing with time. Algorithms are proposed to determine retailer's optimal policy which maximizes his total profit per unit time. Numerical examples and the sensitivity analysis are given to deduce managerial insights.

The proposed model can be extended to allow for time dependent deterioration. It is more realistic if damages during transfer from warehouse to showroom are incorporated.

Assumptions

The following assumptions are used to derive the proposed model.

(1) The inventory system under consideration deals with a single item.

(2) The planning horizon is infinite.

(3) Shortages are not allowed. The lead time is negligible or zero.

(4) The maximum allowable item of displayed stock in the showroom is L.

(5) The time to transfer items from the warehouse to the showroom is negligible or zero.

(6) The units in inventory deteriorate at a constant rate "θ", $0 \leq \theta < 1$. The deteriorated units can neither be repaired nor replaced during the cycle time.

(7) The retailer orders Q-units per order from a supplier and stocks these items in the warehouse. The items are transferred from the warehouse to the showroom in equal size of "q" units until the inventory level in the warehouse reaches to zero. This is known as retailer's order-transfer policy.

Notations

L: The maximum allowable number of displayed units in the showroom

$I(t)$: The inventory level at any instant of time t in the showroom, $I(t) \leq L$

$D(t)$: The demand rate at time t. Consider $D(t) = a(1 - bt)$ where $a, b > 0$, $a \gg b$. a denotes constant demand and $0 < b < 1$ denotes the rate of change of demand due to recession

θ: Constant rate deterioration, $0 \leq \theta < 1$

h_w: The unit inventory carrying cost per annum in the warehouse

h_d: The unit inventory carrying cost per annum in the showroom, with $h_d > h_w$

P: The unit selling price of the item

C: The unit purchase cost, with $C < P$

A: The ordering cost per order

G: The known fixed cost per transfer from the warehouse to the showroom

T: The cycle time in the warehouse, (a decision variable)

n: The integer number of transfers from the warehouse to the showroom per order (a decision variable)

t_1: The cycle time in the showroom (a decision variable)

Q: The optimum procurement units from a supplier (decision variable)

q: The number of units per transfer from the warehouse to the showroom, $0 \leq q \leq L$ (a decision variable)

R: The inventory level of items in the showroom regarding the transfer of q-units from the warehouse to the showroom.

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