A method for rough terrain locomotion based on Divergent Component of Motion

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***Abstract*— For humanoid robots to be used in real world scenarios, there is a need of robust and simple walking controllers. Limitation to flat terrain is a drawback of many walking controllers. We overcome this limitation by extending the concept of Divergent Component of Motion (DCM, also called 'Capture Point') to 3D. Therefor, we introduce the "Enhanced Centroidal Moment Pivot point" (eCMP) and the "Virtual Repellent Point" (VRP), which allow for a very intuitive understanding of the robot's CoM dynamics. Based on eCMP, VRP and DCM, we present a method for real-time planning and control of DCM trajectories in 3D.**

I. INTRODUCTION

Many successful LIP based walking control methods have been presented recently [1]–[3]. The use of the LIP model for bipedal walking control is restricted to horizontal CoM motions ($z = const$). This motivates the derivation of methods for non-constant CoM and floor height. Zhao and Sentis [4] present a method for three-dimensional foot placement planning on uneven ground surfaces. Yet, the lateral foot-placement cannot be predefined, but is dependent on the sagittal dynamics and the desired CoM Surface and the ground surfaces have to be of constant height laterally.

The concept of 'Capture Point' or 'Divergent Component of Motion' (DCM) [5]–[11] splits the CoM dynamics into a stable (CoM) and an unstable (DCM) part and thus simplifies the design of walking controllers. In the presented work and in [12], we derive a method for bipedal gait planning and control on uneven terrain, facilitated by the use of the linear properties of the DCM dynamics and suffering from none of the afore mentioned restrictions.

II. THREE-DIMENSIONAL DCM, ECMP AND VRP

Motivated by the performance of 2D Capture Point (= DCM) control [8], [9], we introduce the three-dimensional Divergent Component of Motion (DCM):

$$\boldsymbol{\xi} = \boldsymbol{x} + b\dot{\boldsymbol{x}}, \tag{1}$$

where $\boldsymbol{\xi} = [\xi_x, \xi_y, \xi_z]^T$ is the DCM, $\boldsymbol{x} = [x, y, z]^T$ and $\dot{\boldsymbol{x}} = [\dot{x}, \dot{y}, \dot{z}]^T$ are the CoM position and velocity and $b > 0$ is the time-constant of the DCM dynamics. By reordering (1), we directly find the CoM dynamics as

$$\dot{\boldsymbol{x}} = -\frac{1}{b}(\boldsymbol{x} - \boldsymbol{\xi}). \tag{2}$$

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This shows that the CoM has a stable first order dynamics for $b > 0$ ($\rightarrow$ it follows the DCM). Additionally, we introduce the so called Enhanced Centroidal Moment Pivot point (eCMP), which encodes the external (e.g. leg-) forces in a linear force law, based on the difference of the CoM and the eCMP:

$$\boldsymbol{F}_{ext} = \frac{m}{b^2}\,(\boldsymbol{x} - \boldsymbol{r}_{ecmp}). \tag{3}$$

The eCMP is closely related to the CMP [13], but not restricted to the foot plane or ground surface. This allows for encoding of not only the direction of the sum of external forces, but also its magnitude. To encode the total sum of forces (including gravity), we introduce the Virtual Repellent Point (VRP):

$$\boldsymbol{r}_{vrp} = \boldsymbol{r}_{ecmp} + \begin{bmatrix}0 & 0 & b^2 g\end{bmatrix}^T = \boldsymbol{r}_{ecmp} + \begin{bmatrix}0 & 0 & \Delta z_{vrp}\end{bmatrix}^T. \tag{4}$$

This leads to following DCM dynamics:

$$\dot{\boldsymbol{\xi}} = \frac{1}{b}\,(\boldsymbol{\xi} - \boldsymbol{r}_{vrp}). \tag{5}$$

For $b > 0$ the DCM has an unstable first order dynamics ("pushed" by the VRP on a straight line). The VRP encodes gravity and external forces in a single point:

$$\boldsymbol{F} = \frac{m}{b^2}(\boldsymbol{x} - \boldsymbol{r}_{vrp}). \tag{6}$$

Figure 1 clarifies the correlations between eCMP, CMP and CoP for general (bipedal) robot dynamics.

III. GENERATION OF DCM REFERENCE

The basic idea - exploiting the first order dynamics of the DCM - is to find a DCM trajectory which corresponds to constant eCMPs in the centers of the preplanned future foot positions $\boldsymbol{r}_{f,i}$, thus fulfilling the ZMP constraints. Given a desired eCMP-to-VRP height difference Δz_{vrp}, we find the according desired VRPs (see fig 2) with (4) as

$$\boldsymbol{r}_{vrp,d,i} = \boldsymbol{r}_{f,i} + \begin{bmatrix}0 & 0 & \Delta z_{vrp}\end{bmatrix}^T \tag{7}$$

With (4), we find the time-constant of the DCM dynamics as $b = \sqrt{\Delta z_{vrp}/g}$. The desired DCM locations at the end of each step are found via recursion:

$$\boldsymbol{\xi}_{d,eos,i-1} = \boldsymbol{\xi}_{d,ini,i} = \boldsymbol{r}_{vrp,d,i} + e^{-\frac{t_{step}}{b}}\,(\boldsymbol{\xi}_{d,eos,i} - \boldsymbol{r}_{vrp,d,i}). \tag{8}$$

For $t < t_{step}$, the desired DCM trajectory in time is

$$\boldsymbol{\xi}_d(t) = \boldsymbol{r}_{vrp,d,1} + e^{\frac{t-t_{step}}{b}}(\boldsymbol{\xi}_{d,eos,1} - \boldsymbol{r}_{vrp,d,1}). \tag{9}$$

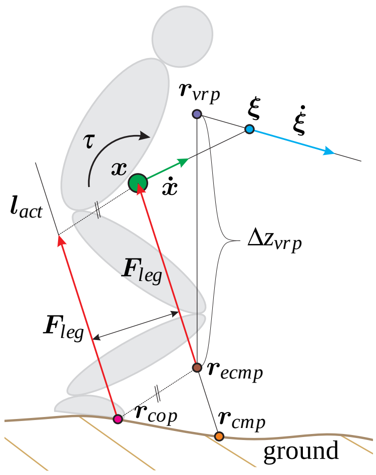

Fig. 1. Point correlations for general robot dynamics. The CMP is at the intersection of the line CoM-to-eCMP with the ground. The leg force line of action $\boldsymbol{l}_{act}$ is shifted by a torque τ around the CoM, so the CoP does generally not coincide with the CMP.

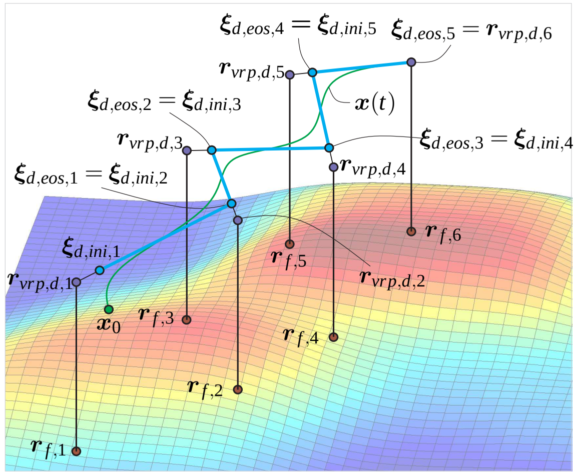

Fig. 2. Planning of DCM trajectory over rough terrain. The DCM reference trajectory (bold lines) and the resulting CoM trajectory (sinusoidal curve, "automatically" follows the DCM) are 3D.

IV. Three-dim. DCM tracking control

The DCM control law used in this work is

$$\boldsymbol{r}_{vrp,c} = \boldsymbol{\xi} + k\, b(\boldsymbol{\xi} - \boldsymbol{\xi}_d) - b\dot{\boldsymbol{\xi}}_d \ , \tag{10}$$

It leads to the following stable closed loop dynamics

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{\xi}} \end{bmatrix} = \underbrace{\begin{bmatrix} -1/b & 1/b \\ 0 & -k \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{\xi} \end{bmatrix}}_{feedback} + \underbrace{\begin{bmatrix} 0 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}_d \\ \dot{\boldsymbol{\xi}}_d \end{bmatrix}}_{feed\,forward} . \tag{11}$$

For $b > 0$ and $k > 0$ the dynamics is stable. The according desired leg-force can be found as

$$\boldsymbol{F}_{leg,c} = \frac{mg}{\Delta z_{vrp}} (\boldsymbol{x} - (\underbrace{\boldsymbol{r}_{vrp,c} - \begin{bmatrix} 0 & 0 & \Delta z_{vrp} \end{bmatrix}^T}_{\boldsymbol{r}_{ecmp,c}})) \tag{12}$$

Note that the only equations that are finally needed are (8) and (9) for three-dimensional DCM trajectory generation and (10) and (12) for force-based DCM tracking control. They can easily be computed in real-time on any computer.

V. Conclusions

In addition to the shown equations, we found methods to guarantee feasibility of the finally commanded forces (not presented in this abstract). The performance of the proposed control framework was tested in point-mass/point-foot (bipedal) simulations and simulations of DLR's humanoid TORO in OpenHRP3. The controller shows high robustness towards external unknown perturbations (constant and impulsive forces) and model inaccuracies.

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