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Influence of Mass Transfer Processes on Couette Flow of Magnetic Fluid

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This article describes the results of a theoretical study of magnetic fluid two-dimensional Couett flow in magnetic fluid seal model in view of mass transfer processes. It has been shown that very inhomogeneous magnetic field in seal gap lead to magnetic particle concentration rearrangement due to magnetophoresis and Brownian diffusion. In turn, it lead to inhomogeneous magnetic fluid viscosity and change in local and integral shearing force at channel walls. Integral shearing force has been shown to depend on magnetic field and magnetic fluid parameters. Closely-packed fluid density distribution conditions have been defined. Proposed theory covers real magnetic fluid seal performance features adequately.

Keywords: Couett flow, Magnetic fluid, Magnetic particles, Brownian diffusion, Magnetophoresis, concentration, Viscosity, Shearing force.

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Magnetic fluids are basically colloidal liquids of solid ferromagnetic particles suspended in a carrier fluid by Brownian motion, i.e. magnetic fluid can be considered as homogenous fluid with magnetic moment (magnetization) M per unit volume. Macroscopic magnetic fluid behavior in an external gradient magnetic field is determined by the force acting on each ferromagnetic particle. Also this force cause Brownian magnetic particles to move relative to the carrier fluid (magnetophoresis phenomenon) resulting in particle concentration rearrangement. Particle volume fraction φ increases along with increasing field strength. In opposition to this, magnetic particles move down the concentration gradient. Both processes result in equilibrium distribution which depends on magnetic field configuration and magnetic fluid properties. As such, the fluid magnetization increases along with the particle concentration. Usually, maximum magnetization regions feature maximum field gradient. It has a significant effect on ponderomotive magnetic force distribution, flotation conditions and all the hydrostatics [1, 2].

These processes are most pronounced in the magnetic fluid seal [3-5] where magnetic field gradient is extremely high and magnetic fluid film thickness is extremely low. Particle concentration rearrangement inter alia results in increased peak differential pressure that the seal holds.

This article also notice that the magnetic fluid viscosity is significantly determined by the concentration. This fact can be successfully analyzed within the framework developed by articles [1, 2]. Therefore magnetophoresis and Brownian diffusion can result in local viscosity inhomogeneities as well as changes in shear elasticity on solid surfaces including temporal changes in seal frictional moment.

Hereinafter, equations from the articles [1, 2] are cited in order to deal the problem of interaction between mass transfer and magnetic fluid viscosity and shear elasticity on Couett flow in magnetic fluid seal model.

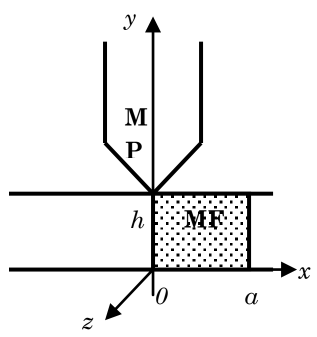

Fig. 1 – Problem geometry

In Fig. 1, Newtonian magnetic fluid of volume (MF) ($0 < x < a$, $0 < y < h$) fills the gap between two flat solid surfaces $y = 0$ and $y = h$ and remains in its place due to inhomogeneous magnetic field $H(x)$ which is created by sharp constant magnet pole (MP). Such magnetic fluid position can be caused by the differential pressure Δp between side surfaces and its value matches the peak differential pressure that the magnetic fluid seal holds. Due to symmetry, obtained results are valid if the magnetic fluid position is symmetric with respect to pole axis. Side surfaces are assumed to be flat. Lower surface (rotating shaft) ($y = 0$) moves relative to the upper one ($y = h$) at a rate of υ_0 along z axis. In such a case, the Couett flow with velocity profile of $\upsilon_z = \upsilon_0(y/h - 1)$ starts. Viscous friction stress τ on solid surfaces equals to $\tau = \eta(\partial\upsilon_z / \partial y) = \eta\upsilon_0 / h$ and depends on dynamic viscosity coefficient η which in turn depends on the particle concentration $\varphi(x)$: $\eta = \eta[\varphi(x)]$.

Therefore the shearing force at fluid film border $F_\tau = w\int_0^a \tau dx = (w\upsilon_0 / h)\int_0^a \eta[\phi(x)]dx$ depends on the particle concentration distribution. (w is fluid film length along the surface move direction z).

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As a matter of convenience, the following non-dimensional values shall be used: coordinate $x' = x/a$, magnetic field strength $H' = H/H_0$, relative concentration $C = \varphi / C_0$, non-dimensional viscosity $\eta' = \eta/\eta_0$, viscous stress $\tau' = \tau h / \eta_0 v_0$, viscous friction $F_\tau' = F_\tau h / wa\eta_0 v_0$ using typical length a, field strength H_0, viscosity η_0, and average absolute concentration C_0.

For classic diffusion [1, 2] processes, static magnetic particle distribution is determined by zero mass flow resulting in the following equation for relative particle concentration (hereinafter, primes will be omitted for non-dimensional values):

$$\frac{\partial C}{\partial x} = UC\Lambda(H)\frac{\partial H}{\partial x} \quad (1)$$

and constant average particle concentration

$$\int_0^1 C(x)dx = 1 \quad (2)$$

Furthermore, $F_\tau = \int_0^1 \eta[\phi(x)]dx$.

The problem is described by non-dimensional parameter $U = \mu_0 m_m H_0 / kT$ (μ_0 – vacuum permeability, mm – particle magnetic moment, k – Boltzmann constant, T – absolute temperature) as well as Langevin function $\Lambda(H) = \mathrm{cth}(UH) - 1/UH$. Magnetic fluid diffusion becomes significant if magnetic parameter $U > 1$. If the particles are of typical magnetic moment of $mm = 2.5 \cdot 10^{-19}$ J/T and the temperature is $T = 300$ K then $U = 1$ takes place for magnetic field strength of about 10 kA/m. In strong high-gradient magnetic fields in magnetic fluid seal gaps this parameter can even achieve two-digit valuesв.

Exact solution of the equations (1), (2) is as follows:

$$C = \frac{\mathrm{sh}(UH)}{H}\left[\int_0^1 \frac{\mathrm{sh}(UH)}{H}dx\right]^{-1} \quad (3)$$

Magnetic force Fm for the entire fluid volume is determined by magnetization M which in turn depends on magnetic particle concentration C and saturation magnetic moment Ms: $M = MsC(x)\Lambda(H)$.

$$F_m = wh\mu_0 \int_a^0 M\frac{dH}{dx}dx = wh\mu_0 M_s \int_a^0 \Lambda C\frac{dH}{dx}dx \; .$$

This equation for the non-dimensional values is as follows:

$$F_m / wh\mu_0 M_s H_0 = \int_1^0 \Lambda[H(x)]C(x)\frac{dH}{dx}dx \quad (4)$$

With regard to (1) the equation can also be as follows:

$$F_m / wh\mu_0 M_s H_0 = (1/U)[C(0) - C(1)] \quad (5)$$

For the homogenous fluid ($C = 1$), this force will be as follows

$$F_m(\tilde{N} = 1) = \int_1^0 \Lambda\frac{dH}{dz} = \frac{1}{U}\ln\left[\frac{C(0)}{C(1)}\right] \quad (6)$$

The interaction between the concentration rearrangement and magnetic force value can be characterized by relative magnetic force coefficient:

$$k_m = \frac{F_m(C)}{F_m(C=1)} = \frac{C(0) - C(1)}{\ln C(0) - \ln C(1)} \quad (7)$$

Similarly, the interaction between the concentration rearrangement and the friction on channel walls shall be characterized by relative friction coefficient: $k_\tau = F_\tau(C) / F_\tau(C=1)$.

At first, it can easily be shown that linear relations between viscosity coefficient and particle concentration (like the Einstein equation) $\eta = (1 + \alpha C)$ result in the following equation

$$F_\tau = \int_0^1 (1 + \alpha C)dx = 1 + \alpha\int_0^1 Cdx = 1 + \alpha = F_\tau(C=1) \text{ and } k_F = 1 \, .$$

Thus, on the assumption of the linear relations between viscosity and particle concentration the friction on the moving surface doesn't depend on the concentration distribution.

If the relations between viscosity and particle concentration are not linear k_τ doesn't equal to 1. Moreover, positive nonlinearity results in $k_\tau > 1$ (increasing friction) and negative nonlinearity results in $k_\tau < 1$ (decreasing friction).

Further approximation of the relations between viscosity and particle concentration is performed using Krieger-Dougherty equation [6]:

$$\eta = (1 - \frac{\varphi}{\varphi_{\max}})^{-2{,}5\phi_{\max}} \, , \quad (8)$$

where φ_{max} is peak allowable concentration which corresponds to some type of close packing.

For $\varphi_{\max} = 0{,}64$ it will be as follows

$$\eta = (1 - 1{,}56\varphi)^{-1{,}6} \, . \quad (9)$$

Magnetic field model approximation $H = H(x)$ describes the fact that field strength quickly decreases along with increasing distance to pole. We can rely on commonly used solution of magnetostatic equations for hyperbolical pole [3]: $H = 1/\sqrt{1 + nx^2}$ or in broader terms:

$$H(x) = 1/(1 + nx^q)^m \quad (10)$$

Calculation results for (3)-(10) are shown below in Fig. 2 and 3 and in Table 1.

Referring to the Fig. 2, it will be seen that the particle concentration in maximum magnetic field strength regions ($x = 0$) can be several times of that in distant regions due to magnetophoresis and Brownian diffusion. The most significant increase in the particle concentration takes place in about third part of the fluid volume immediately under the magnet pole. Maximum concentration increases along with increasing magnetic parameter U up to the value corresponding the close packing as shown in Fig. 2b. Definitely, closely-packed region ($\varphi > \varphi_{max}$) cannot be assumed to be fluid. Closely-packed region volume increases along with the increasing U. The more average particle concentration, the less U is necessary to close packing. Referring to the Table 1, it will be seen that the average concentration of $C_0 = 0.15$ requires $U = 5$ to achieve the close packing while $C_0 = 0.2$ requires $U = 4$.

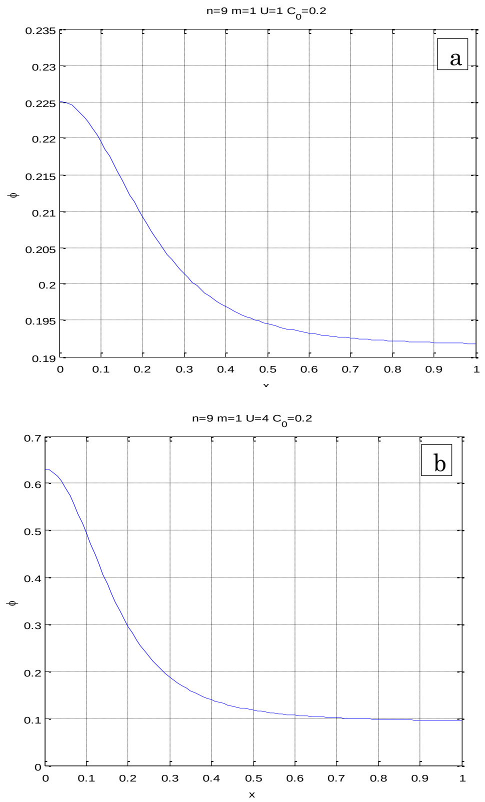

Fig. 2 – Typical concentration distribution φ profiles for the magnetic fluid on the assumption of the average concentration of $C_0 = 0.2$ and two U values: a) $U = 1$, b) $U = 4$. (The magnetic field configuration is $n = 9$, $m = 1$, $q = 2$)

Referring to the Table 1, it will be seen that excess particle concentration on solid surfaces results in increasing friction and $k_\tau > 1$. For the magnetic fluid seal, it means that processes in question result in increasing friction torques. The proposed shear elasticity calculation model is limited by the infinite viscosity of closely-packed regions according to the Krieger-Dougherty equation. Therefore, there are dashes for such regions in the Table 1.

In Fig. 3, the results for equation (7) are shown demonstrating increase in magnetic force coefficient due to magnetic particle concentration rearrangement in the seal gap.

The above steady-state problem implies steady-state particle concentration distribution. Definitely,

Table 1 – Maximum ($C(0)$) and minimum ($C(1)$) concentration and friction coefficient k_τ values for the different average particle concentration C_0 (The magnetic field configuration is $n = 9$, $m = 1$, $q = 2$)

	U	1	2	3	4	5	6
C_0=0.1	$C(0)$	–	0.15	0.22	0.31	0.42	0.55
	$C(1)$	–	0.08	0.07	0.05	0.04	0.02
	k_τ	–	1.00	1.02	1.06	1.19	1.67
C_0=0.15	$C(0)$	0.17	0.22	0.33	0.47	**0.64**	–
	$C(1)$	0.14	0.07	0.1	0.07	0.05	–
	k_τ	1.00	1.02	1.06	1.28	–	–
C_0=0.2	$C(0)$	0.25	0.30	0.45	**0.64**	–	–
	$C(1)$	0.19	0.17	0.14	0.10	–	–
	k_τ	1.00	1.02	1.17	–	–	–

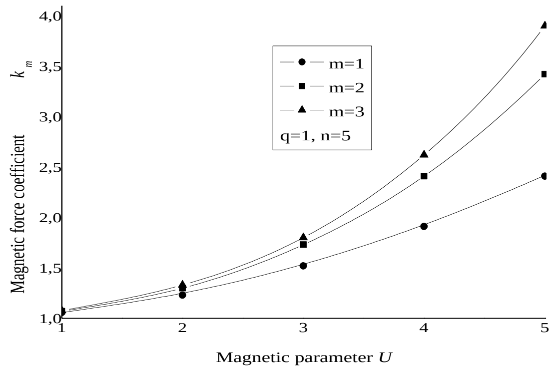

Fig. 3 – The relations between magnetic force coefficient $k_m = F_m/F_m(C = 1)$ and magnetic parameter U in different magnetic field configurations

real magnetic fluid seal performance requires certain amount of time achieve the abovementioned steady-state values and the coefficients k_τ and k_m monotonically increase during this time.

CONCLUSIONS

The above results indicate that the proposed theory adequately describes particle concentration rearrangement in the nanodispersed magnetic fluid due to magnetophoresis and Brownian diffusion and covers real magnetic fluid seal performance features namely increase in friction torque and peak differential pressure that the seal holds.

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