Essays on Interaction between Monetary and Fiscal Policy

Submitted by Yuting Bai to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Economics,

October 2013

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

(Signature) ……………Baiyuting…………………………………………………………

Abstract

This thesis consists of three essays on the discretionary interactions of fiscal and monetary policy authorities when they stabilise a single economy against shocks in the dynamic setting.

In the first essay, I investigate the stabilization bias that arises in a model of non-cooperative monetary and fiscal policy stabilisation of the economy, when the monetary authority implements price level targeting but fiscal authority remains benevolent. I demonstrate that the gain in welfare depends on the level of steady state debt. If the steady state level of the government debt is relatively low, then the monetary price level targeting unambiguously leads to social welfare gains, even if the fiscal authority acts strategically and faces different objectives and has incentives to pursue its own benefit and therefore may offset some or all of monetary policy actions. Moreover, if the fiscal policymaker is able to conduct itself as an intra-period leader then the social welfare gain of the monetary price level targeting regime can be further improved. However, if the economy has a relatively high steady state debt level, the gain of the price level targeting is outweighed by the loss arising from the conflicts between the policy makers, and such policy leads to a lower social welfare than under the cooperative discretionary inflation targeting.

In the second essay I study the macroeconomic effect of the interaction between discretionary monetary policy which re-optimises every period and discretionary fiscal policy which reoptimises less frequently. I demonstrate the existence of two discretionary equilibria if the frequency of fiscal policy re-optimizes annually while monetary policy adjusts quarterly. Following a disturbance to the debt level, the economy can be stabilised either in a 'fast but volatile' or 'slow but smooth' way, where both dynamic paths satisfy the conditions of optimality and time-consistency. I study several delegation regimes and demonstrate that the policy of partial targeting the debt level results in far worse welfare outcomes relative to a strict inflation targeting policy.

In the third essay, I extend the framework developed in the second essay to the case with Blanchard-Yaari type of consumers. This brings in two effects. First, an increase in debt results in higher consumption via the wealth effect, the marginal cost is higher so the need for higher interest rate and higher taxation will increase, therefore the dynamic complementarity between actions of the two policymakers become stronger. Second, higher inflation affects consumption via the average propensity to consume and this effect is likely to weaken the dynamic complementarity. I show that when the households are assigned a mortality rate, overall the first effect dominates the second. The transition paths of the economic variables back to the steady state will be more volatile and the multiple equilibria are more likely to arise.

Contents

Chapter 1

Introduction to Thesis

When a disturbance hits the economy off its steady state growth path, macroeconomic control policies are used to mitigate the negative effects and bring the economy back on track. There was a consensus that stabilization task should be taken by monetary policy while fiscal policy should focus on government debt control (see the discussion in Kirsanova, Leith, and Wren-Lewis, 2009). This was reflected in the literature: much richer research has been done on monetary policy design than on fiscal stabilization policy; the research was done with implicit assumption that fiscal policy is concerned with debt stabilization only, otherwise allowing automatic stabilizers to run.

However the existing empirical literature on monetary-fiscal interactions suggests that fiscal policy does more than simply using automatic stabilizers; see, e.g., Auerbach (2003) and Favero and Monacelli (2005), who analyzed fiscal policy in the United States. Moreover, Davig and Leeper (2005) demonstrate that the debt stabilization in the United States was not the main priority of fiscal policy in the post-war period.

If fiscal policy is not predominantly concerned with the debt stabilization then it may cause adverse effects on the conduct of monetary policy and welfare (Sargent and Wallace, 1981, and Leeper, 1991). Monetary policy may become concerned with debt stabilization instead, and will choose to accommodate inflationary shocks.

At the same time, fiscal policy can also improve the social welfare, in particular in cases when monetary policy is constrained. Stabilizing fiscal policy in a monetary union can substantially mitigate effects of asymmetric shocks, especially in the case of significant inflation persistence (see e.g. Kirsanova, Satchi, and Vines, 2003). Since the creation of the European Monetary Union and the recent recession, when interest rates are close to zero lower bound, there are calls for greater fiscal flexibility.[1] The question of how strategic the fiscal authority should be and to what extent fiscal policy can or should take over the role of monetary policy in stabilizing the economy needs to be discussed.

It has long been known that when both monetary and fiscal policy are considered, because of the interaction between them, the macroeconomic outcomes of the policies are different from those if we only consider effects of one policy in isolation (see Hall and Mankiw, 1994, Woodford, 2001). Since the work of Leeper (1991) the interaction between monetary and fiscal policy begun to receive more attention from macroeconomists. Leith and Wren-Lewis (2000), Benigno and Woodford (2003), Dixit and Lambertini (2003), Lambertini and Rovelli (2003) and Beetsma and Jensen (2005) among many others aimed to set-up a theoretical framework to address various questions of how fiscal policy can be used for the stabilization of the economy. Dixit and Lambertini (2003) used a *static* model to further study the effect of different degrees of pre-commitment of policy makers; they also study the welfare consequences of the leadership structure of these policy interactions. However, if the debt accumulation is to be taken into account then the *dynamic* approach to modelling becomes more suitable. Making models dynamic allows us to make conclusions on the optimal speed of debt stabilization. Schmitt-Grohe and Uribe (2004) studied Ramsey-optimal fiscal and monetary policy in a sticky price model and demonstrates the optimal near-random-walk behavior of the real debt. The policy maker will not inflate the nominal debt to stabilize the real debt, because the cost of increased inflation outweighs the cost of servicing the permanently higher debt level. These results are similar as what Benigno and Woodford (2003) obtained using

[1] Davig and Leeper (2006) argued that during the liquidity trap when interest rate is close to zero lower bound, the fiscal multiplier can be positive; Drautzburg and Uhlig (2011) demonstrated that with stabilising fiscal policy, the financially constrained agents can gain.

the linear-quadratic framework. However, Leith and Wren-Lewis (2014) analyzed discretionary policy-making with sticky prices and overturned this random walk result. They showed that in the New Keynesian model which they study, the debt will be returned to its initial steady state.

Motivated by these considerations, this thesis studies interactions between monetary and fiscal policies in a small stylized New Keynesian model, using the linear-quadratic framework and assuming non-cooperative optimizing authorities. If both authorities use 'sensible' policy objectives, such setup embeds the following transmission mechanism. Suppose the economy is hit by a shock so the debt is above the steady state. A 'sensible' fiscal policy will react to bring debt back to the desired level, but this will typically lead to a cost-push inflation. The 'sensible' monetary policy maker will want to control inflation in one way or another so will most likely raise the interest rate and reduce the marginal costs. A higher interest rate affects the market rate on debt and results in faster debt accumulation. The 'sensible' fiscal policy maker may want to raise taxes even further. Such 'dynamic complementarity' between the actions of monetary and fiscal policy makers – when actions of one policy maker reinforce actions of another policy maker, see Cooper and John (1988) – can have a large effect on the dynamics of the economy. In particular, under discretionary policy when current policy decisions depend on the private sector's expectations of the future policy, multiple policy equilibria-also called expectations traps-can arise (see King and Wolman, 2004) so if the economy is hit by a shock it can converge back following one of several paths with different volatility. The inability of the policy makers to coordinate all agents on the Pareto-preferred adjustment path leads to the higher than desirable volatility and the social welfare loss. In this thesis, therefore, I will assume that discretionary policy makers have non-identical objectives and study implications of such non-cooperative discretionary policymaking for the dynamics of the economy.

Specifically, I make the following three main assumptions about the nature of interactions between the policy makers.

The first assumption is that both policy makers behave in a strategic way instead of following simple rules. Each policy maker is given a policy loss function and policy instruments, which can be changed in order to minimize the corresponding loss. There is a number of important work in the literature assuming that policy makers operate with simple rules (e.g. Taylor, 1985 and Leeper, 1991). Simple rules are easy to analyze and they provide insights into the nature of policy interactions and the transmission mechanism of economic shocks. However, they require pre-commitment, i.e. the policy makers have to credibly promise how they will react to shocks in all situations (Currie and Levine, 1993) and they do not necessary reflect optimizing behavior of the authorities. There is the influential research which argues that the realistic monetary policy is better described by "targeting rules"(see e.g. Svensson, 2003). What we observe as rules is simply the equilibrium outcome of optimization done by the monetary authority. Applying the same assumption to the conduct of fiscal policy may not seem to be currently most empirically relevant approach as many countries may claim that they do not have institutional structure to conduct the fiscal stabilization at a regular basis, but can become relevant in the near future [2]. Therefore, this thesis will treat both policy makers as acting strategically and minimizing its own loss.

The second assumption is that the fiscal policy maker plays a role of intra-period leader. It means that the fiscal policy maker can perceive and exploit the reaction function of the monetary policy makers. The existing institutional structure in many countries makes it more likely that the fiscal policy maker is the intra-period leader: the monetary policy is well studied and predictable; and the fiscal authority in the UK attends monetary policy committee's meetings, which creates bigger chances for the fiscal authority to be able to exploit monetary policy maker's decisions. The recent empirical works done by Fragetta and Kirsanova (2010) and Le Roux and Kirsanova (2014) showed that the fiscal

[2] For example, the Office for Budget Responsibility has recently been established in the UK.

leadership fits the data from the UK and Sweden, better.

The third assumption is about the degree of pre-commitment of the two policy authorities. Some recent empirical evidence shows that both authorities are likely to act under discretion rather than commitment, see Givens (2012) and Chen et al. (2013) on monetary policy and Le Roux and Kirsanova (2014) on monetary and fiscal policy interactions. The full credibility which is required by pre-commitment is unlikely to realize: once the state of the economy is observed, and in order to achieve the best welfare outcome, the benevolent incumbent policy maker should use the policy instrument to coordinate all future actions of all future consequent policy makers and the private sector. In a rational expectations framework with forward looking agents, such commitment policy requires the policy maker to control the expectations of all economic agents. However, the previously announced policy plans, which were optimal at the time of the announcement, may not be optimal in future periods. Therefore the policy makers always have incentive to deviate from the previously chosen plan. The forward looking private sector expects the policy maker to reoptimize and deviate, this lack of credibility leads to the more realistic setup: discretionary policy. Under discretionary policy, policy makers adjust the policy plan every period, and the private sector expects all future policy makers to re-optimise.

In this thesis I only address the following two questions.

First, I study the effect of strategic monetary and fiscal policy interactions assuming that the authorities do not share the same objective. I am particularly interested in the scenario when discretionary monetary policy implements the price level targeting while the fiscal authority minimizes the social loss function. Generally, any discretionary policy regime results in lower social welfare comparing to the same regime under commitment, as level and stabilization biases arise (Barro and Gordon, 1983, Svensson, 1997 and Clarida, Gali, and Gertler (1999)). The stabilization bias, which is studied in this thesis, refers to the situation when the targeted variable converges back to its steady state with volatility higher than that under commitment. This is because the policy maker without a credible commitment technique cannot exploit the expectation of private sector and prefers to eliminate inflation faster, without relying on promises of future policy, and so choosing to react more aggressively in initial periods after the shock. To improve the social outcome of discretionary *monetary* policy, many delegation schemes-which delegate policy decisions to the policy maker with an objective which is different from the social objectives-were proposed. Examples include conservative central banker (Barro and Gordon, 1983, Rogoff, 1985 and Clarida et al., 1999) with a higher weight or lower level of inflation target in monetary policy objectives; price level targeting (Svensson, 1999 and Vestin, 2006) where price level stability is in the policy objective function; speed limiting policy, which targets the change in output gap (Walsh, 2003) rather than the level of output gap; interest smoothing policy (Woodford, 2003b), under which the policy loss function is augmented with a term with the change in the interest rate; nominal GDP targeting which targets the level of economic activity in nominal terms (Hall and Mankiw, 1994), and nominal income growth (Jensen, 2002). It was shown that the stabilization bias can be reduced using these appropriate delegation scheme, however these schemes were studied using models with monetary policy only. What if a strategic *fiscal* policy maker is given objective which does not necessarily coincide with the monetary policy's objective? It is known that such non-cooperative interactions can lead to a fight between policy makers, see Dixit and Lambertini (2003) and Blake and Kirsanova (2011). This thesis studies the Price Level Targeting as it was frequently discussed recently by policy makers as a realistic policy option[3].

Second, I investigate the effect of different frequencies of actions of monetary and fiscal policy on the economy. In reality monetary policy operates monthly while fiscal policy decides on tax and government spending annually or even less frequently. When the economy is hit by shocks then fiscal policy who wants to contribute to stabilization

[3] In recent years Bank of Canada has been discussing extensively about whether economic welfare might be improved by moving from the inflation-targeting framework to some form of price-level targeting (the discussion is reviewed in Ambler and Ambler (2009).)

may have to do *large* infrequent adjustments which will affect monetary policy and the economy. Such changes may strengthen dynamic complementarity between policy instruments discussed above and change results on existence and uniqueness of discretionary policy.

This thesis is organized as follows. Chapter 2 contains the analysis of non-cooperative discretionary policy interactions, between the price level targeting monetary policy maker and the social welfare targeting fiscal policy maker. Using the New Keynesian model with debt accumulation, I study if this type of policy delegation leads to a reduction of the stabilization bias and the gain in the social welfare. I demonstrate that the extent to which the stabilization bias is reduced depends on the steady state level of debt. In the low steady state level of debt case, the monetary price level targeting unambiguously leads to social welfare gains even if the fiscal authority acts strategically, faces different objectives and has incentives to pursue its own benefit and offsets some or all of monetary policy actions. If the fiscal policy maker is able to conduct itself as an intra-period leader the welfare gain of monetary price level targeting is particularly large. However, when the steady state debt level is high, a positive disturbance to the debt results in optimally higher taxes and so leads to the cost push inflation. Because the monetary policy maker wants to bring inflation back while maintaining price stability, such policy results in relatively large movements of the interest rate with large costs for the debt accumulation. This greatly increases the conflict with the fiscal policy maker as with high level of the existing debt the new debt accumulates particularly fast.

Chapter 3 contains the analysis of how the lower frequency of fiscal policy decisions may affect the result of policy interactions. I show that the dynamic complementarity described above is substantially increased if the fiscal policy maker operates only infrequently. This leads to the existence of multiple equilibria (so-called 'expectations traps') and even to the non-existence of discretionary equilibria under certain conditions. I study properties of such economy and investigate which policy delegation scheme are more likely to weaken the complementarity and mitigate the effect of fiscal infrequency on the economy.

Chapter 4 expands the framework developed in Chapter 3 by using Blanchard-Yaari type of households (Blanchard (1985)) who are not infinitely-lived but face positive mortality rate. This makes these consumers non-Ricardian so they treat the government debt as wealth and the government has an extra channel of transmission of policy. I study how this extension affects the described above complementarity of agents' decisions and the welfare implications for the economy. The results show that under the same calibration, the expectation traps and coordination failures are more likely to arise.

Chapter 2

Price Level Targeting with Strategic Fiscal Policy

2.1 Introduction

The purpose of this Chapter is to provide evaluation of price level targeting with an optimizing fiscal policy maker. The last several years have seen the increase of interest in macroeconomic control policies which would reduce the risk of the economy getting into the 'liquidity trap' i.e. approaching the zero low bound for nominal interest rates. Monetary price level targeting and stabilizing fiscal policy have been proposed and discussed.

Price-level targeting has been subjected to an increasing interest in the monetary policy literature. From the literature we know that there are several advantages of price level targeting over inflation targeting:

First, by definition the PLT reduces the uncertainty of future price level, which produces a more certain future purchasing power of the money. After any unexpected disturbance, inflation targeting requires the central bank bring the inflation back on track, however the price will be stabilized on a new level of steady state like a random walk and can drift far away from any given level. Therefore the future price level uncertainty accompanying an inflation targeting policy depends on the sum of realizations of control errors, which can get very large during long period. By definition the price level targeting removes this uncertainty: in the case of a major shock causing the price to deviate from its steady state, price level targeting would provide a stronger guarantee of future price stability around expected level.

Second, the PLT reduces the fluctuations in inflation and output. This contradicts the early research which argued that targeting price-level may add unnecessary short-run inflation fluctuation where a positive inflation shock must be offset by an under target deflation before it comes back to steady state. However, this conclusion was resulted from models with exogenous policy reaction functions and back-ward looking agents. With endogenous decision rules, Svensson (1999) demonstrated that under inflation targeting, the variance of inflation is proportional to the variance in the output gap; but under price-level targeting, the variance of price-level depends on the variance in output gap, which means that the variance of inflation depends on the variance of the change of the variance in output gap. With output persistence, this is smaller than the variance of the output gap, resulting in smaller inflation fluctuation.

Later, Vestin (2006) applied a standard new-Keynesian model with forward-looking elements. Forward-looking households expect the policy maker with a price-level target has stronger incentive to bring the price level back on the steady state, which means the interest rate will react stronger than in the inflation targeting case. Hence, the higher upward pressure of the current inflation, the lower inflation in the following periods will be expected households. Comparing to the case with inflation targeting, given the same shock, the expected future inflation under the PLT deviates less from the steady state level and overshoots when converging back. This helps to reduce the volatility of current inflation and makes the stabilization easier under the price-level targeting than inflation targeting.

Third, the PLT also has potential to reduce the likelihood of sustained deflation or recession when interest rates are close to zero and the traditional monetary policy tools are ineffective. By allowing the monetary policy maker adjust interest rate less to economic disturbance, the PLT lowers the likelihood of hitting the zero lower bound

(Billi, 2008). Even if interest rate is close to zero, the higher inflation expectation reduces the real interest rate which helps to stimulate the economy, this makes the price level targeting particularly appealing when there is deflationary pressures which are commonly associated with recession.

During a period without many shocks hitting the economy, the PLT does not make an obvious difference to the inflation targeting. As we discussed before that the major difference caused by these two policy targets could only be shown when there is a significant deviation of inflation due to economic shocks, such as the recent financial crisis. It appears that current policies were not very effective in improving the economic conditions, the price-level targeting may be an attractive option for the central bankers, especially if the economy is facing the danger of deflation.

Nevertheless, little research has been done in an economy with that fiscal policy also behaves optimally. The literature on fiscal policy as a macroeconomic stabilization tool is relatively new, and policy proposals are typically motivated by the need to design a powerful stabilization instrument in situations when monetary policy is constrained. Before making a decision to change policy to price level targeting, it has to be proved that the appealing features discussed above are still robust in models with more complex features of modern economies. One of the first in line would be a model with optimal fiscal policy.

An institutional implementation of these proposals would create two different mandates for two policy makers. This might result in a conflict between the optimizing policy makers as one of them will try to 'undo' the perceived harm done by the other.[1]

The central questions addressed in this Chapter follow on from this. If the policy makers are unable to precommit, how does the monetary PLT affect the social welfare if fiscal policy is made strategic? What is the value of such delegation? What are the welfare consequences of different intra-period leadership regimes for monetary and fiscal

[1] See e. g. Dixit and Lambertini (2003), Lambertini (2006), Blake and Kirsanova (2011).

policies?

To address these questions we study a version of the familiar sticky price model modified to incorporate debt accumulation (Benigno and Woodford, 2003) with strategic monetary and fiscal policy, pursuing different objectives. This model contains all the features as at the heart of many DSGE models used in policy analysis which makes it a 'representative agent' for our policy analysis. We concentrate on macroeconomic effects of differing policy objectives and the ability of the fiscal policy to conduct itself as an intra-period leader.[2]

To maintain the consistency, we continue using our previous framework but with monetary policy targeting on price level as well as inflation, with different assigned weights. This enables us to compare the result with the literature to see the impact of interactions between the non-cooperative monetary and fiscal policy makers to the previous price level targeting results with only optimal monetary policy, and to compare this result with our previous model with monetary and fiscal policy both targeting inflation, in order to see the impact of the price-level targeting to our original system.

We study a new Keynesian framework modified to incorporate debt accumulation with strategic monetary and fiscal policy, pursuing different objectives, assuming government is using tax rate as a policy instrument to stabilize the economy. We assign different weight to price level and inflation for Central Bank's policy objectives, in order to allow temporary drift in the price level when they these 'intermediate' regimes may be relevant, following the method by Batini and Yates (2003).

We demonstrate that when steady state debt level is low, only a small weight on price level target can make a substantial improvement on social welfare, this finding also provides us a way to avoid some of the cost of changing the completely policy target; Fiscal leadership results much better social welfare than letting government and central bank make the decisions simultaneously; Anther counter-intuitive result is that with

[2]Empirical evidence (Fragetta and Kirsanova, 2010) suggests that in countries without fiscal decentralization, like the UK, the regime of fiscal leadership is the most relevant.

price level targeting, in the low debt case the dynamic complementarity between fiscal and monetary policy makers is weakened, despite their different policy objectives, which further helps the economy stabilization and avoid the multiple equilibriums.

However the results reverse when the steady state level of debt is high. In this case, the conflicts between the two policy makers due to the PLT overweigh the merits PLT brings. In relatively high debt case, the welfare loss in PLT is higher than inflation targeting. PLT has two opposite effects, on one hand it lowers inflation after the disturbance by lowering expected future inflation, which makes the task of stabilization easier; on the other hand an overshot of inflation is required in order to return the price level, which causes a higher volatility of inflation. To maintain a higher steady state government debt level, there will be higher tax, and initially lowered interest rate which cause the realized inflation is almost as high as under inflation targeting; when inflation is at the same level, due to the higher volatility of inflation required to deliver price level, PLT causes higher loss. Steady state debt level matters to whether PLT improves or deteriorate the stabilization results.

This Chapter is organized as follows. In the next section we outline the model and discuss the calibration. Section 2.3 presents the theoretical analysis of the five cases we consider. The first three cases – commitment of benevolent policy makers which delivers the lowest possible loss among all regimes, cooperation of benevolent discretionary policy makers which results in substantial stabilization bias, and the cooperative PLT by both policy makers – describe the three benchmark scenarios, to which we compare and contrast the two cases of our main interest: monetary PLT either in the regime of simultaneous moves or in the regime of fiscal leadership. Section 2.4 discusses the value of delegation as exemplified by the monetary PLT in the regime of simultaneous moves. Section 2.5 discuss the value of leadership, it compares and contrasts the two different non-cooperative regimes. Section 2.7 concludes.

2.2 The Model

We consider the now-mainstream macro policy model modified to take account of the effects of fiscal policy, see e.g. Woodford (2003a) and Benigno and Woodford (2003). It is a closed economy model with two policy makers, the fiscal and monetary authorities. Fiscal policy is assumed to support monetary policy in stabilization of the economy around the non-stochastic steady state.

The economy consists of a representative infinitely-lived household, a representative firm that produces the final good, a continuum of intermediate goods-producing firms and a monetary and fiscal authority. The intermediate goods-producing firms act under monopolistic competition and produce according to a production function that depends only on labor. Goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output. Firms set their prices subject to a Calvo (1983) price rigidity. Households choose their consumption and leisure and can transfer income through time through their holdings of government bonds. We assume that the fiscal authority faces a stream of exogenous public consumption. These expenditures are financed by levying income taxes[3] and by issuing one-period risk-free nominal bonds.

We assume that all public debt consist of riskless one-period bonds. The nominal value $\mathfrak{B}_t$ of end-of-period public debt then evolves according to the following law of motion:

$$\mathfrak{B}_t = (1 + i_{t-1})\,\mathfrak{B}_{t-1} + P_t G_t - \tau_t P_t Y_t, \tag{2.1}$$

where τ_t is the share of national product Y_t that is collected by the government in period t, and government purchases G_t are treated as exogenously given. P_t is aggregate price level and i_t is interest rate on bonds. The national income identity yields

$$Y_t = C_t + G_t, \tag{2.2}$$

where C_t is private consumption. For analytical convenience we introduce $B_t = (1 +$

[3]We could use distortionary consumption taxes to finance the deficit. The transmission mechanism would be the same.

$i_{t-1})\mathfrak{B}_{t-1}/P_{t-1}$ which is a measure of the real value of debt observed at the beginning of period t, so that (2.1) becomes

$$B_{t+1} = (1+i_t)\left(B_t \frac{P_{t-1}}{P_t} - \tau_t Y_t + G_t\right). \tag{2.3}$$

The first-order approximation of equation(2.3) about the non-stochastic zero-inflation and zero-debt steady state yields

$$b_{t+1} = \frac{1}{\beta}\left(b_t + \left(1 - \frac{C}{Y}\right) g_t - \tau\left(\tau_t + y_t\right)\right),$$

where $b_t = \frac{B_t}{Y}$, $c_t = \ln\left(\frac{C_t}{C}\right)$, $\tau_t = \ln\left(\frac{\tau_t}{\tau}\right)$, $g_t = \ln\left(\frac{G_t}{G}\right)$, $y_t = \ln\left(\frac{Y_t}{Y}\right)$ and letters without time subscript denote steady state values of corresponding variables in zero inflation steady state. The private sector's discount factor $\beta = 1/(1+i)$. We have assumed $B = 0$ in order to make the presentation of the model particularly simple. This assumption results in no first-order effects of the interest rate and inflation on debt, so that the final version of the linearized debt accumulation equation can be written as

$$b_{t+1} = \frac{1}{\beta}\left(b_t + (1-\tau)(1-\theta) g_t - \tau\theta c_t - \tau\tau_t\right), \tag{2.4}$$

where we used the linearized (2.2) to substitute out output and denoted $\theta = C/Y$.[4]

The derivation of the appropriate Phillips curve that describes Calvo-type price-setting decisions of monopolistically competitive firms is standard (Benigno and Woodford, 2003, Sec. A.5) and marginal cost is a function of output and taxes. A log-linearization of the aggregate supply relationship around the zero-inflation steady state yields the following New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa\left(\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right) c_t + \frac{(1-\theta)}{\psi} g_t + \frac{\tau}{(1-\tau)}\tau_t\right) + \eta_t, \tag{2.5}$$

where $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{(\psi+\epsilon)}$ is the slope of Phillips curve. Parameter γ is Calvo parameter, parameter ψ is Frisch elasticity of labour supply, σ is elasticity of intertemporal substitution and parameter ϵ is the elasticity of substitution between differentiated goods. Cost push shock η_t follows an autoregressive process.

[4]Because we work with one-period debt only, its proportion in the total stock of debt is not very large.

The social loss is defined by the quadratic loss function[5]

$$L = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda c_t^2\right). \tag{2.6}$$

while the monetary and the fiscal policy makers can have different policy objectives, $L^J = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_J^t Q^J (\pi_t, c_t, \tau_t, b_t, p_t)$, $J \in \{M, F\}$. Each policy maker knows the laws of motion (2.4)-(2.5) of the aggregate economy and takes them into account when formulating policy. The following assumption follows Clarida et al. (1999) and substantially simplifies the exposition of the model.

Assumption 1 (policy instruments) *The monetary policy maker chooses consumption c_t and then, conditional on subsequent optimal evolution of c_t and π_t, decides on the value of interest rate that achieves the desired c_t and π_t. The fiscal policy maker uses the tax rate τ_t as policy instrument.*

In what follows we assume that both policy makers and the private sector know that the decision making is sequential and a different policy maker may be in the office in future periods. We refer to this policy as policy under discretion. Formally, we make the following assumption.

Assumption 2 (policy) *Monetary and fiscal policy mix satisfies the following assumptions.*

(i) Monetary and fiscal authorities act non-cooperatively.

(ii) Both authorities are assumed to optimize sequentially under time-consistency constraint.

(iii) Each policy maker minimizes its loss criterion in the form:

$$L^M = \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left(\left(p_t - (1-\alpha)\, p_{t-1}\right)^2 + \lambda c_t^2\right) \tag{2.7}$$

$$L^F = \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda c_t^2\right) \tag{2.8}$$

[5] The criterion is derived under the assumption of steady state labour subsidy. Here parameter λ is a function of model parameters, $\lambda = \theta\kappa/\epsilon$, and ϵ is the elasticity of substitution between any pair of monopolistically produced goods.

where α measures the degree of price-level-targeting in the monetary policy objectives.

The hybrid price level target monetary policy objective is the same as studied in Roisland (2006) and Batini and Yates (2003). The following assumption substantially simplifies the exposition without the loss of generality.

Assumption 3 *The model is perfect-foresight deterministic.*

In the standard New Keynesian model the only meaningful trade-off for the monetary policy maker is created by cost-push shocks. In contrast, both policy instruments in this model can completely insulate this economy against the cost push shock, but they will face with fiscal consequences of such policy – the effect on debt. In what follows, therefore, we can only consider shocks to debt, if we reinterpret policy instruments as those adjusted for movements needed to eliminate the consequences of cost-push shocks. However, we can go further and only consider the deterministic version of the model where the only disturbance can be generated by initially higher level of debt, b_0. First, because of certainty equivalence in LQ models all results on stability, existence and uniqueness do not depend on the presence of stochastic component, see Anderson, Hansen, McGrattan, and Sargent (1996). Second, in this model the welfare loss generated by either cost push or debt shocks is simply the normalized loss generated by initial state b_0. Because of the transformation is monotonic the welfare analysis for the deterministic model applies to its stochastic counterpart. We illustrate the second point in Section 2.4

To summarize, the law of motion of the deterministic economy can be written as:

$$\pi_t = \beta\pi_{t+1} + \varkappa c_t + \nu\tau_t, \tag{2.9}$$

$$p_t = p_{t-1} + \pi_t, \tag{2.10}$$

$$b_{t+1} = \frac{1}{\beta}\left(b_t - \tau\theta c_t - \tau\tau_t\right), \tag{2.11}$$

and the initial state $\bar{b}$ is known to all agents, and coefficients $\varkappa = \kappa\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)$, $\nu = \kappa\frac{\tau}{(1-\tau)}$. Debt b_t and price p_{t-1} are endogenous predetermined state variables. The objectives of each policy maker coincide and are given by formula (2.6).

This model is highly stylized and involves relatively few parameters. Calibration of $\beta = 0.99, \gamma = 0.75$ and $\theta = 0.75$ correspond to the most frequently estimated values of the steady state annual interest rate of 4%, the average frequency of price changes of one year, and consumption to output share of 75%. We calibrate the Frisch elasticity of labour supply $\psi = 3.0$, consistent with macro-evidence of Peterman (2012) based on the empirical work which matches volatilities of aggregate worked hours and of wages. The empirical evidence for σ is quite far-ranging from near 0.1 reported in e.g. Hall (1988) and Campbell and Mankiw (1989), to above 1 reported in e.g. Rotemberg and Woodford (1997). Attanasio and Weber (1993, 1995) find that the estimate of σ increases from 0.3 for the aggregate data to 0.8 for cohort data, suggesting that the aggregation, which is implicit in the macro data, may cause a significant downward shift in the estimate of σ. Based on this evidence we calibrate intertemporal elasticity $\sigma = 0.3$. The elasticity of substitution between goods, ϵ, determines the monopolistic mark up. Chari et al. (2000) argue for a markup of 11% for the macroeconomy as a whole. Rotemberg and Woodford (1997) obtain elasticity of substitution 7.88, corresponding to a markup of 14.5%. We calibrate $\epsilon = 11.0$.

2.3 Policy

2.3.1 Cooperative Policy-Benchmark Cases

Benevolent Commitment Policy

Benevolent policy under commitment delivers the highest possible welfare, so the performance of all other policies can be naturally compared with it. Commitment policy in a similar class of models is thoroughly investigated, so this section recasts the known from Schmitt-Grohe and Uribe (2004) results for our model.

Proposition 1 *The optimal Ramsey allocation can be written as*

$$\pi_t = \begin{cases} \zeta\frac{\lambda\nu}{\tau}(1-\beta)\,b_0, & t=0 \\ 0, & t=1,2,3... \end{cases}$$
$$b_t = \begin{cases} b_0, & t=0 \\ \zeta(\varkappa-\theta\nu)^2 b_0, & t=1,2,3... \end{cases}$$
$$c_t = \begin{cases} -\zeta\frac{\nu}{\tau}(\varkappa-\theta\nu)(1-\beta)\,b_0, & t=0 \\ -\zeta\frac{\nu}{\tau}(\varkappa-\theta\nu)(1-\beta)\,b_0, & t=1,2,3... \end{cases}$$
$$\tau_t = \begin{cases} \frac{\zeta}{\tau}(1-\beta)(\lambda+\varkappa(\varkappa-\theta\nu))\,b_0, & t=0 \\ \frac{\zeta}{\tau}(1-\beta)\varkappa(\varkappa-\theta\nu)\,b_0, & t=1,2,3... \end{cases}$$

with the associated welfare loss $\lambda(1-\beta)\left(\lambda(1-\beta)+(\varkappa-\theta\nu)^2\right)\left(\frac{\zeta\nu}{\tau}b_0\right)^2$.

Proof. To find commitment policy we write the following Lagrangian

$$\mathcal{L}^c = \sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}\left(\pi_t^2+\lambda c_t^2\right)+\xi_{t+1}\left(\frac{1}{\beta}(b_t-\tau\theta c_t-\tau\tau_t)-b_{t+1}\right)+\psi_{t+1}(\pi_t-\varkappa c_t-\nu\tau_t-\beta\pi_{t+1})\right).$$

The corresponding first order conditions are:

$$\begin{aligned}
0 &= \xi_{t+1}-\xi_t \\
0 &= \pi_t+\psi_{t+1}-\psi_t \\
0 &= \beta\lambda c_t-\tau\theta\xi_{t+1}-\varkappa\beta\psi_{t+1} \\
0 &= -\tau\xi_{t+1}-\nu\beta\psi_{t+1} \\
0 &= \frac{1}{\beta}(b_t-\tau\theta c_t-\tau\tau_t)-b_{t+1} \\
0 &= \pi_t-\varkappa c_t-\nu\tau_t-\beta\pi_{t+1}
\end{aligned}$$

which are simple enough to obtain the following solution

$$\pi_t = \frac{\lambda\nu}{\tau}\zeta(1-\beta)\,b_t+\zeta(\varkappa-\theta\nu)^2\psi_t \tag{2.12}$$
$$c_t = -\frac{\nu}{\tau}\zeta(\varkappa-\theta\nu)(1-\beta)\,b_t+\zeta(\varkappa-\theta\nu)(1-\beta)\,\psi_t \tag{2.13}$$
$$\tau_t = \frac{\zeta}{\tau}(1-\beta)\left(\lambda+\varkappa^2-\theta\varkappa\nu\right)b_t-\frac{\zeta}{\nu}(\varkappa-\theta\nu)(\theta\nu-\beta\varkappa)\,\psi_t \tag{2.14}$$
$$\xi_t = \beta\lambda\zeta\frac{\nu^2}{\tau^2}(1-\beta)\,b_t-\zeta\beta\frac{\lambda\nu}{\tau}(1-\beta)\,\psi_t \tag{2.15}$$
$$b_{t+1} = \zeta(\varkappa-\theta\nu)^2 b_t-\zeta\frac{\tau}{\nu}(\varkappa-\theta\nu)^2\psi_t \tag{2.16}$$
$$\psi_{t+1} = -\zeta\frac{\lambda\nu}{\tau}(1-\beta)\,b_t+\zeta\lambda(1-\beta)\,\psi_t \tag{2.17}$$

where $\zeta = \left((1-\beta)\lambda + (\varkappa - \theta\nu)^2\right)^{-1}$. The claim in proposition immediately follows. ■

Following a disturbance, debt remains permanently high (low) and permanently higher (lower) taxes are used to finance the additional debt.

Benevolent Discretionary Policy

Our discretionary policy is standard, and discussed in e.g. Backus and Driffill (1986), Oudiz and Sachs (1985), Clarida et al. (1999), and Woodford (2003a). We describe the discretionary equilibrium for our model in order to demonstrate that the system under discretionary control of benevolent policy makers demonstrates very different dynamics than the system under commitment.

Proposition 2 *The cooperative discretionary policy of benevolent policy makers results in stationary equilibrium for* b_t, π_t, c_t *and* τ_t.

Proof. private sector's reaction function is a linear function of the state:

$$\pi_t = \pi_b b_t. \tag{2.18}$$

Use equation (2.18) for $t+1$ and substitute equation (2.9) to obtain:

$$\pi_t = \pi_b b_t + (\varkappa - \pi_b \tau\theta) c_t + (\nu - \pi_b \tau) \tau_t. \tag{2.19}$$

The private sector observes policy and the state, and takes into account the 'instantaneous' influence of the policy choice, measured by $(\varkappa - \pi_b \tau\theta)$ and $(\nu - \pi_b \tau)$.

Assuming the quadratic form for the appropriate value function we can write the Bellman equation for the cooperating policy makers:

$$S b_t^2 = \min_{c_t, \tau_t} \left(\pi_t^2 + \lambda c_t^2 + \beta S b_{t+1}^2\right) \tag{2.20}$$

Substitute equation (2.11) and equation (2.19) into equation (2.20)

$$\begin{aligned} S b_t^2 \;\; = \;\; & \min_{c_t, \tau_t} \Big(\left(\pi_b b_t + (\varkappa - \pi_b \tau\theta) c_t + (\nu - \pi_b \tau) \tau_t\right)^2 \\ & + \lambda c_t^2 + \beta S \left(\frac{1}{\beta}\left(b_t - \tau\theta c_t - \tau \tau_t\right)\right)^2 \Big), \end{aligned} \tag{2.21}$$

where we substituted constraints (2.11) and (2.19) written for the appropriate period.

Minimization yields the policy reactions in form of

$$c_t = \tilde{C}_b b_t + \tilde{C}_\tau \tau_t, \quad \tau_t = \tilde{\tau}_b b_t + \tilde{\tau}_c c_t \tag{2.22}$$

with coefficients

$$\begin{aligned}\tilde{C}_b &= -\frac{(\varkappa - \pi_b\tau\theta)\,\pi_b - \frac{\tau\theta}{\beta}S}{\lambda + (\varkappa - \pi_b\tau\theta)^2 + S\frac{\tau^2\theta^2}{\beta}}, \quad \tilde{C}_\tau = -\frac{(\varkappa - \pi_b\tau\theta)\,(\nu - \pi_b\tau) + \frac{\tau^2\theta}{\beta}S}{\lambda + (\varkappa - \pi_b\tau\theta)^2 + S\frac{\tau^2\theta^2}{\beta}}\\ \tilde{\tau}_b &= -\frac{(\nu - \pi_b\tau)\,\pi_b - \frac{\tau}{\beta}S}{(\nu - \pi_b\tau)^2 + \frac{\tau^2}{\beta}S}, \quad \tilde{\tau}_c = -\frac{(\nu - \pi_b\tau)\,(\varkappa - \pi_b\tau\theta) + \frac{\tau^2}{\beta}\theta S}{(\nu - \pi_b\tau)^2 + \frac{\tau^2}{\beta}S}.\end{aligned}$$

In the time-consistent equilibrium the policy reactions can be written as functions of the state only:

$$c_t = C_b b_t \tag{2.23}$$

$$\tau_t = \tau_b b_t \tag{2.24}$$

which yields the following equations for C_b and τ_b :

$$C_b = -\frac{\left((\varkappa - \pi_b\tau\theta)\,(\pi_b + (\nu - \pi_b\tau)\,\tau_b) - \frac{\tau\theta}{\beta}\,(1 - \tau\tau_b)\,S\right)}{\left(\lambda + (\varkappa - \pi_b\tau\theta)^2 + S\frac{\tau^2\theta^2}{\beta}\right)}, \tag{2.25}$$

$$\tau_b = -\frac{\left((\nu - \pi_b\tau)\,(\pi_b + (\varkappa - \pi_b\tau\theta)\,C_b) - \frac{\tau}{\beta}\,(1 - \tau\theta C_b)\,S\right)}{\left((\nu - \pi_b\tau)^2 + \frac{\tau^2}{\beta}S\right)}. \tag{2.26}$$

Equations (2.18), (2.19), (2.23) and (2.24) yield

$$\pi_b = \frac{\varkappa C_b + \nu\tau_b}{\tau\tau_b + \tau\theta C_b}, \tag{2.27}$$

it determines the time-consistent reaction of the private sector in (2.18).

Substitute (2.23) and (2.24) into (2.21) to yield the following equation for the value function

$$S = \pi_b^2 + \lambda C_b^2 + \frac{1}{\beta}\,(1 - \tau\theta C_b - \tau\tau_b)^2\,S. \tag{2.28}$$

The stationary discretionary equilibrium can be described by the set of coefficients $\{\pi_b, C_b, \tau_b, S\}$. Indeed, for a given $b_0 = \bar{b}$, each trajectory $\{b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$ which solves the system of first order conditions (2.11), (2.18), (2.23) and (2.24) we can uniquely map into the set of coefficients $\{\pi_b, C_b, \tau_b, S\}$, satisfying (2.25)-(2.28). Conversely, if the set of coefficients $\{\pi_b, C_b, \tau_b, S\}$ solves (2.25)-(2.28) we can uniquely map it into the trajectory $\{b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$, solving system (2.11), (2.18), (2.23) and (2.24) for given $b_0 = \bar{b}$.

It remains to demonstrate stationary of the discretionary equilibrium. The system of first order conditions (2.25)-(2.28) can be reduced down to two equations in $\{x = C_b, z = \tau_b + \theta C_b\}$:

$$x + \frac{\nu(\varkappa - \theta\nu)}{\lambda\tau z + (\varkappa - \theta\nu)^2} z = 0 \tag{2.29}$$

$$z^2 - \frac{\lambda - (\varkappa - \theta\nu)^2}{\lambda\tau} z - \frac{(1-\beta)(\varkappa - \theta\nu)^2}{\lambda\tau^2} = 0 \tag{2.30}$$

Equation (2.30) only depends on z and always has exactly one positive solution as the free term is negative.

The unique positive root satisfies

$$\frac{1}{\beta}|1 - \tau z| < 1 \tag{2.31}$$

or equivalently $\frac{1-\beta}{\tau} < z < \frac{1+\beta}{\tau}$, so that the equilibrium is stationary. To see this, note that if z_+ is the positive root of (2.30), then $z_- = -\left((1-\beta)(\varkappa-\theta\nu)^2\right)/\left(\lambda\tau^2 z_+\right)$, and $\partial(z_+ + z_-)/\partial z_+ = 1 + (1-\beta)(\varkappa-\theta\nu)^2/\left(\lambda\tau^2 z_+^2\right) > 0$. (i) We show that $z > (1-\beta)/\tau$. Indeed, suppose $\tilde{z}_+ = (1-\beta)/\tau$. $\tilde{z}_+$ is not the positive root of (2.30); if it was the positive root then the negative root would be $\tilde{z}_- = -(\varkappa-\theta\nu)^2/(\lambda\tau)$ and their sum should have been equal to the negative linear coefficient, but $\tilde{z}_+ + \tilde{z}_- < \left(\lambda - (\varkappa-\theta\nu)^2\right)/(\lambda\tau)$. Moreover, any $\tilde{z}_+ < (1-\beta)/\tau$ is not a root of (2.30), because $\partial(z_+ + z_-)/\partial z_+ > 0$. (ii) We show that $z < (1+\beta)/\tau$. Indeed, suppose $\tilde{z}_+ = (1+\beta)/\tau$. $\tilde{z}_+$ is not the positive root to (2.30); if it was the positive root then the negative root would be $\tilde{z}_- = -(1-\beta)(\varkappa-\theta\nu)^2/(\lambda\tau(1+\beta))$ and their sum $\tilde{z}_+ + \tilde{z}_- > \left(\lambda - (\varkappa-\theta\nu)^2\right)/(\lambda\tau)$. Moreover, any $\tilde{z}_+ > (1+\beta)/\tau$ is not a root of (2.30), because $\partial(z_+ + z_-)/\partial z_+ > 0$.

The dynamics of the economy under control can be described by the following two equations

$$\begin{aligned} p_t &= p_{t-1} + \pi_b b_t \\ b_{t+1} &= \frac{1}{\beta}\left(1 - \tau\left(\tau_b + \theta C_b\right)\right) b_t \end{aligned}$$

This system has two eigenvalues, $z_1 = 1$ and $z_2 = \frac{1}{\beta}\left(1 - \tau\left(\tau_b + \theta C_b\right)\right) = \frac{1}{\beta}\left(1 - \tau z\right)$. We proved above $|z_2| = \frac{1}{\beta}|1 - \tau z| < 1$. This implies that b_t once disturbed, converge back to the initial level and so it is the only remaining variable, price level p_t, has unit-root behavior. ■

Discretionary Price Level Targeting

When price level becomes a target, the past price becomes an additional state, so the equilibrium reactions can be written as:

$$\pi_t = \pi_p p_{t-1} + \pi_b b_t \tag{2.32}$$

$$c_t = C_p p_{t-1} + C_b b_t \tag{2.33}$$

$$\tau_t = \tau_p p_{t-1} + \tau_b b_t. \tag{2.34}$$

As before, we lead (2.32) by one period, and substitute the evolution of the economy and the Phillips curve to yield the reaction of the private sector in the following form

$$\pi_t = \frac{\beta \pi_p}{1 - \beta \pi_p} p_{t-1} + \frac{\pi_b}{1 - \beta \pi_p} b_t + \frac{\varkappa - \theta \tau \pi_b}{1 - \beta \pi_p} c_t + \frac{\nu - \tau \pi_b}{1 - \beta \pi_p} \tau_t. \tag{2.35}$$

The private sector observes policy and the state, and takes into account the 'instantaneous' influence of the policy choice, measured by $\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}$ and $\frac{\nu - \tau\pi_b}{1-\beta\pi_p}$.

The policy makers problem in period t can be described by the following Bellman equation (we consider the case $\alpha = 1$):

$$\begin{aligned}
& S_{pp} p_{t-1}^2 + 2S_{pb} b_t p_{t-1} + S_{bb} b_t^2 \qquad (2.36)\\
= & \min_{c_t,\tau_t} \Bigg(\frac{(1+\beta S_{pp})}{(1-\beta\pi_p)^2} \left(p_{t-1} + \pi_b b_t + (\varkappa - \theta\tau\pi_b) c_t + (\nu - \tau\pi_b)\tau_t\right)^2 \\
& + \lambda c_t^2 + \frac{1}{\beta} S_{bb} (b_t - \tau\theta c_t - \tau\tau_t)^2 + 2\frac{S_{pb}}{1-\beta\pi_p} (b_t - \tau\theta c_t - \tau\tau_t) \\
& \times \left(p_{t-1} + \pi_b b_t + (\varkappa - \theta\tau\pi_b) c_t + (\nu - \tau\pi_b)\tau_t\right)\Bigg),
\end{aligned}$$

and the optimization yields the following reactions

$$c_t = \tilde{C}_p p_{t-1} + \tilde{C}_b b_t + \tilde{C}_\tau \tau_t, \qquad (2.37)$$

$$\tau_t = \tilde{\tau}_p p_{t-1} + \tilde{\tau}_b b_t + \tilde{\tau}_c c_t \qquad (2.38)$$

with coefficients

$$\begin{aligned}
\tilde{C}_p &= -\frac{\left((1+\beta S_{pp}) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta\right) \frac{1}{1-\beta\pi_p}}{\left((1+\beta S_{pp}) \left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2 - 2S_{pb}\tau\theta \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} + \tau\theta S_{bb}\frac{1}{\beta}\tau\theta + \lambda\right)} \qquad (2.39)\\
\tilde{C}_b &= -\frac{\left(\left((1+\beta S_{pp}) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta\right) \frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} S_{pb} - \tau\theta S_{bb}\frac{1}{\beta}\right)}{\left((1+\beta S_{pp}) \left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2 - 2S_{pb}\tau\theta \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} + \tau\theta S_{bb}\frac{1}{\beta}\tau\theta + \lambda\right)}\\
\tilde{C}_\tau &= -\frac{\left(\left((1+\beta S_{pp}) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta\right) \frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} S_{pb}\tau + \tau\theta S_{bb}\frac{1}{\beta}\tau\right)}{\left((1+\beta S_{pp}) \left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2 - 2S_{pb}\tau\theta \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} + \tau\theta S_{bb}\frac{1}{\beta}\tau\theta + \lambda\right)}
\end{aligned}$$

$$\begin{aligned}
\tilde{\tau}_p &= -\frac{\left((\nu-\tau\pi_b)(1+\beta S_{pp}) - S_{pb}\tau(1-\beta\pi_p)\right)}{(1+\beta S_{pp})(\nu-\tau\pi_b)^2 + (1-\beta\pi_p)^2 S_{bb}\frac{\tau^2}{\beta} - 2(1-\beta\pi_p)S_{pb}\tau(\nu-\tau\pi_b)} \qquad (2.40)\\
\tilde{\tau}_b &= -\frac{\left((1+\beta S_{pp})(\nu-\tau\pi_b) - \tau S_{pb}(1-\beta\pi_p)\right)\pi_b}{(1+\beta S_{pp})(\nu-\tau\pi_b)^2 + (1-\beta\pi_p)^2 S_{bb}\frac{\tau^2}{\beta} - 2(1-\beta\pi_p)S_{pb}\tau(\nu-\tau\pi_b)} \qquad (2.41)\\
&\quad -\frac{\left((\nu-\tau\pi_b)S_{pb} - S_{bb}\frac{\tau}{\beta}(1-\beta\pi_p)\right)(1-\beta\pi_p)}{(1+\beta S_{pp})(\nu-\tau\pi_b)^2 + (1-\beta\pi_p)^2 S_{bb}\frac{\tau^2}{\beta} - 2(1-\beta\pi_p)S_{pb}\tau(\nu-\tau\pi_b)}
\end{aligned}$$

$$\begin{aligned}
\tilde{\tau}_c &= -\frac{\left((\nu-\tau\pi_b)(1+\beta S_{pp}) - S_{pb}\tau(1-\beta\pi_p)\right)(\varkappa-\theta\tau\pi_b)}{(1+\beta S_{pp})(\nu-\tau\pi_b)^2 + (1-\beta\pi_p)^2 S_{bb}\frac{\tau^2}{\beta} - 2(1-\beta\pi_p)S_{pb}\tau(\nu-\tau\pi_b)} \qquad (2.42)\\
&\quad -\frac{\tau\theta\left(S_{bb}\frac{\tau}{\beta}(1-\beta\pi_p) - S_{pb}(\nu-\tau\pi_b)\right)(1-\beta\pi_p)}{(1+\beta S_{pp})(\nu-\tau\pi_b)^2 + (1-\beta\pi_p)^2 S_{bb}\frac{\tau^2}{\beta} - 2(1-\beta\pi_p)S_{pb}\tau(\nu-\tau\pi_b)}
\end{aligned}$$

In the time-consistent equilibrium, therefore, coefficients of (2.33)-(2.34) satisfy the following equations

$$C_p = -\frac{\left(\left(1+\beta S_{pp}\right)\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}-S_{pb}\tau\theta\right)\frac{1+(\nu-\tau\pi_b)\tau_p}{1-\beta\pi_p}}{\left(1+\beta S_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2-2S_{pb}\tau\theta\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}+\tau\theta S_{bb}\frac{1}{\beta}\tau\theta+\lambda} - \frac{\left(\tau\theta S_{bb}\frac{1}{\beta}\tau-\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}S_{pb}\tau\right)\tau_p}{\left(1+\beta S_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2-2S_{pb}\tau\theta\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}+\tau\theta S_{bb}\frac{1}{\beta}\tau\theta+\lambda} \tag{2.43}$$

$$C_b = -\frac{\left(\left(1+\beta S_{pp}\right)\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}-S_{pb}\tau\theta\right)\frac{\pi_b+(\nu-\tau\pi_b\tau_b)}{1-\beta\pi_p}+S_{pb}\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}-S_{bb}\frac{\tau\theta}{\beta}}{\left(1+\beta S_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2-2S_{pb}\tau\theta\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}+\tau\theta S_{bb}\frac{1}{\beta}\tau\theta+\lambda} - \frac{+\left(S_{bb}\frac{\tau\theta}{\beta}-\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}S_{pb}\right)\tau\tau_b}{\left(1+\beta S_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2-2S_{pb}\tau\theta\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}+\tau\theta S_{bb}\frac{1}{\beta}\tau\theta+\lambda} \tag{2.44}$$

$$\tau_p = -\frac{\left(\left(\nu-\tau\pi_b\right)\left(1+\beta S_{pp}\right)-S_{pb}\tau\left(1-\beta\pi_p\right)\right)\left(1+\left(\varkappa-\theta\tau\pi_b\right)C_p\right)}{\left(1+\beta S_{pp}\right)\left(\nu-\tau\pi_b\right)^2+\left(1-\beta\pi_p\right)^2S_{bb}\frac{\tau^2}{\beta}-2\left(1-\beta\pi_p\right)S_{pb}\tau\left(\nu-\tau\pi_b\right)} - \frac{\tau\theta\left(S_{bb}\frac{\tau}{\beta}\left(1-\beta\pi_p\right)-S_{pb}\left(\nu-\tau\pi_b\right)\right)\left(1-\beta\pi_p\right)C_p}{\left(1+\beta S_{pp}\right)\left(\nu-\tau\pi_b\right)^2+\left(1-\beta\pi_p\right)^2S_{bb}\frac{\tau^2}{\beta}-2\left(1-\beta\pi_p\right)S_{pb}\tau\left(\nu-\tau\pi_b\right)} \tag{2.45}$$

$$\tau_b = -\frac{\left(\left(1+\beta S_{pp}\right)\left(\nu-\tau\pi_b\right)-\tau S_{pb}\left(1-\beta\pi_p\right)\right)\left(\pi_b+C_b\left(\varkappa-\theta\tau\pi_b\right)\right)}{\left(1+\beta S_{pp}\right)\left(\nu-\tau\pi_b\right)^2+\left(1-\beta\pi_p\right)^2S_{bb}\frac{\tau^2}{\beta}-2\left(1-\beta\pi_p\right)S_{pb}\tau\left(\nu-\tau\pi_b\right)} - \frac{\left(\left(\nu-\tau\pi_b\right)S_{pb}-S_{bb}\frac{\tau}{\beta}\left(1-\beta\pi_p\right)\right)\left(1-\theta\tau C_b\right)\left(1-\beta\pi_p\right)}{\left(1+\beta S_{pp}\right)\left(\nu-\tau\pi_b\right)^2+\left(1-\beta\pi_p\right)^2S_{bb}\frac{\tau^2}{\beta}-2\left(1-\beta\pi_p\right)S_{pb}\tau\left(\nu-\tau\pi_b\right)} \tag{2.46}$$

and so it follows from (2.32) that

$$\pi_p = \frac{\beta\pi_p}{1-\beta\pi_p}+\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\tau_p+\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}C_p \tag{2.47}$$

$$\pi_b = \frac{\pi_b}{1-\beta\pi_p}+\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\tau_b+\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}C_b. \tag{2.48}$$

Finally, the value function can be determined from

$$S_{pp} = \left(1+\beta S_{pp}\right)\left(1+\pi_p\right)^2+\lambda C_p^2-2\tau S_{pb}\left(1+\pi_p\right)\left(\theta C_p+\tau_p\right)+\frac{\tau^2}{\beta}S_{bb}\left(\theta C_p+\tau_p\right)^2 \tag{2.49}$$

$$S_{bb} = \left(1+\beta S_{pp}\right)\pi_b^2+\lambda C_b^2+2S_{pb}\pi_b\left(1-\tau\theta C_b-\tau\tau_b\right)+\frac{1}{\beta}S_{bb}\left(1-\tau\theta C_b-\tau\tau_b\right)^2 \tag{2.50}$$

$$S_{pb} = \left(1+\beta S_{pp}\right)\left(1+\pi_p\right)\pi_b+\lambda C_pC_b-\frac{\tau}{\beta}S_{bb}\left(1-\tau\theta C_b-\tau\tau_b\right)\left(\theta C_p+\tau_p\right) +S_{pb}\left(\left(1+\pi_p\right)\left(1-\tau\theta C_b-\tau\tau_b\right)-\pi_b\tau\left(\theta C_p+\tau_p\right)\right) \tag{2.51}$$

We summarize these results in the form of the following proposition.

Proposition 3 *The cooperative PLT discretionary equilibrium can be described by the set of coefficients* $\{C_p, C_b, \tau_p, \tau_b, \pi_p, \pi_b, S_{pp}, S_{bb}, S_{pb}\}$.

Proof. Indeed, for a given $b_0 = \bar{b}, p_0 = \bar{p}$ there is one-to-one mapping between each trajectory $\{p_t, b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$ which solves the system of first order conditions (2.10)-(2.11), (2.32)-(2.34), (2.36) and the set of coefficients $\{C_p, C_b, \tau_p, \tau_b, \pi_p, \pi_b, S_{pp}, S_{bb}, S_{pb}\}$, satisfying (2.43)-(2.51). ■

2.3.2 Non-Cooperative Policy: Monetary Price Level Targeting under Discretion

Policy Reactions in the Regime of Simultaneous Moves

If the monetary policy maker moves simultaneously with the fiscal policy maker then it does not take into account any effect of fiscal policy on the economy. The optimization problem of the monetary policy maker can be written as:

$$\begin{aligned} S_{pp}p_{t-1}^2 + 2S_{pb}b_t p_{t-1} + S_{bb}b_t^2 &= \min_{c_t} \left(\left(p_t - (1-\alpha)\, p_{t-1}\right)^2 + \lambda c_t^2 \right. \\ &\left. + \beta \left(S_{pp}p_t^2 + 2S_{pb}b_{t+1}p_t + S_{bb}b_{t+1}^2 \right) \right), \end{aligned} \tag{2.52}$$

subject to constraints (2.10)-(2.11), (2.34) and (2.35). As a result, the optimal monetary policy reaction function of the monetary policy maker can be written in the form (2.37) with

$$\tilde{C}_b = -\frac{\left((1+\beta S_{pp}) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta \right) \frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} S_{pb} - S_{bb}\frac{\tau\theta}{\beta}}{\lambda + (\beta S_{pp}+1) \frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{1}{\beta}(\tau\theta)^2 S_{bb}} \tag{2.53}$$

$$\tilde{C}_p = -\frac{(\alpha + \beta S_{pp} + (1-\alpha)\beta\pi_p) \frac{(\varkappa-\theta\tau\pi_b)}{1-\beta\pi_p} - \tau\theta S_{pb}}{\lambda + (1+\beta S_{pp}) \frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{(\tau\theta)^2}{\beta} S_{bb}} \frac{1}{1-\beta\pi_p} \tag{2.54}$$

$$\tilde{C}_\tau = -\frac{\left((\beta S_{pp}+1) \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta \right) \frac{\nu-\tau\pi_b}{1-\beta\pi_p} + \tau \left(S_{bb}\frac{\tau\theta}{\beta} - \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p} S_{pb} \right)}{\lambda + (1+\beta S_{pp}) \frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{(\tau\theta)^2}{\beta} S_{bb}} \tag{2.55}$$

so that the coefficients of (2.33)-(2.34) satisfy

$$C_b = -\frac{\left((1+\beta S_{pp})\frac{(\varkappa-\theta\tau\pi_b)}{(1-\beta\pi_p)} - \tau\theta S_{pb}\right)\frac{(\pi_b+(\nu-\tau\pi_b)\tau_b)}{1-\beta\pi_p}}{\lambda+(1+\beta S_{pp})\frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{(\tau\theta)^2}{\beta}S_{bb}} - \frac{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}S_{pb} - S_{bb}\frac{\tau\theta}{\beta}\right)(1-\tau\tau_b)}{\lambda+(1+\beta S_{pp})\frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{(\tau\theta)^2}{\beta}S_{bb}} \tag{2.56}$$

$$C_p = -\frac{\left(\frac{(\alpha+\beta S_{pp})(\varkappa-\theta\tau\pi_b)}{(1-\beta\pi_p)} - \tau\theta S_{pb}\right)\frac{(1+(\nu-\tau\pi_b)\tau_p)}{1-\beta\pi_p} + S_{bb}\frac{\tau^2\theta}{\beta}\tau_p}{\lambda+(1+\beta S_{pp})\frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{(\tau\theta)^2}{\beta}S_{bb}} - \frac{\left(\frac{(1-\alpha)(\beta\pi_p+(\nu-\tau\pi_b)\tau_p)}{(1-\beta\pi_p)} - \tau\tau_p S_{pb}\right)\frac{(\varkappa-\theta\tau\pi_b)}{1-\beta\pi_p}}{\lambda+(1+\beta S_{pp})\frac{(\varkappa-\theta\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\tau\theta S_{pb} + \frac{(\tau\theta)^2}{\beta}S_{bb}}. \tag{2.57}$$

Similarly, the Bellman equation for the fiscal policy maker can be written as:

$$V_{pp}p_{t-1}^2 + 2V_{pb}b_t p_{t-1} + V_{bb}b_t^2 = \min_{\tau_t}\left(\pi_t^2 + \lambda c_t^2 + \beta\left(V_{pp}p_t^2 + 2V_{pb}b_{t+1}p_t + V_{bb}b_{t+1}^2\right)\right) \tag{2.58}$$

subject to constraints (2.10)-(2.11), (2.33) and (2.35). The coefficients in the fiscal policy reaction function (2.38)

$$\tilde{\tau}_p = -\frac{\left((1+\beta V_{pp})\frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \tau V_{pb}\right)\frac{1}{1-\beta\pi_p}}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} + \beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau + \frac{\tau^2}{\beta}V_{bb}} \tag{2.59}$$

$$\tilde{\tau}_b = -\frac{\left((1+\beta V_{pp})\frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \tau V_{pb}\right)\frac{\pi_b}{1-\beta\pi_p} + \left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}V_{pb} - \frac{\tau}{\beta}V_{bb}\right)}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} + \beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau + \frac{\tau^2}{\beta}V_{bb}} \tag{2.60}$$

$$\tilde{\tau}_c = -\frac{\left((1+\beta V_{pp})\frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \tau V_{pb}\right)\frac{(\varkappa-\theta\tau\pi_b)}{1-\beta\pi_p} + \left(\frac{\tau}{\beta}V_{bb} - \frac{\nu-\tau\pi_b}{1-\beta\pi_p}V_{pb}\right)\tau\theta}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} + \beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau + \frac{\tau^2}{\beta}V_{bb}} \tag{2.61}$$

so that the coefficients in (2.34) satisfy:

$$\tau_b = -\frac{\left((1+\beta V_{pp})\frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \tau V_{pb}\right)\frac{\pi_b+(\varkappa-\theta\tau\pi_b)C_b}{1-\beta\pi_p}}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} + \beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau + \frac{\tau^2}{\beta}V_{bb}} - \frac{\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}V_{pb} - \frac{\tau}{\beta}V_{bb}\right)(1-\tau\theta C_b)}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} + \beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau + \frac{\tau^2}{\beta}V_{bb}} \tag{2.62}$$

$$\tau_p = -\frac{\left((1+\beta V_{pp})\frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \tau V_{pb}\right)\frac{1+(\varkappa-\theta\tau\pi_b)C_p}{1-\beta\pi_p} + \left(\frac{\tau}{\beta}V_{bb} - \frac{\nu-\tau\pi_b}{1-\beta\pi_p}V_{pb}\right)\tau\theta C_p}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} + \beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2} - 2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau + \frac{\tau^2}{\beta}V_{bb}}. \tag{2.63}$$

Policy Reactions in the Regime of Intra-period Fiscal Leadership

The assumption of fiscal intra-period leadership is motivated by the observation that the monetary policy reaction function is much more transparent and predictable, so the fiscal policy maker is able to take it into account when formulating policy. Using the interest rate as an instrument implies that consumption and price-setting decisions are made simultaneously, while in this model they are consecutive decisions taken by relevant agents. This makes no difference for our results.

If the monetary policy maker acts as an intra-period follower, it treats the state of fiscal policy parametrically, when choosing optimal policy. The policy maker exploits the reaction of the private sector (2.35), as the private sector is the ultimate follower.

The monetary policy maker's objective is (2.52), with constraints are (2.10)-(2.11) and (2.35). As before, optimization with respect to c_t yields the monetary policy maker reaction function (2.37) with coefficients (2.53)-(2.55).

The fiscal policy maker's Bellman equation is (2.58) where constraints (2.10)-(2.11), (2.35), and (2.37) are applied in any period t. Optimization yields the following coefficients in (2.34)

$$\begin{aligned}\tau_p &= -\left(\lambda\tilde{C}_\tau\tilde{C}_p+\tau\left(\theta\tilde{C}_\tau+1\right)\left(\theta\tau\tilde{C}_pV_{bb}-V_{pb}\frac{1+(\varkappa-\theta\tau\pi_b)\tilde{C}_p}{1-\beta\pi_p}\right)\right. \\ &\left.+\frac{\nu-\tau\pi_b+(\varkappa-\theta\tau\pi_b)\tilde{C}_\tau}{1-\beta\pi_p}\left((1+\beta V_{pp})\frac{\left(1+(\varkappa-\theta\tau\pi_b)\tilde{C}_p\right)}{1-\beta\pi_p}-1-\tau\theta V_{pb}\tilde{C}_p\right)\right) \\ &\times\left(\lambda\tilde{C}_\tau^2+\tau^2\left(\theta\tilde{C}_\tau+1\right)^2V_{bb}+(1+\beta V_{pp})\left(\frac{\nu-\tau\pi_b+(\varkappa-\theta\tau\pi_b)\tilde{C}_\tau}{1-\beta\pi_p}\right)^2\right. \\ &\left.-2\tau\left(\theta\tilde{C}_\tau+1\right)\frac{(\varkappa-\theta\tau\pi_b)\tilde{C}_\tau+\nu-\tau\pi_b}{1-\beta\pi_p}V_{pb}\right)^{-1}\end{aligned} \tag{2.64}$$

$$
\begin{aligned}
\tau_b \;=\; & -\left(\lambda\tilde{C}_\tau\tilde{C}_b-\tau\left(\theta\tilde{C}_\tau+1\right)\left(V_{pb}\frac{\pi_b+(\varkappa-\theta\tau\pi_b)\,\tilde{C}_b}{1-\beta\pi_p}+V_{bb}\left(1-\tau\theta\tilde{C}_b\right)\right)\right. \qquad (2.65)\\
& \left.+\frac{(\varkappa-\theta\tau\pi_b)\,\tilde{C}_\tau+\nu-\tau\pi_b}{1-\beta\pi_p}\left((1+\beta V_{pp})\,\frac{\pi_b+(\varkappa-\theta\tau\pi_b)\,\tilde{C}_b}{1-\beta\pi_p}+V_{pb}\left(1-\tau\theta\tilde{C}_b\right)\right)\right)\\
& \times\left(\lambda\tilde{C}_\tau^2+\tau^2\left(\theta\tilde{C}_\tau+1\right)^2V_{bb}+(1+\beta V_{pp})\left(\frac{\nu-\tau\pi_b+(\varkappa-\theta\tau\pi_b)\,\tilde{C}_\tau}{1-\beta\pi_p}\right)^2\right.\\
& \left.-2\tau\left(\theta\tilde{C}_\tau+1\right)\frac{(\varkappa-\theta\tau\pi_b)\,\tilde{C}_\tau+\nu-\tau\pi_b}{1-\beta\pi_p}V_{pb}\right)^{-1}
\end{aligned}
$$

In the regime of fiscal leadership, therefore, the monetary policy reaction function can also be written in the form of (2.33) with

$$C_p \;=\; \tilde{C}_p+\tilde{C}_\tau\tau_p \qquad (2.66)$$

$$C_b \;=\; \tilde{C}_b+\tilde{C}_\tau\tau_b \qquad (2.67)$$

Value Functions

In both leadership regimes value functions can be described by

$$
\begin{aligned}
S_{pp} \;=\; & (\pi_p+\alpha)^2+\lambda C_p^2+\beta S_{pp}(1+\pi_p)^2-2S_{pb}\tau(\theta C_p+\tau_p)(1+\pi_p) \qquad (2.68)\\
& +S_{bb}\frac{\tau^2}{\beta}(\theta C_p+\tau_p)^2\\
S_{pb} \;=\; & (\pi_p+\alpha+\beta S_{pp}(1+\pi_p)-S_{pb}\tau(\theta C_p+\tau_p))\,\pi_b+\lambda C_pC_b \qquad (2.69)\\
& +\left(S_{pb}(1+\pi_p)-S_{bb}\frac{\tau}{\beta}(\theta C_p+\tau_p)\right)(1-\tau(\theta C_b+\tau_b))\\
S_{bb} \;=\; & (1+\beta S_{pp})\,\pi_b^2+\lambda C_b^2+2S_{pb}\pi_b(1-\tau(\theta C_b+\tau_b)) \qquad (2.70)\\
& +S_{bb}\frac{1}{\beta}(1-\tau(\theta C_b+\tau_b))^2\\
V_{pp} \;=\; & \pi_p^2+\lambda C_p^2+\beta V_{pp}(1+\pi_p)^2-2V_{pb}\tau(\theta C_p+\tau_p)(1+\pi_p) \qquad (2.71)\\
& +V_{bb}\frac{\tau^2}{\beta}(\theta C_p+\tau_p)^2
\end{aligned}
$$

$$V_{pb} = (\pi_p + \beta V_{pp}(1+\pi_p) - V_{pb}\tau(\theta C_p + \tau_p))\pi_b + \lambda C_p C_b \tag{2.72}$$
$$+\left(V_{pb}(1+\pi_p) - V_{bb}\frac{\tau}{\beta}(\theta C_p + \tau_p)\right)(1-\tau(\theta C_b + \tau_b))$$
$$V_{bb} = (1+\beta V_{pp})\pi_b^2 + \lambda C_b^2 + 2V_{pb}\pi_b(1-\tau(\theta C_b + \tau_b)) \tag{2.73}$$
$$+V_{bb}\frac{1}{\beta}(1-\tau(\theta C_b + \tau_b))^2$$

with π_p and π_b satisfying

$$\pi_p = \frac{\beta\pi_p + (\varkappa - \theta\tau\pi_b)C_p + (\nu - \tau\pi_b)\tau_p}{1-\beta\pi_p} \tag{2.74}$$
$$\pi_b = \frac{\pi_b + (\varkappa - \theta\tau\pi_b)C_b + (\nu - \tau\pi_b)\tau_b}{1-\beta\pi_p} \tag{2.75}$$

We summarize our results in the form of two propositions.

Proposition 4 *The non-cooperative simultaneous-moves discretionary equilibrium can be described by the set of coefficients* $\{C_p, C_b, \tau_p, \tau_b, \pi_p, \pi_b, S_{pp}, S_{bb}, S_{pb}, V_{pp}, V_{bb}, V_{pb}\}$.

Proof. Indeed, for a given $b_0 = \bar{b}, p_0 = \bar{p}$ there is one-to-one mapping between each trajectory $\{p_t, b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$ which solves the system of first order conditions (2.10)-(2.11), (2.32)-(2.34), (2.52)-(2.58) and the set of coefficients $\{C_p, C_b, \tau_p, \tau_b, \pi_p, \pi_b, S_{pp}, S_{bb}, S_{pb}, V_{pp}$, V_{bb}, $V_{pb}\}$, satisfying (2.56)-(2.57), (2.62)-(2.63), (2.68)-(2.75). ■

Proposition 5 *The non-cooperative discretionary equilibrium with fiscal intra-period leadership can be described by the set of coefficients* $\{\tilde{C}_p, \tilde{C}_b, \tilde{C}_\tau, \tau_p, \tau_b, \pi_p, \pi_b, S_{pp}, S_{bb}, S_{pb}$, V_{pp}, $V_{bb}, V_{pb}\}$.

Proof. Indeed, for a given $b_0 = \bar{b}, p_0 = \bar{p}$ there is one-to-one mapping between each trajectory $\{p_t, b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$ which solves the system of first order conditions (2.10)-(2.11), (2.32)-(2.34), (A.62)-(2.58) and the set of coefficients $\{\tilde{C}_p, \tilde{C}_b, \tilde{C}_\tau, \tau_p, \tau_b, \pi_p, \pi_b, S_{pp}, S_{bb}, S_{pb}$, V_{pp}, V_{bb}, $V_{pb}\}$, satisfying (2.53)-(2.67), (2.68)-(2.75). ■

2.4 Value of Delegation

Figure 2.1 plots dynamic responses of the economy to a unit-increase in the level of government debt as well as the social loss as function of the degree of PLT (where relevant) for the following four scenarios.

I: Cooperation of benevolent policy makers under commitment. This is the first of benchmark scenarios which we discussed in Section 2.3.1. As a result of initial disturbance, there is an unexpected raise in government debt. To stabilize the economy, policy makers face two options: balancing the budget at a new higher level of debt, by a permanently increased tax rate which leads to a permanently lowered consumption; or bringing the debt back to its initial level, at the cost of much higher inflation volatility. Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004) showed that under commitment, inflation stability outweighs the cost of sustaining a permanently higher debt level. Therefore, the debt under commitment exhibits random walk behavior. To achieve the minimum social welfare loss, government debt level is chosen as the absorber to the shock, rather than inflation. We confirmed this in our impulse responses shown in Figure (2.1): the level of debt remains permanently higher, tax rate is permanently raised but only to a level which is enough to serve the new higher level of debt, so inflation is pushed-up only slightly, and is brought back within one period; consumption is permanently below its initial level, see Proposition 1. This regime delivers the lowest level of social loss. The first column in Table 2.1 which presents the welfare loss in terms of compensating consumption – the permanent fall in the steady state consumption level, as percentage of steady state consumption, that would balance the welfare gain from eliminating the volatility of consumption and leisure (Lucas, 1987).

II: Cooperation of benevolent policy makers under discretion. In presence of an endogenous state, Markov-perfect discretionary policy is unable to manipulate the

Table 2.1: Social Welfare Loss

	Absolute loss, % of steady state consumption		Relative Loss, normalized
type of disturbance →	initial debt	cost-push shock	any
Commitment of benevolent policy makers	0.0028	0.0117	1.000
Discretion of benevolent policy makers	0.0471	0.1938	16.625
Cooperative PLT under discretion	0.0317	0.1304	11.183
Monetary PLT, simultaneous moves ($\alpha = 1$)	0.0075	0.0308	2.6454
Monetary PLT, fiscal leadership ($\alpha = 1$)	0.0056	0.0232	1.9915

private agents' expectation anymore. When a disturbance to debt level happens, the households expect the government is tempted to adjust tax rates until debt return to its original steady state. The change in tax rate will be significantly larger than in the commitment case. Now the private agents' decision on inflation depends on the level of debt, for a higher debt, households expect a higher inflation. The optimal time-consistent policy has to stabilize debt at the original steady state; the expectation of the households is validated. Proposition 2 proves that stationary equilibrium exists and unique, all relevant economic variables are brought back to the steady state. The base line calibration suggests that a unit-change in consumption is about 10 times more effective than a unit change in tax rate in terms of stabilizing inflation, but nearly equally effective in their effects on debt. Therefore the optimal policy bring debt to the steady state by changing tax rates, and stabilize inflation by reducing demand to reduce the marginal costs. The stabilization is relatively slow. Inflation goes back to steady state gradually while price level reaches a new high level and stays. The loss of this policy in consumption equivalent is about 15 times greater than the loss under commitment, see Table 2.1. There is no price stability in this regime.

III: Cooperation on price level targeting of discretionary policy makers. If we are to delegate a different target to the monetary policy maker, it is imperative to look at the cooperative delegation first, as usually different targets increase the conflicts between the policy makers and offset the effects of each other's action. This scenario illustrates some important properties of PLT policymaking, which are likely to realize under any type of price-level targeting, whether cooperative or not. (The equilibrium is defined in Proposition 3.)

We assume pure price level targeting ($\alpha = 1$) for both monetary and fiscal policy. When the price level becomes explicit policy target the private sector recognize that, if inflation is higher than steady state level, the future policy makers will have to arrange for a negative inflation to achieve price stability. No commitment policy plan is required, just the presence of price stability among the targets will lead the future policy makers to generate the path of future endogenous states required for a negative inflation to happen.

In this case, given the same initial conditions, expected future inflation (π_{t+1}) is lower than in the scenario of cooperative inflation targeting discussed above. Adjusting their expectation on next period price with smaller scale, the households react less to the same level of disturbance, which results in less violent current inflation. This is represented by a smaller feedback of inflation on state variables in our system. Moreover, as past period price level is a state variable which will be considered when the firms setting new price, inflation becomes more persistent.

The fiscal policy maker can exploit this to stabilize debt faster, by implementing a tax rate higher than that under inflation targeting, without causing extra increase in current inflation. Due to the lowered future inflation expectation, every same sized increase in tax corresponds to lower level of inflation. Similarly, the monetary policy maker who implements the same reduction in demand, will engineer lower inflation, due to the reduced future inflation expectation. Figure 2.1 suggests that the transmission path of consumption in this regime is the same as before-only partially pull down inflation. Although changing demand has stronger impact on inflation, volatility of consumption is costly. As the fiscal policy maker cooperates towards the same price level targeting, it is more beneficial to change tax rate for the common target-price stabilization. Figure

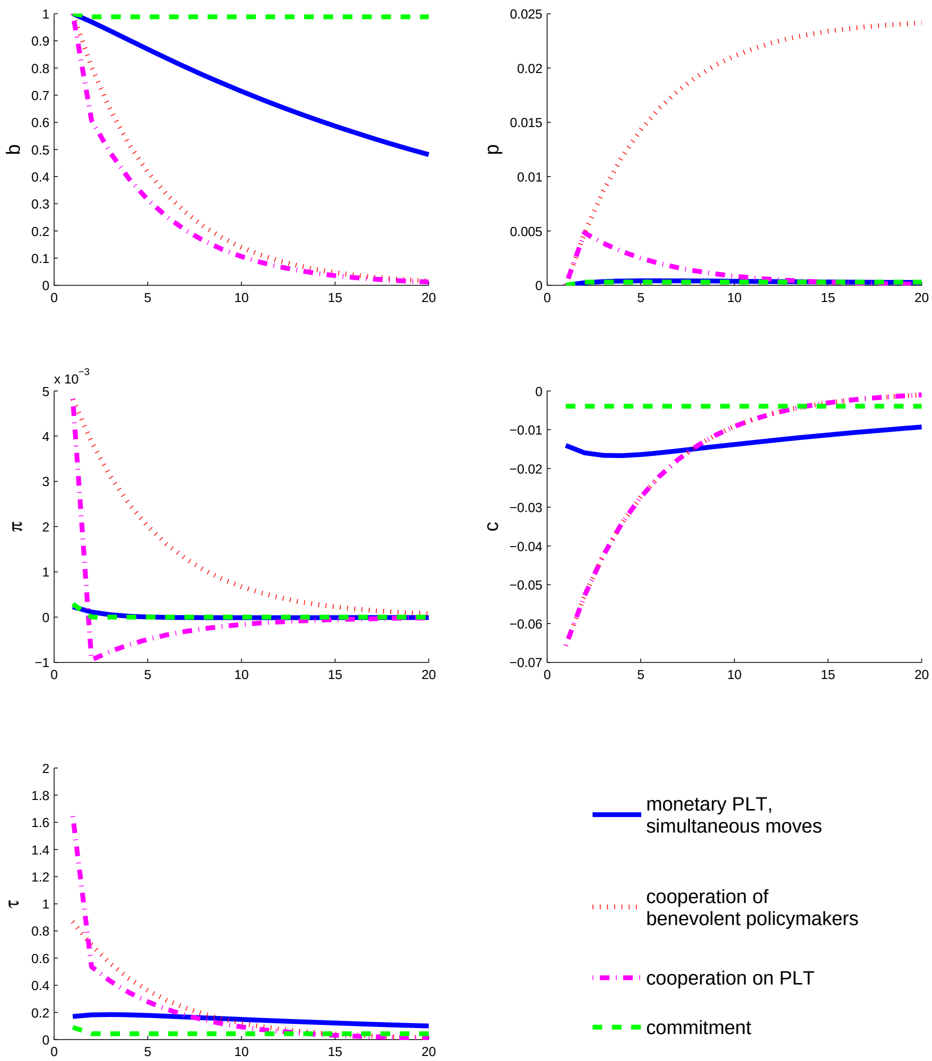

Figure 2.1: Impulse Responses of an unexpected initial debt level deviation

2.1 shows that, compare to the benevolent discretionary case, it is optimal for the fiscal policy maker to increase tax rate initially by more to ensure the first-period reduction in the level of debt, while its cost-push effect on inflation is mitigated by the lower expected inflation; then tax rate drops sharply to a lower level, to deliver the required inflation overshooting baseline. Then tax rate gradually reduces to base-line level, with inflation also gradually increases back. When inflation overshoots, the price level is reduced. With inflation gradually goes back to steady state, the price level converges to its steady state.

The economy converges back to the steady state at the same speed under the price level targeting and under the cooperation of benevolent policy makers. This is apparent from the identical reaction of consumption in both cases taking into account that there is unique rate of convergence of all variables in linear models. The half-lives of both processes are identical.[6] Here the welfare gain can only be obtained because of the difference in the magnitude of inflation deviation from the steady state. Table 2.2 reports numerical values of policy and the private sector reaction functions. The equilibrium feedback of demand on debt is the same in these two scenarios, while consumption feedback negatively on last period price level as well in the latter scenario. The same dynamic path for demand is only possible if debt level is lower during the whole convergence process with price level targeting than with inflation targeting, otherwise consumption would be even lower due to the required action on positive price level. This lower debt is realized optimally by tax rate increase.

Two conclusions follow. First, cooperation on PLT improves the social welfare. By definition price level targeting delivers the long-term price stability. Even if social welfare still only cares inflation stability not price stability, welfare still gains from this regime due to smaller inflation volatility. The essential difference which leads to this improvement comparing to inflation targeting, is that the expectation of future inflation is lowered when policy makers target price level. Although discretionary policy makers cannot

[6] We can check this numerically, and the result holds for all calibrations of the model.

Table 2.2: Agent's reactions in discretionary equilibrium

	structural form	reduced form
Benevolent cooperation	$c_t = 0.000p_{t-1} - 0.033b_t - 0.037\tau_t$	$c_t = 0.000p_{t-1} - 0.066b_t$
	$\tau_t = 0.000p_{t-1} + 0.192b_t - 10.269c_t$	$\tau_t = 0.000p_{t-1} + 0.867b_t$
	$\pi_t = 0.000p_{t-1} + 0.005b_t + 0.065c_t + 0.005\tau_t$	$\pi_t = 0.000p_{t-1} + 0.005b_t$
PLT cooperation	$c_t = -3.931p_{t-1} - 0.044b_t - 0.013\tau_t$	$c_t = -2.680p_{t-1} - 0.066b_t$
	$\tau_t = -117.32p_{t-1} + 1.123b_t - 7.943c_t$	$\tau_t = -96.036p_{t-1} + 1.645b_t$
	$\pi_t = -0.443p_{t-1} + 0.003b_t + 0.036c_t + 0.003\tau_t$	$\pi_t = -0.804p_{t-1} + 0.005b_t$
Monetary PLT, simu-ltaneous	$c_t = -7.145p_{t-1} - 0.007b_t - 0.042\tau_t$	$c_t = -10.222p_{t-1} - 0.014b_t$
	$\tau_t = -36.259p_{t-1} + 0.020b_t - 10.701c_t$	$\tau_t = 73.123p_{t-1} + 0.170b_t$
	$\pi_t = -0.320p_{t-1} + 0.000b_t + 0.045c_t + 0.004\tau_t$	$\pi_t = -0.476p_{t-1} + 0.000b_t$
Monetary PLT, fiscal leadership	$c_t = -5.448p_{t-1} - 0.006b_t - 0.032\tau_t$	$c_t = -5.751p_{t-1} - 0.010b_t$
	$\tau_t = 9.475p_{t-1} + 0.142b_t$	$\tau_t = 9.475p_{t-1} + 0.142b_t$
	$\pi_t = -0.359p_{t-1} + 0.000b_t + 0.042c_t + 0.002\tau_t$	$\pi_t = -0.565p_{t-1} + 0.000b_t$
Benevolent commitment*	$c_t = 0.000p_{t-1} - 0.0040b_t + 0.1612\psi_t$	
	$\tau_t = 0.000p_{t-1} + 0.0897b_t + 159.4472\psi_t$	
	$\pi_t = 0.000p_{t-1} + 0.0003b_t + 0.9882\psi_t$	

Note: * Commitment coefficients correspond to equations (2.12)-(2.17) affect expectations directly, they can affect the states to achieve their targets. Knowing that the policy makers target price level stability, the private sector expects declining prices following an above steady state inflation, and a negative inflation will happen in the future in order to drive price back, even if there will be another new policy maker re-optimize in the next periods, as long as they still undertake price level targeting. This results in lower expected inflation set by price-setters, which improves the welfare.

Second, fiscal policy plays important role in achieving price stability. The required inflation overshooting is generated by a tax cut, together with the reduction in consumption. This is only possible if the fiscal policy maker also targets the price. Additionally, the initial-period increase in taxes leads to lower level of debt in all consequent periods, which helps to lower inflation and prevents consumption to fall more. As shown in Table2.2, inflation is positively related to the debt level while consumption reacts to debt negatively. The parameters in these two regimes are very close; therefore a lower debt level leads to smaller volatility in both consumption and inflation, which means higher welfare. The initial-period increase of inflation due to higher taxes is outweighed by a reduction of inflation in the following periods caused by expectations of negative future inflation. The initial inflation is not higher than that in the inflation targeting case, hence no extra cost occurs.

Following the intuition from Blake and Kirsanova (2011), that non-coincide policy objectives may lead to one strategic policy maker tries to offset some actions of the other policy maker, we would expect the welfare worsened comparing to the case with only central bank is assigned price level targeting. However, in the following analysis, we show that our case counters this conventional intuition: different targeting can improve social welfare.

IV: Monetary price level targeting. In this Section we study the case when the policy makers have different objectives: the discretionary fiscal policy maker remains benevolent, while the discretionary monetary policy maker targets the price level to some degree (the degree is measured by parameter α in objective (2.7)).We firstly focus on comparing the delegation merits, so we consider the case of pure price level targeting ($\alpha = 1$) and both policy makers optimize simultaneously. Table 2.1 demonstrates that the social welfare loss with pure PLT ($\alpha = 1$) is greatly reduced: it is only 25% of the loss under the cooperative price level targeting. This is despite the main component of the social loss is the inflation volatility, and the reduction of inflation volatility was helped by fiscal policy maker who now does not have an incentive to do the same.

It is instructive to compare all the impulse responses in this case to those in benevolent discretion and under discretionary PLT cooperation. We plotted the dynamic responses of the economy to an initial increase in debt, see Figure 2.1. The monetary price level targeting is assumed to be strict, with $\alpha = 1$. All dynamic responses of monetary policy targeting price level with inflation targeting fiscal policy are plotted using the solid line. Figure 2.1 shows that when monetary policy targets price level and fiscal policy targets inflation, the transition path is the closest to the benchmark commitment case.

Recall that in the time-consistent equilibrium the future inflation is a function of future states. As soon as any policy maker targets the price level, the price-setting private sector knows that any positive current inflation will be followed by a dynamic path of predetermined states such that demand will be sufficiently lowered to make it optimal for a firm to set negative inflation in the future. This expectation results in optimal firms decide to set initial inflation lower than inflation targeting regime, for same level of marginal cost, cet. par.

Table 2.2 shows that when only monetary policy targets price level, demand is far more sensitive to price level deviation. Fiscal policy maker knows that cost-push inflation due to tax raise will cause a strong monetary reaction. To ensure price stability, an inflation overshooting is required, which will cause larger inflation volatility and demand reduction, both harm the inflation targeting fiscal policy's benefit. Fiscal policy maker's expectation of monetary policy's stronger reaction to cost-push inflation makes it optimal to set a smaller feedback of tax on debt deviation, leading to a slower return of debt. The resulted cost-push inflation is relatively small and price is stabilized by the monetary policy maker. To bring the future inflation below the base line to achieve the price-level target, monetary policy maker reduces demand by a relatively smaller amount but keeps consumption below the steady state level for a sufficiently longer time. Due to the expectations effect and moderate cost-push inflation, the required fall in demand is not too large. The tax rate has to stay high for a long time, to offset the effect of demand on debt and ensure the debt stabilization.

In this regime, the different targets actually prevent policy makers' volatile behaviors which would lead to large fluctuations. Each policy authority expects the other will react to its own action stronger, causing significant loss to them. It is optimal to avoid the large volatility by moving the policy instruments carefully and gently. To reduce fluctuation in inflation, it is better for fiscal policy to adjust tax rate only at a small scale, to avoid a big overshooting of the inflation. Households with rational expectations understand these, and set a lower inflation accordingly. The policy makers and the economic variables behave more like under commitment regime. As a result, there is a substantial welfare gain, as inflation and consumption remain close to the base line for the whole period of adjustment.

In this case, expected future inflation can still be lowered due to monetary policy targeting price level. On top of this beneficial influence, the inflation targeting fiscal policy maker prefers to avoid inflation overshooting by not change tax as violently as before. Therefore, this further gain of welfare, comparing to the case with both policy makers targeting price level, is due to fiscal expectation towards monetary policy. Therefore an inflation targeting fiscal policy maker tries to avoid the inflation variance due to overshooting by not starting the fight with the monetary policy maker, who will be determined to produce the overshooting large enough for price stability.

2.5 Value of Leadership

In the previous section we demonstrated that the large welfare gain observed under the monetary PLT was merely the result of expectations of the price setters that any current inflation will have to be followed by a period of low demand and (optimally set) negative inflation. The substantial gain was achieved despite the non-cooperative behavior with different objectives of the two policy makers, and potential incentives of the two policy makers to offset each other's actions in the regime of simultaneous moves. In this section we investigate the effect of intra-period leadership on social welfare, with different weight on price level and inflation to check the intermediate regimes.

Figure 2.2 plots the social welfare loss for three regimes, cooperative PLT regime and two monetary PLT regimes as function of the degree of price level targeting, α. One of these two monetary PLT regimes is with the intra-period fiscal leadership; the other characterizes both policy makers making decisions simultaneously. The loss values are renormalized so that the zero loss level corresponds to the loss under commitment, while the level of one corresponds to the loss under discretionary cooperation of benevolent policy makers.

Two observations are apparent. First, the regime of fiscal leadership substantially outperforms all other PLT regimes. Second, the graph of loss in the regime of simultaneous shows that, with a positive α the initial gain of introducing PLT is quickly increasing until $\alpha \simeq 0.05$, then the increase in gain is substantially slowed down until $\alpha \simeq 0.1$ before the loss is relatively quickly and steadily reduced achieving its global minimum in the strict PLT regime with $\alpha = 1$.

In order to understand these results, we plot dynamic responses to a unit-increase level of debt in the initial period in Figure 2.3. We plot responses for three values of $\alpha = \{0.05, 0.1, 1.0\}$. Each panel contains impulse responses for three regimes: cooperation

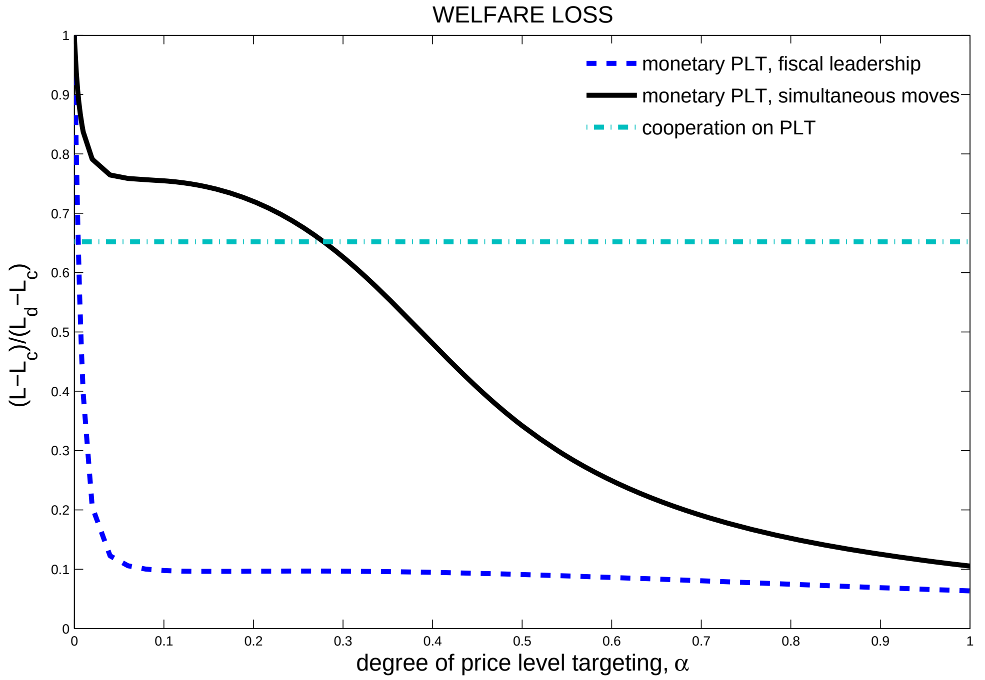

Figure 2.2: Welfare loss under different leadership with different degree of PLT

of benevolent policy makers, monetary PLT under fiscal leadership, monetary PLT with simultaneous moves.

There are two effects of higher degree of PLT, α: 1) the expectation effect of price level targeting and 2) the effect of different policy objectives of the two policy makers.

We discuss the monetary PLT with simultaneous moves first. With small α, the monetary policy maker has to deliver the overshoot of the inflation eventually, but with little motivation to sacrifice with dramatic inflation fluctuation. Both policy makers expect the other behave similar to under cooperation benevolent case, as the similar objectives: to stabilize the economy the fiscal policy maker raises taxes slightly less than in the cooperation case, because the fiscal policy maker takes into account the expectations of the private sector which are affected by the future negative inflation.

Once the degree of the price level targeting α increases further, the incentives of the two policy maker to offset each other's actions increase in the regime of simultaneous

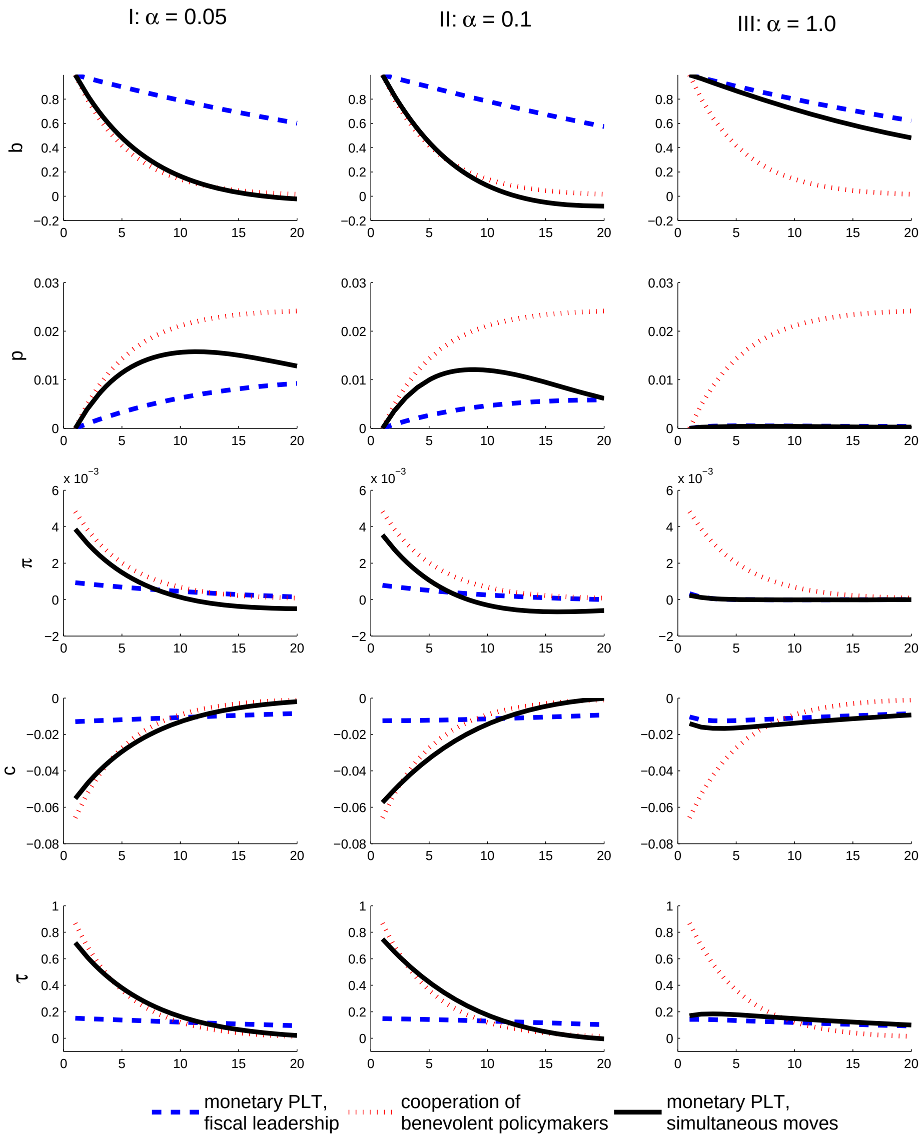

Figure 2.3: Impulse Responses after initial debt deviation with different degree of PLT

moves. With higher α, the fiscal policy maker raises taxes by more and keeps them high, the monetary policy maker generates lower demand and keeps it low. As a result, inflation does not rise by much in the initial moment, but a long period with large negative inflation is generated. There is practically no extra welfare gain relative to the case with $\alpha = 0.05$, as the high welfare loss of the prolonged period of negative inflation and low consumption nearly outweighs the gain of the lower initial-period inflation.

With further increase in α the fight between the two policy makers in the regime of simultaneous moves becomes counterproductive. Stronger monetary and fiscal response would create a greater negative inflation, and smaller initial increase in inflation, but to keep the price level stability the integral under the graph of inflation should be equal to zero. In order to engineer this, and given temporarily higher taxes, the monetary policy maker has an incentive to lower initial demand less but keep it positive and small in the future for infinitely long time. Conversely, with not too low current demand, the fiscal policy maker will choose not to increase taxes too high; a moderate increase in taxes will not create inflation and is sufficient to stabilize the debt.

However when the fiscal policy takes action before monetary policy, the tax rate is raised by much less in all these cases with different values of α. Because the fiscal policy maker takes monetary reactions into account and knows that higher taxes will only result in lower demand, but similar inflation, while a moderate tax can avoid demand fall by too much. The fiscal leadership can completely avoids the fight between policy makers we described before in the simultaneous case, and takes most of the advantage of PLT from a small α, when increasing α this gain stays. When the fiscal policy maker targets inflation but not price level at all, fiscal instrument will not be used to deliver inflation overshooting, and it is the monetary policy maker who will have to deliver the future negative inflation. Moreover, the fiscal policy maker is unwilling to raise taxes as high as in the case of benevolent cooperative policy makers. Such an increase would result in cost-push inflation, the monetary policy maker would have to reduce demand by more than in the case of inflation targeting cooperation, so the overall result for the fiscal policy maker would be worse.

However, stabilization of inflation in a time-consistent way requires stabilization of debt, so the fiscal policy maker does raise the tax rate. This increase in taxes results in debt reduction with relatively small consequent increase in inflation. The monetary policy maker responses to this raise in inflation by more than in the benevolent case, to reduce demand and bring the future inflation below the steady state line. Expectations of this result in lower present inflation.

As a result, inflation does go up, but by much smaller amount than in other regimes. The monetary policy maker does generate negative inflation, but with a substantial delay; inflation remains negative for a long time, but the size of this negative bias is relatively small. Therefore, there is a large social welfare gain.

Figure 2.2 demonstrates that although the maximum welfare gain is achieved with strict PLT ($\alpha = 1$) the loss is relatively flat in α. This shape of the loss is easy to understand. Once $\alpha > 0$ the negative inflation is inevitable and the described above mechanism is at work. This leads to a sharp immediate reduction in the strength of policy responses and in large welfare gains.

When with fully price targeting monetary policy, fiscal leadership delivers better welfare than simultaneous moves. In the former case, fiscal policy knows that if tax rate does not increase a lot, monetary policy as the follower has no incentive to generate a large reduction in consumption to let inflation overshoot more. However in the case of simultaneous moves, fiscal policy expects the monetary policy expecting a higher tax rate so will reduce the demand in order to pull the inflation to a negative level, it is optimal for fiscal policy to validate this expectation by increase tax rate by more than the value in the fiscal leadership case.

To summarize, the fiscal policy maker acts as an intra-period leader and anticipates the relatively low future demand which will be generated in response of current high taxes, it therefore decides not to raise taxes by much and avoids the fight with the monetary policy maker despite the difference in targets.

The price level targeting brings two major effects to the economy: long term price stabilization and short term inflation fluctuation in the form of overshooting. With rational expectations, a pure price level targeting can actually lower this inflation fluctuation due to the lowered expected future inflation. Two factors decide the volatility: how forceful the policy maker is to deliver the overshooting for price stability, which increases with the weight on price level target; and how much of the future inflation expectation can be lowered, which is also increasing with the weight of price level target. They increase at different rate. In simultaneous movement setting, in a small range of alpha, the loss of fluctuation from overshooting can go up with increasing alpha, until the expectation effect catches up (which lowers the fluctuation). While under fiscal leadership, the benefit of price level targeting can be captured with only a small weight.

2.6 Impact of steady state level of debt

Up till now we have analyzed the impact of PLT under different regime but all under the assumption of zero steady state debt level, which is far-fetched in current economy. In order to understand if the merit of price level targeting remains with higher level of steady state debt, in this Section we compare the impulse responses of economic variables with three different positive level of debt to our previous results.

When steady state debt level is positive, the law of motion of the deterministic economy has to include the Euler equation and the monetary policy tool is interest rate. The

the system of Equation 2.9 to Equation 2.11 is transformed to:

$$\pi_t = \beta\pi_{t+1} + \varkappa c_t + \nu\tau_t \tag{2.76}$$

$$p_t = p_{t-1} + \pi_t \tag{2.77}$$

$$b_{t+1} = \frac{1}{\beta}\left(b_t - \chi\pi_t - \tau\theta c_t - \tau\tau_t\right) + \chi i_t \tag{2.78}$$

$$c_t = c_{t+1} - \sigma\left(i_t - \pi_{t+1}\right) \tag{2.79}$$

and the initial state $\bar{b}$ is known to all agents, and coefficients $\varkappa = \kappa\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)$, $\nu = \kappa\frac{\tau}{(1-\tau)}$, $\kappa = \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}$. Debt b_t and price p_{t-1} are endogenous.i_t is nominal interest rate. Assuming steady state level of debt is B, $\chi = \frac{B}{Y}$ is the proportion of debt to GDP in steady state. We normalize $Y = 1$, Figure 2.4 illustrates the impulse responses to an initially higher than steady state level of debt, under different regimes with different level of B.(Both monetary PLT is under fiscal leadership)

The first column shows the cases we discussed before with $B = 0$. Then we increase B by a small value ($B = 0.25$), the transmission paths behave similar as before but fluctuate to a larger scale: the pure monetary PLT delivers the results closest to the commitment benchmark case (shown in A.1) comparing to all the other regimes we considered, followed by monetary PLT with $\alpha = 0.05$ showing that even just a small degree of PLT can help to reduce most of the stabilization bias.

However, when B is raised to 0.7 and higher in our framework, the direction of the impact of monetary fiscal interactions start to change. See the last two columns in Figure 2.4. If the fiscal policy maker does not raise taxes it knows that the monetary policy maker will be forced to stabilize debt in order to deliver price stability. This can be achieved with 'passive' monetary policy when interest rate is lowered in the immediate response to the higher debt but then raised to fight the consequent inflation. Moreover, because of the requirement of price stability the movements of interest rate are likely to be large so that inflation overshoots the base line. (Keeping interest rate high for a long time may not be optimal because of the confliction with debt stabilization.) Such

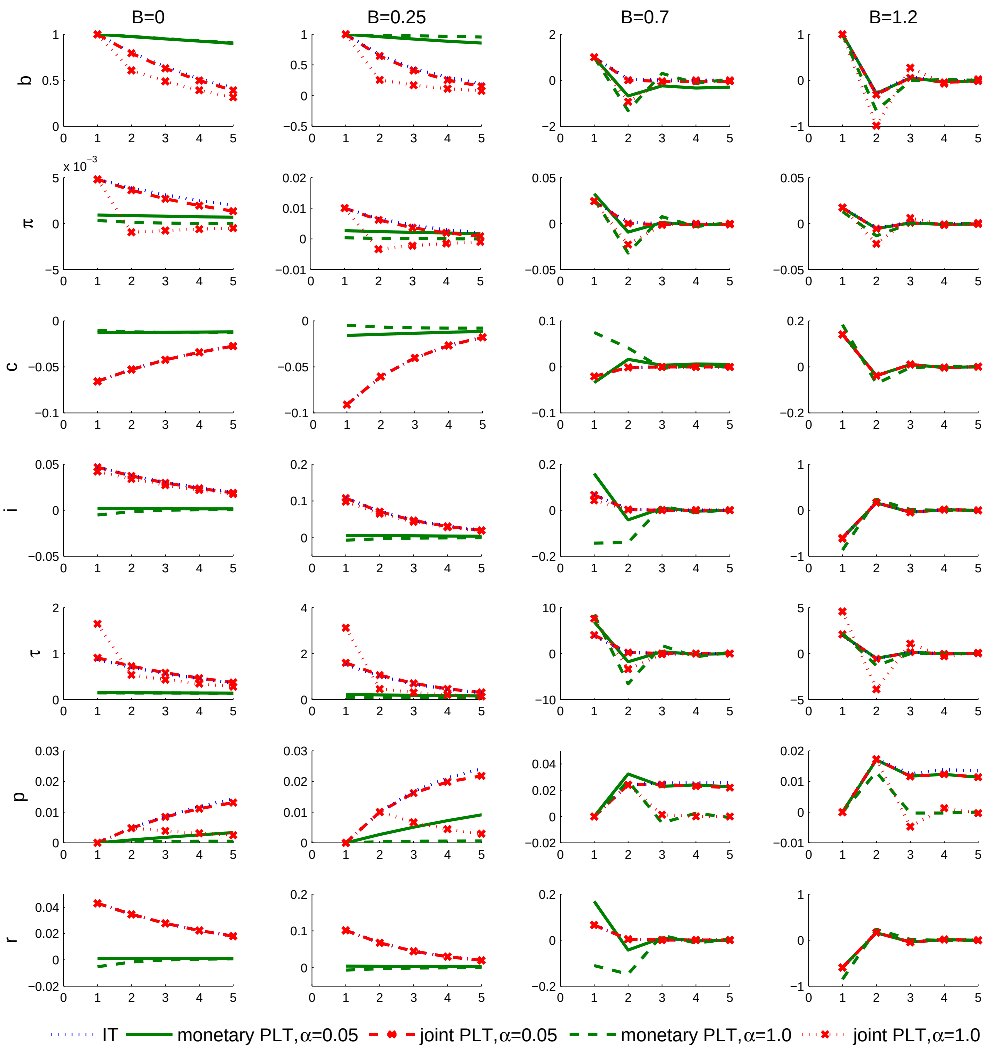

Figure 2.4: Impulse Responses of different discretionary policy with different steady state debt level

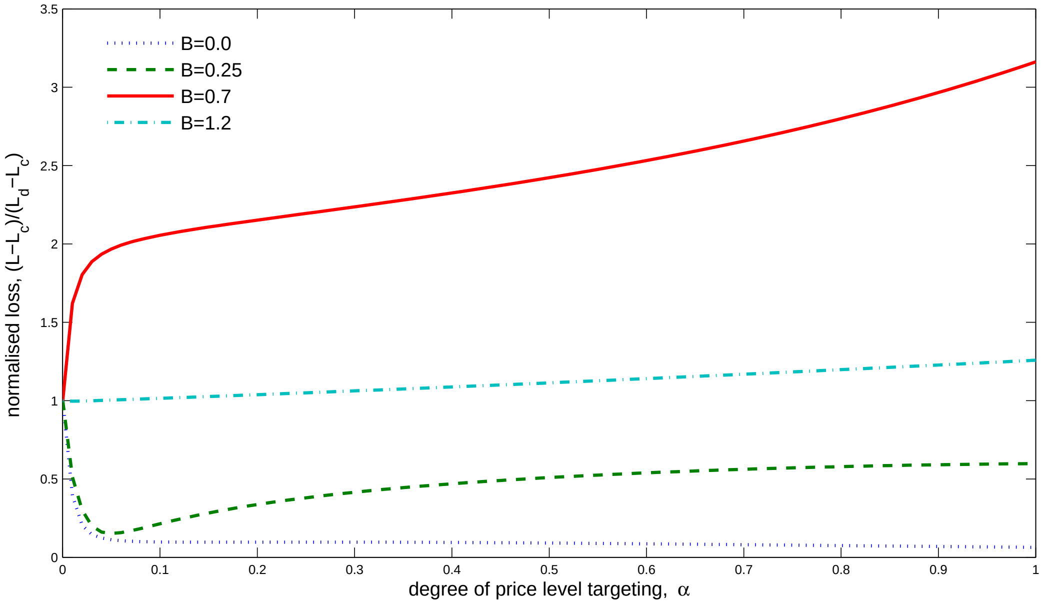

Figure 2.5: Welfare Loss under fiscal leadership with different steady state debt level and degree of PLT

policy naturally leads to high volatility of inflation, which the fiscal policy maker would like to avoid. Therefore, it becomes optimal for the fiscal policy maker to intervene. The third column in Figure 2.4 demonstrates that with higher debt and small degree of PLT ($\alpha = 0.05$, solid line) taxes are raised nearly as high as under the joint PLT, so the fiscal policy maker effectively tries to coordinate with monetary policy maker rather than to offset its actions. With higher degree of PLT, the fiscal policy maker moves taxes more violently, but initially higher taxes are not high enough to ensure debt stabilization so the monetary policy maker operates in 'passive' way: interest rate is lowered when $\alpha = 1$.

With further increase in the steady state level of debt all cooperative and non-cooperative regimes become very similar. The monetary policy maker finds it very difficult to stabilize debt and deliver overshooting of inflation, so the fiscal policy is forced to react more aggressively to debt and to inflation in the similar way as in all cooperative regimes.

When $B = 0$, Monetary PLT makes fiscal policy behaves like under Commitment as well. This is due to the fiscal policy maker's also expects Monetary policy maker will forcefully reduce and negatively overshoot the inflation if tax creates any, which does no good for the inflation targeting fiscal policy.

With a higher B interest rate has a direct effect on the accumulation of debt, the increase in debt will induce a higher tax, which leads to higher inflation. Below certain threshold, further rise in B creates more problems for policy makers if monetary policy raises the interest rate in response to an increase in inflation. The higher interest rate will put upward pressure on debt accumulation which makes fiscal policy less effective on debt stabilization. When B is below this threshold, the gains from less volatility of targeted variables outweigh the losses from the slow stabilization of the economy. When B is above this threshold, it becomes welfare improving to stabilize debt quicker. As the first-order effect of interest rates on debt is large, a fall in interest rates reduces the level of domestic debt. Therefore, it becomes optimal to lower the interest rate in the first period after the shock and raise it in the second period. This policy leads to faster debt stabilization, also curtails inflation.

The impulse responses show that under PLT, this threshold is lower than that under Inflation targeting regime: in the case of $B = 0.7$, interest rate is lowered instead of increased after the initial disturbance. In order to stabilize price, Central Bank chooses to help tax to bring debt down by lowering interest rate to increase consumption. This way the lowered debt leads to a reduced tax which can deliver the negative overshoot of the inflation.

If steady state debt level goes up even more, e.g.$B = 1.2$, Central bank lowers interest by more, therefore tax does not need to raise as much as before. Fiscal Policy maker knows that monetary policy, the follower, will have to lower interest by larger size to pull back debt in order to achieve the price stability. All regimes are more similar in the case where $B = 1.2$.

Figure 2.5 shows that $B = 1.2$ has lower loss than $B = 0.7$ in all discretionary regimes with different α in our framework. This demonstrates that the social loss is a non-monotonic function of B, which is consistent with the striking change in the way the stabilization policy works.

In relatively high debt case (e.g. $B = 0.7$, $B = 1.2$), the welfare loss in PLT is higher than inflation targeting. PLT has two opposite effects, on one hand, it lowers inflation after the disturbance by lowering expected future inflation, which makes the task of stabilization easier; on the other hand, an overshoot of inflation is required in order to return the price level to baseline, which causes a higher volatility of inflation. With higher B, there will be higher tax, and initially lowered interest rate which cause the realized inflation almost as high as under inflation targeting; when inflation is at the same level, due to the higher volatility of inflation required to deliver price level stability, PLT causes higher loss.

2.7 Conclusion

This Chapter revisited the idea that the PLT delegation scheme can reduce the stabilization bias in monetary policy models. We present a detailed account of discretionary monetary and fiscal policy interactions assuming that the monetary policy maker implements the PLT while the fiscal policy maker remains benevolent and has incentives to pursue its own benefit and offset some or all of monetary policy actions. If steady state debt level is low, we demonstrate that delegating PLT to the monetary policy maker results in substantial reduction of the social welfare loss even in case of strategic fiscal policy. A comparison to the joint PLT suggests that fiscal policy should be prevented from maintaining price stability – unilateral monetary PLT substantially outperforms the joint PLT regime. The ability of the fiscal policy maker to conduct itself as an intra-period leader results in greater welfare gain.

First we compared the case that both policy makers target price level and the case both of them target inflation. Our study confirms the appealing features of price level policy on general: price-level targeting lead the discretionary results closer to the best commitment results, in which: 1) price level will come back to initial path; 2) the volatility in inflation and consumption (output) is smaller 3) tax rate no longer increase dramatically as a response to a positive shock to debt , the dynamic complementarity between fiscal and monetary policy is weakened 4) welfare loss is reduced with a price level targeting.

Then we assign price level targeting to monetary policy only, and let fiscal policy targets inflation. Surprisingly, this non-cooperative setting-up can further reduce the welfare loss. Monetary PLT makes fiscal policy behaves like under Commitment as well. This is due to the fiscal policy maker expects Monetary policy maker to forcefully reduce and negatively overshoot the inflation in order to sustain price level, if any created by tax. This does no good for the inflation targeting fiscal policy. The conflict between the policy makers is lessened rather than increased.

Later, we compare the case of fiscal leadership with both policy makers making decision simultaneously. We found out that fiscal leadership improves the social welfare further, and just a small weight on price target can demonstrate the appealing features of price level, the economic variables behave similar to those under commitment regime, while if policy makers re-optimize at the same time, the interactions between them lead to a relatively higher cost at lower level of price target weight, and it behaves more like inflation targeting discretionary policy makers.

In both monetary PLT regimes the maximum welfare is achieved under strict price level targeting, i.e. in the case when the inflation stabilization term in policy objective is replaces by the price stabilization term.

However, when the steady state debt level is high, the strengthened dynamic complementarity between policy makers causes a higher inflation even under PLT, and the loss from volatility of inflation required by price level targeting dominates. This shows that PLT will deteriorate the economy and cause more violent fluctuation rather than improves the social welfare. PLT may only be a good alternative to inflation targeting when the steady state debt level is relatively low.

Despite demonstrating these results using a particular model, this model is at the core of more general and empirically relevant DSGE models widely used in policy analysis. Our results are likely to remain valid for this wide class of models.

Chapter 3

Infrequent fiscal policy

3.1 Introduction

Fiscal and monetary policies face different institutional restrictions and operate at different frequencies. Monetary policy makers set interest rate every month and the decision process can arguably be described as (constrained) optimization with the clear aim to stabilize short run fluctuations.[1] In contrast, fiscal decisions are often taken annually, and the policy of contemporary fiscal authorities can rarely be described as aiming to stabilize the economy in the short run. This situation is likely to change, however, if fiscal policy is given a more active short run stabilization role: not only the fiscal policy becomes more focussed on stabilization, but also the decision process becomes more regular. This Chapter contributes to the discussion on the institutional design of stabilizing fiscal policy, which operates at a lower frequency than monetary policy, uses distortionary taxes as a policy instrument and acts without implementation lags.

This institutional design has important implications for the dynamics of the economy. With a longer fiscal cycle the optimal fiscal adjustments are bigger. They impact more on the monetary policy maker and escalate the conflict between the two authorities when the fiscal policy maker uses distortionary taxes. Indeed, optimal actions of the monetary and the fiscal policy makers are dynamic complements in the sense of Cooper and John

[1] There is an extensive literature on the subject, see e.g. King (1997), Svensson (2010).

(1988). Higher tax rate, set by the fiscal policy maker in response to a higher debt level, generates cost-push inflation, which increases the marginal return to a monetary policy decision to raise the interest rate and contribute to debt accumulation. In standard quarterly models this reinforcement mechanism is weak. We demonstrate that it is greatly amplified if discretionary fiscal policy operates only infrequently.

We show that the gain from monetary and fiscal policy stabilization of macroeconomic fluctuations can be greatly overestimated, if it is evaluated using models with frequent fiscal policy stabilization. These models fail to account for arising expectations traps (King and Wolman, 2004) with implications of excessive volatility of welfare-relevant economic variables; these models fail to demonstrate the necessity to constrain the fiscal policy maker, as time-consistent policy may not exist.

We study interactions of monetary and fiscal policies in the Blanchard and Kahn (1980) class of infinite horizon non-singular discrete-time linear dynamic models that is typically used to study aggregate fluctuations in macroeconomics. We use the standard New Keynesian model with monopolistic competition and sticky prices to demonstrate the results. The economy is controlled by monetary and fiscal policy makers which act non-cooperatively at different frequencies. The monetary policy maker optimizes every period while the fiscal policy maker optimizes less frequently, choosing the distortionary tax rate once every several periods. After the tax rate is chosen, it stays at this level until the next fiscal optimization, which happens with certainty after the given finite number of periods. The fiscal policy maker can be characterized as having intra-period leadership. In other words, the monetary policy maker observes fiscal policy in each period, and the fiscal policy maker knows that the monetary policy maker optimizes every period and takes this into account when formulating policy.

More specifically, we demonstrate the existence of expectations traps in the case of longer fiscal cycles. However, we also find that these traps are unlikely to present a problem for a policymaking as we invoke coordination mechanisms. Following Dennis and Kirsanova (2012) we investigate stability properties of these equilibria and find that the agents are likely to coordinate on the Pareto-preferred equilibrium in all cases that we study. More importantly, we demonstrate that discretionary equilibria may not exist, once the fiscal cycle is sufficiently long – one year in our model – and the reinforcement mechanism between the optimal actions of the two policy makers becomes particularly strong. We demonstrate that these adverse effects can be mitigated if the fiscal policy maker is constrained in its actions. We use a number of policy scenarios to illustrate our findings which include the scenario with debt stabilization faster than socially optimal and the scenario with constrained fiscal policy maker.

This research contributes to the literature on optimal monetary and fiscal policies in linear-quadratic (LQ) rational expectations (RE) models, as exemplified by e.g. Leeper (1991), Dixit and Lambertini (2003), Schmitt-Grohe and Uribe (1997); Schmitt Grohe and Uribe (2000); Schmitt-Grohe and Uribe (2004), Linnemann (2006), Leith and von Thadden (2008) and Schabert and von Thadden (2009). It draws on the literature on time-consistent policy with expectations traps (King and Wolman, 2004; Blake and Kirsanova, 2012) and on coordination in RE models, see Evans (1986), Guesnerie and Woodford (1992); Evans and Guesnerie (1993, 2003, 2005); Evans and Honkapohja (2001), Ellison and Pearlman (2011), Dennis and Kirsanova (2012). The design of policy which we study is similar in spirit to the limited commitment framework (Schaumburg and Tambalotti, 2007; Debertoli and Nunes, 2010), but differs crucially by assumptions regarding the number of policy makers, the certainty of reoptimizations and the finite number of periods between reoptimizations.

The Chapter is organized as follows. In the next Section we present a model of monetary and fiscal policy interactions. Section 3.3 presents the general framework with infrequent stabilization. Section 3.5 discusses policy implications in three special cases: quarterly, biannual and annual fiscal stabilization. Section 3.6 concludes.

3.2 The Model with Government Debt

We consider the now-mainstream macro policy model, discussed in Woodford (2003a), modified to take account of the effects of fiscal policy.[2] It is a closed economy model with two policy makers, the fiscal and monetary authorities. Fiscal policy is assumed to support monetary policy in stabilization of the economy around the non-stochastic steady state.

The economy consists of a representative infinitely-lived household, a representative firm that produces the final good, a continuum of intermediate goods-producing firms and a monetary and fiscal authority. The intermediate goods-producing firms act under monopolistic competition and produce according to a production function that depends only on labor. Goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output. Firms set their prices subject to a Calvo (1983) price rigidity. Households choose their consumption and leisure and can transfer income through time through their holdings of government bonds. We assume that the fiscal authority faces a stream of exogenous public consumption. These expenditures are financed by levying income taxes[3] and by issuing one-period risk-free nominal bonds.

3.2.1 Government debt

We assume that all public debts consist of riskless one-period bonds. The nominal value $\mathfrak{B}_t$ of end-of-period public debt then evolves according to the following law of motion:

$$\mathfrak{B}_t = (1 + i_{t-1})\,\mathfrak{B}_{t-1} + P_t G_t - \tau_t P_t Y_t, \tag{3.1}$$

where τ_t is the share of national product Y_t that is collected by the government in period t, and government purchases G_t are treated as exogenously given and time-invariant. P_t is aggregate price level and i_t is interest rate on bonds. The national income identity

[2] See e.g. Benigno and Woodford (2003).

[3] We could use distortionary consumption taxes to finance the deficit. The transmission mechanism would be the same.

yields

$$Y_t = C_t + G_t, \tag{3.2}$$

where C_t is private consumption. For analytical convenience we introduce $B_t = (1 + i_{t-1})\mathfrak{B}_{t-1}/P_{t-1}$ which is a measure of the real value of debt observed at the beginning of period t, so that (3.1) becomes

$$B_{t+1} = (1 + i_t)\left(B_t \frac{P_{t-1}}{P_t} - \tau_t Y_t + G_t\right). \tag{3.3}$$

The first-order approximation of (3.3) about the non-stochastic zero-inflation and zero-debt steady state yields

$$b_{t+1} = \frac{1}{\beta}\left(b_t + \left(1 - \frac{C}{Y}\right) g_t - \tau\left(\tau_t + y_t\right)\right),$$

where $b_t = \frac{B_t}{Y}$, $c_t = \ln\left(\frac{C_t}{C}\right)$, $\tau_t = \ln\left(\frac{\tau_t}{\tau}\right)$, $g_t = \ln\left(\frac{G_t}{G}\right)$, $y_t = \ln\left(\frac{Y_t}{Y}\right)$ and letters without time subscript denote steady state values of corresponding variables in zero inflation steady state. The private sector's discount factor $\beta = 1/(1+i)$. We have assumed $B = 0$ in order to make the presentation of the model particularly simple. This assumption results in no first-order effects of the interest rate and inflation on debt, so that the final version of the linearized debt accumulation equation can be written as

$$b_{t+1} = \frac{1}{\beta}\left(b_t + (1 - \tau)(1 - \theta) g_t - \tau\theta c_t - \tau\tau_t\right), \tag{3.4}$$

where we used the linearized (3.2) to substitute out output and denoted $\theta = C/Y$.[4]

3.2.2 Private Sector

The derivation of the appropriate Phillips curve that describes Calvo-type price-setting decisions of monopolistically competitive firms is standard (Benigno and Woodford, 2003, Sec. A.5) and marginal cost is a function of output and taxes. A log-linearization of the

[4]Because we work with one-period debt only, its proportion in the total stock of debt is not very large. We discuss implications of this assumption for policy in Section 3.3.

aggregate supply relationship around the zero-inflation steady state yields the following New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(\left(\frac{1}{\sigma} + \frac{\theta}{\psi} \right) c_t + \frac{(1-\theta)}{\psi} g_t + \frac{\tau}{(1-\tau)} \tau_t \right) + \eta_t, \tag{3.5}$$

where $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma} \frac{\psi}{(\psi+\epsilon)}$ is the slope of Phillips curve. Parameter γ is Calvo parameter, parameter ψ is Frisch elasticity of labour supply, σ is elasticity of intertemporal substitution and parameter ϵ is the elasticity of substitution between differentiated goods. Cost push shock η_t follows an autoregressive process.

3.2.3 Social Loss Function

The social loss is defined by the quadratic loss function[5]

$$L = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda c_t^2 \right). \tag{3.6}$$

while the monetary and the fiscal policy makers can have different policy objectives, $L^J = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_J^t Q^J (\pi_t, c_t, \tau_t, g_t, b_t)$, $J \in \{M, F\}$. Each policy maker knows the laws of motion (3.4)-(3.5) of the aggregate economy and takes them into account when formulating policy. The following assumption follows Clarida, Gali, and Gertler (1999) and substantially simplifies the exposition of the model.

3.2.4 Assumptions

Assumption 4 (policy instruments) *The monetary policy maker chooses consumption c_t and then, conditional on subsequent optimal evolution of c_t and π_t, decides on the value of interest rate that achieves the desired c_t and π_t. The fiscal policy maker uses the tax rate τ_t as policy instrument and keeps government spending constant $g_t = 0$.*

Apart from making the exposition clear, keeping fiscal spending constant allows us to focus on the particular transmission mechanism of monetary and fiscal policy.

[5] The criterion is derived under the assumption of steady state labour subsidy. Here parameter λ is a function of model parameters, $\lambda = \theta\kappa/\epsilon$, and ϵ is the elasticity of substitution between any pair of monopolistically produced goods.

Despite the simplicity of the model, finding time-consistent optimal policy is not trivial. Of course, the economy can be completely insulated against shocks if the two policy instruments are adjusted to offset the effect of shocks on inflation and debt. However, such policy would be time-inconsistent as it would need to offset the effect of expectations $\mathbb{E}_t\pi_{t+1}$ on current inflation. In what follows we assume that both policy makers and the private sector know that the decision making is sequential and a different policy maker may be in the office in future periods. We refer to this policy as policy under discretion. Formally, we make the following assumption.

Assumption 5 (policy) *Monetary and fiscal policy mix satisfies the following assumptions.*

(i) Monetary and fiscal authorities act non-cooperatively.

(ii) Both authorities are assumed to optimize sequentially under time-consistency constraint.

(iii) The monetary policy maker optimizes every period, but the fiscal policy maker optimizes once every N periods, $N \geq 1$.

(iv) The fiscal authority has intra-period leadership.

The assumption of fiscal intra-period leadership is motivated by the observation that the monetary policy reaction function is much more transparent and predictable, so the fiscal policy maker is able to take it into account when formulating policy.[6] Using the interest rate as an instrument implies that consumption and price-setting decisions are made simultaneously, while in this model they are consecutive decisions taken by relevant agents. This makes no difference for our results.

The assumption of time-consistency prevents the complete and instantaneous stabilization of the economy. Moreover, the relatively large adjustments of infrequent fiscal

[6] Simultaneous moves of the two policy makers could be another possibility. Empirical evidence (Fragetta and Kirsanova, 2010) suggests that in countries without fiscal decentralization, like the UK, the regime of fiscal leadership is the most relevant.

policy may create more difficulties for monetary policy to offset the effect of disturbances on the economy. Smooth stabilization may not be possible any more.

The infrequency of fiscal decisions can be interpreted as fiscal commitment to the policy of fixed tax rates in all periods between the optimization. Such policymaking, however, remains sequential, without the ability to manipulate the expectations of the private sector beyond the periods between fiscal reoptimizations.

Assumption 6 (policy objectives) *Both policy makers are benevolent.*

Different objectives of the two policy makers are likely to result in a conflict between the policy makers as one policy maker tries to 'undo' the perceived harm done by the other.[7] We shall demonstrate that a similar conflict exists even if both policy makers are benevolent but operate at different frequencies. The assumption of different frequencies also makes the leadership structure important. If both policy makers are benevolent and face identical constraints, then the intra-period leadership does not play any role. In our case the policy makers face different constraints, so the leadership does matter. In this Chapter we have chosen to study fiscal leadership as arguably most empirically relevant.

Finally, we make the assumption which is crucial for clear exposition without the loss of generality.

Assumption 7 *The model is perfect-foresight deterministic.*

If the stochastic model is linear-quadratic then the stochastic component of the solution can be obtained in the unique way once the deterministic component is known.[8] We are interested in issues of existence and uniqueness of the time-consistent policy and these properties are unaffected by the introduction of stochastic components in the LQ framework.

---

[7] See e. g. Dixit and Lambertini (2003), Lambertini (2006).

[8] See Anderson, Hansen, McGrattan, and Sargent (1996).

Table 3.1: Calibration

Parameters		calibration	
		base	range
Discount factor	β	0.99	–
Calvo parameter	γ	0.75	–
Consumption share	θ	0.75	–
Intertemporal elasticity	σ	0.3	[0.1, 1.3]
Frisch elasticity of labour supply	ψ	3.0	[0.3,4]
Elasticity of substitution between goods	ϵ	11.0	[4,11]

To summarize, the law of motion of the deterministic economy can be written as:

$$\pi_t = \beta\pi_{t+1} + \varkappa c_t + \nu\tau_t, \tag{3.7}$$

$$b_{t+1} = \frac{1}{\beta}\left(b_t - \tau\theta c_t - \tau\tau_t\right), \tag{3.8}$$

and the initial state $\bar{b}$ is known to all agents, and coefficients $\varkappa = \kappa\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)$, $\nu = \kappa\frac{\tau}{(1-\tau)}$. Debt b_t is the only endogenous predetermined state variable. The objectives of each policy maker coincide and are given by formula (3.6).

3.2.5 Calibration

This model is highly stylized and involves relatively few parameters. Table 3.1 reports the baseline calibration of parameters. Calibration of β, γ and θ is relatively straightforward, they correspond to the most frequently estimated values of the steady state annual interest rate of 4%, the average frequency of price changes of one year, and consumption to output share of 75%. Estimation and the consequent calibration of the remaining three parameters varies across studies.

Estimates of the Frisch elasticity of labour supply ψ vary widely, depending on whether macro- or micro-evidence is used. Peterman (2012) reports values of $\psi \in [2.9, 3.1]$ from the empirical work which matches volatilities of aggregate worked hours and of wages. This range is consistent with values used by macroeconomists to calibrate general equilibrium models but greater than the estimates of $\psi \in [0.3, 0.8]$ which are obtained in microeconomic studies even if decisions on labour participation are taken into account, see Chetty, Guren, Manoli, and Weber (2011). The main source of this difference lies in the heterogeneity of the workforce's reservation wages. When a larger proportion of the workforce's reservation wage is about the market wage, a small change in the market wage leads to a large change in the labour force participation, see Chang and Kim (2005) and Gourio and Noual (2009). However the density of marginal workers can only be observed at the macro-level; the effect is larger in countries with higher involuntary unemployment which leads to higher aggregate elasticity of labour supply. This effect is not identified at the micro-level where a small change in the market rate often does not lead to a noticeable change in the participation status of an individual. So we consider values between 0.3 and 4 plausible for ψ.

Similarly, estimates of the intertemporal elasticity σ vary depending on the wealth of the representative households and the proportion of nondurable goods in their consumption bundle, see Atkeson and Ogaki (1996), Rotemberg and Woodford (1997). The empirical evidence for σ is quite far-ranging from near 0.1 reported in e.g. Hall (1988) and Campbell and Mankiw (1989), to above 1 reported in e.g. Rotemberg and Woodford (1997). Attanasio and Weber (1993, 1995) find that the estimate of σ increases from 0.3 for the aggregate data to 0.8 for cohort data, suggesting that the aggregation, which is implicit in the macro data, may cause a significant downward shift in the estimate of σ.

The elasticity of substitution between goods, ϵ, determines the monopolistic mark-up. Chari et al. (2000) argue for a mark-up of 11% for the macro economy as a whole. Rotemberg and Woodford (1997) obtain elasticity of substitution 7.88, corresponding to a markup of 14.5%. Different industries have different mark-ups, Berry, Levinsohn, and Pakes (1995) and Nevo (2001) report mark ups of 27-45% for automobiles and branded cereals industries.

In all numerical exercises we use the base line values of parameters as reported in the first column in Table 3.1. However, but we shall also investigate the robustness of our

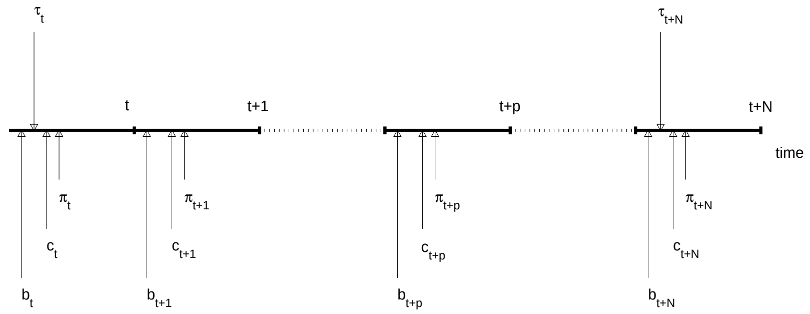

Figure 3.1: Timing of events

results to the range of alternative calibrations given in the second column in Table 3.1.

3.3 Discretionary Equilibrium

In this Section we define discretionary equilibrium in which the monetary policy maker reoptimizes every period while the fiscal policy maker decides once every N periods, $1 \leq N < \infty$. We refer to the period between fiscal reoptimizations as the fiscal cycle. We denote the set of numbers p congruent to a modulo N as $[p]_N$. There are exactly N different sets $[p]_N$. We shall identify these sets with the corresponding residue: $[p]_N = p$, so p denotes the time period after the latest fiscal reoptimization. Both the monetary and fiscal policy makers optimize in period $0 = [0]_N$. Only the monetary policy maker optimizes in periods $[1]_N$,..,$[N-1]_N$, which are labelled $p = 1, .., N-1$. The timing of events is illustrated in Figure 3.1.

Suppose the monetary and fiscal policy maker both optimize at period t. private sector's reaction function is a linear function of the state:

$$\pi_{t+p} = \pi_b^p b_{t+p}, \quad p = 0, .., N-1. \tag{3.9}$$

Use (3.9) for $p+1$ and substitute (3.7) for the appropriate period to obtain:

$$\pi_{t+p} = \pi_b^{p+1} b_{t+p} + \left(\varkappa - \pi_b^{p+1}\tau\theta\right) c_{t+p} + \left(\nu - \pi_b^{p+1}\tau\right)\tau_{t+p} \tag{3.10}$$

The private sector observes policy and the state, and takes into account the 'instantaneous' influence of the policy choice, measured by $\left(\varkappa - \pi_b^{p+1}\tau\theta\right)$ and $\left(\nu - \pi_b^{p+1}\tau\right)$.

The monetary policy maker's problem in period $p = 0,..,N-1$ can be described by the following Bellman equation, where the value function depends on the number of periods passed since the last fiscal optimization. Assuming the quadratic form for the appropriate value function we can write the Bellman equation for the monetary policy maker in period p:

$$S^p b_{t+p}^2 = \min_{c_{t+p}} \left(\pi_{t+p}^2 + \lambda c_{t+p}^2 + \beta S^{p+1} b_{t+1}^2\right) \tag{3.11}$$

$$\begin{aligned} S^p b_{t+p}^2 = \min_{c_{t+p}} \Bigg(&\left(\pi_b^{p+1} b_{t+p} + \left(\varkappa - \pi_b^{p+1}\tau\theta\right) c_{t+p} + \left(\nu - \pi_b^{p+1}\tau\right)\tau_{t+p}\right)^2 \\ &+\lambda c_{t+p}^2 + \beta S^{p+1} \left(\frac{1}{\beta}\left(b_{t+p} - \tau\theta c_{t+p} - \tau\tau_{t+p}\right)\right)^2 \Bigg), \end{aligned} \tag{3.12}$$

where we substituted constraints (3.8) and (3.10) written for the appropriate period.

Minimization with respect c_{t+p}^p yields the following monetary policy reaction function:

$$c_{t+p} = c_b^p b_{t+p} + c_\tau^p \tau_{t+p} \tag{3.13}$$

where

$$c_b^p = -\frac{\left(\varkappa - \tau\theta\pi_b^{p+1}\right)\pi_b^{p+1} - \frac{\tau\theta}{\beta} S^{p+1}}{\left(\varkappa - \tau\theta\pi_b^{p+1}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta} S^{p+1}}, \tag{3.14}$$

$$c_\tau^p = -\frac{\left(\varkappa - \tau\theta\pi_b^{p+1}\right)\left(\nu - \tau\pi_b^{p+1}\right) + \frac{\tau^2\theta}{\beta} S^{p+1}}{\left(\varkappa - \tau\theta\pi_b^{p+1}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta} S^{p+1}}, \tag{3.15}$$

and $p = 1,..,N-1$. The monetary policy maker observes fiscal policy, and takes into account its 'instantaneous' influence, measured by c_τ^p.

The fiscal policy maker only optimizes in periods $[0]_N$. Suppose the optimization happens at time t. The Bellman equation which describes the fiscal policy decision can be written as:

$$Vb_t^2 = \min_{\tau_t} \left(\sum_{p=0}^{N-1} \beta^p \left(\pi_{t+p}^2 + \lambda c_{t+p}^2 \right) + \beta^N V b_{t+N}^2 \right) \tag{3.16}$$

where constraints (3.8), (3.10), and (3.13) are applied in any period $p = 0,..,N-1$, because the state in period $[N]_N \equiv [0]_N$ depends on fiscal policy in all intermediate periods, τ_{t+p}, $p = 0,..,N-1$.

We assume that the fiscal policy maker, when chooses τ_t also sets τ_{t+p} $p = 1,..,N-1$ such that

$$\tau_{t+p} = \tau_t. \tag{3.17}$$

This policy has the following representation

$$\tau_{t+p} = \tau_b^p b_{t+p}, \quad p = 1,..,N-1. \tag{3.18}$$

Indeed, take (3.18) one period forward and use (3.17) to obtain

$$\begin{aligned} \tau_{t+p+1} &= \tau_b^{p+1} b_{t+p+1} = \tau_b^{p+1} \frac{1}{\beta} \left(1 - \tau\theta c_b^p - \tau \left(1 + \theta c_\tau^p\right) \tau_b^p\right) b_{t+p} \\ &= \tau_b^p b_{t+p} \end{aligned}$$

from where

$$\tau_b^{p+1} = \frac{\beta \tau_b^p}{1 - \tau\theta c_b^p - \tau \left(1 + \theta c_\tau^p\right) \tau_b^p}, \quad p = 0,..,N-2.$$

Using recursive substitution we can write the complete set of constraints as

$$\pi_{t+p} = \Pi_b^{p,0} b_t + \Pi_\tau^{p,0} \tau_t, \quad c_{t+p} = \mathcal{C}_b^{p,0} b_t + \mathcal{C}_\tau^{p,0} \tau_t, \quad b_{t+p+1} = \mathcal{B}_b^{p,0} b_t + \mathcal{B}_\tau^{p,0} \tau_t$$

where the coefficients $\Pi_b^{p,0}$, $\Pi_\tau^{p,0}$, $\mathcal{C}_b^{p,0}$, $\mathcal{C}_\tau^{p,0}$, $\mathcal{B}_b^{p,0}$ and $\mathcal{B}_\tau^{p,0}$ are defined by

$$\Pi_b^{p,0} = \frac{\pi_b^{p+1} + \left(\varkappa - \pi_b^{p+1}\tau\theta\right) c_b^p}{\beta^p} \prod_{k=1}^{p} \left(1 - \tau\theta c_b^{p-k}\right) \tag{3.19}$$

$$\Pi_\tau^{p,0} = \left(\varkappa - \pi_b^{p+1}\tau\theta\right) c_\tau^p - \tau\pi_b^{p+1} + \nu - \tau\left(\pi_b^{p+1} + \left(\varkappa - \pi_b^{p+1}\tau\theta\right) c_b^p\right) \times \sum_{j=1}^{p} \frac{\theta c_\tau^{p-j} + 1}{\beta^{j-1}} \prod_{k=1}^{j-1} \left(1 - \tau\theta c_b^{p-k}\right) \tag{3.20}$$

$$\mathcal{C}_b^{p,0} = \frac{c_b^p}{\beta^p} \prod_{k=1}^{p} \left(1 - \tau\theta c_b^{p-k}\right) \tag{3.21}$$

$$\mathcal{C}_\tau^{p,0} = c_\tau^p - \tau c_b^p \sum_{j=1}^{p} \frac{\theta c_\tau^{p-j} + 1}{\beta^{j-1}} \prod_{k=1}^{j-1} \left(1 - \tau\theta c_b^{p-k}\right) \tag{3.22}$$

$$\mathcal{B}_b^{p,0} = \frac{1}{\beta^{p+1}} \prod_{k=0}^{p} \left(1 - \tau\theta c_b^{p-k}\right) \tag{3.23}$$

$$\mathcal{B}_\tau^{p,0} = -\tau \sum_{j=0}^{p} \frac{\theta c_\tau^{p-j} + 1}{\beta^{j-1}} \prod_{k=0}^{j-1} \left(1 - \tau\theta c_b^{p-k}\right). \tag{3.24}$$

Substitute these constraints into the Bellman equation (3.16) and differentiate with respect to τ_t to yield:

$$\tau_t = -\frac{\sum_{p=0}^{N-1} \beta^p \left(\Pi_\tau^{p,0}\Pi_b^{p,0} + \lambda \mathcal{C}_\tau^{p,0}\mathcal{C}_b^{p,0}\right) + \beta^N \mathcal{B}_\tau^{N,0} V \mathcal{B}_b^{N,0}}{\sum_{p=0}^{N-1} \beta^p \left(\left(\Pi_\tau^{p,0}\right)^2 + \lambda\left(\mathcal{C}_\tau^{p,0}\right)^2\right) + \beta^N V \left(\mathcal{B}_\tau^{N,0}\right)^2} b_t \tag{3.25}$$

From (3.10), (B.21) and (B.25) it follows

$$\pi_b^p = \pi_b^{p+1} + \left(\varkappa - \tau\theta\pi_b^{p+1}\right) c_b^p + \left(\left(\varkappa - \tau\theta\pi_b^{p+1}\right) c_\tau^p + \left(\nu - \tau\pi_b^{p+1}\right)\right) \tau_b^p \tag{3.26}$$

which determines the time-consistent reaction of the private sector in (3.10).

The resulting transition of the economy for $p = 0, .., N-1$ can be written as:

$$c_{t+p} = C_b^p b_{t+p} \tag{3.27}$$

$$\pi_{t+p} = \pi_b^p b_{t+p} \tag{3.28}$$

$$b_{t+p+1} = B_b^p b_{t+p} \tag{3.29}$$

where

$$B_b^p = \frac{1}{\beta}\left(1 - \tau\theta c_b^p - \tau\left(\theta c_\tau^p + 1\right)\tau_b^p\right)$$

$$C_b^p = c_b^p + c_\tau^p \tau_b^p$$

Substitute them into (3.12) and (3.16) to yield

$$S^p = (\pi_b^p)^2 + \lambda (C_b^p)^2 + \beta S^{p+1} (B_b^p)^2, \quad p = 0, .., N-1 \tag{3.30}$$

and

$$V = \sum_{p=0}^{N-1} \beta^p \left((\pi_b^p)^2 + \lambda (C_b^p)^2 \right) \prod_{j=0}^{p} \left(B_b^{j-1}\right)^2 + \beta^N V \prod_{p=0}^{N-1} (B_b^p)^2 \tag{3.31}$$

It follows that $V = S^0 = S^N$: in periods when both benevolent policy makers reoptimize their value functions are the same.

Proposition 6 *Given Assumptions 5-7 the stationary discretionary equilibrium with intra-period fiscal leadership can be described by the set of coefficients* $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$.

Proof. For a given $b_0 = \bar{b}$, each trajectory $\{b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$ which solves the system of first order conditions (3.8), (3.10), (3.13), and (3.18) we can uniquely map into the set of coefficients $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$, satisfying (3.14), (3.15), (3.25), (3.26), (3.30) and (3.31). Conversely, if the set of coefficients $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$ solves (3.14), (3.15), (3.25), (3.26), (2.28) and (3.31) we can uniquely map it into the trajectory $\{b_t, \pi_t, c_t, \tau_t\}_{t=0}^{\infty}$, solving system (3.24), (3.10), (3.13) for given $b_0 = \bar{b}$. ■

3.4 Coordination Mechanisms

Discretionary policy may result in multiple policy equilibria, see Albanesi, Chari, and Christiano (2003), King and Wolman (2004), Blake and Kirsanova (2012); in our case it implies that system (3.14), (3.15), (3.25), (3.26), (2.28) and (3.31) may have several distinct solutions. Current policy decisions depend on the forecast of future policy, which are made by the private sector. If there are dynamic complementarities between actions of economic agents then multiple equilibria might arise and coordination failures occur. Not all equilibria are empirically relevant: economic agents may coordinate on some equilibria more likely than on others. Following Evans (1986) and drawing on the large literature which employs learning to analyze coordination in rational expectations (RE) models[9] Dennis and Kirsanova (2012) develop and apply several iteration expectations (IE) stability criteria for LQ RE discretionary policy models with one policy maker. In this Section we extend these criteria to the case of two policy makers. This allows us to focus on empirically-relevant discretionary equilibria.

Specifically, we consider learning by the private sector, the joint learning by followers, the private sector and the monetary policy maker, the joint learning of all economic agents and learning by the leader. We label these types of learning PS-, JF-, J- and L-learning correspondingly.

3.4.1 Learning by Private Agents

In this section we investigate the IE-stability of RE private sector equilibria, in which the private sector rationally responds to the given policy rules of both policy makers. The given pair of rules represents *equilibrium* discretionary policy.

Discretionary equilibrium is fully characterized by the set $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$. We want to examine whether private agents can learn their equilibrium reaction $\{\pi_b^p\}_{p=0}^{N-1}$, given policies which are described by $V \cup \{c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$.

Suppose the private agents know that the policy makers implement (3.13) and (3.18) within each fiscal cycle. They know that the fiscal policy maker changes the tax rate at periods $[p]_N = 0$, so that all policies and reactions have a 'seasonality' component. The private sector starts the learning process and forms the expectation of the whole vector of responses within the fiscal cycle

$$\pi_{t+p} = \bar{\pi}_b^p b_{t+p}, \quad p = 0, .., N-1.$$

Here and below we denote the guessed values with bars. This perceived reaction of the private sector will be consistent with a RE equilibrium if it is supported by the evolution

[9] See Guesnerie and Woodford (1992), Evans and Guesnerie (1993), Evans and Guesnerie (2003), Evans and Guesnerie (2005), Evans and Honkapohja (2001).

of the economy. The evolution of the economy (2.9)-(3.24) implies

$$\pi_{t+p} = \left(\bar{\pi}_b^{p+1} + \left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right) C_b^p + \left(\nu - \tau\bar{\pi}_b^{p+1}\right) \tau_b^p\right) b_{t+p},$$

where $C_b^p = c_b^p + c_\tau^p \tau_b^p$. Equating coefficients yields

$$\pi_b^p = \left(1 - \tau\left(\tau_b^p + \theta C_b^p\right)\right) \bar{\pi}_b^{p+1} + \varkappa C_b^p + \nu\tau_b^p, \quad p = 0, .., N-1. \tag{3.32}$$

Definition 7 *Equations (3.32) define revision mapping* $\mathbb{T}_{PS}$ *from the initial guess of the decision rule* $\bar{\pi} = \{\bar{\pi}_b^p\}_{p=0}^{N-1}$ *to the revised decision rule* $\pi = \{\pi_b^p\}_{p=0}^{N-1}$*, summarized by* $\pi = \mathbb{T}_{PS}(\bar{\pi})$.

Definition 8 *Fix-point* $\pi^* = \{\pi_b^{p*}\}_{p=0}^{N-1}$ *of the* $\mathbb{T}_{PS}-$ *map,* $\pi = \mathbb{T}_{PS}(\bar{\pi})$ *is said to be locally IE-stable under private sector learning if*

$$\lim_{k\to\infty} \mathbb{T}_{PS}^k (\bar{\pi}) = \pi^*$$

for all $\bar{\pi}$ *in a neighborhood of* π^*, $\bar{\pi} \neq \pi^*$.

It follows that π^* is locally IE-stable if and only if it is a locally stable fix-point of the system of difference equations

$$\pi_{k+1} = \mathbb{T}_{PS}(\pi_k)$$

where the index k denotes the step of the updating process. A fix-point of this mapping results in a perceived law of motion for the economy which is consistent with the economy's actual law-of-motion in a RE equilibrium.

3.4.2 Joint Learning by Followers

Suppose that the monetary policy maker is also learning. The monetary policy maker and the private sector take the fiscal policy decisions as given. The monetary policy maker and the private sector jointly learn their equilibrium reactions $\{\pi_b^p, c_b^p, c_\tau^p, S^p\}_{p=0}^{N-1}$, given fiscal policy which is described by $\{V, \tau_b^0\}$.

Recall that the fiscal policy maker chooses the policy once, at the beginning of the fiscal cycle, and keeps the tax rate *level* constant until the next reoptimization. The state-dependent *representation* of this policy within the fiscal cycle depends on the state, which is affected by decisions of other agents. We assume that the private sector and the monetary policy maker treat the intra-cycle fiscal policy parametrically, as given, but once they revise their expectations they realize the effect of the revision on the representation of the intra-cycle fiscal reaction function, and revise the representation. In what follows we treat $\tau_b^p = \bar{\tau}_b^p,\ p = 2, .., N-1$, but we omit this notation to avoid writing each equation twice, once for $p = 0$, and once for all other periods.

The monetary policy maker and the private sector form expectations about the RE equilibrium. The perceived reaction of the private sector should be supported by the evolution of the economy in order to be consistent with a RE equilibrium. The evolution of the economy (2.9)-(2.11) implies

$$\pi_{t+p} = \left(\left(1 - \tau\tau_b^p\right)\bar{\pi}_b^{p+1} + \nu\tau_b^p\right) b_{t+p} + \left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right) c_{t+p}, \tag{3.33}$$

for $p = 0, .., N-1$.

The perceived reaction of the monetary policy maker should also be consistent with implementing the best response to the guessed reaction of the private sector:

$$\begin{aligned} S^p b_{t+p}^2 &= \min_{c_{t+p}} \Bigg(\left(\left(\bar{\pi}_b^{p+1} - \bar{\pi}_b^{p+1}\tau\tau_b^p + \nu\tau_b^p\right) b_{t+p} + \left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right) c_{t+p}\right)^2 \\ &\quad + \lambda c_{t+p}^2 + \beta\bar{S}^{p+1}\left(\frac{1}{\beta}\left(\left(1 - \tau\tau_b^p\right) b_{t+p} - \tau\theta c_{t+p}\right)\right)^2 \Bigg), \end{aligned}$$

where $p = 0, .., N-1$. The revised reaction rules $c_{t+p} = C_b^p b_{t+p}$ with coefficients

$$C_b^p = -\frac{\left(\left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right)\left(\left(1 - \tau\tau_b^p\right)\bar{\pi}_b^{p+1} + \nu\tau_b^p\right) - \bar{S}^{p+1}\frac{\tau\theta}{\beta}\left(1 - \tau\tau_b^p\right)\right)}{\left(\frac{(\tau\theta)^2}{\beta}\bar{S}^{p+1} + \left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right)^2 + \lambda\right)} = C_b^p\left(\bar{S}, \bar{\pi}\right) \tag{3.34}$$

implement the best policy response. The revised vector of value functions $\{S^p\}_{p=0}^{N-1}$ can be written as

$$S^p = \left(\left(\left((1-\tau\tau_b^p)\,\bar{\pi}_b^{p+1} + \nu\tau_b^p\right) + \left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right) C_b^p\left(\bar{S},\bar{\pi}\right)\right)^2 + \lambda\left(C_b^p\left(\bar{S},\bar{\pi}\right)\right)^2 + \frac{1}{\beta}\bar{S}^{p+1}\left((1-\tau\tau_b^p) - \tau\theta C_b^p\left(\bar{S},\bar{\pi}\right)\right)^2\right) \tag{3.35}$$

where $\left\{C_b^p\left(\bar{S},\bar{\pi}\right)\right\}_{p=0}^{N-1}$ are determined in (2.25). The revision process of the private sector described by (3.33) can be written as $\pi_{t+p} = \pi_b^p b_{t+p}, \quad p = 0,..,N-1$, where

$$\pi_b^p = \left((1-\tau\tau_b^p)\,\bar{\pi}_b^{p+1} + \nu\tau_b^p + \left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right) C_b^p\left(\bar{S},\bar{\pi}\right)\right) = \pi_b^p\left(\bar{S},\bar{\pi}\right). \tag{3.36}$$

Finally, the representation of intra-cycle fiscal policy is updated according to

$$\tau_b^{p+1} = \frac{\beta\tau_b^p}{1-\tau\theta C_b^p\left(\bar{S},\bar{\pi}\right) - \tau\tau_b^p}, \quad p = 0,..,N-2. \tag{3.37}$$

Definition 9 *Equations (3.34)-(3.37) define the revision mapping* $\mathbb{T}_{JF}$ *from the initial guess of the reaction* $\bar{x} = \left\{\bar{\pi},\bar{c},\bar{S}\right\}$ *to the updated reaction* $x = \{\pi, c, S\}$, $x = \mathbb{T}_{JF}(\bar{x})$.

Definition 10 *A fix-point,* $x^* = (\pi^*, c^*, S^*)$ *of the* $\mathbb{T}_{JF}-$ *map,* $x = \mathbb{T}_{JF}(\bar{x})$ *is said to be locally IE-stable under JF-learning if*

$$\lim_{k\to\infty} \mathbb{T}_{JF}^k(\bar{x}) = x^*$$

for all $\bar{x}$ *in a neighborhood of* x^*, $\bar{x} \neq x^*$.

By construction, the fix-point of the revision mapping results in the law of motion of the economy which is consistent with the RE equilibrium. As before, the fix-point of the mapping needs to be locally stable to allow the private sector *and* the monetary policy maker to learn the RE equilibrium.

3.4.3 Joint Learning

We now assume that all agents learning the equilibrium described by $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$. Both policy makers and the private sector make their guess about the RE equilibrium.

The perceived reaction of the private sector should be supported by the evolution of the economy in order to be consistent with a RE equilibrium. The evolution of the economy (2.9)-(2.11) implies

$$\pi_{t+p} = \bar{\pi}_b^{p+1} b_{t+p} + \left(\varkappa - \bar{\pi}_b^{p+1}\tau\theta\right) c_{t+p} + \left(\nu - \bar{\pi}_b^{p+1}\tau\right)\tau_{t+p}$$

for $p = 0, .., N-1$.

The perceived reaction of the monetary policy maker should be consistent with implementing the best response to the guessed reaction rule of the private sector:

$$\begin{aligned} S^p b_{t+p}^2 &= \min_{c_{t+p}} \Bigg(\left(\bar{\pi}_b^{p+1} b_{t+p} + \left(\varkappa - \bar{\pi}_b^{p+1}\tau\theta\right) c_{t+p} + \left(\nu - \bar{\pi}_b^{p+1}\tau\right)\tau_{t+p}\right)^2 \\ &\quad + \lambda c_{t+p}^2 + \beta \bar{S}^{p+1} \left(\frac{1}{\beta}\left(b_{t+p} - \tau\theta c_{t+p} - \tau\tau_{t+p}\right)\right)^2 \Bigg) \end{aligned}$$

where $p = 0, .., N-1$. The revised reaction rules $c_{t+p} = c_b^p b_{t+p} + c_\tau^p \tau_{t+p}, \quad p = 0, .., N-1$, with coefficients

$$c_b^p = -\frac{\left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right)\bar{\pi}_b^{p+1} - \frac{\tau\theta}{\beta}\bar{S}^{p+1}}{\left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}\bar{S}^{p+1}} = c_b^p\left(\bar{S}, \bar{\pi}\right) \tag{3.38}$$

$$c_\tau^p = -\frac{\left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right)\left(\nu - \tau\bar{\pi}_b^{p+1}\right) + \frac{\tau^2\theta}{\beta}\bar{S}^{p+1}}{\left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}\bar{S}^{p+1}} = c_\tau^p\left(\bar{S}, \bar{\pi}\right) \tag{3.39}$$

implement the best policy response. Consistent with the revised reactions, the intra-cycle representation of fiscal policy $\tau_b^p\left(\bar{S}, \bar{\pi}\right)$ satisfies

$$\tau_b^{p+1} = \frac{\beta\tau_b^p}{1 - \tau\theta\left(c_b^p\left(\bar{S}, \bar{\pi}\right) + c_\tau^p\left(\bar{S}, \bar{\pi}\right)\tau_b^p\right) - \tau\tau_b^p} = \tau_b^{p+1}\left(\bar{S}, \bar{\pi}\right), \quad p = 0, .., N-2 \tag{3.40}$$

The perceived reaction of the fiscal policy maker should be consistent with implementing the best response to the guessed reaction rules of the private sector and of the monetary policy maker:

$$V b_t^2 = \min_{\tau_t} \left(\sum_{p=0}^{N-1} \beta^p \left(\pi_{t+p}^2 + \lambda c_{t+p}^2\right) + \beta^N \bar{V} b_{t+N}^2 \right)$$

subject to constraints

$$\pi_{t+p} = \bar{\Pi}_b^{p,0} b_t + \bar{\Pi}_\tau^{p,0}\tau_t, \quad c_{t+p} = \bar{\mathcal{C}}_b^{p,0} b_t + \bar{\mathcal{C}}_\tau^{p,0}\tau_t, \quad b_{t+p+1} = \bar{\mathcal{B}}_b^{p,0} b_t + \bar{\mathcal{B}}_\tau^{p,0}\tau_t$$

where the functional form of $\bar{\Pi}$, $\bar{\mathcal{C}}$ and $\bar{\mathcal{B}}$ can be determined from

$$
\begin{aligned}
\bar{\Pi}_b^{p,0} &= \frac{\bar{\pi}_b^{p+1} + \left(\varkappa - \bar{\pi}_b^{p+1}\tau\theta\right) c_b^p\left(\bar{S},\bar{\pi}\right)}{\beta^p} \prod_{k=1}^{p}\left(1 - \tau\theta c_b^{p-k}\left(\bar{S},\bar{\pi}\right)\right) \\
\bar{\Pi}_\tau^{p,0} &= \left(\varkappa - \bar{\pi}_b^{p+1}\tau\theta\right) c_\tau^p\left(\bar{S},\bar{\pi}\right) - \tau\bar{\pi}_b^{p+1} + \nu - \tau\left(\bar{\pi}_b^{p+1} + \left(\varkappa - \bar{\pi}_b^{p+1}\tau\theta\right) c_b^p\left(\bar{S},\bar{\pi}\right)\right) \\
&\quad \times \sum_{j=1}^{p} \frac{\theta c_\tau^{p-j}\left(\bar{S},\bar{\pi}\right) + 1}{\beta^{j-1}} \prod_{k=1}^{j-1}\left(1 - \tau\theta c_b^{p-k}\left(\bar{S},\bar{\pi}\right)\right) \\
\bar{\mathcal{C}}_b^{p,0} &= \frac{c_b^p\left(\bar{S},\bar{\pi}\right)}{\beta^p} \prod_{k=1}^{p}\left(1 - \tau\theta c_b^{p-k}\left(\bar{S},\bar{\pi}\right)\right) \\
\bar{\mathcal{C}}_\tau^{p,0} &= c_\tau^p\left(\bar{S},\bar{\pi}\right) - \tau c_b^p\left(\bar{S},\bar{\pi}\right) \sum_{j=1}^{p} \frac{\theta c_\tau^{p-j}\left(\bar{S},\bar{\pi}\right) + 1}{\beta^{j-1}} \prod_{k=1}^{j-1}\left(1 - \tau\theta c_b^{p-k}\left(\bar{S},\bar{\pi}\right)\right) \\
\bar{\mathcal{B}}_b^{p,0} &= \frac{1}{\beta^{p+1}} \prod_{k=0}^{p}\left(1 - \tau\theta c_b^{p-k}\left(\bar{S},\bar{\pi}\right)\right) \\
\bar{\mathcal{B}}_\tau^{p,0} &= -\tau \sum_{j=0}^{p} \frac{\theta c_\tau^{p-j}\left(\bar{S},\bar{\pi}\right) + 1}{\beta^{j-1}} \prod_{k=0}^{j-1}\left(1 - \tau\theta c_b^{p-k}\left(\bar{S},\bar{\pi}\right)\right)
\end{aligned}
$$

with $c_b^p\left(\bar{S},\bar{\pi}\right)$ and $c_\tau^p\left(\bar{S},\bar{\pi}\right)$ determined in (3.38)-(3.39). Therefore, $\bar{\Pi}$, $\bar{\mathcal{C}}$ and $\bar{\mathcal{B}}$ also depend on guessed values of $\bar{S}$ and $\bar{\pi}$.

The revision of τ_t can be written as $\tau_t = \tau_b^0 b_t$ with coefficients

$$
\tau_b^0 = -\frac{\sum_{p=0}^{N-1} \beta^p \left(\bar{\Pi}_\tau^{p,0}\bar{\Pi}_b^{p,0} + \lambda\bar{\mathcal{C}}_\tau^{p,0}\bar{\mathcal{C}}_b^{p,0}\right) + \beta^N \bar{\mathcal{B}}_\tau^{N,0}\bar{V}\bar{\mathcal{B}}_b^{N,0}}{\sum_{p=0}^{N-1} \beta^p \left(\left(\bar{\Pi}_\tau^{p,0}\right)^2 + \lambda\left(\bar{\mathcal{C}}_\tau^{p,0}\right)^2\right) + \beta^N \bar{V}\left(\bar{\mathcal{B}}_\tau^{N,0}\right)^2} = \tau_b^0\left(\bar{S},\bar{\pi}\right). \tag{3.41}
$$

Therefore, the consistent with RE equilibrium revision of the private sector reaction function can be written as $\pi_{t+p} = \pi_b^p b_{t+p}, \quad p = 0,..,N-1$, with

$$
\begin{aligned}
\pi_b^p &= \bar{\pi}_b^{p+1} + \left(\varkappa - \bar{\pi}_b^{p+1}\tau\theta\right) c_b^p\left(\bar{S},\bar{\pi}\right) \\
&\quad + \left(\left(\varkappa - \tau\theta\bar{\pi}_b^{p+1}\right) c_\tau^p\left(\bar{S},\bar{\pi}\right) + \left(\nu - \bar{\pi}_b^{p+1}\tau\right)\right)\tau_b^p\left(\bar{S},\bar{\pi}\right) = \pi_b^p\left(\bar{S},\bar{\pi}\right)
\end{aligned} \tag{3.42}
$$

Finally, consistent with RE equilibrium revision of value functions is

$$
\begin{aligned}
S^p &= \left(\pi_b^p\left(\bar{S},\bar{\pi}\right)\right)^2 + \lambda\left(c_b^p\left(\bar{S},\bar{\pi}\right) + c_\tau^p\left(\bar{S},\bar{\pi}\right)\tau_b^p\left(\bar{S},\bar{\pi}\right)\right)^2 \\
&\quad + \frac{1}{\beta}\bar{S}^{p+1}\left(1 - \tau\theta c_b^p\left(\bar{S},\bar{\pi}\right) - \tau\left(\theta c_\tau^p\left(\bar{S},\bar{\pi}\right) + 1\right)\tau_b^p\left(\bar{S},\bar{\pi}\right)\right)^2
\end{aligned} \tag{3.43}
$$

for $p = 0, .., N-1$, and

$$V = \sum_{p=0}^{N-1} \beta^p \left(\left(\pi_b^p \left(\bar{S}, \bar{\pi}\right)\right)^2 + \lambda \left(c_b^p \left(\bar{S}, \bar{\pi}\right) + c_\tau^p \left(\bar{S}, \bar{\pi}\right) \tau_b^p \left(\bar{S}, \bar{\pi}\right)\right)^2 \right) \tag{3.44}$$
$$\times \prod_{j=0}^{p} \left(\frac{1}{\beta} \left(1 - \tau\theta c_b^{j-1} \left(\bar{S}, \bar{\pi}\right) - \tau \left(\theta c_\tau^{j-1} \left(\bar{S}, \bar{\pi}\right) + 1\right) \tau_b^{j-1} \left(\bar{S}, \bar{\pi}\right)\right) \right)^2$$
$$+\beta^N \bar{V} \prod_{p=0}^{N-1} \left(\frac{1}{\beta} \left(1 - \tau\theta c_b^p \left(\bar{S}, \bar{\pi}\right) - \tau \left(\theta c_\tau^p \left(\bar{S}, \bar{\pi}\right) + 1\right) \tau_b^p \left(\bar{S}, \bar{\pi}\right)\right) \right)^2.$$

Definition 11 *Equations (3.38), (3.39), (3.40), (3.41), (3.42), (3.43), (3.44) define the revision mapping* $\mathbb{T}_J$ *from the initial guess of the reaction* $\bar{x} = \left\{\bar{\pi}, \bar{c}, \bar{S}, \bar{\tau}, \bar{V}\right\}$ *to the updated reaction* $x = \{\pi, c, S, \tau, V\}$, $x = \mathbb{T}_J(\bar{x})$.

Definition 12 *A fix-point,* $x^* = (\pi^*, c^*, S^*, \tau^*, V^*)$ *of the* $\mathbb{T}_J-$ *map,* $x = \mathbb{T}_J(\bar{x})$ *is said to be locally IE-stable under J-learning if*

$$\lim_{k \to \infty} \mathbb{T}_J^k(\bar{x}) = x^*$$

for all $\bar{x}$ *in a neighborhood of* x^*, $\bar{x} \neq x^*$.

By construction, the fix-point of this natural revision mapping results in the law of motion of the economy which is consistent with the RE equilibrium. The fix-point of the mapping needs to be locally stable to allow all agents to learn the RE equilibrium jointly.

3.4.4 Learning by the Leader

We now assume that only the fiscal policy maker learns the RE equilibrium policy $\{\tau_b^0, V\}$, knowing the reaction of all agents $\{\pi_b^p, c_b^p, c_\tau^p\}_{p=0}^{N-1}$. The perceived reaction of the fiscal policy maker should be consistent with implementing the best response to the known reaction rule of the private sector and of the monetary policy maker:

$$V b_t^2 = \min_{\tau_t} \left(\sum_{p=0}^{N-1} \beta^p \left(\pi_{t+p}^2 + \lambda c_{t+p}^2\right) + \beta^N \bar{V} b_{t+N}^2 \right) \tag{3.45}$$

subject to constraints

$$\pi_{t+p} = \Pi_b^{p,0} b_t + \Pi_\tau^{p,0} \tau_t, \quad c_{t+p} = \mathcal{C}_b^{p,0} b_t + \mathcal{C}_\tau^{p,0} \tau_t, \quad b_{t+p+1} = \mathcal{B}_b^{p,0} b_t + \mathcal{B}_\tau^{p,0} \tau_t$$

where coefficients Π, $\mathcal{C}$ and $\mathcal{B}$ are the same as in (3.19)-(3.24).

It is straightforward to see that optimization problem (3.45) is equivalent to the standard discounted LQ problem, described in e.g. Lancaster and Rodman (1995) and Kwakernaak and Sivan (1972). The revised reaction rule $\tau_b^0\left(\bar{\tau}_b^0, \bar{V}\right)$ in

$$\tau_t = \tau_b^0 b_t = \tau_b^0\left(\bar{\tau}_b^0, \bar{V}\right) b_t$$

and the corresponding update of the value function $V = V\left(\bar{\tau}_b^0, \bar{V}\right)$ are consistent with RE equilibrium by construction.

The corresponding revision map $\tau_b^0 = \mathbb{T}_L\left(\bar{\tau}_b^0\right)$ has at most one[10] stationary fixed point which is always locally stable, see Lancaster and Rodman (1995), see also Dennis and Kirsanova (2012) where the same fact is proved for the LQ RE models with single policy maker.

3.5 Policy Interactions

In this section we study how an increase in the length of the fiscal cycle affects the economy under discretionary policy. Dynamic complementarities play a crucial role in shaping the dynamics of the economy once the fiscal cycle becomes longer.

We start with the known case of frequent monetary and fiscal policy stabilization.[11] We use this example to discuss the transmission mechanisms of monetary and fiscal policy interactions.

We continue with the case of biannual fiscal optimization, which enables us to demonstrate how the dynamic complementarity between the optimal actions of *consequent* monetary policy makers within the fiscal cycle results in multiple discretionary equilibria and potential expectation traps. We also demonstrate that the agents are likely to coordinate on the best equilibrium.

[10] Because of discounting the stationary solution may not exist.
[11] See Blake and Kirsanova (2011) for a general form solution to this class of problems.

These two cases help us to investigate the more complex case of annual fiscal optimization, which is arguably the most empirically relevant case. We demonstrate how the dynamic complementarity between the optimal actions of monetary and fiscal policy makers leads to expectation traps. Although we demonstrate that in this case the coordination problem is likely to be resolved as well, as all agents are more likely to coordinate on the best equilibrium, we also show that the existence of these equilibria is very sensitive to the parameterization of the model and to the length of fiscal cycle. We argue that actions of the fiscal policy maker should be restricted to some extent, as this ensures the existence of good equilibrium outcome for a wide range of parameterization of the model and policy scenarios, as well as for longer fiscal cycle.

3.5.1 Quarterly Fiscal Stabilization

In the standard case of frequent stabilization both policy makers operate at the same quarterly frequency. The model is simple enough to prove the following proposition.

Proposition 13 *If* $0 < \beta < 1$, $\tau > 0$, $\lambda > 0$ *then a stationary discretionary equilibrium exists and unique.*

Proof. The system of first order conditions (2.53), (2.55), (3.25), (2.27), (2.28) and (3.31) can be written as follows (where we omit the index p):

$$c_b = -\frac{(\varkappa - \pi_b \tau\theta)\pi_b - \frac{\tau\theta}{\beta}V}{(\varkappa - \tau\theta\pi_b)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}V} \tag{3.46}$$

$$c_\tau = -\frac{(\varkappa - \pi_b \tau\theta)(\nu - \pi_b\tau) + \frac{\tau^2\theta}{\beta}V}{(\varkappa - \tau\theta\pi_b)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}V} \tag{3.47}$$

$$\begin{aligned} \tau_b = & -\frac{((\varkappa - \tau\theta\pi_b)c_\tau + \nu - \tau\pi_b)(\pi_b + (\varkappa - \tau\theta\pi_b)c_b) + \lambda c_\tau c_b}{((\varkappa - \tau\theta\pi_b)c_\tau + \nu - \tau\pi_b)^2 + \lambda c_\tau^2 + \frac{\tau^2}{\beta}(1+\theta c_\tau)^2 V} \\ & -\frac{1}{\beta}\frac{\tau(1+\theta c_\tau)(1-\tau\theta c_b)V}{((\varkappa - \tau\theta\pi_b)c_\tau + \nu - \tau\pi_b)^2 + \lambda c_\tau^2 + \frac{\tau^2}{\beta}(1+\theta c_\tau)^2 V} \end{aligned} \tag{3.48}$$

$$\pi_b = (\pi_b + (\varkappa - \pi_b\tau\theta)c_b + ((\varkappa - \pi_b\tau\theta)c_\tau + (\nu - \pi_b\tau))\tau_b) \tag{3.49}$$

$$V = \pi_b^2 + \lambda(c_b + c_\tau\tau_b)^2 + \frac{1}{\beta}V(1 - \tau\theta c_b - \tau(1+\theta c_\tau)\tau_b)^2 \tag{3.50}$$

Introduce new variable, $C_b = c_b + c_\tau \tau_b$. Using several substitutions we transform the system of first order conditions (3.46)-(3.50) into the system of two equations in $\{C_b, \tau_b + \theta C_b\}$:

$$C_b\left(C_b, \tau_b + \theta C_b\right) = C_b + \frac{\nu\left(\varkappa - \theta\nu\right)}{\lambda\tau\left(\tau_b + \theta C_b\right) + \left(\varkappa - \theta\nu\right)^2}\left(\tau_b + \theta C_b\right) = 0 \tag{3.51}$$

$$\tau_b\left(C_b, \tau_b + \theta C_b\right) = \left(\tau_b + \theta C_b\right)^2 - \frac{\lambda - \left(\varkappa - \theta\nu\right)^2}{\lambda\tau}\left(\tau_b + \theta C_b\right) - \frac{\left(1-\beta\right)\left(\varkappa - \theta\nu\right)^2}{\lambda\tau^2} \tag{3.52}$$

Equation (3.52) only depends on $z = \tau_b + \theta C_b$ and always has exactly one positive solution as the free term is negative.

The unique positive root satisfies

$$\frac{1}{\beta}\left|1 - \tau\left(\theta C_b + \tau_b\right)\right| < 1 \tag{3.53}$$

or equivalently $\frac{1-\beta}{\tau} < \tau_b + \theta C_b < \frac{1+\beta}{\tau}$ so that the equilibrium is stationary. To see this, note that if z_+ is the positive root, then $z_- = -\left(\left(1-\beta\right)\left(\varkappa - \theta\nu\right)^2\right)/\left(\lambda\tau^2 z_+\right)$, and $\partial\left(z_+ + z_-\right)/\partial z_+ = 1 + \left(1-\beta\right)\left(\varkappa - \theta\nu\right)^2/\left(\lambda\tau^2 z_+^2\right) > 0$. (i) We show that $\tau_b + \theta C_b > \left(1-\beta\right)/\tau$. Indeed, suppose $\tilde{z}_+ = \left(1-\beta\right)/\tau$. $\tilde{z}_+$ is not the positive root to (3.52); if it was the positive root then the negative root would be $\tilde{z}_- = -\left(\varkappa - \theta\nu\right)^2/\left(\lambda\tau\right)$ and their sum should have been equal to the negative linear coefficient, but $\tilde{z}_+ + \tilde{z}_- < \left(\lambda - \left(\varkappa - \theta\nu\right)^2\right)/\left(\lambda\tau\right)$. Moreover, any $\tilde{z}_+ < \left(1-\beta\right)/\tau$ is not a root of (3.52), because $\partial\left(z_+ + z_-\right)/\partial z_+ > 0$. (ii) We show that $\tau_b + \theta C_b < \left(1+\beta\right)/\tau$. Indeed, suppose $\tilde{z}_+ = \left(1+\beta\right)/\tau$. $\tilde{z}_+$ is not the positive root to (3.52); if it was the positive root then the negative root would be $\tilde{z}_- = -\left(1-\beta\right)\left(\varkappa - \theta\nu\right)^2/\left(\lambda\tau\left(1+\beta\right)\right)$ and their sum $\tilde{z}_+ + \tilde{z}_- > \left(\lambda - \left(\varkappa - \theta\nu\right)^2\right)/\left(\lambda\tau\right)$. Moreover, any $\tilde{z}_+ > \left(1+\beta\right)/\tau$ is not a root of (3.52), because $\partial\left(z_+ + z_-\right)/\partial z_+ > 0$. ■

Panel I in Figure 3.2 presents constraints (3.51)-(3.52) in $\{C_b, \tau_b + \theta C_b\}$ space. Solution to equation (3.52) is plotted with the dashed line, and solution to equation (3.51) is plotted with solid line. The unique equilibrium is labelled A in Panel I in Figure 3.2 and its characteristics are given in Table 3.2.

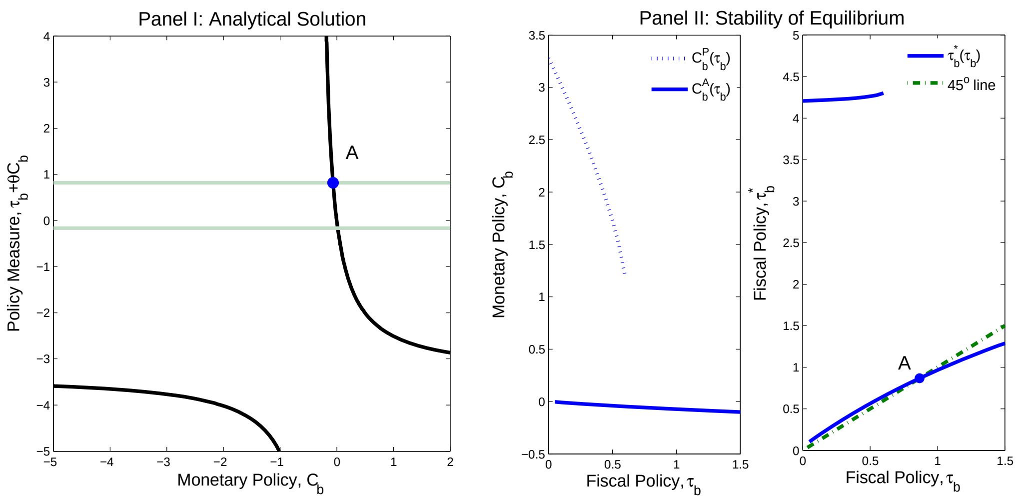

Figure 3.2: Unique Equilibrium in Frequent Fiscal Optimization Model

Table 3.2: Characteristics of Equilibria

		Eq. A	Eq. B
Frequent Fiscal Stabilization			
Fiscal Policy	$[\tau_b]$	$[0.8671]$	–
Monetary Policy	$[C_b]$	$[-0.0657]$	–
Private Sector	$[\pi_b]$	$[0.0048]$	–
Normalized Loss	L	1.0000	–
IE-stability	[PS,JF,J]	[Y,Y,Y]	–
Biannual Fiscal Stabilization			
Fiscal Policy	$[\tau_b^0; \tau_b^1]$	$[0.79; 0.96]$	$[3.57; 34.72]$
Monetary Policy	$[C_b^0; C_b^1]$	$-[6.41; 6.87] \times 10^{-2}$	$[2.72; 48.36] \times 10^{-2}$
Private Sector	$[\pi_b^0; \pi_b^1]$	$[4.7; 5.0] \times 10^{-3}$	$[23.8; 0.7] \times 10^{-3}$
Normalized Loss	L	0.9956	18.8413
IE-stability	[PS,JF,J]	[Y,Y,Y]	[Y,N,N]
Annual Fiscal Stabilization			
Fiscal Policy	$[\tau_b^0; \tau_b^1; \tau_b^2; \tau_b^3]$	$[0.71; 0.84; 1.04; 1.36]$	$[1.23; 1.72; 2.87; 9.02]$
Monetary Policy	$[C_b^0; C_b^1; C_b^2; C_b^3]$	$-[6.2; 7.0; 7.9; 8.3] \times 10^{-2}$	$-[0.10; 0.13; 0.17; 0.35]$
Private Sector	$[\pi_b^0; \pi_b^1; \pi_b^2; \pi_b^3]$	$[4.6; 5.2; 5.7; 6.0] \times 10^{-3}$	$-[0.8; 1.0; 1.4; 2.3] \times 10^{-2}$
Normalized Loss	L	1.0256	2.0068
IE-stability	[PS,JF,J]	[Y,Y,Y]	[Y,Y,N]

Equilibrium A is IE-stable under all types of learning discussed in Section 3.4, and we report this in Table 3.2.

It is easy to see that because equilibrium A is stationary, i.e. $\frac{1}{\beta}\left|1-\tau\left(\tau_b+\theta C_b\right)\right|<1$, then equation (3.32) implies that the fix-point of $\mathbb{T}_{PS}$ is locally stable under the PS-learning.

IE-stability under the JF-learning plays an important role in the analysis of cases with longer fiscal cycle. Using the fact that all equilibria are IE-stable under the L-learning, and replicating the steps of the revision process of all agents who are learning, helps us to discover RE equilibria in this and more complex cases with longer fiscal cycle. We illustrate this process in Panel II of Figure 3.2. Suppose the fiscal policy maker considers implementing policy τ_b, which is not necessarily optimal. In response to this policy the followers learn their optimal response $\{C_b, S, \pi_b\}$. Their learning problem is equivalent to the joint learning in the single-policymaker setting, which is discussed in details in Blake and Kirsanova (2012) and Dennis and Kirsanova (2012). If $\tau_b = 0$ then the fiscal policy maker does not respond to debt and there is unique set $\{C_b, S, \pi_b\}^P$ which describes the case in which the monetary policy maker and the private sector coordinate on the reaction so that in response to higher debt the monetary policy maker generates high demand and accommodates high inflation so that debt is quickly steered back to its equilibrium level. The corresponding positive C_b^P is plotted in the left chart in Panel II. If $\tau_b > 0$ and sufficiently large then the process of debt stabilization is tightly controlled by fiscal policy and in response to higher debt the monetary policy maker and the private sector coordinate on the response $\{C_b, S, \pi_b\}^A$ in which the demand is lowered and inflation is not accommodated. The corresponding positive C_b^A is plotted in the left chart in Panel II.[12] If τ_b is moderate, both types of responses of the monetary policy maker and the private sector exist, as shown in the left chart in Panel II. In response to each $\{C_b, S, \pi_b\}^j, j \in \{A, P\}$ the fiscal policy maker can learn its optimal response

[12] There is close resemblance between these two partial equilibria and 'active' and 'passive' monetary policy described in Leeper (1991).

$\tau_b^{*j}, j \in \{A, P\}$. For each set $\{C_b, S, \pi_b\}$ the response τ_b^* is unique if it exists, see Section 3.4.4. Therefore, for an initial guess τ_b we find the update in the revision process of the fiscal policy maker $\tau_b^*(\tau_b)$. We plot $\tau_b^* = \tau_b^*(\tau_b)$ for a range of initial guesses and for each equilibrium reaction $\{C_b, S, \pi_b\}^j, j \in \{A, P\}$ in the right chart in Panel II in Figure 3.2 with the solid line. By construction, all points of intersection of this line with the 45^o line are the points of discretionary equilibria which are IE-stable under the JF-learning by construction.

The result of Proposition 13 on the uniqueness of the equilibrium is not obvious if the model has dynamic complementarities between action of the economic agents (Cooper and John, 1988). Optimal actions of the monetary authority and of the aggregated private sector can be dynamic complements. Suppose the reaction of fiscal policy is given and fixed at τ_b^*. For a given reaction of the private sector π_b in $\pi_t = \pi_b b_t$ the monetary policy finds the optimal response by solving the corresponding Bellman equation, taking into account its intra-period leadership. If π_b is sufficiently high (low) then in response to higher-than-steady-state debt the monetary policy maker optimally raises demand. Greater tax base leads to higher tax collection and reduces the level of debt towards the steady state. Inflation starts moving back to the steady state. We plot this U-shaped optimal reaction function $C_b = C_b(\pi_b)$ in the left hand side chart of Panel I in Figure 3.3 with the solid line. In its turn, the optimal reaction of the private sector $\pi_b = \pi_b(C_b)$ is increasing in C_b. If the debt is higher than its steady state level and the monetary policy maker generates higher demand, the total effect of the higher demand on marginal costs is always positive, as the tax rate is fixed to τ_b^*. We plot the positively sloped reaction function $\pi_b = \pi_b(C_b)$ in the left hand side chart of Panel I in Figure 3.3 with the dashed line. Both lines are positively sloped in the area with relatively large π_b, and this can result in multiplicity of partial equilibria, i.e. in multiplicity of optimal responses of the monetary authorities and the private sector. Indeed, if we reduce (e.g. halve) the fiscal feedback τ_b then there are three points of intersection of optimal reactions of monetary authorities and the private sector, see the right hand side chart. The case in the left hand side chart in Panel I corresponds to multiple discretionary equilibria discussed in Blake and Kirsanova (2012) where the fiscal feedback on debt of *non-strategic* fiscal policy was relatively small to guarantee multiplicity of equilibria.

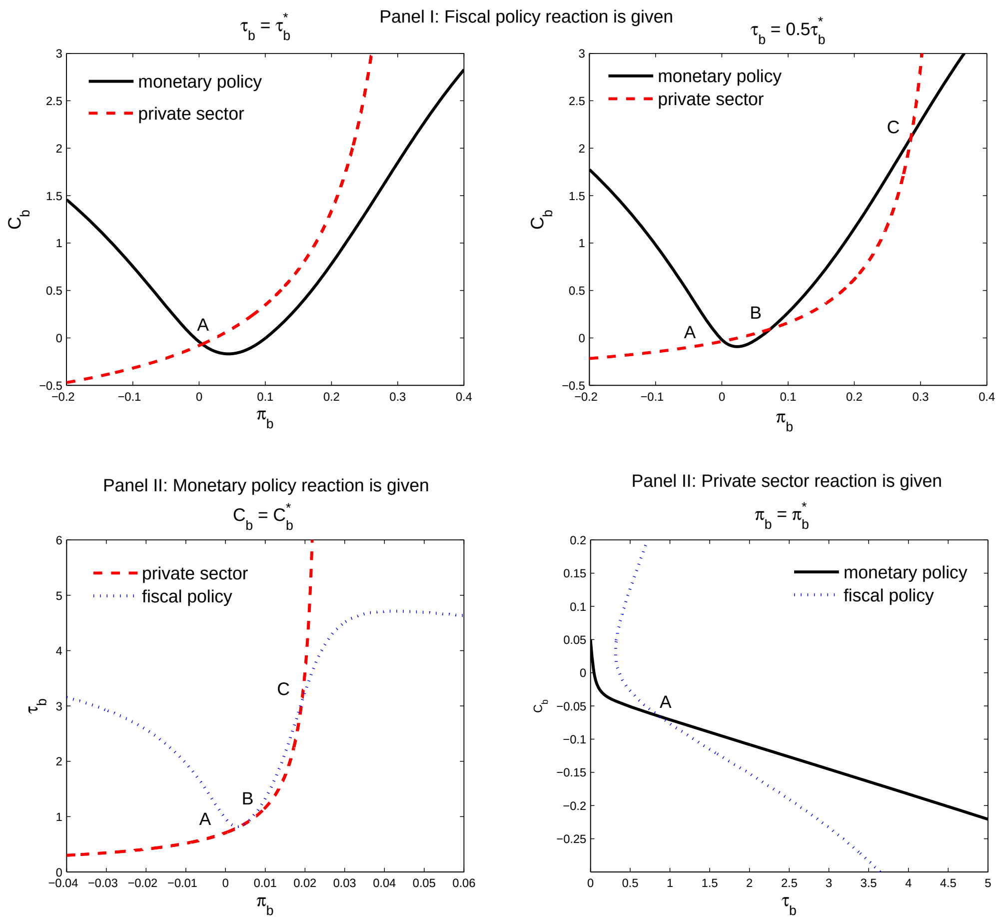

Figure 3.3: Dynamic complementarities between agents' actions

Optimal actions of fiscal policy and the private sector can be dynamic complements too. Keeping the monetary policy reaction fixed at the optimal level $C_b^* < 0$ produces reactions of the fiscal authority $\tau_b = \tau_b(\pi_b)$ and of the private sector $\pi_b = \pi_b(\tau_b)$ plotted in Panel II in Figure 3.3. The reaction of the private sector is positively sloped as higher tax rate set by the fiscal authority in response to higher debt τ_b always results in higher prices set by firms. The reaction curve of the fiscal policy maker is also positively sloped but only for moderately positive response of inflation to debt, π_b. To understand this, suppose the debt is higher than its steady state level, due to $C_b^* < 0$ the demand is automatically reduced as well as the marginal costs. Lower demand also contributes to faster debt accumulation. If the response of inflation to debt is only moderately positive, then the optimal response of taxes to debt rises with stronger response of inflation. This will keep debt under control, and will not compromise inflation stabilization. As a result, the reaction curves of the private sector and the fiscal policy maker are both positively sloped, but only in a relatively narrow area of responses of the private sector. For our baseline calibration, and given C_b^*, there are three jointly optimal discretionary responses of fiscal policy maker and the private sector. However, only one of them results in discretionary equilibrium in the model, as the other two partial equilibria require different optimal policy response once the monetary policy becomes strategic.

Finally, and most importantly for our study, optimal actions of the monetary and the fiscal policy makers can also be dynamic complements. Higher tax rate, set by the fiscal policy maker in response to a high debt level, τ_b , generates greater cost-push inflation, which increases the marginal return to a monetary policy decision to reduce demand and contribute to the debt accumulation. The monetary policy reaction function $C_b = C_b(\tau_b)$ is negatively sloped, see Panel III in Figure 3.3. Conversely, a reduction in response of demand to debt, C_b, makes it optimal to raise taxes in order to prevent too fast accumulation of debt. As a result, the fiscal policy reaction function $\tau_b = \tau_b(C_b)$ is also negatively sloped in wide area, see Panel III in Figure 3.3.

The presence of dynamic complementarities is a necessary condition for the multiplicity of discretionary equilibria, see King and Wolman (2004) and Blake and Kirsanova (2012). However this condition is not sufficient and, as we argue next, the interaction of the two mechanisms in the model with frequent fiscal optimization results in the uniqueness of the equilibrium.

First, the complementarity between optimal decisions of the private sector and of the monetary policy maker may result in multiplicity only if fiscal policy *optimally* responds to debt only weakly, see Panel I, the right hand chart. The optimal fiscal response τ_b^* even to the weak initial guess τ_b is strong enough to rule out the equilibrium with passive monetary policy.

Second, although the optimal decision of the fiscal policy maker is increasing in the optimal decision of the monetary policy maker, this dynamic complementarity between optimal decisions of the two policy makers in case of frequent fiscal optimization is not strong enough to create the multiplicity.

The following two cases demonstrate how the longer fiscal cycle increases the strength of dynamic complementarities in the model and how this shapes the optimal outcome of monetary and fiscal policy interactions.

3.5.2 Biannual Fiscal Stabilization

Suppose that both policy makers optimize in even periods, and we index all such periods with index 0. Only the monetary policy maker optimizes in odd periods, we index such periods with index 1. To save on notation we use the period index $p \in \{0, 1\}$ and use $-p$ to indicate odd periods if $p = 0$, and even periods if $p = 1$.

Despite we cannot prove analytically the existence and multiplicity of equilibria, we can find *all* discretionary equilibria numerically.

Proposition 14 *For the base line calibration of the model two discretionary equilibria exist.*

Proof. The first order conditions derived in Section 3.3 can be written as

$$\tau_b^0 = -\frac{\Pi_\tau^0\Pi_b^0 + \lambda c_\tau^0 c_b^0 + \beta\Pi_\tau^{1,0}\Pi_b^{1,0} + \beta\lambda\mathcal{C}_\tau^{1,0}\mathcal{C}_b^{1,0} + \beta^2\mathcal{B}_\tau^{2,0}\mathcal{B}_b^{2,0}V}{(\Pi_\tau^0)^2 + \lambda(c_\tau^0)^2 + \beta\left(\Pi_\tau^{1,0}\right)^2 + \beta\lambda\left(\mathcal{C}_\tau^{1,0}\right)^2 + \beta^2 V\left(\mathcal{B}_\tau^{2,0}\right)^2} \quad (3.54)$$

$$\tau_b^1 = \frac{\beta\tau_b^0}{1-\tau\left(\left(1+\theta c_\tau^0\right)\tau_b^0 + \theta c_b^0\right)} \quad (3.55)$$

$$c_b^p = -\frac{\left(\varkappa - \tau\theta\pi_b^{-p}\right)\pi_b^{-p} - \frac{\tau\theta}{\beta}S_{-p}}{\left(\varkappa - \tau\theta\pi_b^{-p}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}S_{-p}} \quad (3.56)$$

$$c_\tau^p = -\frac{\left(\varkappa - \tau\theta\pi_b^{-p}\right)\left(\nu - \tau\pi_b^{-p}\right) + \frac{\tau^2\theta}{\beta}S_{-p}}{\left(\varkappa - \tau\theta\pi_b^{-p}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}S_{-p}} \quad (3.57)$$

$$S_0 = V \quad (3.58)$$

$$S_1 = \left(\pi_b^1\right)^2 + \lambda\left(C_b^1\right)^2 + \beta S_0\left(B_b^1\right)^2 \quad (3.59)$$

$$V = \left(\pi_b^0\right)^2 + \lambda\left(C_b^0\right)^2 + \beta\left(\left(\pi_b^1\right)^2 + \lambda\left(C_b^1\right)^2\right)\left(B_b^0\right)^2 + \beta^2 V\left(B_b^0\right)^2\left(B_b^1\right)^2 \quad (3.60)$$

After multiple substitutions the system of first order conditions (3.54)-(3.60) can be reduced to the polynomial system of two equations $C_b^0\left(C_b^0, \tau_b^0\right) = 0$ and $\tau_b^0\left(C_b^0, \tau_b^0\right) = 0$, although at the expense of much complexity. We plot solutions to these equations in Panel I in Figure 3.4. The curves intersect in two points with $V = S_0 > 0, S_1 > 0$, and $|B_b^0 B_b^1| < 1$.We label these points of intersection as equilibria A and B. ■

Multiplicity of discretionary equilibria implies that following a disturbance, for example a higher initial debt level, the economy can follow one of multiple paths, each of which satisfies conditions of optimality and time-consistency. Each of these paths is associated with different monetary and fiscal policies; see Figure 3.5 which plots two different adjustment paths following the same initial increase in the debt level. For comparison, the Figure also includes responses in case of frequent fiscal stabilization.

Suppose the level of debt is above the steady state and fiscal policy raises the tax rate for two periods. Following the high marginal cost inflation will rise and stay above the steady state for these two periods. The monetary policy maker will find it optimal to intervene. The monetary policy maker at time 0 takes into account monetary policy in period 1. There is a dynamic complementarity between the actions of the two consequent

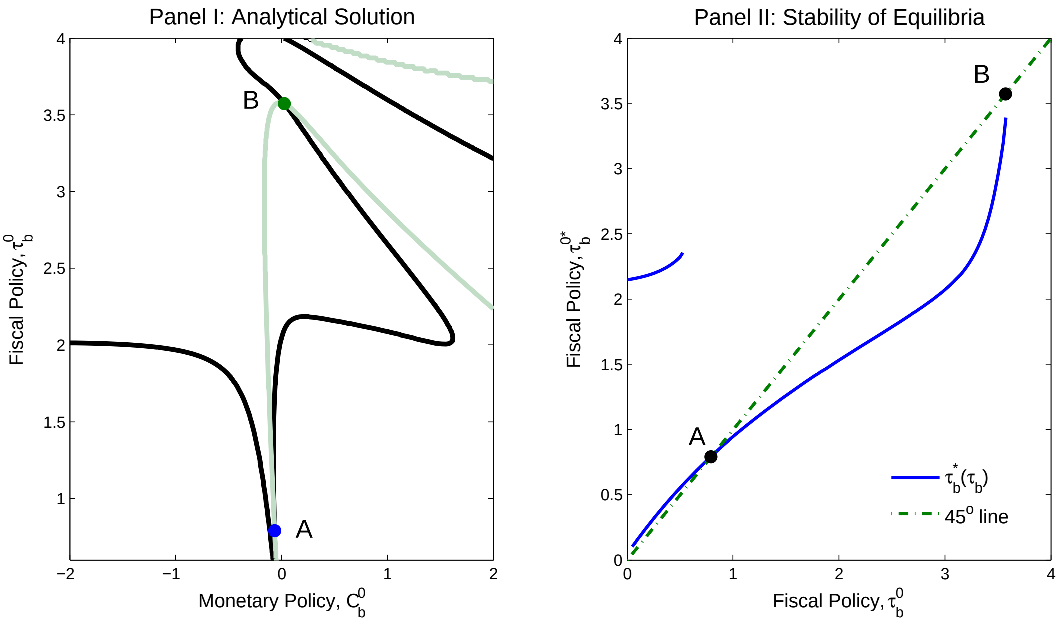

Figure 3.4: Discretionary Equilibria in the Biannual Fiscal Stabilization Model

monetary policy makers within the fiscal cycle: the deeper is the future cut in demand, the bigger payoff the current monetary policy maker gets from engineering high demand today. A high demand today results in optimal reduction of demand in the future, within the same fiscal cycle. Two point-in-time equilibria arise. In the first such equilibrium, the period-0 monetary policy maker will keep the current demand low and the period-1 monetary policy maker does not generate a big cut in demand. In the second equilibrium, the period-0 monetary policy maker stimulates high demand in anticipation that the period-1 monetary policy maker will implement a cut in demand. The fiscal policy maker when choosing policy in period 0, perceives the both possibilities. The optimal fiscal response in the first point-in-time equilibrium response is to raise the tax rate less than in the second equilibrium. The strong response of the tax rate in the second equilibrium generates a 'zig-zag' pattern of adjustment: with low two-period-average demand, the increase in the tax rate generates substantial fall in the stock of debt so that the second half year cycle 'mirrors' the first half year one, but with the opposite sign. Figure 3.5 also demonstrates that in equilibrium A the paths of all variables 'approximate' the optimal paths of the corresponding variables under frequent optimization, and we shall call equilibrium A 'approximating'. We call equilibrium B 'zig-zag'.

Despite the clearly increased inflation volatility, the loss in the approximating equilibrium is slightly lower than it is in the unique equilibrium under frequent optimization, see Table 3.2. This is mainly due to faster stabilization of the economy in this equilibrium. The two-period tax rate increase predominantly determines the two-period speed of debt adjustment $|B_b^0 B_b^1|_A = 0.64 < 0.96 = |B_b|^2$. This welfare gain of faster stabilization is slightly higher than the welfare loss of higher volatility. The loss in the 'zig-zag' equilibrium is much higher than in the 'approximating' equilibrium. Not only it generates the relatively slow speed of adjustment, as $|B_b^0 B_b^1|_B = 0.79 > |B_b^0 B_b^1|_A$, but it also induces very high volatility of economic variables.

Finally, equilibrium A is IE-stable under all types of learning we consider in this Chapter. Equilibrium B is not IE-stable under both JF-and J-learning. Panel II in Figure 3.4 illustrates this.

To summarize, the main conclusion from the example of biannual fiscal stabilization is the demonstration of the existence of the approximating equilibrium. Although the other equilibrium exists, the approximating equilibrium delivers the best possible outcome under the infrequent discretionary fiscal stabilization, and is also the only equilibrium which is IE-stable under all types of learning we study in this Chapter. In this equilibrium the monetary policy maker can offset most adverse effects of fiscal infrequency on welfare-related macroeconomic variables. In the next example we argue that we should not take this result for granted once the fiscal cycle becomes longer.

3.5.3 Annual Fiscal Stabilization

Multiplicity of Discretionary Policy Equilibria

Building on results in the previous section we present the third example of infrequent fiscal stabilization. Arguably, this is the most empirically relevant setup in which the

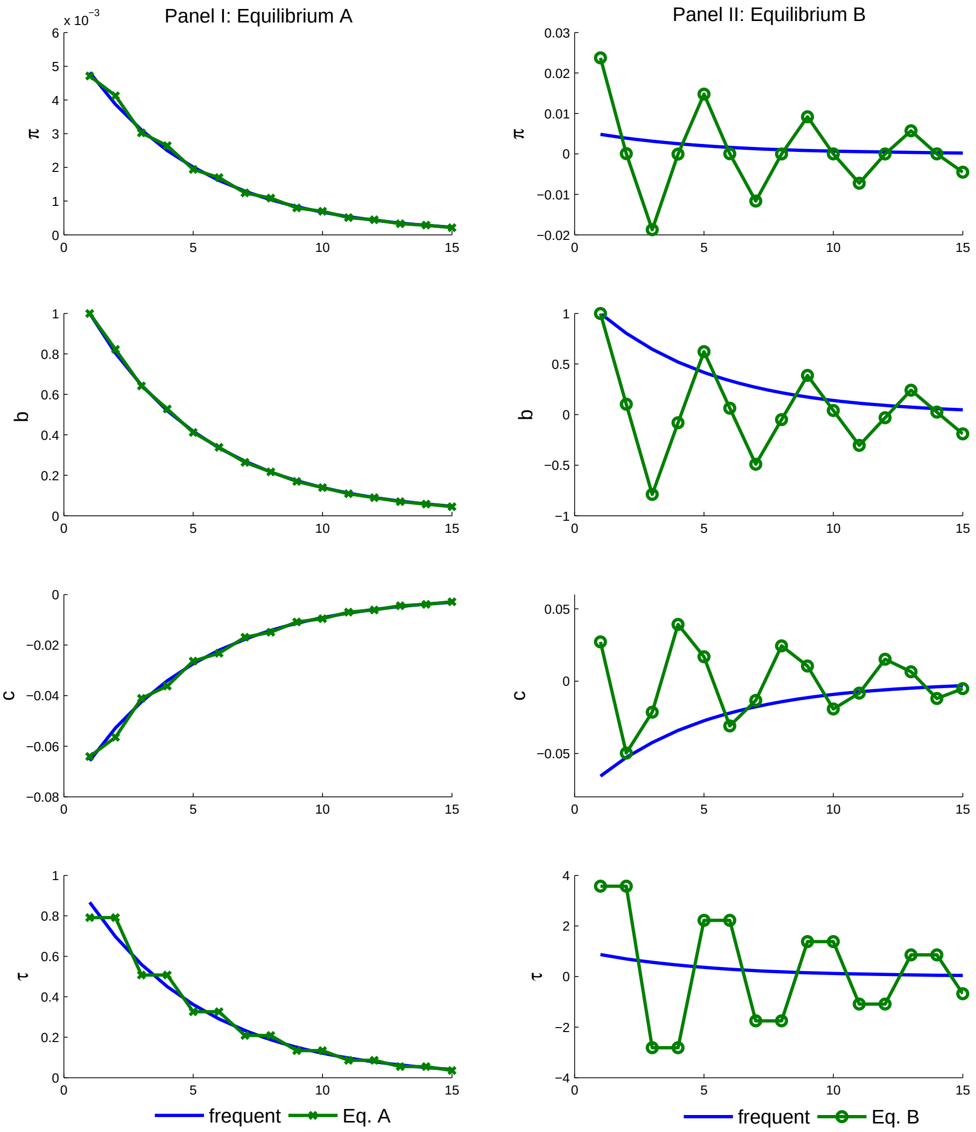

Figure 3.5: Impulse responses and counterfactual simulations. Fiscal policy optimizes every other period.

monetary policy maker reoptimizes every quarter, but the fiscal policy maker reoptimizes only at the beginning of every four quarters.

In this model we are unable to present the system of first order conditions as a system of two polynomial equations and use the graphical method of finding solutions. We have to resort to numerical methods and the stability properties to find discretionary equilibria of interest.

Proposition 15 *If the monetary policy maker takes decisions quarterly and the fiscal policy maker optimizes annually then for the base line calibration of the model there are two discretionary equilibria which are IE-stable under the JF-learning.*

Proof. The proof relies on the use of numerical methods. As discussed in Section 3.5.1 we search for equilibria which are IE-stable under the JF-learning by replicating the steps of the revision process of all agents who are learning. For every, not necessarily optimal τ_b^0 we find all $\lim_{k\to\infty} \mathbb{T}_{JF}^k(\bar{x}) = x^*$ where $\bar{x}$ is an initial guess of $x = (\pi, c, S)$, as explained in Section 3.4.2.[13] For every x^* we find $\tau_b^{0*} = \lim_{k\to\infty} \mathbb{T}_L^k(\bar{\tau}_b^0)$ for the initial guess $\bar{\tau}_b^0$. We, therefore obtain the mapping $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0)$ which is plotted in Panel I in Figure 3.6 with the solid line. The curve $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0)$ intersects the 45^o line in two points, labelled A and B. By construction, these points are the points of discretionary equilibria which are IE-stable under the JF-learning. With further increase in τ_b^0 no further equilibria were discovered.[14] ■

The dynamic complementarity between the optimal actions of the two policy makers is responsible for the multiplicity of equilibria. An optimal response of monetary policy reinforces the action of fiscal policy: higher levels of taxation have a cost-push effect and so the optimal monetary response is to reduce demand and the tax base. Smaller tax base requires a higher tax rate to ensure the desired speed of debt stabilization. Both policy makers can coordinate on either slow or fast correction of the level of debt towards

[13] The limit is computed numerically with tolerance $|x_{k+1} - x_k| < 10^{-13}$.

[14] In the area of discontinuity in Panel I the time-consistent representation of the fixed tax rate policy requires infinitely large feedback on debt in the last quarter τ_b^3. No discretionary equilibria exist there.

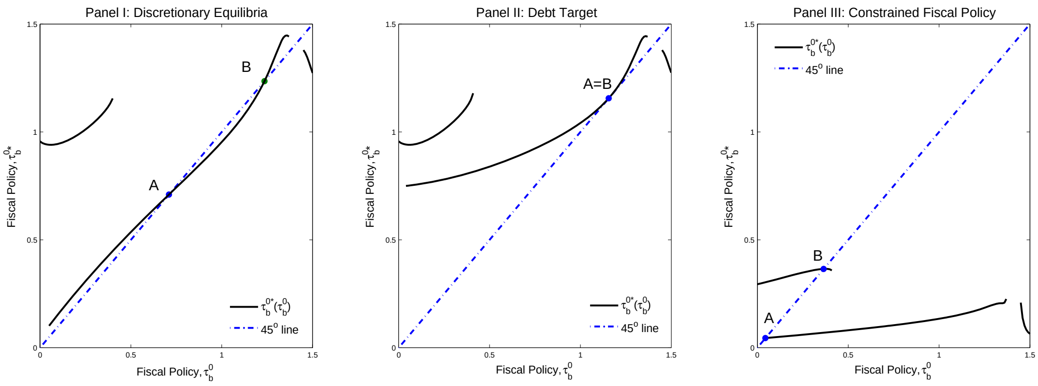

Figure 3.6: Discretionary Equilibria

the target. Figure 3.7 illustrates these interactions. Consider equilibrium A. Suppose the initial debt is higher than in the steady state and the tax rate is kept high for four periods. This implies a steep reduction in debt. The effect of future high tax rates and high marginal cost creates expectations of high future inflation. If monetary policy does not offset the effect of fiscal 'infrequency' then debt and consumption adjust in a linear way between the periods of fiscal optimization. The effect of lower consumption is smaller than the effect of higher tax rate and inflation stays above the frequent optimization solution benchmark. The tax rate remains high for the four periods and, by the end of the fourth period, it is much higher than it would be if optimization happened every period. The tax correction in the fifth period brings inflation down. Figure 3.7 demonstrates that the ability of the optimal monetary policy to reduce inflation volatility is limited. Indeed, it is clear from the picture that consumption should go down first and then up in the first four periods if the inflation humps in first two periods to be eliminated. Such stabilization results in sub-optimally high volatility of consumption. In what follows we call equilibrium A 'slow approximating'. This equilibrium is IE-stable under all types of learning we consider in this Chapter, $\lim_{n\to\infty} \mathbb{T}_j^n\left(\tau_b^0\right) = \tau_b^{0A}$, $j \in \{PS, JF, J\}$

Under discretionary equilibrium B the tax rate is initially kept above the frequent-optimization benchmark. This generates a much steeper reduction of the level of debt

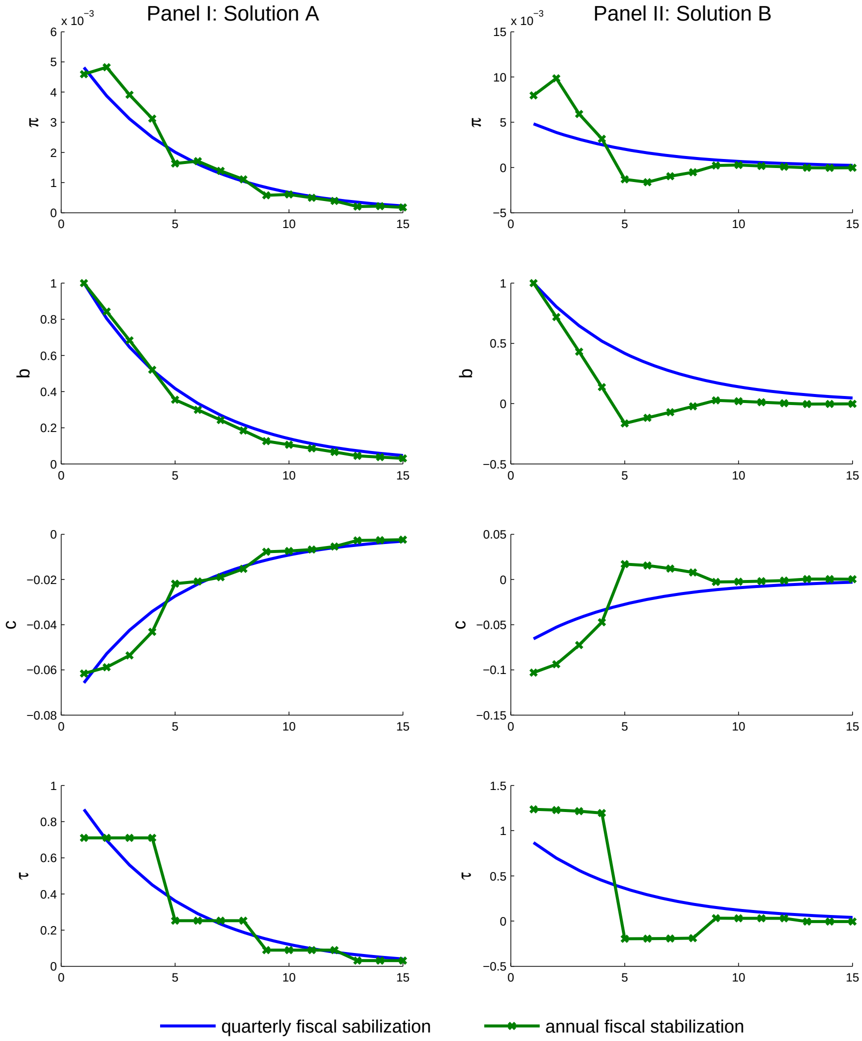

Figure 3.7: Impulse responses and counterfactual simulations. Fiscal policy optimizes once a year.

than is observed in the slow approximating equilibrium A. The higher tax rate results in a higher level of inflation and lower consumption. We call equilibrium B 'fast approximating'. This equilibrium is not IE-stable under the J-learning but is stable under the private sector learning and the JF-learning. The IE-stability under the JF-learning allowed us to locate it, see Figure 3.6.

To summarize, in case of the annual fiscal cycle there are at least two discretionary policy equilibria. Only two equilibria are IE-stable under the JF-learning. Their existence is a result of the strong dynamic complementarity between optimal actions under the two policies, monetary and fiscal, given that fiscal policy uses distortionary taxes as the policy instrument. However, their existence is likely to be non-robust to the model specification; this is suggested by Panel I in Figure 3.6. Indeed, equilibria A and B are located on the same curve $\tau_b^{0*}\left(\tau_b^0\right)$, which may not intersect the 45^o line at all. In the next section we discuss why this may occur.

Existence of Approximating Policy Equilibrium

We argue that the existence of the approximating equilibrium is not robust to changes in model calibration and to policy scenarios. To communicate the argument we present several examples.

Fast Stabilization of Debt Consider a policy scenario in which the fiscal policy maker is assigned an additional target to stabilize debt faster than socially optimal, such as the new European Fiscal Treaty. Suppose the monetary policy maker is benevolent, but the fiscal authority's objective function is modified to include a debt target

$$L^F = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + \lambda c_t^2 + \mu_b b_t^2\right).$$

If the fiscal policy maker is benevolent, as studied above, then $\mu_b = 0$.

The strength of the dynamic complementarity depends on the calibration of μ. Both approximating equilibria do not exist if $\mu_b > 0$ and is sufficiently high.[15] In order to

[15] For the benchmark calibration of model parameters this threshold value of $\mu = 0.0003$.

understand this result, consider the familiar scenario of high initial debt. Suppose that both policy makers are benevolent and we are in the slow approximating equilibrium A, see the left panel in Figure 3.7. If we impose a debt target for fiscal policy, i.e. start increasing $\mu_b \geq 0$, the fiscal policy maker will try to speed up the debt stabilization with an increase in the tax rate relative to the benchmark case of $\mu_b = 0$. The cost-push effect will increase inflation more and so the monetary policy maker will choose to engineer a bigger fall in consumption. This, of course, will slow down the speed of debt stabilization and require an even higher tax rate. The process converges: each additional reduction in demand requires a smaller increase in the tax rate. Equilibrium exists, and in this equilibrium the debt is reduced faster than is plotted in the left panel in Figure 3.7.

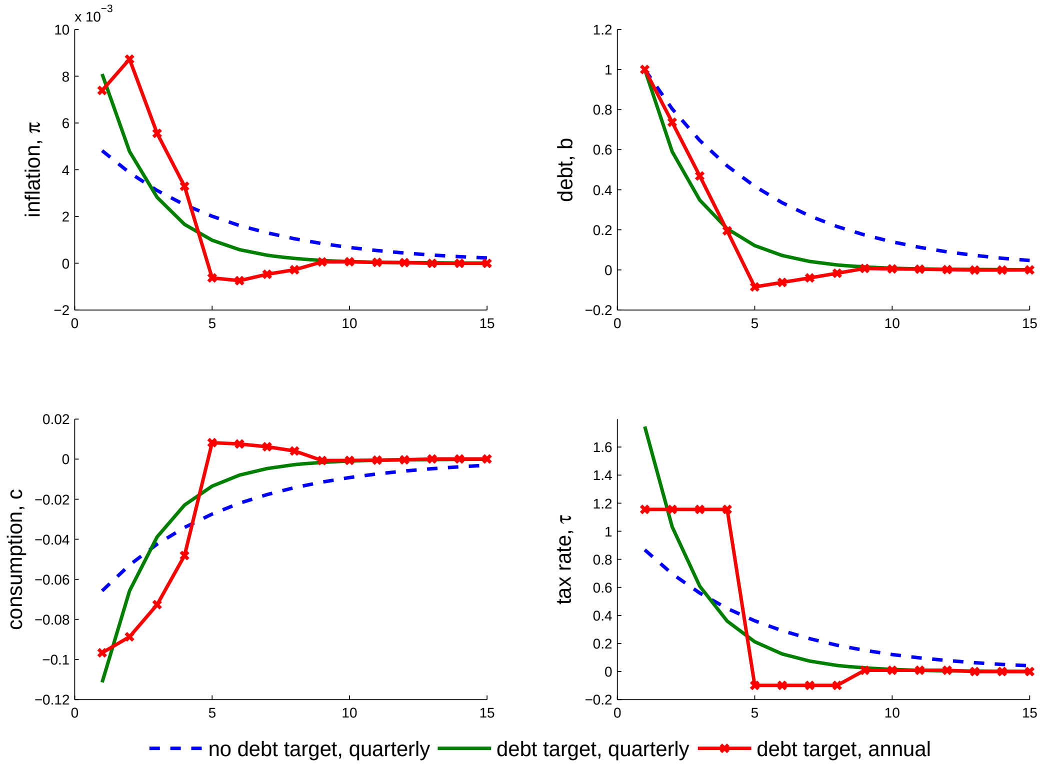

Figure 3.8: The Limiting Case of the Unique Approximating Equilibrium in the Annual Fiscal Optimization Model

This contrasts with the effect of introducing the debt target in equilibrium B. Suppose debt is higher than in the steady state by one unit, policy makers are benevolent and we are in the fast approximating equilibrium B, see the right panel in Figure 3.7. Note that in this equilibrium debt is stabilized with an observed overshooting after the first year. If we impose a debt target, i.e. start increasing $\mu_b \geq 0$, then raising the tax rate in the first several periods becomes counterproductive. If the tax rate is raised higher than in the $\mu_b = 0$ case, this results in even bigger overshooting of debt, which works towards *destabilizing* the debt. In order to ensure faster debt convergence the tax rate has to rise less and monetary policy has to engineer a smaller fall in consumption. The fiscal policy maker anticipates that demand will not respond much and will lower the tax rate. This process converges: each additional reduction in the size of demand cut requires a smaller reduction in the size of the tax rate increase.

To summarize, with the increasing weight on the debt target equilibria A and B move towards each other so that the dynamics of the economy in equilibria A and B become similar. The dynamic of the economy in response to the higher debt level in the limiting case $A = B$ is plotted in Figure 3.8. For comparison we also plot the result of frequent stabilization without the debt target.

If the debt target becomes even stronger, then no approximating equilibrium exists. Any proposed increase of the tax rate τ_b^0 results in a strong optimal IE-stable under the JF-learning response of the other agents within the fiscal cycle. To counteract the perceived response requires the bigger initial rise $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0) > \tau_b^0$. We can summarize this outcome in the form of the following proposition.

Proposition 16 *For the base line calibration of the model and with sufficiently high weight on the debt target of fiscal authorities the approximating discretionary equilibrium does not exist.*

Proof. The proof is numerical. Our iterative approach finds two equilibria under the base line calibration with $\mu_b = 0$. With an increase in parameter μ_b the two equilibria eventually coincide and disappear as $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0)$ does not intersect the 45^o degree line.

■

This result does not imply that there is no discretionary equilibrium if equilibria A and B do not exist. Yet another equilibrium might exist. In particular, Panel II in Figure 3.6 and its similarity with Panel II in Figure 3.4 suggests that 'zig-zag' equilibrium might exist.[16] Strategic complementarity between the actions of subsequent monetary policy makers may lead to a zig-zag adjustment of demand within the fiscal cycle. These adjustments might be 'fine tuned' such that the annual average magnitude of them is not large enough to provoke the destabilizing increase in the tax rate. However, such equilibrium is not IE-stable under the JF- and J-learning. Moreover, it is difficult to call such equilibrium 'approximating'.

The existing algorithms for finding solutions are not suited to obtaining *all* possible equilibria in a complex case with many states. We could only do this for the quarterly and the biannual models. However, Figure 3.6 makes it clear that the approximating equilibrium will disappear if the debt target is sufficiently strong, rather than we suddenly became unable to locate it numerically. We can be reasonably sure that if an additional equilibrium exists in the annual optimization model, this equilibrium will not be IE-stable under the JF- and J-learning, it will also generate a very low level of social welfare because of the high volatility in macroeconomic variables.

Calibration of the Model The existence of the approximating equilibrium is also sensitive to the calibration of the model. Calibrations of the model which result in stronger reactions of monetary and fiscal policies are likely to lead to non-existence of the approximating equilibrium.

Both approximating equilibria do not exist if the Frisch elasticity labour supply ψ is reduced to 2.3, or if the baseline value for the elasticity of intertemporal substitution σ

[16] We cannot use the 'continuity' argument as the reaction function of an agent is described by a rational function, not by a polynomial function.

is only slightly increased to 0.35, or if the elasticity of substitution between goods ϵ is reduced to 5.7 which corresponds to an increase of the mark up to 21%.

All these threshold values of parameters are completely plausible and are within the range of estimates which are often obtained in empirical studies of aggregated data, as we argue in Section 2.2.

High Level of Debt The main case discussed in Section 2.2 assumes zero level of the steady state debt. This assumption reflects the relatively small proportion of the short-term debt in a typical developed economy, and allows us to present many results in an analytical way. However, high and persistent level of debt is not uncommon.

We can rewrite our model in the more general form, using interest rate as the monetary policy instrument, and retaining the possibility to study implications of higher level of steady state debt. We can demonstrate that the size of the steady state level of debt affects the strength of the dynamic complementarity. The effect of the nominal interest rate and inflation on the process of debt accumulation rises linearly with the steady state level of debt. In response to high inflation the optimal monetary policy will raise interest rate; both the high (real) interest rate and the consequently low tax base increase the rate of debt accumulation, and this effect is stronger with higher steady state level of debt.

The numerical analysis of this scenario produces diagrams that are remarkably similar to the case of the debt target. If the steady state debt to output ratio reaches approximately 0.25 – which corresponds to short-term debt to annual output ratio of 0.07 – then discretionary equilibria A and B coincide. With higher debt to output ratio the approximating equilibrium does not exist.

Frequency of Fiscal Optimization The strength of the dynamic complementarity depends on the frequency of fiscal optimization. The longer the period between the reoptimizations the longer the tax rate remains fixed, and the stronger action of monetary policy is required in order to offset the adverse effect on inflation when the tax is adjusted. The approximating discretionary equilibrium may not exist.

Constraining the Fiscal Policy maker In order to preserve the approximating equilibrium the strength of the complementarity should be reduced. One way to achieve this is to constrain the fiscal policy maker by imposing penalty μ_τ on the excessive movement of fiscal instrument

$$L^F = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + \lambda c_t^2 + \mu_\tau \tau_t^2\right).$$

If μ_τ is not too large then this policy results in the unique IE-stable under the JF-learning equilibrium. However, if μ_τ is sufficiently large then the complementarity between the monetary policy maker and the private sector's actions leads to multiplicity, very similar to the result in Dennis and Kirsanova (2012). Panel III in Figure 3.6 demonstrates the outcome when $\mu_\tau = 0.1$. There are two stationary equilibria, both of which are IE-stable under all types of learning which we consider. If fiscal policy does not react to debt sufficiently strongly – in this case because it is constrained – then the agents can either coordinate on equilibrium A in which the private sector does not expect the monetary policy maker reacts to debt but stabilized inflation and the monetary policy maker validates these expectations, or they can coordinate on equilibrium B in which the private sector expects the monetary policy maker accommodates inflationary shocks but ensures fast stabilization of domestic debt and the monetary policy maker validates these expectations.

3.6 Conclusion

This Chapter studies the implications of infrequent discretionary fiscal optimization for the stabilization of the economy, assuming dynamic interactions of monetary and fiscal policy makers where both policy makers are benevolent and the fiscal policy maker uses distortionary taxes to stabilize the economy. We demonstrate the presence of dynamic complementarity between the optimal monetary and fiscal policies. A higher tax rate, which is required to stabilize higher debt, will have a cost-push effect. The optimal monetary policy response to this will generate a reduction in demand and in the tax base, and faster debt accumulation. Anticipating this, the fiscal authorities will wish to raise tax rates further.

If both policies operate with the same frequency, this reinforcement mechanism is weak and does not lead to adverse effects. However, with longer fiscal cycle, the effect of this mechanism is greatly amplified. If the length of fiscal cycle is not too long then expectation traps arise. With more periods between fiscal reoptimizations and with stronger reinforcement mechanism an (IE-stable) discretionary equilibrium may not exist.

We demonstrate the latter outcome for many practical scenarios. We argue, therefore, that the fiscal policy maker who reoptimizes only infrequently should be constrained. A moderate penalty on variability of the fiscal instrument can be sufficient to reduce the degree of dynamic complementarity between the actions of the two policy makers.

Chapter 4

Infrequent fiscal policy with Blanchard Yaari consumers

In this Chapter, by employing Blanchard-Yaari type of overlapping generations instead of infinite living households, I extend the framework of optimal policy design with infrequent fiscal policy, which was developed in Chapter 3, to investigate the interactions among central bank, government and aggregated private sector, as well as the dynamic macroeconomic effects of monetary and fiscal policy.

To focus on the impact of infrequent fiscal policy and overlapping generation with the tax as fiscal instrument and interest rate as monetary instrument, we continue to assume that government spending G is exogenous and constant.

4.1 Introduction

In Chapter Two, we analyzed the macroeconomic impacts of infrequently optimized fiscal policy. We demonstrated that the longer fiscal period strengthened the conflicts between monetary and fiscal policy, which makes the economy more sensitive to the debt increase, the existence of learnable time-consistent equilibrium cannot be guaranteed.

These results were found under the assumption of infinitely lived households. Yet this limiting case eliminates the direct effect of debt on the households' consumption pattern. It is of interest to investigate how the consequences of interactions between monetary and infrequent fiscal policy, with presence of higher-than-steady-state debt, discussed in the previous Chapter will change if the households have mortality instead. In this case government debt affects aggregate consumption dynamics. Due to this property, theoretical studies of fiscal policy and its interaction with monetary policy under the overlapping generations' assumption have been increasing. This Chapter builds the new Keynesian framework with infrequent fiscal policy into a Perpetual Youth model to assess how inter-generational redistributions of wealth and the frequency of policy optimization impact the economic consequences of accumulated government debt. The representative households are assigned an identical constant mortality rate, regardless of age.

It is known that a higher steady state debt-GDP ratio reinforces the dynamic complementarity between fiscal and monetary policies, makes the economy more difficult to be stabilized. A positive mortality rate introduces direct impacts of changes in debt, inflation and propensity of consumption on aggregate consumption. We modified the set-up of overlapping generations which was modelled through the Blanchard-Yaari (Yaari (1965); Blanchard (1985)) version. Two opposite effects on the conflicts between policy makers emerge: positive effect of debt increase on aggregate consumption strengthening the conflicts between fiscal and monetary policy, and negative impact of inflation on aggregation consumption weakening dynamic complementarities of these two policy makers. Whether the transmission paths of the economic variables back to the steady states will be more volatile or not depends on which effect dominates. Using standard calibration, our impulse responses results show that the conflicting effect dominates the stabilizer effects.

The remainder of this Chapter proceeds as follows. Section 2 develops the economic model. Section 3 establishes the existence of steady states and summarizes the dynamic linearized system around these steady states. Section 4 discusses the analytical intuition of the model. Section 5 listed the calibration of the structural parameters. Section 6 delivers the numerical results and discussion of the main results in this Chapter. Technical parts and proofs are delegated to the Appendix.

4.2 The Model

Under a Blanchard-Yaari framework, in which individuals have an exogenous, identical, constant death probability every period and households enjoy a perpetual youth, we modified the now-mainstream macro policy model discussed in Woodford (2003a) to take account of the fiscal policy and monetary policy effects. It is a closed economy model with two policy makers, the fiscal and monetary authorities. Here fiscal policy is allowed to support monetary policy in stabilization of the economy around the steady state, and re-optimize with a less frequent rate. The labor supply decision is endogenous. Consumption goods are produced by monopolistic competitive firms who set price in a Calvo sticky price style. Households are benefit from an exogenous government spending and receive lump-sum transfers from the government who finance them by nominal debt and distortionary tax on households income.

4.2.1 Households

Individuals

The economy is inhabited by a large number of households who specialize in the production of a differentiated good (indexed by z) which cost them $h(z)$ of effort. They consume a basket of goods and derive utility from per capita government consumption. Each household faces a constant death probability p. β is time preference rate, which is assumed the same value as the intertemporal discount factor applied by infinite living households. In order to focus on the impact of distortionary tax, government spending and lump-sum transfers are assumed to be constant and excluded from households utility function for simplicity. When $p = 0$ the model reduces to the infinitely lived representative households setup in the last Chapter.

At time t, the representative household born at time s chooses consumption of goods C_t^s and the efforts, which is measured by the working hours $N_t^s(z)$, put to produce the goods to maximize their expected lifetime utility:

$$\max \mathcal{E}_t \sum_{v=t}^{\infty} \left[\frac{\beta}{1+p}\right]^{v-t} [u(C_v^s, \xi_v) - v(N_v^s(z), \xi_v)] \tag{4.1}$$

subject to the intertemporal budget constraint:

$$\sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v} P_v C_v^s) \leq \mathcal{A}_t^s + \mathcal{H}_t^s \tag{4.2}$$

where $u(.)$ is the utility the household derived from consumption, and $v(.)$ is the disutility caused by working. $Q_{t,v}$ is the stochastic discount factor by which the value of nominal income in time T is determined in time t. For simplicity, we assume the particular utility functions as: .

$$u(C_v^s, \xi_v) = \frac{(C_v^s \xi_v)^{1-1/\sigma}}{1-1/\sigma} \tag{4.3}$$

$$v(N_v^s(z), \xi_v) = \varkappa \frac{(N_v^s \xi_v)^{1+1/\psi}}{1+1/\psi} \tag{4.4}$$

Here ξ_v is stochastic shocks. σ is the elasticity of intertemporal substitution, i.e. the inverse of the household's relative risk aversion. σ influences the decision on delaying consumption, it can be seen as a measure of the responsiveness of the growth rate of the future consumption to the compensated real interest rate; ψ is the Frisch elasticity of labor supply, which indicates the sensitivity of labor supply to real wage. Nominal human capital $\mathcal{H}^s$ is the nominal present value of the expected total future after-tax labor income. $\mathcal{H}_t^s = \sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v}^s \int_0^1 (1-\tau_v)\,(W_v(z) N_v^s(z) + D_v(z))\, dz + T)$, P is the aggregate price level, $P_t C_t = \int_0^1 p(z) c(z) dz$, $p(z)$ is the price of a differentiated good z while $c(z)$ is the consumption level of good z. $\mathcal{A}_t^s$ is the household's nominal financial assets, we assume no physical capital in this model, so $\mathcal{A}_t = \mathcal{B}_t$ the government debt. τ_v is the proportional income tax rate[1]. $W_v(z)$ is the nominal wage rate the household

[1] it would make no difference if assuming the share of total output that the government collected is from revenue tax instead of labor income tax, as income tax will be transfered to firm's total revenue via wage level required. The final impact of income tax and revenue tax on the price setting decision would be the same.

takes, $D_v(z)$ is the profit the household receives from the firms invested, T is a constant lump-sum tax or subsidy from the government. $Q^s_{t,v}$ is a stochastic discount factor which determines the compensation the household in time t requires if postpone consumption and carry the state-contingent amount $\mathcal{A}_t$ of wealth from time t to time v. *As was pointed out by Yaari (1965),* expected future utility is discounted both because of pure time preference and life-time uncertainty.

First-order conditions for household's utility maximisation show that:

$$W_v(z) = \frac{\varkappa \left(C^s_v \xi_v\right)^{1/\sigma} \left(N_v \xi_v\right)^{1/\psi} P_v}{(1-\tau_v)} \tag{4.5}$$

And that for the generation born at time s, the intertemporal rule for consumption :

$$C^s_v = \left[\frac{1+p}{\beta}\frac{P_{v+1}}{P_v} Q^s_{v,v+1}\right]^{\sigma} C^s_{v+1} \frac{\xi_{v+1}}{\xi_v} \tag{4.6}$$

Where $Q^s_{v,v+1}$[2] is the nominal individual stochastic discount factor of time $v+1$ at time v

$$Q^s_{v,v+1} = \frac{Q^s_{v+1}}{Q^s_v} = \frac{\beta}{1+p} \frac{u_C(C^s_{v+1}, \xi_{v+1})}{u_C(C^s_v, \xi_v)} \frac{P_v}{P_{v+1}} \tag{4.7}$$

Therefore

$$C^s_v = C^s_t \prod_{k=0}^{v-t-1} \left[\frac{1+p}{\beta} \Pi_{t+k+1} Q^s_{t+k,t+k+1}\right]^{-\sigma} \frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$P_v = P_t \prod_{k=0}^{v-t-1} (1+\Pi_{t+k+1})$$

where $\Pi_{t+k+1} = \frac{P_{t+k+1}}{P_{t+k}}$ is the ratio of price levels at two successive times.

[2] There are other ways to define this nominal stochastic discount factor. For example, Farmer, Nourry and Venditti (2011) derived an expression for the pricing kernel in Blanchard-Yaari economy. They set Q_t as the price of the security that pays one unit of the consumption commodity to price the cash flows. Their results allow for aggregate shocks in an internally consistent way. However the main part of this Chapter focuses on deterministic case, for simplicity of the model, I assume the aggregated $Q_{t.t+1}$ is the same as the individual stochastic discount factor.

An *individual's* consumption and wealth for a generation born at time s :

$$(\mathcal{A}_t^s + \mathcal{H}_t^s) = P_t C_t^s + \sum_{v=t+1}^{\infty} Q_{t,v}^s P_v C_v^s = P_t C_t^s \Phi_t \tag{4.8}$$

where

$$\begin{aligned}\Phi_t &= 1 + \sum_{v=1}^{\infty} \left(\frac{\beta}{1+p}\right)^{v\sigma} \prod_{k=0}^{v-1} \left(\frac{P_{t+k+1}}{P_{t+k}} Q_{t+k,t+k+1}^s\right)^{1-\sigma} \frac{\xi_{t+k}}{\xi_{t+k+1}} \\ &= 1 + \left(\frac{\beta}{1+p}\right)^{\sigma} \left(\frac{P_{t+1}}{P_t} Q_{t,t+1}^s\right)^{1-\sigma} \frac{\xi_t}{\xi_{t+1}} \Phi_{t+1}\end{aligned} \tag{4.9}$$

We define propensity to consumption as the proportion of total resources, composed of human capital and financial assets, the household spend on consumption. Φ_t represents the reverse of propensity to consumption. Positive probability of death, p, reduces the present value of future income but increases current generations' propensity to consume $\frac{1}{\Phi_t}$.

Aggregation

Applying the same method as Blandchard, we assume that at any time the fraction of total population of a new-born generation s is p, which implied that the size of generation s at time t is $\frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s}$ We aggregate all relationships across all generations by summing their weighted relative sizes. The size of total population at time t is $\frac{p}{(1+p)} \sum_{s=-\infty}^{t} \left(\frac{1}{1+p}\right)^{t-s} = 1$. Therefore.

$$\begin{aligned}C_t^a &= \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} C_t^s, \\ \mathcal{A}_t^a &= \sum_{s=-\infty}^{t} \frac{p}{(1+p)} \left(\frac{1}{1+p}\right)^{t-s} \mathcal{A}_t^s = \frac{1}{Q_{t,t+1}^s} \frac{1}{1+p} \left(\mathcal{A}_{t-1}^a + (1-\tau) P_t Y_t - P_t C_t^a\right)\end{aligned}$$

and we define aggregate nominal human capital as:

$$\mathcal{H}_t^a = \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^s \left((1-\tau_v) Y_v P_v + T_v^a\right) = \frac{1}{Q_{t,t+1}^s} (\mathcal{H}_{t-1}^a - (1-\tau) Y_t P_t) \tag{4.10}$$

where

$$Y_v P_v = \sum_{s=-\infty}^{t} \frac{p}{(1+p)} (\frac{1}{1+p})^{t-s} \int_0^1 \left(W_v(z) N_v^s(z) + D_v^s(z)\right) dz$$

Aggregating relationship (4.8) yields:

$$P_t C_t^a = \frac{1}{\Phi_t} \left(\mathcal{A}_t^a + \mathcal{H}_t^a \right).$$

To derive a dynamic Euler equation for aggregate consumption, we move this equation one period forward and substitute $\mathcal{H}_{t+1}^a$ out using (4.10) then taking expectations, we obtain

$$C_t^a = \mathcal{E}_t \left(\left(\frac{1+p}{\beta} \frac{P_{t+1}}{P_t} Q_{t,t+1} \right)^{\sigma} \left(C_{t+1}^a + \frac{p\mathcal{A}_{t+1}^a}{P_{t+1}\Phi_{t+1}} \right) \frac{\xi_{t+1}}{\xi_t} \right) \tag{4.11}$$

with

$$\mathcal{A}_{t+1}^a = \frac{1}{Q_{t,t+1}^s} \frac{1}{1+p} \left(\mathcal{A}_t^a + (1-\tau_t) P_t Y_t - P_t C_t^a \right)$$

so when p is a positive number, higher real government debt pushes the financing of government expenditures onto future generation, which has a positive wealth effect on consumption of living generations. This relationship between real government debt and aggregate consumption, which only exist with finite-living households, is critical for understanding the aggregate consequences of debt accumulation with the infrequent fiscal policy.

4.2.2 Firms

The market is monopolistically competitive, the representative firm in the production sector employs labour and produces goods z, at time t, it discounts future profit with Q_t^f , Nominal wages W_t are equalized across all firms. A firm chooses employment and prices to maximize profit:

$$\max_{\{N_t(i), p_s^*(i)\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^f \left(y_v(z) \, p_v(z) - W_v N_v(z) \right).$$

subject to the production function:

$$y_t(z) = Z_t N_t(z),$$

Each producer of good z understands that sales depend on the demand it faces $y_t^a(z)$, which is a function of prices, elasticity of demand ϵ and aggregate output Y_t^a. Intra-temporal consumption optimization implies

$$y_t^a(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} Y_t^a$$

and each period firms are able to reoptimize their prices with probability $1-\gamma$, so that their prices remain fixed as last period with probability γ:

$$\begin{aligned} p_t(z) &= p_t^*(z) \\ p_{t+1}(z) &= \begin{cases} p_t^*(z), & \text{with prob } \gamma \\ p_{t+1}(z), & \text{with prob1} - \gamma \end{cases} \end{aligned}$$

Profit maximisation problem can be split into to separate problems: decision on hours of labour required to minimize cost intra-temporally and choice on prices to maximize present value of expect future profit.

Employment

Firm z choose how much labor it will employ to minimize nominal cost:

$$\min_{N_t(z)} (W_t N_t(z))$$

subject to the production constraint

$$y_t(z) = Z_t N_t(z).$$

we denote $\Delta_t = \int \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} di$ as price dispersion, which is the variation in prices across suppliers of the same good.

Aggregation yields

$$N_t = \int N_t(z)\, di = \int \frac{y_t(z)}{Z_t} di = \int \frac{Y_t^a}{Z_t}\left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} di = \frac{Y_t^a}{Z_t}\Delta_t$$

$$N_t(z) = \frac{Y_t^a}{Z_t}\left(\frac{p_t(z)}{P_t}\right)^{-\epsilon_t}$$

Therefore the real marginal cost is

$$mc_t = \frac{W_t}{P_t Z_t} = \frac{w_t}{Z_t} = \frac{\varkappa \left(C_t^s \xi_t\right)^{1/\sigma} \left(N_t \xi_t\right)^{1/\psi}}{Z_t(1-\tau_t)}$$

Price setting

Firm z chooses a price at time t to maximize the expected discounted present value of profit, with wages independent on index (z), as labour is assumed to be perfectly mobile and so all wages are equalized across all firms.

$$\max_{\{p_s^*(i)\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^f \left(y_v(z)\, p_v(z) - W_v N_v(z)\right)$$

subject to:

$$\begin{aligned} y_t(z) &= Y_t^a \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} \\ p_t(z) &= p_t^*(z) \\ p_{t+1}(z) &= \begin{cases} p_t^*(z), & \text{with prob } \gamma \\ p_{t+1}(z), & \text{with prob } 1-\gamma \end{cases} \end{aligned}$$

Assume firms expect to live forever:

$$Q_{t,v}^f = \beta^{v-t} \left(\frac{C_t^a}{C_v^a} \frac{\xi_t}{\xi_v}\right)^{\frac{1}{\sigma}} \frac{P_t}{P_v}$$

In a equilibrium, all producers make identical decisions, first order condition with respect to $p_t^*(z)$ yields:

$$\left(\frac{p_t^*(z)}{P_t}\right)^{\frac{\psi+\epsilon}{\psi}} = \frac{\mathcal{E}_t \sum_{v=t}^{\infty} (\gamma\beta)^{v-t} \frac{\epsilon}{\epsilon-1} \varkappa \frac{(1+1/\psi)}{(1-\tau_v)} \left(\frac{Y_v^a}{Z_v}\right)^{1+1/\psi} \left(\frac{P_v}{P_t}\right)^{\epsilon(1+1/\psi)}}{\mathcal{E}_t \sum_{v=t}^{\infty} (\gamma\beta)^{v-t} C_v^{-\frac{1}{\sigma}} Y_v^a \left(\frac{P_v}{P_t}\right)^{\epsilon-1}}$$

$$\frac{\left(1-\gamma \Pi_t^{\epsilon-1}\right)}{(1-\gamma)} = \left(\frac{K_t}{F_t}\right)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)}$$

where is $\Pi_t = \frac{P_t}{P_t-1}$ is the ratio of price level at time t and time (t-1) and

$$\begin{aligned}
K_t &= k_t + \gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon(1+1/\psi)}K_{t+1} \\
F_t &= f_t + \gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon-1}F_{t+1} \\
k_s &= \varkappa\mu\frac{(1+1/\psi)}{(1-\tau_s)}\left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi} \\
f_s &= (C_s^s\xi_s)^{-\frac{1}{\sigma}}Y_s^a \\
\Delta_t &= (1-\gamma)\left(\frac{1-\gamma\Pi_t^{\epsilon-1}}{1-\gamma}\right)^{\frac{\epsilon}{\epsilon-1}} + \gamma\Pi_t^{\epsilon}\Delta_{t-1}
\end{aligned}$$

4.2.3 Government Debt

The government finances its deficit by short-term nominal bonds, The evolution of the nominal debt $\mathcal{B}_t$ accumulation is:

$$\mathcal{B}_{t+1} = (1+i_t)(\mathcal{B}_t + G_t - \tau_t Y_t P_t) \tag{4.12}$$

in which G_t is assumed to be constant in real term so it is not used to balance the budget or stabilize the economy; τ_t is set to ensure that any increases in the market value of debt are met with the expectation that future taxes will raise by enough to match the higher debt and retire it back to its stationary level; and i_t is responsible to inflation stabilization. b_t is the only state variable in our model.

4.2.4 Market clearing condition

Equilibrium requires the national total output equals to household consumption plus government spending each period:

$$Y_t = C_t + G_t$$

as we assume no physical investment. The equilibrium is characterized by a sequence of prices, $\{\Pi_t, W_t,\}_{t=0}^{\infty}$ and quantities, $\left\{C_t, N_t, \mathcal{B}_t, Y_t, Q_{t,t+1}^f,\right\}_{t=0}^{\infty}$

4.2.5 Steady States and Linearized System

We assume at steady state, price is constant, i.e. $\Pi = 1$, $\chi \neq 0$. The steady state value for the variables are:

$$\begin{aligned}
\Phi &= \frac{1}{\left(1 - \frac{\beta^{\sigma}(1+i)^{\sigma-1}}{(1+p)}\right)} \\
C &= \left(\left(\frac{1}{\beta}\frac{1}{(1+i)}\right)^{\sigma}\left(C + pB\frac{1}{\Phi}\frac{1}{(1+\pi)}\right)\right) \\
K &= \varkappa\mu\frac{(1+1/\psi)(C+G)^{1/\psi+1}}{(1-\tau)(1-\gamma\beta)} \\
F &= \frac{C^{-\frac{1}{\sigma}}(C+G)}{(1-\gamma\beta)} \\
\Delta &= 1 \\
B &= -\frac{(1+i)(G-\tau(C+G))}{i}
\end{aligned}$$

$$\chi = \frac{B}{Y} = -\frac{(1+i)(1-\theta-\tau)}{i} \tag{4.13}$$

$$i = \frac{p(1-\theta-\tau)\beta^{\sigma} - p(1-\theta-\tau)(1+p)(1+i)^{1-\sigma}}{\beta^{\sigma}\theta(1+p) - (1+i)^{-\sigma}\theta(1+p)} \tag{4.14}$$

where $\frac{C}{Y} = \theta$ and $\frac{G}{Y} = (1-\theta)$ due to $Y = C + G$.

To focus on the impact of the dynamic complementarity which is caused by discretionary tax and interest rate on the economy where the fiscal policy maker can only set the tax rate less frequent than monetary policy can decide on the interest rate, we assume the government spending is constant. Therefore, linearization around $\Pi = 1$ and $\chi \neq 0$ steady states yields the deterministic system:

$$
\begin{aligned}
b_{t+1} &= (1+i)\left(b_t - \chi\hat{\pi}_t - \tau\theta\hat{C}_t - \tau\hat{\tau}_t\right) + \chi\hat{\imath}_t && (4.15)\\
\hat{\pi}_t &= \beta\hat{\pi}_{t+1} + \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}\left(\left(\frac{\theta}{\psi}+\frac{1}{\sigma}\right)\hat{C}_t + \frac{\tau}{(1-\tau)}\hat{\tau}_t\right) && (4.16)\\
\hat{C}_t &= [\beta(1+i)]^{-\sigma}\frac{p}{\theta\Phi}\left(b_{t+1} - \chi\hat{\pi}_{t+1} - \chi\hat{\Phi}_{t+1}\right) + [\beta(1+i)]^{-\sigma}\hat{C}_{t+1} && (4.17)\\
& -\sigma\left(\hat{\imath}_t - \pi_{t+1}\right) && (4.18)\\
\hat{\Phi}_t &= \frac{(\beta(1+i))^{\sigma}}{(1+p)(1+i)}\hat{\Phi}_{t+1} - \frac{(1-\sigma)(\beta(1+i))^{\sigma}}{(1+p)(1+i)}\left(\hat{\imath}_t - \pi_{t+1}\right) && (4.19)
\end{aligned}
$$

When $\chi = 0$ and $p = 0$,this final system reduces to be the same as in the previous Chapter, where an increase in debt leads to higher tax, followed by higher inflation therefore higher interest rate to depress consumption, and unavoidably reduces tax-base and effect of tax on inflation, which induces an even stronger tax rate. When $\chi = 0$ and $p > 0$,the debt will influence household consumption dynamics, however, this extra impact can be offset completely by a corresponding higher interest rate alone. It is not the case with $\chi > 0$,when interest rate changes the debt accumulation directly as well as through consumption. The inter-generational wealth transfer is of most interest when the households have financial assets, hence we loose up our previous assumption of zero debt steady state and investigate the more usual case when steady state level of government debt is positive, i.e. χ is no longer zero.

4.2.6 Policy Makers

There are two policy makers in our economy: a monetary authority setting the nominal interest rate i_t and a fiscal authority determining the rate of proportional tax on household income τ_t, as we assume government spending and transfers are constant in real terms so they are not used to balance the budget or stabilize the economy. In our framework the monetary policy maker reoptimizes every period while the fiscal policy maker decides once every N periods, $1 \leq N < \infty$. We refer to the period between fiscal reoptimizations as the fiscal cycle. We denote the set of numbers p congruent to a modulo N as $[p]_N$. There are exactly N different sets $[p]_N$. We shall identify these sets with the corresponding residue: $[p]_N = p$, so p denotes the time period after the latest fiscal reoptimization. Both the monetary and fiscal policy makers optimize in period $0 = [0]_N$, and fiscal policy maker makes decision before monetary policy maker, i.e. fiscal leadership. Only the monetary policy maker optimizes in periods $[1]_N$,..,$[N-1]_N$, which are labelled $p = 1, .., N-1$.

As quantitative impact of the perpetual youth set-up depends on the planning horizon of households, the deviation of welfare metric from that in standard Ramsey set-up may be non-trivial. We assume the benevolent policy makers take into account the welfare loss of the future unborn generations as well as living generations. (Calvo and Obstfeld, 1988) defined the social welfare function consisting two parts: the total utility of representative unborn generations and the total utility of currently living households. Both of them are discounted back to the time when the current generations were born, instead of the current period. By treating generations symmetrically, this method can avoid the time inconsistency in preferences, also spit the problem into an intratemporal one across generations at a given point in time, and an intertemporal one over time.

However, in this Chapter the prime interest is the macroeconomic impact of the intergenerational wealth transfer of debt, so I focus on the intertemporal problem. Assuming a weighting scheme to equalize the welfare of aggregated overlapping generations and the infinitely lived households can be implemented, the policy makers ignore the distribution of variables across generation at a given period.

Therefore the benevolent policy makers are assumed to minimize the discounted sum of all future losses of social welfare, which is the same social welfare loss function as in the previous Chapter, and expressed as the sum of deviation from inflation and output targets:

$$\min_{\{i_v,\tau_v\}_{v=t}^{\infty}} L = \frac{1}{2}\mathcal{E}_t \sum_{v=t}^{\infty} \beta^{v-t} \left(\pi_{t+p}^2 + \lambda C_{t+p}^2\right)$$

Where $C_{t+p} = \theta Y_{t+p}$ and λ is chosen as if the household will live forever.

Following the same set-up as on page 67-68 in the last Chapter, The monetary policy maker's problem in period $p = 0, .., N-1$ can be described by the same Bellman equation as Equation (3.11), where the value function depends on the number of periods passed since the last fiscal optimization:

$$S^p b_{t+p}^2 = \min_{i_t} \left(\left(\pi_{t+p}^2 + \lambda c_{t+p}^2 \right) + \beta V b_{t+1}^2 \right) \tag{4.20}$$

While the fiscal policy maker only optimizes in periods $[0]_N$, at the time t when optimization happens, the infrequently adjusted fiscal policy aims to solve the Bellman equation:

$$V b_t^2 = \min_{\tau_t} \left(\sum_{p=0}^{N-1} \beta^p \left(\pi_{t+p}^2 + \lambda c_{t+p}^2 \right) + \beta^N V b_{t+N}^2 \right) \tag{4.21}$$

N is the length of fiscal cycle, at the beginning of which the tax rate is set, then it stays at the constant level until the fiscal period finishes. As leader fiscal policy maker will take into account both the reaction of private sector and the monetary policy to fiscal policy.

As there is only one state variable-the government debt, the policy actions can be expressed as feedback coefficients on the debt:

$$c_{t+p} = c_b^p b_{t+p} + c_\tau^p \tau_{t+p} = \left(c_b^p + c_\tau^p \tau_b^p \right) b_{t+p} \tag{4.22}$$

$$\tau_{t+p} = \tau_t = \tau_b b_t = \tau_b^p b_{t+p} \tag{4.23}$$

Where $p = 1, .., N-1$.

4.3 Analytical Intuition

The critical difference brought in by Blanchard-Yaari type of consumers is the intergenerational wealth transfer, which makes the debt level, inflation and propensity to consume relevant to households' consumption decision.

We discussed the existence of dynamic complementarity between the action of central bank optimally concentrate on inflation stabilization and the action of government focusing on stabilizing debt in the last Chapter: increased tax rate induced by above steady state debt level has cost-push effect on prices, to bring the inflation down, a higher interest rate is needed, but this reduces tax base and weakens the effectiveness of fiscal policy. A positive steady state debt level introduces a direct impact from interest rate change and inflation on debt accumulation; the higher χ, the stronger positive effect of interest rate on debt.

The system 4.15-4.19 shows that higher mortality rate has two opposite effects on this conflict we described.

On the one hand, positive probability of death increases the conflicts of the two authorities which lead to a higher degree of dynamic complementarity. The higher the mortality rate, the more likely that the living generations, who benefit from increased government debt, will die before taxes come due. Due to the expected shift of the tax burden onto future generations, debt as a financial asset owned by households has a positive wealth effects on living generations consumption at time t. Equation (4.17) shows that a higher debt generates a higher consumption for the living generation. This pushed-up aggregate demand will increase the inflation, on top of the cost-push effect of raised tax rate aiming to reduce debt.

In the positive steady state debt case, a positive p makes the task of inflation stabilization more difficult not just by causing higher inflation but also by reducing the effectiveness of the interest rate. Higher interest rate is imposed to lower the inflation, by depressing current consumption through intertemporal substitution effect. This policy needs to enlarge the gap between interest rate and expected future inflation, shown in Equation (4.17). In the meantime this gap also raises the propensity of consumption (see the last term of Equation (4.19)) and consumption (via the term $-\chi\hat{\Phi}_{t+1}$ in Equation (4.17)). The household with financial assets can benefit from higher interest rate and have higher propensity to consume in current period. This cancels out some of the effect from the interest rate to consumption. Therefore, in order to achieve the same reduction in the consumption (via the last term $-\sigma\left(\hat{\imath}_t - \pi_{t+1}\right)$ in Equation (4.17))as in the infinite agents' case, Central bank has to set an even higher interest rate.

With $\chi \neq 0$, this higher interest rate has stronger effect on debt which leads to stronger reaction of the fiscal policy, tax raises by more. Followed by this larger cost-push, the task of inflation stabilization becomes heavier, interest rate will rise higher. The dynamic complementarity between monetary policy and fiscal policy is strengthened. This magnifies the volatility introduced by an infrequently optimized fiscal policy, and further weakens the power of policy instruments on stabilizing the economy.

On the other hand, Blanchard-Yaari consumers may lessen the conflicts we described. There are two stabilizer effects emerged due to a positive p. The first one is between debt and consumption. Although higher debt increases consumption and output, a higher tax base helps to reduce the debt. The second one is between inflation and consumption. When $\chi \neq 0$, the finitely lived agents feel poorer when higher inflation happens, because it reduces real financial wealth, which imposes negative wealth effects. The shorter the planning horizon, the stronger this negative wealth effect from positive inflation is. The living generations will reduce their consumption, which contributes to stabilize inflation. These two mechanisms may take some pressure off debt and inflation stabilization of the policy makers' shoulders.

We investigate which effect dominates in the following .

Table 4.1: Calibration

Parameters		calibration	
		base	range
Discount rate of pure time preference	β	0.99	–
Calvo parameter	γ	0.75	–
Consumption share in national income	θ	0.75	–
Intertemporal elasticity	σ	0.3	[0.1, 1.3]
Frisch elasticity of labour supply	ψ	3.0	[0.3,4]
Elasticity of substitution between goods	ϵ	11.0	[6,11]

4.4 Calibration and Solution Technique

The model is calibrated the same as in the previous Chapter except for debt-output ratio and mortality rate, summarized in Table 4.1. The model is highly stylized and involves relatively few parameters. We take the monetary policy frequency to be quarterly and calibrate the model at a quarter frequency. Our debt level, $\chi = 0.1$, corresponds to 2.5% of annual output, which is less than the level of debt in a number of European economies. However, we only consider one-period debt, so the figure of 2.5% is large enough to demonstrate qualitative difference with $\chi = 0$, which corresponds to 0% of annual output and which we treat as benchmark case. Morality rate p measures households' life expectancy by definition, which is around 0.5% if we consider 50 years of working time as a reasonable number. However, households can be myopic and only make decisions considering shorter planning horizon. Therefore a higher value for p is also justified to examine how greater deviations from Ricardian equivalence due to shorter planning horizon impact equilibrium outcomes on top of infrequent fiscal policy. We considered $p = 0.05$ and $p = 0.1$ to investigate the sensitivity of p.

4.5 Numerical Results

In this section, our numerical results show that the presence of inter-generational redistributions of wealth increase the volatility of debt and inflation and further compromise the effectiveness of the macro control policies with infrequent fiscal policy. The consequential danger of this is that the existence of equilibrium become even more fragile.

4.5.1 Benchmark case

We assume that in our benchmark case households live infinitely and steady state level of debt is zero. The corresponding impulse responses of economic variables following an surprisingly increased debt are shown by solid line in Figure (4.1). A higher than steady state debt causes the action of fiscal authority, the tax rate is increased. This has a cost-push effect on price level, to combat higher inflation, central bank raises interest rate to press the consumption down. Although a lower aggregate demand stabilize inflation, the shrinking tax base reduces the tax effect on debt which may lead to even higher tax. However as we discussed in the last Chapter, with two policy makers re-optimise at the same period, This complementarity is not strong enough to result in multiple equilibrium. Figure (4.1) illustrates the impulse responses after a sudden high level debt. We use this as a benchmark to investigate the influence of positive steady state debts and finite lived households brought on the transition paths to steady states.

4.5.2 Positive Steady State Level of Debt

First, we keep mortality rate constant, and check how the changes in debt can influence the transition path of the variables to steady state, after an initially higher than steady state debt happens. Comparing with the benchmark case, a positive steady state debt imposes a positive influence of interest rate and negative effect of inflation on debt accumulation. Every level of interest rate now leads to higher level of debt, as more future debt needs to be financed due to the interest rate payment. Fiscal policy maker will increase tax rate by more, this produces higher inflation which has to be reduced by a higher interest rate, which adds extra burden on debt stabilization. However, the positive inflation caused by distortionary tax reduces the real value of debt. But the interest rate is higher enough to reduce inflation, so the positive effect of an increased interest rate dominates the negative impact of a positive inflation on debt. This added burden on

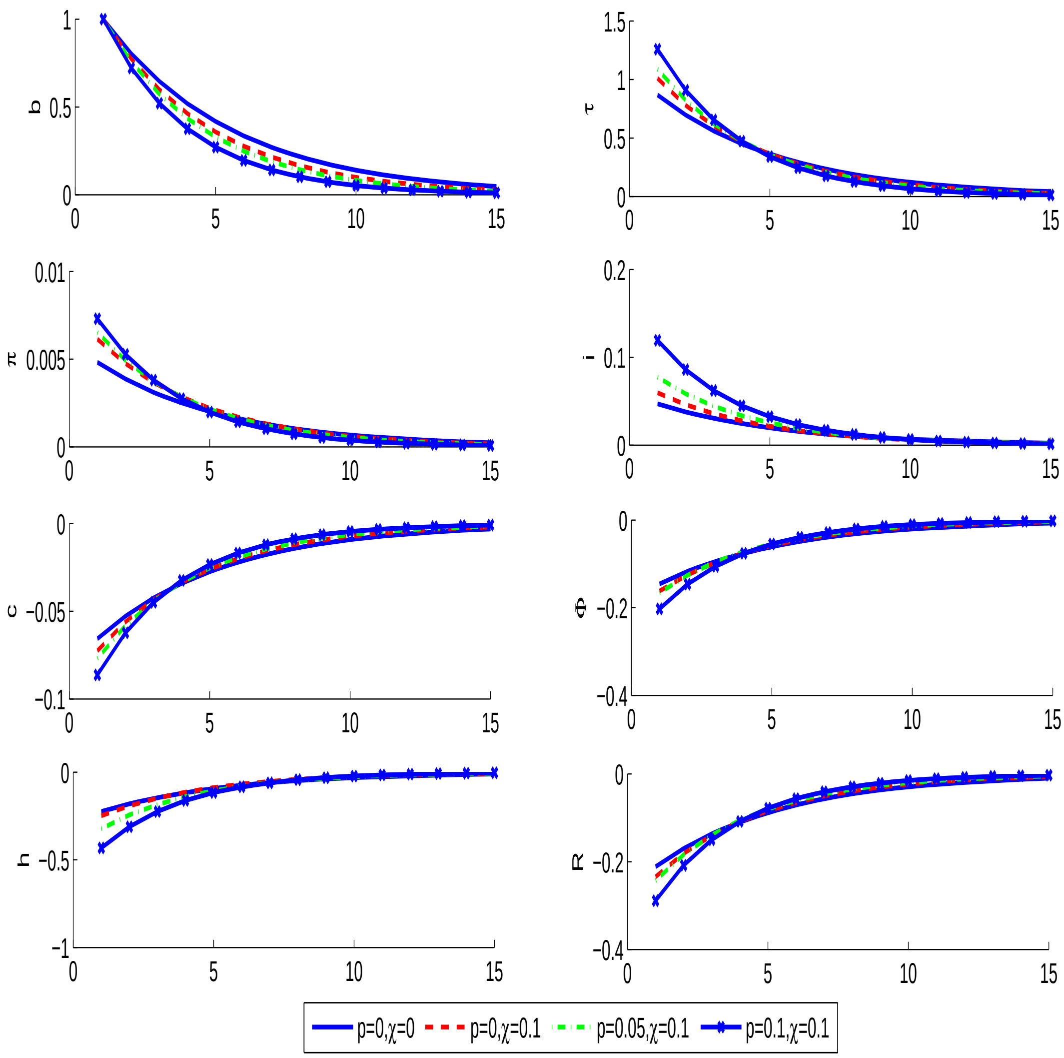

Figure 4.1: Impulse responses to an initial higher than steady state level of debt with quarterly fiscal optimization.

debt will induce a higher tax rate for stabilization. As we discussed before, this reinforce the monetary action of increasing interest rate more. Therefore, interest rate can influence inflation via two different path: directly through demand reduction, and indirectly through helping fiscal policy reduce debt to result a lower tax rate and a lower overall marginal cost. With relatively moderate debt level, optimal monetary policy maker raises interest rate because it is the more effective method to achieve targeted inflation. The impulse responses of the case with infinitely lived households and positive steady state debt, $\chi = 0.1$, is represented by the dashed line in Figure (4.1), showing that the policies are implemented with a larger magnitudes, responded by stronger changes with faster return to steady states in the other economic variables.

4.5.3 Increased Mortality Rate

If steady state level of debt is zero, a higher p requires a higher interest rate as agents discount future more; a higher portion of total wealth will be consumed; as current consumption has no change, the total human resources as well as total resources are lowered. However the inter-generational wealth transfer has no effect on transition path of tax, debt, inflation and consumption at all. Therefore, to facilitate the analysis of the mortality rate impact on transition path toward the steady state, we fix steady state level of debt at $\chi = 0.1$, then simulate impulse responses with different length of planning horizon, the results are illustrated by the crossed line with a lower p=0.05 and the dotted line with a higher p=0.1 in Figure (4.1).

Although by strict definition value of p is around 0.02, higher values for the probability of death can account for agents being myopic. Higher values for p are also justified to examine how greater deviations from Ricardian equivalence due to shorter planning horizon impact equilibrium outcomes. When mortality rate is larger, any fluctuations from target are stronger and the monetary authority's ability to control inflation is weakened, regardless of the presence of an infrequent fiscal policy. Increased volatility stems from the wealth effects created by unexpected raised debt. When agents restrict their planning horizons, government liabilities are seen as additional wealth.

Figure (4.1). shows that, in this case a shorter planning horizon of the households induces stronger policy feedbacks on debt deviation. With a positive probability of death (and non-zero debt steady state value), debt, inflation and the propensity to consumption become determinants of current consumption change $\hat{C}_t$, and their influences on $\hat{C}_t$ are increasing with mortality rate p. The reverse of the proportion of current consumption in total resources $\hat{\Phi}_t$ has a negative relationship with the difference between interest rate and the expected inflation in the next period, this relationship is weakened with a higher p.

We illustrated in the previous Chapter that the dynamic complementarities between monetary policy and fiscal policy after a surprising raise of debt plays an important role in equilibrium analysis. Fiscal authority will set a higher tax in order to stabilize debt, which will induce a higher marginal cost followed by mounting inflation. Monetary policy takes action to reduce inflation by increasing interest rate so as to lower aggregate demand. However, higher interest rate in a positive steady state level of debt together with a lowered tax base contributes to debt accumulation and may significantly weaken the impact of fiscal policy on debt reduction. Fiscal authority increases its action, i.e. higher feedback of tax rate on debt, as a response to the monetary authority's increased action.

With Blanchard-Yaari consumers, this dynamic complementarity is strengthened due to the stronger influence of debt on inflation. When households have a shorter planning horizon, debt as a financial asset owned by households increases the current generations' wealth. The debt influences inflation dynamics not just via fiscal policy but also via increased consumption. Equation 4.17 shows that a higher debt push up aggregate demand which put extra up-ward pressure on inflation leading to a stronger monetary reaction.

In order to reduce the inflation, central bank chooses to control the demand by raising interest rate; with positive steady state debt level, this has a direct impact on future debt: the real value of future debt will have to increase. Moreover, a higher interest rate enlarges the gap between itself and the expected next-period inflation, $\hat{\Phi}_t$ is reduced, although the direct effect from interest rate to consumption dominates here, the effectiveness of contracting monetary policy on current consumption is weakened. Therefore, a higher than in the benchmark interest rate is required. So the presence of mortality rates makes it more difficult for the policy makers to control the economic variables, the dynamic complementarities get stronger. From the Figure 4.1 we can see that a higher p causes larger magnitudes of the changes in policy instruments and private sectors actions, with a faster return to steady states.

As we discussed before, higher expectation of inflation makes the finitely lived agents feel poorer, as inflation reduces real financial wealth, which imposes negative wealth effects. Current generations will reduce their consumption which in turn may help stabilizing inflation. Another stabilization effect appears under finite lived households assumption is that the consumption increased with debt may help reduce debt. However, these effects are relatively weak and were dominated.

When agents face a higher probability of death, their expected lifetimes become further misaligned with the government's infinite planning horizon and wealth effects are magnified. We next investigate how this affects the responses in the infrequent fiscal policy framework.

4.5.4 Infrequent fiscal policy

Figure 4.2 shows the impulse responses when $\chi = 0.1$ and $p = 0.05$ with different length of fiscal cycle. The impact of longer fiscal cycle to the transition paths is similar comparing to Panel I in 3.7: Tax rate is fixed during the fiscal period, it is lower than the required level of tax comparing with the benchmark case when tax rate can be optimized every period, and higher than the needed level during the latter part of the fiscal period; when

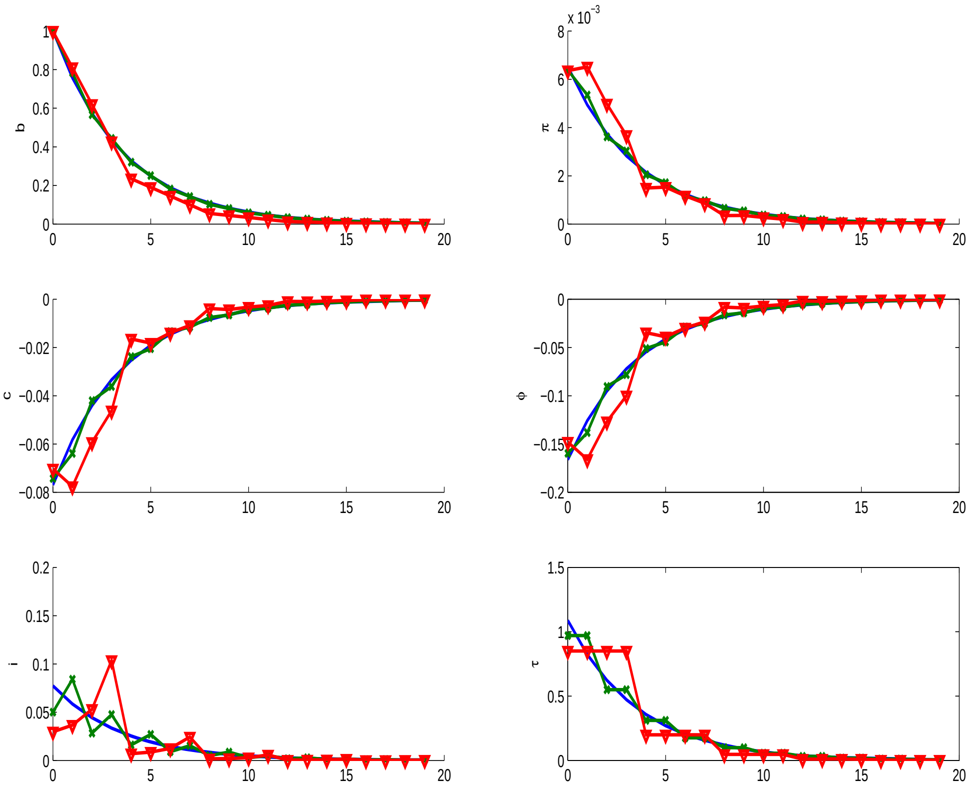

Figure 4.2: Impulse responses with infrequent fiscal policy

the new fiscal period starts, the gap between the tax rate implemented by an infrequent fiscal policy maker and the tax rate in our benchmark case is relatively large. While tax rate is not high enough to peg down debt, monetary authority is forced to help stabilize debt by allowing inflation rises higher than the level in the bench mark case. When the fixed tax rate is higher than the optimal level in the benchmark case, the interest rate is raised with a rather large amount to keep inflation under control. Here with a positive p, in the annual case the consumption dropped further in the second period before it increases, due to the anticipated future higher inflation and lower debt comparing with the benchmark case.

As 4.1 shows, when the mortality rate is larger, any fluctuations from target are stronger and the monetary authority's ability to control inflation is weakened. With

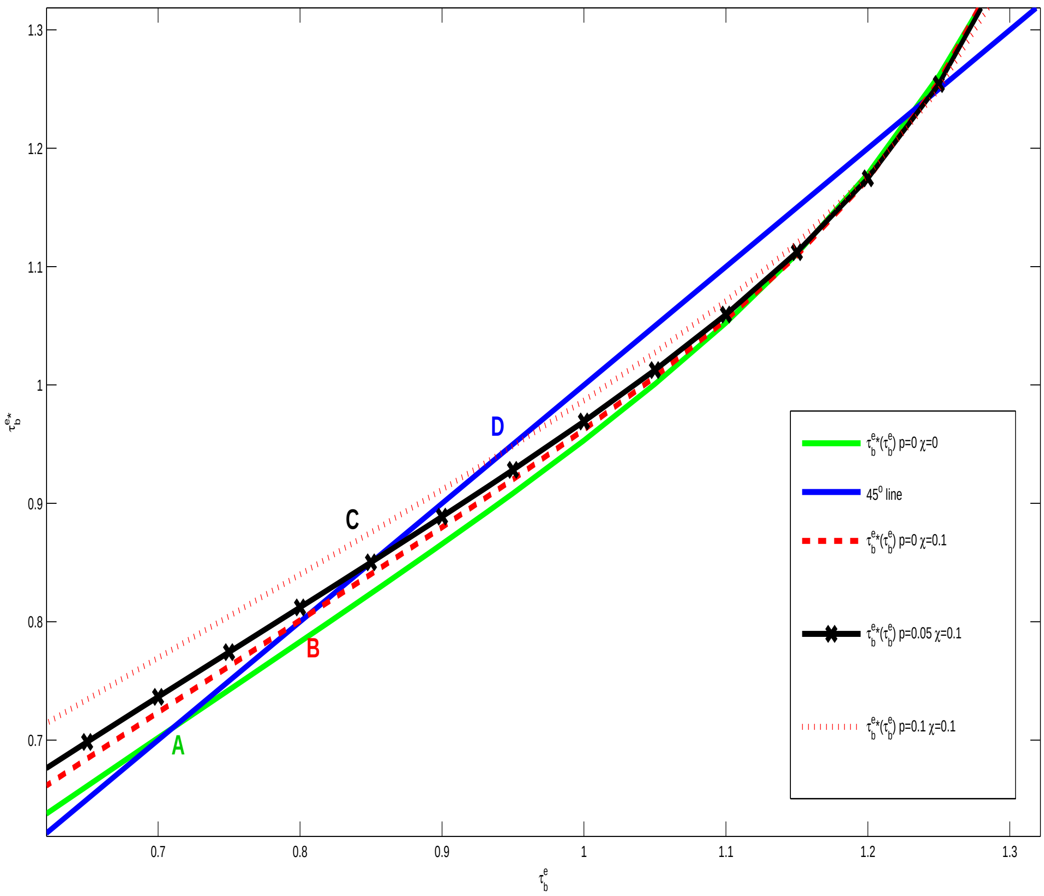

Figure 4.3: Equilibriums when Fiscal Policy Re-optimise Annually (fiscal cycle = 4 periods)

the presence of an infrequent fiscal policy, the existence of the equilibrium becomes more fragile. We take annual fiscal stabilization as an example. In this case, $N = 4$ in Equation (4.21). Due to the dynamic complementarity between monetary and fiscal policy makers that we described, there are two equilibrium. Figure 4.3 illustrates the impact of a higher steady state debt, and increased mortality rates on these two equilibrium. The impulse responses in Figure 4.2 corresponds to Point C in Figure 4.3.

Figure 4.3 demonstrates the responses of fiscal policy to debt fluctuation τ_b in both equilibrium become stronger with increasing p and χ. Higher feedback on debt is to compensate the weaker controlling power of fiscal policy on debt stabilization. This extra controlling difficulty stems from the wealth effects created by unexpected increased debt, mainly due to that when agents restrict their planning horizons, government liabilities are seen as additional wealth. We label the equilibrium with smaller τ_b as good equilibrium as it characterizes smoother transition pathes back to the steady states, approximating our benchmark case and resulting a smaller social loss. The variables in the other equilibrium behave as a zig-zag way which cause larger consequential loss, we call it the bad equilibrium. The sensitivity of τ_b in these two equilibriums to p is different. Although they both move up with increasing p, the good equilibrium is much more sensitive than the bad equilibrium, as shown in Figure 4.3, it moves up faster. Hence, these two equilibrium are getting closer with increasing p. A higher τ_b in the good equilibrium leads to both stronger fiscal and monetary policy actions. Economy is stabilized faster at the cost of a larger volatility.

The sensitivity of τ_b to p in the good equilibrium is increasing with p, which means τ_b moves faster and faster. Eventually τ_b causes over-shoot of the variables and behaves more like the zig-zag equilibrium, until both equilibrium disappear, leave the economy unstable.

Correspondingly, due to the complementarity between the policy makers, the current monetary policy maker may choose different path to stabilize the inflation, depending on his expectation of the next monetary policy maker's decision. When facing an expected higher than bench mark case tax rate, monetary authority can choose to reduce the consumption relatively smoothly over period or increase consumption while during lower-than-benchmark-tax-rate period, then pull it down sharply. Moreover, when the households have a shorter planning horizon and steady state debt is higher than *certain threshold level*, the inflation caused by distortionary tax has a positive net impact on consumption. This further weakens the effectiveness of monetary policy, and may lead to stagflation where consumption is low with a high inflation. The longer the fiscal period is, the more violently the interest rate fluctuates, so do the policy targets: consumption and inflation. The longer the tax stays higher than needed, the more severe of the stagflation risk.

4.6 Conclusions

This Chapter extended our previous framework with infrequent fiscal policy to investigate the impact of finite living households. The shorter planning horizon may strengthen the dynamic complementarity between monetary and fiscal policy due to a positive effect from debt on consumption to inflation, or may weaken this complementarity due to automatic stabilizer created by wealth impact of inflation.

However our results show that with finite living households the problems observed in the last Chapter get worse. The former impact dominates. Higher mortality rate, or more myopic households, reduce the effectiveness of policies. Therefore stronger policies have to be implemented which causes more volatile responses of economic variables. Multiple equilibriums still exist, the relative good equilibrium is more sensitive to the mortality rate and both equilibriums disappear with higher p, leaves the economy unstable and a commitment technique is needed to stabilize the economy.

Chapter 5

Conclusion

There has been a long history in analyzing the design of optimal macro control policies aiming to maintain economic stabilization and achieve the social welfare maximization. Comparing with the rich literature devoted to the impacts of monetary policy on economic activities, the study of strategic fiscal policy is a relatively new area. Analysis of using fiscal policy as a macroeconomic stabilization tool has been receiving more and more attraction especially since this recent recession. Fiscal policy proposals are typically motivated by the need to design a powerful stabilization instrument in situations when monetary policy is constrained. Since the rational expectations revolution in the 1970s, one important line of research has been the study of the interactions between policy makers as well as private agents. It is well understood that discretionary policy making can result in expectation traps and multiple equilibria due to the policy makers inability to control the expectation of the private sector. There is no doubt that fiscal policy maker can behave as another strategic player, that the monetary policy maker will take its action into account while setting monetary policies. This interaction between monetary and fiscal policy makers can change the policy impacts concluded from previous literature. If the dynamic complementarity between their actions strengthened due to different re-optimization frequency, debt level or different policy targets, the policy regimes which were considered to be optimal in the absence of them may worse off the situation and even lead economy into a turmoil.

This thesis contributes to the research agenda by providing a better understanding of the interactions between monetary and fiscal policy when the assumptions are changed and the resulted economic impacts.

Chapter 2 investigates the stabilization bias that arises in a model of monetary and fiscal policy stabilisation of the economy, when monetary authority implements price level targeting. We demonstrate that in the low debt case, the monetary price level targeting unambiguously leads to social welfare gains even if the fiscal authority acts strategically, faces different objectives and has incentives to pursue its own benefit and offset some or all of monetary policy actions. If the fiscal policymaker is able to conduct itself as an intra-period leader the welfare gain of monetary price level targeting is particularly large. However, when steady state debt level is high, price level targeting is not as effective as in the low debt case in terms of achieving a low inflation expectation; therefore it brings more volatility when delivering the inflation overshooting. The level of steady state debt is crucial to the impact of PLT.

In Chapter 3 we demonstrated how the different re-optimization frequency between the policy makers can influence the impact of the policies on economy. We study discretionary non-cooperative monetary and fiscal policy stabilization in a New Keynesian model, where the fiscal policymaker uses a distortionary tax as the policy instrument and operates with long periods between optimal time-consistent adjustments of the instrument. We demonstrate that longer fiscal cycles result in stronger complementarities between the optimal actions of the monetary and fiscal policymakers. When the fiscal cycle is not very long, the complementarities lead to expectation traps. However, with a sufficiently long fiscal cycle -one year in our model - no learnable time-consistent equilibrium exists. Moreover, we show that constraining the fiscal policymaker in its actions may help to avoid these adverse effects.

Chapter 4 extended the framework in Chapter 3, to study the case of finite living households. We assigned a mortality rate to the representative households who have perpetual youth; this assumption brings in the intergenerational wealth transfer due to the government debt. We found out that in this case the dominating effect makes the dynamic complementarity between the policy makers more serious, which leads to an even worse situation. An infrequent fiscal policy brings more difficulties for stabilization problem when the households are myopic.

Appendix A

Appendix to Chapter 2

A.1 Model Derivation

A.1.1 Private Sector

Household consumption decision

In this section we derive the aggregate supply function needed for Phillips Curve.

The economy is inhabited by a large number of households who specialize in the production of a differentiated good (indexed by z) which cost them $h(z)$ of effort. They consume a basket of goods and derive utility from per capita government consumption. β is time preference rate, which is assumed the same value as the intertemporal discount factor applied by infinite living households. In order to focus on the impact of distortionary tax, government spending and lump-sum transfers are assumed to be constant and excluded from households utility function for simplicity. When $p = 0$ the model reduces to the infinitely lived representative households setup in the last Chapter.

At time t, the representative household at time t chooses consumption of goods C_t and the efforts, which is measured by the working hours $N_t(z)$, put to produce the goods to maximize their expected lifetime utility:

$$\max \mathcal{E}_t \sum_{v=t}^{\infty} [\beta]^{v-t} \left[u(C_v, \xi_v) - v(N_v(z), \xi_v)\right] \tag{A.1}$$

subject to the intertemporal budget constraint:

$$\sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v} P_v C_v) \leq \mathcal{A}_t + \mathcal{H}_t \tag{A.2}$$

where $u(.)$ is the utility the household derived from consumption, and $v(.)$ is the disutility caused by working. $Q_{t,v}$ is the stochastic discount factor by which the value of nominal income in time T is determined in time t. For simplicity, we assume the particular utility functions as: .

$$u(C_v, \xi_v) = \frac{(C_v \xi_v)^{1-1/\sigma}}{1-1/\sigma} \tag{A.3}$$

$$v(N_v(z), \xi_v) = \varkappa \frac{(N_v \xi_v)^{1+1/\psi}}{1+1/\psi} \tag{A.4}$$

Here ξ_v is stochastic shocks. σ is the elasticity of intertemporal substitution, i.e. the inverse of the household's relative risk aversion. σ influences the decision on delaying consumption, it can be seen as a measure of the responsiveness of the growth rate of the future consumption to the compensated real interest rate; ψ is the Frisch elasticity of labor supply, which indicates the sensitivity of labor supply to real wage. Nominal human capital $\mathcal{H}^s$ is the nominal present value of the expected total future after-tax labor income. $\mathcal{H}_t = \sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v} \int_0^1 (1-\tau_v)\left(W_v(z)N_v(z) + D_v(z)\right) dz + T)$, P is the aggregate price level, $P_t C_t = \int_0^1 p(z)c(z)dz$, $p(z)$ is the price of a differentiated good z while $c(z)$ is the consumption level of good z. $\mathcal{A}_t$ is the household's nominal financial assets, we assume no physical capital in this model, so $\mathcal{A}_t = \mathcal{B}_t$ the government debt. τ_v is the proportional income tax rate[1]. $W_v(z)$ is the nominal wage rate the household takes, $D_v(z)$ is the profit the household receives from the firms invested, T is a constant lump-sum tax or subsidy from the government. $Q_{t,v}$ is a stochastic discount factor which determines the compensation the household in time t requires if postpone consumption

[1] it would make no difference if assuming the share of total output that the government collected is from revenue tax instead of labor income tax, as income tax will be transfered to firm's total revenue via wage level required. The final impact of income tax and revenue tax on the price setting decision would be the same.

and carry the state-contingent amount $\mathcal{A}_t$ of wealth from time t to time v. *As was pointed out by Yaari (1965),* expected future utility is discounted both because of pure time preference and life-time uncertainty.

First-order conditions for household's utility maximisation show that:

$$W_v(z) = \frac{\varkappa \left(C_v \xi_v\right)^{1/\sigma} \left(N_v \xi_v\right)^{1/\psi} P_v}{(1-\tau_v)} \tag{A.5}$$

And that for the generation born at time s, the intertemporal rule for consumption :

$$C_v = \left[p \frac{P_{v+1}}{P_v} Q^s_{v,v+1}\right]^{\sigma} C_{v+1} \frac{\xi_{v+1}}{\xi_v} \tag{A.6}$$

Where $Q^s_{v,v+1}$ is the nominal individual stochastic discount factor of time $v+1$ at time$v$

$$Q_{v,v+1} = \frac{Q_{v+1}}{Q^s_v} = \beta \frac{u_C(C_{v+1}, \xi_{v+1})}{u_C(C^s_v, \xi_v)} \frac{P_v}{P_{v+1}} \tag{A.7}$$

Firms

A firm in monopolistic competition market employs labour and produces goods z, it discount future profit with Q^f , Nominal wages W are equalized across all firms. A firm chooses employment and prices to maximize profit, we consider these two parts separately in the following.

Employment First we consider the firm z chooses how much labor it will employ to minimize nominal cost:

$$\min_{N_t(z)} \left(W_t N_t(z)\right) \tag{A.8}$$

subject to the production constraint

$$y_t(z) = Z_t N_t(z).$$

The Lagrangian can be written as:

$$L = W_t N_t(z) - P_t \lambda_{wt} \left(Z_t N_t(z) - y_t(z)\right)$$

$$\frac{\partial L}{\partial N_t(z)} = W_t - Z_t P_t \lambda_{wt} = 0$$

From where

$$\lambda_{wt} = \frac{w_t}{Z_t} = mc_t$$

and

$$w_t = \frac{W_t}{P_t}$$

we denote $\Delta_t = \int \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} di$ as price dispersion, price dispersion is variation in prices across suppliers of the same good.

Aggregation yields

$$N_t = \int N_t(z)\, di = \int \frac{y_t(z)}{Z_t} di = \int \frac{Y_t^a}{Z_t} \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} di = \frac{Y_t^a}{Z_t} \Delta_t$$

$$y_t^a(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon_t} Y_t^a$$

from here we have

$$N_t(z) = \frac{Y_t^a}{Z_t} \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon_t}$$

recall that

$$W_v(z) = \frac{\varkappa (C_v^s \xi_v)^{1/\sigma} (N_v \xi_v)^{1/\psi} P_v}{(1-\tau_v)} \tag{A.9}$$

Price setting Next the firm z chooses a price to maximize the expected profit, with wages independent on index z, as labour is assumed to be perfectly mobile and so all wages are equalized across all firms.

$$\max_{\{p_s^*(z)\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} Q_{t,s}^f \left(y_s(z)\, p_s(z) - W_s N_s(z)\right) \tag{A.10}$$

subject to:

$$y_t(z) = Y_t^a \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} \tag{A.11}$$

$$p_t(z) = p_t^*(z)$$

$$p_{t+1}(z) = \begin{cases} p_t^*(z), & \text{with prob } \gamma \\ p_{t+1}(z), & \text{with prob } 1-\gamma \end{cases} \tag{A.12}$$

We only consider profits of those firms that fix price at time t. Additional discount factor applies. The problem for the optimal prices setting at time t can, equivalently, be written as

$$\begin{aligned}
&\max_{\{p_s^*(z)\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \left(y_s(z) p_t^*(z) - W_s N_s(z)\right) \\
&= \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \left(y_s(z) p_t^*(z) - \frac{\varkappa (C_s)^{1/\sigma} (N_s)^{1+1/\psi} P_s}{(1-\tau_s)} \right) \\
&= \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \left(Y_s^a \left(\frac{p_t(z)}{P_s}\right)^{-\epsilon} p_t^*(z) - \left(Y_s^a \left(\frac{p_t(z)}{P_s}\right)^{-\epsilon} \right)^{(1+1/\psi)} \frac{\varkappa (C_s)^{1/\sigma} P_s}{(1-\tau_s)(Z_s)^{1+1/\psi}} \right) \\
&= \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \left(Y_s^a P_s^{\epsilon} p_t^*(z)^{1-\epsilon} - \frac{\varkappa (C_s)^{1/\sigma}}{(1-\tau_s)} \left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi} P_s^{\epsilon(1+1/\psi)+1} p_t^*(z)^{-\epsilon(1+1/\psi)} \right)
\end{aligned}$$

First order condition with respect to $p_t^*(z)$:

$$0 = \frac{\partial}{\partial p_t^*(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \left(Y_s^a P_s^{\epsilon} p_t^*(z)^{-\epsilon} - \frac{\varkappa (C_s)^{1/\sigma}}{(1-\tau_s)} \left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi} P_s^{\epsilon(1+1/\psi)+1} p_t^*(z)^{-\epsilon(1+1/\psi)} \right)$$

$$\begin{aligned}
&\left(\frac{p_t^*(z)}{P_t}\right)^{-\epsilon} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f Y_s^a \left(\frac{P_t}{P_s}\right)^{-\epsilon} \\
&= \left(\frac{p_t^*(z)}{P_t}\right)^{-\epsilon(1+1/\psi)-1} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \frac{\epsilon}{\epsilon-1} (1+1/\psi) \frac{\varkappa (C_s)^{1/\sigma}}{(1-\tau_s)} \left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi} \left(\frac{P_t}{P_s}\right)^{-\epsilon(1+1/\psi)-1}
\end{aligned}$$

$$\left(\frac{p_t^*(z)}{P_t}\right)^{-\epsilon+\epsilon(1+1/\psi)-1} = \frac{\mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f \frac{\epsilon}{\epsilon-1} (1+1/\psi) \frac{\varkappa (C_s)^{1/\sigma}}{(1-\tau_s)} \left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi} \left(\frac{P_t}{P_s}\right)^{-\epsilon(1+1/\psi)-1}}{\mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s}^f Y_s^a \left(\frac{P_t}{P_s}\right)^{-\epsilon}}$$

Note this $Q_{t,s}^f$is a stochastic individual discount factor as function of individual consumption

From the household maximization Equation (A.6) and (A.7) we got:

$$C_t = \mathcal{E}_t \left(\left(\frac{1+p}{\beta} \frac{P_{t+1}}{P_t} Q_{t,t+1}^s \right)^{\sigma} (C_{t+1}) \frac{\xi_{t+1}}{\xi_t} \right) \tag{A.13}$$

$$Q_{t,v}^f = \beta^{v-t} \left(\frac{C_t}{C_v} \frac{\xi_t}{\xi_v} \right)^{\frac{1}{\sigma}} \frac{P_t}{P_v}$$

Substitute $Q_{t,s}^{f}$.

$$\begin{aligned}
\left(\frac{p_t^*(z)}{P_t}\right)^{\frac{\psi+\epsilon}{\psi}} &= \frac{\mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t}\beta^{s-t}\left(\frac{C_t^a}{C_s^a}\right)^{\frac{1}{\sigma}} \frac{P_t}{P_s}\frac{\epsilon}{\epsilon-1}\left(1+1/\psi\right)\varkappa\frac{(C_s)^{1/\sigma)}}{(1-\tau_s)}\left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi}\left(\frac{P_t}{P_s}\right)^{-\epsilon(1+1/\psi)-1}}{\mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t}\beta^{s-t}\left(\frac{C_t^a}{C_s^a}\right)^{\frac{1}{\sigma}} \frac{P_t}{P_s} Y_s^a \left(\frac{P_t}{P_s}\right)^{-\epsilon}} \\
&= \frac{\mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t}\frac{\epsilon}{\epsilon-1}\varkappa\frac{(1+1/\psi)}{(1-\tau_s)}\left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi}\left(\frac{P_s}{P_t}\right)^{\epsilon(1+1/\psi)}}{\mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} C_s^{-\frac{1}{\sigma}} Y_s^a \left(\frac{P_s}{P_t}\right)^{\epsilon-1}} = \frac{K_t}{F_t}
\end{aligned}$$

Therefore

$$\frac{p_t^*(z)}{P_t} = \left(\frac{K_t}{F_t}\right)^{\frac{\psi}{\psi+\epsilon}}$$

assume that $\mu = \frac{\epsilon}{\epsilon-1}$

$$\begin{aligned}
K_t &= \mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t}\mu\varkappa\frac{(1+1/\psi)}{(1-\tau_s)}\left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi}\left(\frac{P_s}{P_t}\right)^{\epsilon(1+1/\psi)} \\
&= \mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} k_s \left(\frac{P_s}{P_t}\right)^{\epsilon(1+1/\psi)} \\
k_s &= \varkappa\mu\frac{(1+1/\psi)}{(1-\tau_s)}\left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi}
\end{aligned}$$

$$\begin{aligned}
F_t &= \mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} C_s^{-\frac{1}{\sigma}} Y_s^a \left(\frac{P_s}{P_t}\right)^{\epsilon-1} \\
&= \mathcal{E}_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} f_s \left(\frac{P_s}{P_t}\right)^{\epsilon-1} \\
f_s &= C_s^{-\frac{1}{\sigma}} Y_s^a
\end{aligned}$$

It follows that

$$\begin{aligned}
K_t &= k_t + \gamma\beta\mathcal{E}_t \Pi_{t+1}^{\epsilon(1+1/\psi)} K_{t+1} \\
F_t &= f_t + \gamma\beta\mathcal{E}_t \Pi_{t+1}^{\epsilon-1} F_{t+1}
\end{aligned}$$

Price in the sector is determined as $P_t = \left[(1-\gamma)\left(p_t^*\right)^{1-\epsilon} + \gamma P_{t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$

From where

$$\begin{aligned}
\Pi_t^{1-\epsilon} &= \left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} = (1-\gamma)\left(\frac{p_t^*}{P_t}\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} + \gamma \\
&= (1-\gamma)\left(\frac{p_t^*}{P_t}\right)^{1-\epsilon}\Pi_t^{1-\epsilon} + \gamma
\end{aligned}$$

$$\left(\frac{p_t^*}{P_t}\right)^{1-\epsilon} = \frac{\Pi_t^{1-\epsilon}-\gamma}{(1-\gamma)\,\Pi_t^{1-\epsilon}} = \frac{1-\gamma\Pi_t^{\epsilon-1}}{1-\gamma} = \left(\frac{p_t^*}{P_t}\right)^{1-\epsilon} = \left(\frac{K_t}{F_t}\right)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)}$$

Finally

$$\begin{aligned}
\frac{\left(1-\gamma\Pi_t^{\epsilon-1}\right)}{(1-\gamma)} &= \left(\frac{K_t}{F_t}\right)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} && \text{(A.14)} \\
K_t &= k_t + \gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon(1+1/\psi)}K_{t+1} && \text{(A.15)} \\
F_t &= f_t + \gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon-1}F_{t+1} && \text{(A.16)} \\
k_s &= \varkappa\mu\frac{(1+1/\psi)}{(1-\tau_s)}\left(\frac{Y_s^a}{Z_s}\right)^{1+1/\psi} \\
f_s &= \left(C_s^s\xi_s\right)^{-\frac{1}{\sigma}}Y_s^a \\
\Delta_t &= (1-\gamma)\left(\frac{1-\gamma\Pi_t^{\epsilon-1}}{1-\gamma}\right)^{\frac{\epsilon}{\epsilon-1}} + \gamma\Pi_t^{\epsilon}\Delta_{t-1} && \text{(A.17)}
\end{aligned}$$

The Steady States of the System

In steady states, $x_t = x_{t+1}$, therefore for our dynamic system the

$$\begin{aligned}
\Pi &= \beta(1+i) \\
K^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} &= \frac{\left(1-\gamma\Pi^{\epsilon-1}\right)}{(1-\gamma)}F^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} \\
K &= \varkappa\mu\frac{(1+1/\psi)\,(C+G)^{1/\psi+1}}{(1-\tau)\left(1-\gamma\beta\Pi^{\epsilon(1+1/\psi)}\right)} \\
F &= \frac{(C)^{-\frac{1}{\sigma}}(C+G)}{\left(1-\gamma\beta\Pi^{\epsilon-1}\right)} \\
\Delta &= \frac{(1-\gamma)}{(1-\gamma\Pi^{\epsilon})}\left(\frac{1-\gamma\Pi^{\epsilon-1}}{1-\gamma}\right)^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}$$

with government debt:

$$B = (1+i)(B\frac{1}{\Pi} + G - \tau\,(C+G) - T)$$

At zero inflation steady state i.e.$\Pi = 1$

$$\begin{aligned}
\beta &= \frac{1}{(1+i)} && \text{(A.18)}\\
K^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} &= F^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} \\
K &= \varkappa\mu\frac{(1+1/\psi)\,(C+G)^{1/\psi+1}}{(1-\tau)\,(1-\gamma\beta)} && \text{(A.19)}\\
F &= \frac{(C)^{-\frac{1}{\sigma}}\,(C+G)}{(1-\gamma\beta)} && \text{(A.20)}\\
\Delta &= 1 && \text{(A.21)}\\
B &= (1+i)(B+G-\tau\,(C+G)-T) && \text{(A.22)}
\end{aligned}$$

A.1.2 Log-linearization

Now we linearize the system around zero inflation steady states A.18 to A.22

Philip's curve

from A.14 to A.16

$$\begin{aligned}
(F_t)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)}\left(1-\gamma\Pi_t^{\epsilon-1}\right) &= (1-\gamma)\,(K_t)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} \\
K_t &= \frac{\varkappa\mu\,(1+1/\psi)}{(1-\tau_s)Z_s^{1+1/\psi}}\,(C_t^a+G_t)^{1+1/\psi} && \text{(A.23)}\\
&\quad +\gamma\beta\mathcal{E}_t\Pi_{t+1}^{(1+1/\psi)\epsilon}K_{t+1} \\
F_t &= (C_s^s\xi_s)^{-\frac{1}{\sigma}}\,(C_t^a+G_t)+\gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon-1}F_{t+1} && \text{(A.24)}
\end{aligned}$$

K Here we assume:

$$\begin{aligned}
(1-\tau_t) &= (1-\tau)(1-\hat{\tau}_t) \\
\hat{\tau}_t &= \ln\frac{(1-\tau_t)}{(1-\tau)}
\end{aligned}$$

Given A.19 and A.23

$$\begin{aligned} & K\left(1+\hat{K}_t\right) \\ = & \left[\frac{\varkappa\mu\left(1+1/\psi\right)}{(1-\tau_s)Z_s^{1/\psi+1}}\right]\left(Y\left(1+\hat{Y}_t\right)\right)^{1/\psi+1}\xi\left(1+\hat{\xi}_t\right)^{1/\psi}+\gamma\beta\mathcal{E}_t\left(1+\hat{\pi}_{t+1}\right)^{(1+1/\psi)\epsilon}K\left(1+\hat{K}_{t+1}\right) \end{aligned}$$

$$\begin{aligned} \hat{K}_t &= (1-\gamma\beta)\left(\left(\frac{1}{\psi}+1\right)\hat{Y}_t+\frac{\tau}{(1-\tau)}\hat{\tau}_t+\frac{1}{\psi}\hat{\xi}_t\right) \\ &\quad +\gamma\beta\mathcal{E}_t\left(\epsilon\left(1+1/\psi\right)\hat{\pi}_{t+1}+\hat{K}_{t+1}\right) \end{aligned} \tag{A.25}$$

delta Assuming that

$$\hat{\Delta}_t = a\hat{\pi}_t^2+b\hat{\Delta}_{t-1}$$

from A.17

$$\begin{aligned} \left(1+\hat{\Delta}_t\right) &= (1-\gamma)^{\frac{-1}{\epsilon-1}}\left(1-\gamma\left(1+(\epsilon-1)\hat{\pi}_t\right)\right)^{\frac{\epsilon}{\epsilon-1}}+\gamma\left(1+\hat{\pi}_t\right)^{\epsilon}\left(1+\hat{\Delta}_{t-1}\right) \\ 1+\hat{\Delta}_t &= (1-\gamma)^{\frac{-1}{\epsilon-1}}-(1-\gamma)^{\frac{-1}{\epsilon-1}}\gamma^{\frac{\epsilon}{\epsilon-1}}-(1-\gamma)^{\frac{-1}{\epsilon-1}}\gamma^{\frac{\epsilon}{\epsilon-1}}\epsilon\hat{\pi}_t+\gamma\left(1+\epsilon\hat{\pi}_t+\hat{\Delta}_{t-1}\right) \end{aligned}$$

F From A.24 and A.20

$$F=\frac{C^{-\frac{1}{\sigma}}\left(C+G\right)}{(1-\gamma\beta)}$$

$$\begin{aligned} F\left(1+\hat{F}_t\right) &= \left(C_t^s\xi_t\right)^{-\frac{1}{\sigma}}\left(Y_t\right)+\gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon-1}F_{t+1} \\ \hat{F}_t &= (1-\gamma\beta)\left(\hat{Y}_t-\frac{1}{\sigma}\hat{C}_t-\frac{1}{\sigma}\hat{\xi}_t\right)+\gamma\beta\mathcal{E}_t\left((\epsilon-1)\hat{\pi}_{t+1}+\hat{F}_{t+1}\right) \end{aligned}$$

Inflation Given above linearized $\hat{K}_t$, $\hat{F}_t$, $\hat{\Delta}_t$ and A.14 to A.17

$$\begin{aligned}(F_t)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)}\left(1-\gamma\Pi_t^{\epsilon-1}\right) &= (1-\gamma)\,(K_t)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} \\ \left(1+\frac{\psi}{\psi+\epsilon}(1-\epsilon)\,\hat{F}_t\right)\left((1-\gamma)-\gamma(\epsilon-1)\,\hat{\pi}_t\right) &= (1-\gamma)+(1-\gamma)\frac{\psi}{\psi+\epsilon}(1-\epsilon)\,\hat{K}_t\end{aligned}$$

$$\begin{aligned}\hat{K}_t &= (1-\gamma\beta)\left(\left(\frac{1}{\psi}+1\right)\hat{Y}_t+\frac{\tau}{(1-\tau)}\hat{\tau}_t+\frac{1}{\psi}\hat{\xi}_t\right)+\gamma\beta\mathcal{E}_t\left(\epsilon\,(1+1/\psi)\,\hat{\pi}_{t+1}+\hat{K}_{t+1}\right) \\ \hat{F}_t &= (1-\gamma\beta)\left(\hat{Y}_t-\frac{1}{\sigma}\hat{C}_t-\frac{1}{\sigma}\hat{\xi}_t\right)+\gamma\beta\mathcal{E}_t\left((\epsilon-1)\,\hat{\pi}_{t+1}+\hat{F}_{t+1}\right)\end{aligned}$$

$$\begin{aligned}\hat{\pi}_t &= \frac{(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}\left(\hat{K}_t-\hat{F}_t\right) \\ &= \frac{(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}\left((1-\gamma\beta)\left(\left(\frac{1}{\psi}+1\right)\hat{Y}_t+\frac{\tau}{(1-\tau)}\hat{\tau}_t+\frac{1}{\psi}\hat{\xi}_t\right)\right. \\ &\quad \left.+\gamma\beta\mathcal{E}_t\left(\epsilon\,(1+1/\psi)\,\hat{\pi}_{t+1}+\hat{K}_{t+1}\right)\right) \\ &\quad -\frac{(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}\left((1-\gamma\beta)\left(\hat{Y}_t-\frac{1}{\sigma}\hat{C}_t-\frac{1}{\sigma}\hat{\xi}_t\right)+\gamma\beta\mathcal{E}_t\left((\epsilon-1)\,\hat{\pi}_{t+1}+\hat{F}_{t+1}\right)\right) \\ &= \frac{(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}\left(\gamma\beta\left(\frac{1}{1-\gamma}\right)\left(\frac{\psi+\epsilon}{\psi}\right)\mathcal{E}_t\hat{\pi}_{t+1}\right) \\ &\quad +\frac{(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}\left((1-\gamma\beta)\left(\frac{1}{\psi}\hat{Y}_t+\frac{1}{\psi}\hat{\xi}_t+\frac{1}{\sigma}\hat{C}_t+\frac{1}{\sigma}\hat{\xi}_t+\frac{\tau}{(1-\tau)}\hat{\tau}_t\right)\right) \\ &= \beta\mathcal{E}_t\hat{\pi}_{t+1}+\frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}\left(\frac{1}{\psi}\hat{Y}_t+\frac{1}{\sigma}\hat{C}_t+\frac{\tau}{(1-\tau)}\hat{\tau}_t+\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\hat{\xi}_t\right)\end{aligned}$$

with $\theta=\frac{\hat{C}_t}{\hat{Y}_t}$ yields $\hat{Y}_t=\theta\hat{C}_t+(1-\theta)\hat{G}_t$

So the Phillips curve in our model is

$$\hat{\pi}_t=\beta\mathcal{E}_t\hat{\pi}_{t+1}+\kappa\left(\left(\frac{1}{\sigma}+\frac{\theta}{\psi}\right)\hat{C}_t+\frac{(1-\theta)}{\psi}\hat{G}_t+\frac{\tau}{(1-\tau)}\hat{\tau}_t\right)+\hat{\xi}_t \tag{A.26}$$

where $\kappa=\frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}$

Debt

We know that

$$B_{t+1} = (1+i_t)(B_t\Pi_t^{-1} + G_t - \tau_t Y_t - T)$$

And from A.22

$$B = -\frac{(1+i)\,(G-\tau Y)}{i}$$

Therefore

$$\begin{aligned}
B(1+\hat{B}_{t+1}) &= (1+i)(1+\hat{\imath}_t)(B(1+\hat{B}_t)(1+\hat{\pi}_t)^{-1} + G(1+\hat{G}_t) - \tau(1+\hat{\tau}_t)Y(1+\hat{Y}_t)) \\
1+\hat{B}_{t+1} &= (1+i)(1+\hat{\imath}_t)((1+\hat{B}_t)(1-\hat{\pi}_t) + \frac{G(1+\hat{G}_t) - \tau Y(1+\hat{\tau}_t)(1+\hat{Y}_t)}{B}) \\
\hat{B}_{t+1} &= \hat{\imath}_t + (1+i)\hat{B}_t - (1+i)\hat{\pi}_t - \frac{i\left((1-\tau)\,(1-\theta)\,\hat{G}_t - \tau\theta\hat{C} - \tau\hat{\tau}_t\right)}{(1-\theta-\tau)} \\
b_{t+1} &= \chi i + (1+i)b_t - \chi(1+i)\hat{\pi}_t \\
&\quad + \frac{(1+i)(1-\theta-\tau)}{i}\frac{i\left((1-\tau)\,(1-\theta)\,\hat{G}_t - \tau\theta\hat{C} - \tau\hat{\tau}_t\right)}{(1-\theta-\tau)} \\
b_{t+1} &= (1+i)b_t - \chi(1+i)\hat{\pi}_t - (1+i)\tau\theta\hat{C} + (1+i)\,(1-\tau)\,(1-\theta)\,\hat{G}_t + \chi i - (1+i)\tau\hat{\tau}_t
\end{aligned}$$

Where χ is the proportion of steady state debt level to GDP:

$$\begin{aligned}
b_t &= \chi\hat{B}_t \\
\chi &= \frac{B}{Y} = -\frac{(1+i)(1-\theta-\tau)}{i}
\end{aligned}$$

When $\chi = 0$, $\hat{G}_t = 0$ as government spending is assumed to be exogenous, substitute $(1+i) = \frac{1}{\beta}$ in:

$$b_{t+1} = \frac{1}{\beta}\left(b_t - \tau\theta\hat{C} - \tau\hat{\tau}_t\right)$$

The final deterministic system for private sector is

Assuming exogenous government spending:

$$\begin{aligned} b_{t+1} &= \frac{1}{\beta}\left(b_t - \tau\theta\hat{C} - \tau\hat{\tau}_t\right) \\ \hat{\pi}_t &= \beta\mathcal{E}_t\hat{\pi}_{t+1} + \kappa\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)\hat{C}_t \end{aligned}$$

with $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}$

A.1.3 Objective function

The quadratic loss function for the benevolent policy objective is a second-order approximation of expected utility derived around the steady state allocation. Frequent use is made of the following second-order approximation of relative deviations in terms of log deviations:

$$\frac{X_t - X}{X} = \hat{x}_t + \frac{1}{2}\hat{x}_t^2$$

$\hat{x}_t = x_t - x$ is the log deviation from steady state for x_t.

Therefore the second-order Taylor expansion of the representative households utility around a steady state (C, N) yields:

$$u_t = u + u_c C(\frac{C_t - C}{C}) + \frac{1}{2}u_{cc}C^2(\frac{C_t - C}{C})^2 + u_N N(\frac{N_t - N}{N}) + \frac{1}{2}u_{NN}N^2(\frac{N_t - N}{N})^2$$

when market clears, $\hat{y}_t = \hat{c}_t$

$$u_t - u = u_c C(\hat{y}_t + \frac{1 - \frac{1}{\sigma}}{2}\hat{y}_t^2) + u_N N(\hat{n}_t + \frac{1}{2}\left(1 + \frac{1}{\psi}\right)\hat{n}_t^2)$$

Because

$$\begin{aligned} N_t &= \frac{Y_t}{Z_t}\int_0^1 \left(\frac{p_t}{P_t}\right)^{-\epsilon} di \\ \hat{n}_t &= \hat{y}_t - z_t + \ln\int_0^1 \left(\frac{p_t\,(i)}{P_t}\right)^{-\epsilon} di \end{aligned}$$

Using the the definition of P_t, a second order approximation to the expression is

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{1-\epsilon} di = \int_0^1 \left(1 + (1-\epsilon)\hat{p}_t + \frac{(1-\epsilon)^2}{2}\hat{p}_t^2\right) \\ E(\hat{p}_t) &= \frac{(\epsilon-1)}{2} E\left(\hat{p}_t^2\right) \end{aligned}$$

second order approximation to $\left(\frac{p_t(i)}{P_t}\right)^{-\epsilon}$ is:

$$\begin{aligned} \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} &= 1 - \epsilon\hat{p}_t + \frac{\epsilon^2}{2}\hat{p}_t{}^2 \\ &= 1 + \left(\frac{\epsilon}{2}\right)\hat{p}_t{}^2 \\ \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} di &= 1 - \epsilon E(\hat{p}_t) + \frac{\epsilon^2}{2} E\left(\hat{p}_t{}^2\right) \\ &= 1 + \left(\frac{\epsilon(1-\epsilon)}{2} + \frac{\epsilon^2}{2}\right) E\left(\hat{p}_t{}^2\right) \\ &= 1 + \frac{\epsilon}{2} E\left(\hat{p}_t{}^2\right) = 1 + \frac{\epsilon}{2} var(p_t) \\ \ln\left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} &= \ln\left(1 + \frac{\epsilon}{2} var(p_t)\right) = \frac{\epsilon}{2} var(p_t) \end{aligned}$$

Now, the period t utility can be rewritten as

$$\begin{aligned} u_t - u &= u_c C(\hat{y}_t + \frac{1-\frac{1}{\sigma}}{2}\hat{y}_t^2) + u_N N(\hat{y}_t + \frac{\epsilon}{2} var\left(p_t{}^2\right) + \frac{1}{2}\left(1 + \frac{1}{\psi}\right)(\hat{y}_t - z_t)^2) + t.i.p. \\ \frac{u_t - u}{u_c C} &= -\frac{1}{2}\left(\epsilon var(p_t) - \frac{1-\frac{1}{\sigma}}{2}\hat{y}_t^2 + \frac{1}{2}\left(1 + \frac{1}{\psi}\right)(\hat{y}_t - z_t)^2\right) + t.i.p. \\ &= -\frac{1}{2} + t.i.p. \end{aligned}$$

Therefore the second-order approximation to the households' welfare losses can be expressed as a fraction of steady state consumption as:

$$\begin{aligned} L_t &= -E_t \sum_{s=t}^{\infty} \beta^{s-t}\left(\frac{u_t - u}{u_c C}\right) \\ &= \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t}\left(\epsilon var(p_t) + \left(\frac{1}{\sigma} + \frac{1}{\psi}\right)\hat{y}_t^2\right) \end{aligned}$$

Because

$$\sum_{s=t}^{\infty}\beta^{s-t}var\left(p_t\right)=\sum_{s=t}^{\infty}\beta^{s-t}\frac{\gamma}{\left(1-\gamma\beta\right)\left(1-\gamma\right)}\frac{\psi+\epsilon}{\psi}\pi_t^2$$

The loss function can be expressed as :

$$\begin{aligned}
L_t &= \frac{1}{2}E_t\sum_{s=t}^{\infty}\beta^{s-t}\left(\pi_t^2+\left(\frac{1}{\sigma}+\frac{1}{\psi}\right)\hat{y}_t^2\right)\\
L_t &= \frac{1}{2}\frac{\epsilon}{\kappa}E_t\sum_{s=t}^{\infty}\beta^{s-t}\left(\pi_t^2+\frac{\kappa}{\epsilon}\left(\frac{1}{\sigma}+\frac{1}{\psi}\right)\hat{y}_t^2\right)\\
L_t &= \frac{1}{2}E_t\sum_{s=t}^{\infty}\beta^{s-t}\left(\pi_t^2+\lambda\hat{c}_t^2\right)
\end{aligned}$$

Where $\kappa=\frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}$ and $\lambda=\frac{\kappa}{\epsilon}\left(\frac{1}{\sigma}+\frac{1}{\psi}\right)$

A.2 Discretionary Policies in Deterministic LQ RE Models

In this section, we demonstrate the general optimal discretionary policy problems in Linear Quaratic Rational Expectation models of the type described by Blanchard and Kahn (1980).

The policy objective function is an intertemporal loss function:

$$L_t=\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}g_s'Qg_s=\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(z_s'\mathcal{Q}z_s+2z_s'\mathcal{P}u_s+u_s'\mathcal{R}u_s\right). \tag{A.27}$$

The matrix $\mathcal{Q}$and $\mathcal{R}$ are assumed to be symmetric and positive semi-definite.

The linearized constraints can be represented in general from by the following dynamic system:

$$\begin{bmatrix} y_{t+1} \\ x_{t+1}\end{bmatrix}=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix} y_t \\ x_t\end{bmatrix}+\begin{bmatrix} B_1 \\ B_2\end{bmatrix}\left[u_t\right], \tag{A.28}$$

Where y_t is an n_1-vector of predetermined variables with initial conditions y_0 given, x_t is n_2-vector of forward-looking (or jump) variables, and u_t is a k-vector of policy instruments (control variables). For notational convenience we define the n-vector $z_t = (y_t', x_t')'$ where $n = n_1 + n_2$. The matrices A and B are constant functions of structural model parameters. We assume A_{22} is invertible.

Under discretion the policy maker is maximizing its objective function in each period of time with respect to u_t, taking private sector expectations and time-consistent reaction x_t as given, and recognizing dependence of x_t on policy u_t. The private sector expects that future policy makers will implement the same decision process in subsequent periods. Hence, expected future variables are taken as given and at any time t the policy maker and the private sector respond only to the current states. Therefore both the private sector response and policy response can be written as a feedback on the state variables:

$$\begin{aligned} x_{t+1} &= N_{t+1} y_{t+1} \\ u_{t+1} &= F_{t+1} y_{t+1} \end{aligned}$$

From here

$$\begin{aligned} x_{t+1} &= N_{t+1} \left(A_{11} y_t + A_{12} x_t + B_1 u_t \right) \\ &= A_{21} y_t + A_{22} x_t + B_2 u_t \end{aligned}$$

results in

$$\begin{aligned} x_t &= \left(N_{t+1} A_{12} - A_{22} \right)^{-1} \left(A_{21} - N_{t+1} A_{11} \right) y_t + \left(N_{t+1} A_{12} - A_{22} \right)^{-1} \left(B_2 - N_{t+1} B_1 \right) u_t \\ &= -J y_t - K u_t \end{aligned}$$

Therefore the constraints A.28 can be written as linear rules

$$y_{t+1} = A_{11} y_t + A_{12} x_t + B_1 u_t = \left(A_{11} - A_{12} J \right) y_t + \left(B_1 - A_{12} K \right) u_t \tag{A.29}$$

$$= A^* y_t + B^* u_t \tag{A.30}$$

$$x_t = -J y_t - K u_t = -N y_t \tag{A.31}$$

where $u_t = -Fy_t$and $N = J - KF$

We define a constrained welfare loss function as:

$$w = \mathcal{E}_t \sum_{s=t}^{\infty} H_s,$$

where

$$H_s = \frac{1}{2}\beta^{s-t}(z_s'\mathcal{Q}z_s+2z_s'\mathcal{P}u_s+u_s'\mathcal{R}u_s)+\lambda_{s+1}'(A_{11}y_s+A_{12}x_s+B_1u_s-y_{s+1})+\mu_s'\left(x_s + Jy_s + Ku_s\right),$$

with λ_{s+1} is a vector of (non-predetermined) Lagrangian multipliers, the FOCs are:

$$\frac{\partial H_s}{\partial u_s} = \beta^{s-t}\left(\mathcal{P}'z_s + \mathcal{R}u_s\right) + B_1'\lambda_{s+1} + K'\mu_s = 0, \tag{A.32}$$
$$\frac{\partial H_s}{\partial y_s} = \beta^{s-t}\left(\mathcal{Q}_{11}y_s + \mathcal{Q}_{12}x_s + \mathcal{P}_1u_s\right) + A_{11}'\lambda_{s+1} - \lambda_s + J'\mu_s = 0, \tag{A.33}$$
$$\frac{\partial H_s}{\partial x_s} = \beta^{s-t}\left(\mathcal{Q}_{12}'y_s + \mathcal{Q}_{22}x_s + \mathcal{P}_2u_s\right) + A_{12}'\lambda_{s+1} + \mu_s = 0 \tag{A.34}$$
$$\frac{\partial H_s}{\partial \lambda_{s+1}} = \left(A_{11}y_s + A_{12}x_s + B_1u_s - y_{s+1}\right) = 0 \tag{A.35}$$
$$\frac{\partial H_s}{\partial \mu_{s+1}} = x_s + Jy_s + Ku_s = 0 \tag{A.36}$$

$$0 = P^{*\prime}y_s + R^*u_s + B^{*\prime}\beta\lambda_{s+1}, \tag{A.37}$$

$$0 = Q^*y_s + P^*u_s - \lambda_s + A^{*\prime}\beta\lambda_{s+1}, \tag{A.38}$$

$$0 = (\ A^*y_s + B^*u_s - y_{s+1}) \tag{A.39}$$

where

$$Q^* = \mathcal{Q}_{11} - \mathcal{Q}_{12}J - J'\mathcal{Q}_{21} + J'\mathcal{Q}_{22}J,\ P^* = J'\mathcal{Q}_{22}K - \mathcal{Q}_{12}K + \mathcal{P}_1 - J'\mathcal{P}_2, \tag{A.40}$$

$$R^* = K'\mathcal{Q}_{22}K + \mathcal{R} - K'\mathcal{P}_2 - \mathcal{P}_2'K,\ A^* = A_{11} - A_{12}J,\ B^* = B_1 - A_{12}K. \tag{A.41}$$

This system needs to be solved given initial conditions $y_0 = \bar{y}$ and subject to terminal (transversality) conditions in the form $\lim_{t\to\infty}\beta^t y_t < \infty$.[2]

[2] As all other variables will necesserily be linear finctions of y_t, such terminal conditions impliy non-explosiveness of all economic variables.

First order conditions (A.32)–(A.35) treat the intra-period reaction function of the second player explicitly. The optimization is conditional: we are looking for an extremum *on the other player's reaction function.* This defines a Stackelberg equilibrium. We now solve system (A.32)–(A.35) for instrument as linear function of state variables y_t, $u_t = -Fy_t$.

The first order conditions (A.32)–(A.35) can be written in a matrix form as follows:

$$\begin{array}{cccc} I & 0 & 0 & y_{t+1} \\ 0 & 0 & B^{*\prime}\beta & u_{s+1} \\ 0 & 0 & A^{*\prime}\beta & \lambda_{s+1} \end{array} = \begin{array}{cccc} A^* & B^* & 0 & y_t \\ -P^{*\prime} & -R^* & 0 & u_s \\ -Q^* & -P^* & I & \lambda_s \end{array}$$

$$\begin{bmatrix} I & 0 \\ 0 & \Phi_{22} \end{bmatrix} \begin{bmatrix} y_{t+1} \\ \tilde{u}_{t+1} \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} y_t \\ \tilde{u}_t \end{bmatrix}, \tag{A.42}$$

where we used $\mu_s = \beta^{s-t}\lambda_s$, denoted $\tilde{u}_t = [u_t', \mu_t']'$ and used the reaction function of the follower, $x_t = -Jy_t - Ku_t$. The matrix coefficients are:

$$\Phi_{22} = \begin{bmatrix} 0 & \beta B^{*\prime} \\ 0 & \beta A^{*\prime} \end{bmatrix}, \quad \Psi_{21} = \begin{bmatrix} -P^{*\prime} \\ -Q^* \end{bmatrix}, \quad \Psi_{22} = \begin{bmatrix} -R^* & 0 \\ -P^* & I \end{bmatrix},$$
$$\Psi_{11} = A^*, \quad \Psi_{12} = \begin{bmatrix} B^* & 0 \end{bmatrix}.$$

where Q^*, P^*, R^*, A^* and B^* are given by (A.40)-(A.41), and matrices $\mathcal{Q}$, $\mathcal{P}$ and $\mathcal{R}$ are partitioned conformally with $z_s = [y_s', x_s']'$ and u_s.

A solution to linear system (A.28) will necessarily have a linear form of

$$\tilde{u}_t = \begin{bmatrix} u_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} -F \\ S \end{bmatrix} y_t \tag{A.43}$$

It is easy to show that system matrices in (A.43) satisfy the following equations

$$S = Q^* + \beta A^{*\prime} S A^* - (P^{*\prime} + \beta B^{*\prime} S A^*)(R^* + \beta B^{*\prime} S B^*)^{-1}(P^{*\prime} + \beta B^{*\prime} S A^*) \tag{A.44}$$

$$F = (R^* + \beta B^{*\prime} S B^*)^{-1}(P^{*\prime} + \beta B^{*\prime} S A^*) \tag{A.45}$$

where the first equation is a symmetric discrete algebraic Riccati equation (A.44) for S. Therefore, all solutions S of system (A.28) are among solutions of (A.44). The following two results were shown in the literature:

1. There is a *unique* symmetric solution to (A.44) if matrix pair (A^*, B^*) is controllable, i.e. if the controllability matrix, $[B^*, A^*B^*, A^{*2}B^*, ..., A^{*n_1-1}B^*]$, has full row rank.

2. Policy F, $k*n_{1,}$ which is *uniquely* determined from (A.45) if S is given, is *stabilizing,* i.e. all eigenvalues of matrix M that defines the evolution of the dynamic system under control

$$\begin{aligned} y_{t+1} &= A_{11}y_t + A_{12}x_t + B_1u_t = (A_{11} - A_{12}J)\,y_t + (B_1 - A_{12}K)\,u_t \\ &= (A^* - B^*F)\,y_t = My_t, \end{aligned} \tag{A.46}$$

are strictly inside the unit circle.

Such solution satisfies all boundary conditions for system (A.28) and, therefore, is a unique solution of (A.28). Practically, the solution can be found with either generalized Schur decomposition of (A.28) or with some iterative procedure that solves (A.44).[3] The equilibrium of the system of first order conditions to A.28-A.27 are matrices $\{N, S, F\}$,which can be solved from the following system:

[3]See results on symplectic matrix pencil in e.g. Wimmer (2006); they guarantee that the combination of explosive and non-explodive generalised eigenvalues of matrix pair (Φ, Ψ) is always suitable for the generalised Schur decomposition, as there are k infinitely large eigenvalues, n_1 non-explosive eigenvalues and n_1 finite explosive eigenvalues.

$$
\begin{aligned}
S &= Q^* + \beta A^{*\prime} S A^* \\
&\quad - \left(P^{*\prime} + \beta B^{*\prime} S A^*\right)\left(R^* + \beta B^{*\prime} S B^*\right)^{-1}\left(P^{*\prime} + \beta B^{*\prime} S A^*\right) \\
F &= \left(R^* + \beta B^{*\prime} S B^*\right)^{-1}\left(P^{*\prime} + \beta B^{*\prime} S A^*\right) \\
N &= \left(A_{22} + N A_{12}\right)^{-1}\left(\left(A_{21} - B_2 F\right) + N\left(A_{11} - B_1 F\right)\right) \\
Q^* &= \mathcal{Q}_{11} - \mathcal{Q}_{12} J - J' \mathcal{Q}_{21} + J' \mathcal{Q}_{22} J \\
P^* &= J' \mathcal{Q}_{22} K - \mathcal{Q}_{12} K + \mathcal{P}_1 - J' \mathcal{P}_2 \\
R^* &= K' \mathcal{Q}_{22} K + \mathcal{R} - K' \mathcal{P}_2 - \mathcal{P}_2' K \\
A^* &= A_{11} - A_{12} J,\ B^* = B_1 - A_{12} K \\
J &= \left(N A_{12} + A_{22}\right)^{-1}\left(A_{21} + N A_{11}\right) \\
K &= \left(N A_{12} + A_{22}\right)^{-1}\left(B_2 + N_{t+1} B_1\right)
\end{aligned}
$$

A.3 Discretionary Policy (Simultaneous)

A.3.1 Monetary Policy Maker's Reaction Function (Simultaneous)

The monetary policy maker's optimization problem in period t can be explained by the following Bellman equation:

$$
\mathcal{S}(b_t) = \min_{c_t}\left(p_t^2 + \lambda c_t^2 + \beta^m \mathcal{S}(b_{t+1})\right). \tag{A.47}
$$

Because of the linear-quadratic nature of the problem we assume that the value function $\mathcal{S}(b_t, \eta_t)$ is quadratic in state and substitute (2.35) and (2.11). We arrive to the following form:

$$
\tau_t = \tau_b b_t + \tau_p p_{t-1}
$$

$$
S_{pp} p_{t-1}^2 + 2 S_{pb} b_t p_{t-1} + S_{bb} b_t^2
$$

$$\left(S_{pp}p_{t-1}^2 + 2S_{pb}b_t p_{t-1} + S_{bb}b_t^2\right) = \min_{c_t} (p_t)^2 + \lambda (c_t)^2 + \beta \left(S_{pp}p_t^2 + 2S_{pb}b_{t+1}p_t + S_{bb}b_{t+1}^2\right)$$

$$\begin{aligned} S_{pp}p_{t-1}^2 + 2S_{pb}b_t p_{t-1} + S_{bb}b_t^2 &= \min_{c_t} \left((1-\alpha)\pi_t^2 + \alpha p_t^2 + \lambda c_t^2 \right. \\ &\quad \left. + \beta \left(S_{pp}p_t^2 + 2S_{pb}b_{t+1}p_t + S_{bb}b_{t+1}^2\right)\right), \end{aligned} \tag{A.48}$$

$$\begin{aligned} 0 &= 2\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}(\alpha + \beta S_{pp}) \left(\frac{1 + (\nu - \tau\pi_b)\tau_p}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b + (\nu - \tau\pi_b)\tau_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t\right) \\ &\quad + 2\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}(1-\alpha)\left(\frac{\beta\pi_p + (\nu - \tau\pi_b)\tau_p}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b + (\nu - \tau\pi_b)\tau_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t\right) \\ &\quad + 2\lambda c_t - \beta 2 S_{pb}\left(\frac{1}{\beta}\tau\theta\right)\left(\frac{1 + (\nu - \tau\pi_b)\tau_p}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b + (\nu - \tau\pi_b)\tau_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t\right) \\ &\quad + \beta 2 S_{pb}\left(\frac{1}{\beta}\left((1-\tau\tau_b) b_t - \tau\theta c_t - \tau\tau_p p_{t-1}\right)\right)\left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}\right) \\ &\quad - \frac{1}{\beta}\tau\theta 2\beta S_{bb}\left(\frac{1}{\beta}\left((1-\tau\tau_b) b_t - \tau\theta c_t - \tau\tau_p p_{t-1}\right)\right) \end{aligned} \tag{A.49}$$

from where:

$$c_t = C_b b_t + C_p p_{t-1} \tag{A.50}$$

where

$$\begin{aligned} C_b &= -\frac{(\alpha + \beta S_{pp})\frac{(\varkappa - \theta\tau\pi_b)(\pi_b + (\nu - \tau\pi_b)\tau_b)}{(1-\beta\pi_p)^2} + (1-\alpha)\frac{(\varkappa - \theta\tau\pi_b)(\pi_b + (\nu - \tau\pi_b)\tau_b)}{(1-\beta\pi_p)^2}}{\frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(\alpha + \beta S_{pp}) - 2S_{pb}\tau\theta\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} + \frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(1-\alpha) + \lambda + \frac{(\tau\theta)^2}{\beta}S_{bb}} \\ &\quad + \frac{S_{pb}\left(\frac{(\varkappa - \theta\tau\pi_b)(1-\tau\tau_b) - \tau\theta(\pi_b + (\nu - \tau\pi_b)\tau_b)}{1-\beta\pi_p}\right) - S_{bb}\frac{\tau\theta}{\beta}(1-\tau\tau_b)}{\frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(\alpha + \beta S_{pp}) - 2S_{pb}\tau\theta\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} + \frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(1-\alpha) + \lambda + \frac{(\tau\theta)^2}{\beta}S_{bb}} \\ C_p &= -\frac{\left((\alpha + \beta S_{pp})(1 + (\nu - \tau\pi_b)\tau_p) + (1-\alpha)(\beta\pi_p + (\nu - \tau\pi_b)\tau_p)\right)\frac{(\varkappa - \theta\tau\pi_b)}{(1-\beta\pi_p)^2}}{\frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(\alpha + \beta S_{pp}) + \frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(1-\alpha) + \lambda - 2S_{pb}\tau\theta\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} + \frac{(\tau\theta)^2}{\beta}S_{bb}} \\ &\quad - \frac{S_{pb}\left(\frac{\tau\theta(1 + (\nu - \tau\pi_b)\tau_p) + (\varkappa - \theta\tau\pi_b)\tau\tau_p}{1-\beta\pi_p}\right) - S_{bb}\frac{\tau^2\theta}{\beta}\tau_p}{\frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(\alpha + \beta S_{pp}) + \frac{(\varkappa - \theta\tau\pi_b)^2}{(1-\beta\pi_p)^2}(1-\alpha) + \lambda - 2S_{pb}\tau\theta\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} + \frac{(\tau\theta)^2}{\beta}S_{bb}} \end{aligned}$$

A.3.2 Fiscal Policy Maker's Reaction Function (Simultaneous)

The Bellman equation which describes the policy decision in period t can be written as:

$$Vb_t^2 = \min_{\tau_t} \left(\pi_t^2 + \lambda_f c_t^2 + \beta^f \left(Vb_{t+1}^2 \right) \right) \tag{A.51}$$

and constraints are (A.64) and:

$$Vb_t^2 = \min_{\tau_t} \left(\pi_t^2 + \lambda c_t^2 + \beta \left(V_{pp} p_t^2 + 2V_{pb} b_{t+1} p_t + V_{bb} b_{t+1}^2 \right) \right) \tag{A.52}$$

and constraints are (A.64) and:

$$\pi_t = \frac{\beta\pi_p + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} + \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t \tag{A.53}$$

$$b_{t+1} = \frac{1}{\beta} \left((1 - \tau\theta C_b) b_t - \tau\theta C_p p_{t-1} - \tau\tau_t \right) \tag{A.54}$$

$$p_t = \frac{1 + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} + \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t \tag{A.55}$$

Substitute these equation into the Bellman equation

$$Vb_t^2 = \min_{\tau_t} \left(\begin{array}{c} \left(\frac{\beta\pi_p + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} + \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t \right)^2 + \lambda \left(C_b b_t + C_p p_{t-1} \right)^2 \\ + \beta V_{pp} \left(\frac{1 + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} + \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t \right)^2 \\ + 2\beta V_{pb} \frac{1}{\beta} \left((1 - \tau\theta C_b) b_t - \tau\theta C_p p_{t-1} - \tau\tau_t \right) \left(\frac{1 + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t \right. \\ \left. + \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t \right) + \beta V_{bb} \left(\frac{1}{\beta} \left((1 - \tau\theta C_b) b_t - \tau\theta C_p p_{t-1} - \tau\tau_t \right) \right)^2 \end{array} \right) \tag{A.56}$$

Differentiate with respect to τ_t we obtain:

$$0 = \left(\begin{array}{c} \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \frac{\beta\pi_p + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \beta V_{pp} \frac{1 + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} \\ + \beta V_{pb} \left(\frac{1}{\beta} (-\tau) \right) \frac{1 + (\varkappa - \theta\tau\pi_b) C_p}{1 - \beta\pi_p} p_{t-1} - \left(\frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \right) V_{pb} \tau\theta C_p p_{t-1} \\ + \tau V_{bb} \frac{1}{\beta} \tau\theta C_p p_{t-1} + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t \\ + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \beta V_{pp} \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t + \beta V_{pb} \left(\frac{1}{\beta} (-\tau) \right) \frac{\pi_b + (\varkappa - \theta\tau\pi_b) C_b}{1 - \beta\pi_p} b_t \\ + \left(\frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \right) V_{pb} (1 - \tau\theta C_b) b_t - \tau V_{bb} \frac{1}{\beta} (1 - \tau\theta C_b) b_t \\ + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t + \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \beta V_{pp} \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t \\ + \beta V_{pb} \left(\frac{1}{\beta} (-\tau) \right) \frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \tau_t - \left(\frac{\nu - \tau\pi_b}{1 - \beta\pi_p} \right) V_{pb} \tau\tau_t + \tau V_{bb} \frac{1}{\beta} \tau\tau_t \end{array} \right) \tag{A.57}$$

$$\tau_t = \tau_b b_t + \tau_p p_{t-1} \tag{A.58}$$

where

$$\tau_b = -\frac{\frac{(\nu-\tau\pi_b)(\pi_b+(\varkappa-\theta\tau\pi_b)C_b)}{(1-\beta\pi_p)^2}+\beta\frac{(\nu-\tau\pi_b)(\pi_b+(\varkappa-\theta\tau\pi_b)C_b)}{(1-\beta\pi_p)^2}V_{pp}}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}+\beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}-2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau+\frac{\tau^2}{\beta}V_{bb}} - \frac{\frac{(\nu-\tau\pi_b)(1-\tau\theta C_b)-\tau(\pi_b+(\varkappa-\theta\tau\pi_b)C_b)}{1-\beta\pi_p}V_{pb}-\frac{\tau}{\beta}(1-\tau\theta C_b)V_{bb}}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}+\beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}-2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau+\frac{\tau^2}{\beta}V_{bb}}$$

$$\tau_p = -\frac{\frac{(\nu-\tau\pi_b)(\beta\pi_p+(\varkappa-\theta\tau\pi_b)C_p)}{(1-\beta\pi_p)^2}+\frac{\beta(\nu-\tau\pi_b)(1+(\varkappa-\theta\tau\pi_b)C_p)}{(1-\beta\pi_p)^2}V_{pp}}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}+\beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}-2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau+\frac{\tau^2}{\beta}V_{bb}} + \frac{\frac{(1+(\varkappa-\theta\tau\pi_b)C_p+(\nu-\tau\pi_b)\theta C_p)}{1-\beta\pi_p}\tau V_{pb}+\frac{\theta\tau^2}{\beta}V_{bb}C_p}{\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}+\beta V_{pp}\frac{(\nu-\tau\pi_b)^2}{(1-\beta\pi_p)^2}-2\left(\frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)V_{pb}\tau+\frac{\tau^2}{\beta}V_{bb}}$$

A.3.3 Transition of the Economy (Simultaneous)

The transition of the economy can be now described by (A.71) and

$$c_t = C_b b_t \tag{A.59}$$

$$\pi_t = (\pi_b + (\varkappa - \pi_b\tau\theta)C_b + (\nu - \pi_b\tau)T_b)b_t$$

$$= \Pi_b b_t \tag{A.60}$$

$$b_{t+1} = \frac{1}{\beta}(1 - \tau\theta C_b - \tau T_b)b_t$$

$$= B_b b_t \tag{A.61}$$

A.3.4 Value Functions (Simultaneous)

Substitute (B.10)-(B.12) into (A.62) to obtain:

$$S = \Pi_b^2 + \lambda C_b^2 + \beta S B_b^2$$

Similarly, substitute (B.10)-(B.12) into (A.51) to obtain:

$$V = \Pi_b^2 + \lambda C_b^2 + \beta V B_b^2$$

so $S = V$

A.3.5 Final System (Simultaneous)

The final system

$$
\begin{aligned}
S &= V = \pi_b^2 + \lambda C_b^2 + \beta\left(\frac{1}{\beta}\left(1-\tau\theta C_b - \tau\tau_b\right)\right)^2 V \\
\tau_b &= -\frac{\left(\left(\nu-\pi_b\tau\right)\left(\pi_b + \left(\varkappa - \pi_b\tau\theta\right)C_b\right) - \tau V\frac{1}{\beta}\left(1-\tau\theta C_b\right)\right)}{\left(\left(\nu-\pi_b\tau\right)^2 + V\frac{\tau^2}{\beta}\right)} \\
\pi_b &= \left(\pi_b + \left(\varkappa - \pi_b\tau\theta\right)C_b + \left(\nu - \pi_b\tau\right)\tau_b\right) \\
C_b &= -\frac{\left(\left(\varkappa-\pi_b\tau\theta\right)\left(\pi_b + \left(\nu-\pi_b\tau\right)\tau_b\right) - \frac{\tau\theta S}{\beta}\left(1-\tau\tau_b\right)\right)}{\left(\lambda + V\frac{\tau^2\theta^2}{\beta} + \left(\varkappa-\pi_b\tau\theta\right)^2\right)}
\end{aligned}
$$

A.4 Discretionary Equilibrium (Fiscal leadership)

A.4.1 Monetary Policy Maker's Reaction Function (Follower)

The monetary policy maker's optimization problem in period t can be explained by the following Bellman equation:

$$
\mathcal{S}(b_t) = \min_{c_t}\left(p_t^2 + \lambda c_t^2 + \beta\mathcal{S}(b_{t+1})\right). \tag{A.62}
$$

Because of the linear-quadratic nature of the problem we assume that the value function $\mathcal{S}(b_t)$ is quadratic in state and substitute (2.11) and (2.35). We arrive to the following form:

$$
\left(S_{pp}p_{t-1}^2 + 2S_{pb}b_tp_{t-1} + S_{bb}b_t^2\right) = \min_{c_t}\left(p_t\right)^2 + \lambda\left(c_t\right)^2 + \beta\left(S_{pp}p_t^2 + 2S_{pb}b_{t+1}p_t + S_{bb}b_{t+1}^2\right)
$$

$$
\begin{aligned}
S_{pp}p_{t-1}^2 + 2S_{pb}b_tp_{t-1} + S_{bb}b_t^2 &= \min_{c_t}\left(\left(1-\alpha\right)\pi_t^2 + \alpha p_t^2 + \lambda c_t^2\right. \\
&\quad \left. + \beta\left(S_{pp}p_t^2 + 2S_{pb}b_{t+1}p_t + S_{bb}b_{t+1}^2\right)\right),
\end{aligned} \tag{A.63}
$$

$$
\begin{aligned}
Sb_t^2 &= \min_{c_t}\left(1-\alpha\right)\pi_t^2 + \left(\alpha + \beta S_{pp}\right)\left(p_{t-1}+\pi_t\right)^2 + \lambda\left(c_t\right)^2 \\
&\quad + \beta 2S_{pb}\left(\frac{1}{\beta}\left(b_t - \tau\theta c_t - \tau\tau_t\right)\right)\left(p_{t-1}+\pi_t\right) + \beta S_{bb}\left(\frac{1}{\beta}\left(b_t - \tau\theta c_t - \tau\tau_t\right)\right)^2
\end{aligned}
$$

$$
\begin{aligned}
Sb_t^2 &= \min_{c_t} (\alpha + \beta S_{pp}) \left(\frac{1}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \tau_t \right)^2 \\
&\quad + (1-\alpha) \left(\frac{\beta\pi_p}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \tau_t \right)^2 + \lambda c_t^2 \\
&\quad + 2S_{pb} (b_t - \tau\theta c_t - \tau\tau_t) \left(\frac{p_{t-1}}{1-\beta\pi_p} + \frac{\pi_b b_t}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \tau_t \right) \\
&\quad + \beta S_{bb} \left(\frac{1}{\beta} (b_t - \tau\theta c_t - \tau\tau_t) \right)^2
\end{aligned}
$$

from where:

$$
\begin{aligned}
0 &= 2(\alpha + \beta S_{pp}) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \left(\frac{1}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \tau_t \right) + 2\lambda c_t \\
&\quad + 2(1-\alpha) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \left(\frac{\beta\pi_p}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \tau_t \right) \\
&\quad + \beta 2S_{pb} \left(\frac{1}{\beta} (-\tau\theta) \right) \left(\frac{1}{1-\beta\pi_p} p_{t-1} + \frac{\pi_b}{1-\beta\pi_p} b_t + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_t + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \tau_t \right) \\
&\quad + \beta 2S_{pb} \left(\frac{1}{\beta} (b_t - \tau\theta c_t - \tau\tau_t) \right) \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right) - \tau\theta 2 \frac{1}{\beta} \beta S_{bb} \left(\frac{1}{\beta} (b_t - \tau\theta c_t - \tau\tau_t) \right)
\end{aligned}
$$

$$
c_t = c_p p_{t-1} + c_b b_t + c_\tau \tau_t \tag{A.64}
$$

where

$$
\begin{aligned}
c_p &= -\frac{\left((\alpha + \beta S_{pp} + (1-\alpha)\beta\pi_p) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta \right)}{(1-\beta\pi_p) \left(\lambda + (\beta S_{pp} + 1) \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right)^2 - 2 \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right) \tau\theta S_{pb} + \frac{1}{\beta} (\tau\theta)^2 S_{bb} \right)} \\
c_b &= -\frac{\left(\left((1 + \beta S_{pp}) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta \right) \frac{\pi_b}{1-\beta\pi_p} + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right) S_{pb} - \tau\theta S_{bb} \frac{1}{\beta} \right)}{\left(\lambda + (\beta S_{pp} + 1) \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right)^2 - 2 \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right) \tau\theta S_{pb} + \frac{1}{\beta} (\tau\theta)^2 S_{bb} \right)}, \\
c_\tau &= -\frac{\left(\left((\beta S_{pp} + 1) \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} - S_{pb}\tau\theta \right) \frac{\nu - \tau\pi_b}{1-\beta\pi_p} - \left(\left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right) S_{pb} - \tau\theta S_{bb} \frac{1}{\beta} \right) \tau \right)}{\left(\lambda + (\beta S_{pp} + 1) \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right)^2 - 2 \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} \right) \tau\theta S_{pb} + \frac{1}{\beta} (\tau\theta)^2 S_{bb} \right)}.
\end{aligned}
$$

A.4.2 Fiscal Policy Maker's Reaction Function (Leader)

The Bellman equation which describes the policy decision in period t can be written as:

$$
Vb_t^2 = \min_{\tau_t} \left(\pi_t^2 + \lambda c_t^2 + \beta \left(V_{pp} p_t^2 + 2V_{pb} b_{t+1} p_t + V_{bb} b_{t+1}^2 \right) \right) \tag{A.65}
$$

and constraints are (A.64) and:

$$\pi_t = \left(\frac{\beta\pi_p}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}c_p\right)p_{t-1} + \left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}c_b\right)b_t \quad \text{(A.66)}$$

$$+ \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p}\right)\tau_t \quad \text{(A.67)}$$

$$b_{t+1} = \frac{1}{\beta}\left(\left(1-\tau\theta c_b\right)b_t - \tau\theta c_p p_{t-1} - \tau\left(\theta c_\tau + 1\right)\tau_t\right) \quad \text{(A.68)}$$

$$p_t = \left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}c_p\right)p_{t-1} + \left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}c_b\right)b_t \quad \text{(A.69)}$$

$$+ \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p}\right)\tau_t \quad \text{(A.70)}$$

Substitute these equation into the Bellman equation A.65 and differentiate with respect to τ_t we obtain:

$$\tau_t = \tau_p p_{t-1} + \tau_b b_t \quad \text{(A.71)}$$

where

$$\tau_p = -\frac{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(\frac{\beta\pi_p}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_p\right) + \beta V_{pp}\left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_p\right) - V_{pb}\tau\theta c_p\right)}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(1+\beta V_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau\left(\theta c_\tau + 1\right)V_{pb}\right) + \lambda c_\tau^2 + \tau^2\left(\theta c_\tau + 1\right)^2 V_{bb}}$$

$$+\frac{\lambda c_\tau c_p - \tau\left(\theta c_\tau + 1\right)V_{pb}\left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_p\right) + \tau\left(\theta c_\tau + 1\right)V_{bb}\tau\theta c_p}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(1+\beta V_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau\left(\theta c_\tau + 1\right)V_{pb}\right) + \lambda c_\tau^2 + \tau^2\left(\theta c_\tau + 1\right)^2 V_{bb}}$$

$$\tau_b = -\frac{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_b\right) + \beta V_{pp}\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_b\right) + \beta V_{pb}\frac{1}{\beta}\left(1-\tau\theta c_b\right)\right)}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(1+\beta V_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau\left(\theta c_\tau + 1\right)V_{pb}\right) + \lambda c_\tau^2 + \tau^2\left(\theta c_\tau + 1\right)^2 V_{bb}}$$

$$+\frac{\lambda c_\tau c_b - \tau\left(\theta c_\tau + 1\right)V_{pb}\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_b\right) - \tau\left(\theta c_\tau + 1\right)V_{bb}\left(1-\tau\theta c_b\right)}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(1+\beta V_{pp}\right)\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau\left(\theta c_\tau + 1\right)V_{pb}\right) + \lambda c_\tau^2 + \tau^2\left(\theta c_\tau + 1\right)^2 V_{bb}}$$

A.4.3 Transition of the Economy (Fiscal leadership)

The transition of the economy can be now described by (A.71) and

$$c_t = (c_p + c_\tau \tau_p) p_{t-1} + (c_b + c_\tau \tau_b) b_t = C_p p_{t-1} + C_b b_t \tag{A.72}$$

$$\begin{aligned} \pi_t &= \left(\left(\frac{\beta \pi_p}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_p \right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \right) \tau_p \right) p_{t-1} \\ &\quad + \left(\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_b \right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \right) \tau_b \right) b_t \\ &= \pi_p p_{t-1} + \pi_b b_t \end{aligned} \tag{A.73}$$

$$\begin{aligned} b_{t+1} &= \frac{1}{\beta} \left((1 - \tau\theta c_b) - \tau (\theta c_\tau + 1) \tau_b \right) b_t - \tau \frac{1}{\beta} \left(\theta c_p + (\theta c_\tau + 1) \tau_p \right) p_{t-1} \\ &= B_p p_{t-1} + B_b b_t \end{aligned} \tag{A.74}$$

$$\begin{aligned} p_t &= \left(\left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_p \right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \right) \tau_p \right) p_{t-1} \\ &\quad + \left(\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_b \right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p} \right) \tau_b \right) b_t \\ &= P_p p_{t-1} + P_b b_t \end{aligned} \tag{A.75}$$

A.4.4 Value Functions (Fiscal leadership)

Equation (A.62) can be written as:

$$\begin{aligned} & \left(S_{pp} p_{t-1}^2 + 2 S_{pb} b_t p_{t-1} + S_{bb} b_t^2 \right) \\ =\ & \left(P_p p_{t-1} + P_b b_t \right)^2 + \lambda \left(C_p p_{t-1} + C_b b_t \right)^2 + \beta \left(S_{pp} p_t^2 + 2 S_{pb} b_{t+1} p_t + S_{bb} b_{t+1}^2 \right) \end{aligned}$$

$$\begin{aligned} S_{pp} &= \left(P_p^2 + \lambda C_p^2 + \beta S_{pp} P_p^2 + 2\beta S_{pb} B_p P_p + \beta S_{bb} B_p^2 \right) \\ S_{pb} &= P_p P_b + \lambda C_p C_b + \beta S_{pp} P_p P_b + \beta S_{pb} B_p P_b + \beta S_{pb} B_b P_p + \beta S_{bb} B_p B_b \\ S_{bb} &= P_b^2 + \lambda C_b^2 + \beta S_{pp} P_b^2 + 2\beta S_{pb} P_b B_b + \beta S_{bb} B_b^2 \end{aligned}$$

Similarly, Equation (A.65) can be written as:

$$\begin{aligned} & \left(V_{pp} p_{t-1}^2 + 2 V_{pb} b_t p_{t-1} + V_{bb} b_t^2 \right) \\ =\ & \left(\pi_p p_{t-1} + \pi_b b_t \right)^2 + \lambda \left(C_p p_{t-1} + C_b b_t \right)^2 + \beta \left(V_{pp} p_t^2 + 2 V_{pb} b_{t+1} p_t + V_{bb} b_{t+1}^2 \right) \end{aligned}$$

$$
\begin{aligned}
V_{pp} &= \left(\pi_p^2 + \lambda C_p^2 + \beta V_{pp} P_p^2 + 2\beta V_{pb} B_p P_p + \beta V_{bb} B_p^2\right) \\
V_{pb} &= \pi_p \pi_b + \lambda C_p C_b + \beta V_{pp} P_p P_b + \beta V_{pb} B_p P_b + \beta V_{pb} B_b P_p + \beta V_{bb} B_p B_b \\
V_{bb} &= \pi_b^2 + \lambda C_b^2 + \beta V_{pp} P_b^2 + 2\beta V_{pb} P_b B_b + \beta V_{bb} B_b^2
\end{aligned}
$$

A.4.5 Final System (Fiscal leadership)

For Stackelberg Equilibrium the complete system is

$$
\begin{aligned}
V_{pp} &= \left(\pi_p^2 + \lambda C_p^2 + \beta V_{pp} P_p^2 + 2\beta V_{pb} B_p P_p + \beta V_{bb} B_p^2\right) \\
V_{pb} &= \pi_p \pi_b + \lambda C_p C_b + \beta V_{pp} P_p P_b + \beta V_{pb} B_p P_b + \beta V_{pb} B_b P_p + \beta V_{bb} B_p B_b \\
V_{bb} &= \pi_b^2 + \lambda C_b^2 + \beta V_{pp} P_b^2 + 2\beta V_{pb} P_b B_b + \beta V_{bb} B_b^2 \\
S_{pp} &= \left(P_p^2 + \lambda C_p^2 + \beta S_{pp} P_p^2 + 2\beta S_{pb} B_p P_p + \beta S_{bb} B_p^2\right) \\
S_{pb} &= P_p P_b + \lambda C_p C_b + \beta S_{pp} P_p P_b + \beta S_{pb} B_p P_b + \beta S_{pb} B_b P_p + \beta S_{bb} B_p B_b \\
S_{bb} &= P_b^2 + \lambda C_b^2 + \beta S_{pp} P_b^2 + 2\beta S_{pb} P_b B_b + \beta S_{bb} B_b^2 \\
C_p &= (c_p + c_\tau \tau_p) \\
C_b &= (c_b + c_\tau \tau_b) \\
B_p &= -\tau \frac{1}{\beta} \left(\theta c_p + (\theta c_\tau + 1)\, \tau_p\right) \\
B_b &= \frac{1}{\beta} \left((1 - \tau\theta c_b) - \tau \left(\theta c_\tau + 1\right) \tau_b\right) \\
P_p &= \left(\left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_p\right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p}\right) \tau_p\right) \\
P_b &= \left(\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_b\right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p}\right) \tau_b\right) \\
\pi_p &= \left(\left(\frac{\beta\pi_p}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_p\right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p}\right) \tau_p\right) \\
\pi_b &= \left(\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_b\right) + \left(\frac{\varkappa - \theta\tau\pi_b}{1-\beta\pi_p} c_\tau + \frac{\nu - \tau\pi_b}{1-\beta\pi_p}\right) \tau_b\right)
\end{aligned}
$$

$$\tau_p = -\frac{\left(\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(\frac{\beta\pi_p}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_p\right) + \beta V_{pp}\left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_p\right) - V_{pb}\tau\theta c_p\right)\right)}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left((1+\beta V_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau(\theta c_\tau + 1)V_{pb}\right) + \lambda c_\tau^2 + \tau^2(\theta c_\tau + 1)^2 V_{bb}}$$

$$+\frac{\lambda c_\tau c_p - \tau(\theta c_\tau + 1)V_{pb}\left(\frac{1}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_p\right) + \tau(\theta c_\tau + 1)V_{bb}\tau\theta c_p}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left((1+\beta V_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau(\theta c_\tau + 1)V_{pb}\right) + \lambda c_\tau^2 + \tau^2(\theta c_\tau + 1)^2 V_{bb}}$$

$$\tau_b = -\frac{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left(\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_b\right) + \beta V_{pp}\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_b\right) + \beta V_{pb}\frac{1}{\beta}(1-\tau\theta c_b)\right)}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left((1+\beta V_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau(\theta c_\tau + 1)V_{pb}\right) + \lambda c_\tau^2 + \tau^2(\theta c_\tau + 1)^2 V_{bb}}$$

$$+\frac{\lambda c_\tau c_b - \tau(\theta c_\tau + 1)V_{pb}\left(\frac{\pi_b}{1-\beta\pi_p} + \frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_b\right) - \tau(\theta c_\tau + 1)V_{bb}(1-\tau\theta c_b)}{\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right)\left((1+\beta V_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}c_\tau + \frac{\nu-\tau\pi_b}{1-\beta\pi_p}\right) - 2\tau(\theta c_\tau + 1)V_{pb}\right) + \lambda c_\tau^2 + \tau^2(\theta c_\tau + 1)^2 V_{bb}}$$

$$c_p = -\frac{(1+\beta S_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right) - S_{pb}\tau\theta}{(1-\beta\pi_p)\left((1+\beta S_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2 - 2\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)\tau\theta S_{pb} + (\tau\theta)^2 S_{bb}\frac{1}{\beta} + \lambda\right)}$$

$$c_b = -\frac{(1+\beta S_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)\frac{\pi_b}{1-\beta\pi_p} - S_{pb}\frac{\tau\theta\pi_b}{1-\beta\pi_p} + \left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)S_{pb} - S_{bb}\frac{\tau\theta}{\beta}}{(1+\beta S_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2 - 2\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)\tau\theta S_{pb} + (\tau\theta)^2 S_{bb}\frac{1}{\beta} + \lambda}$$

$$c_\tau = -\frac{\left((1+\beta S_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right) - S_{pb}\tau\theta\right)\frac{\nu-\tau\pi_b}{1-\beta\pi_p} - \left(\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)S_{pb} - \tau\theta S_{bb}\frac{1}{\beta}\right)\tau}{(1+\beta S_{pp})\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)^2 - 2\left(\frac{\varkappa-\theta\tau\pi_b}{1-\beta\pi_p}\right)\tau\theta S_{pb} + (\tau\theta)^2 S_{bb}\frac{1}{\beta} + \lambda}$$

A.5 Dynamic responses in cooperative regimes

The impulse responses with different level of steady state debt are plotted in A.1 below, to show the difference between different cooperative regimes and the commitment regime which results the best outcome. Figure A.1 demonstrated that under all levels of B we assumed, without risk premier, debt level goes up to absorb all the disturbance, and tax rate is raised while consumption falls permanently but only up to the amount which is enough to sustain the higher steady state debt level. This results in much smaller fluctuation in targeted variables and social welfare loss.

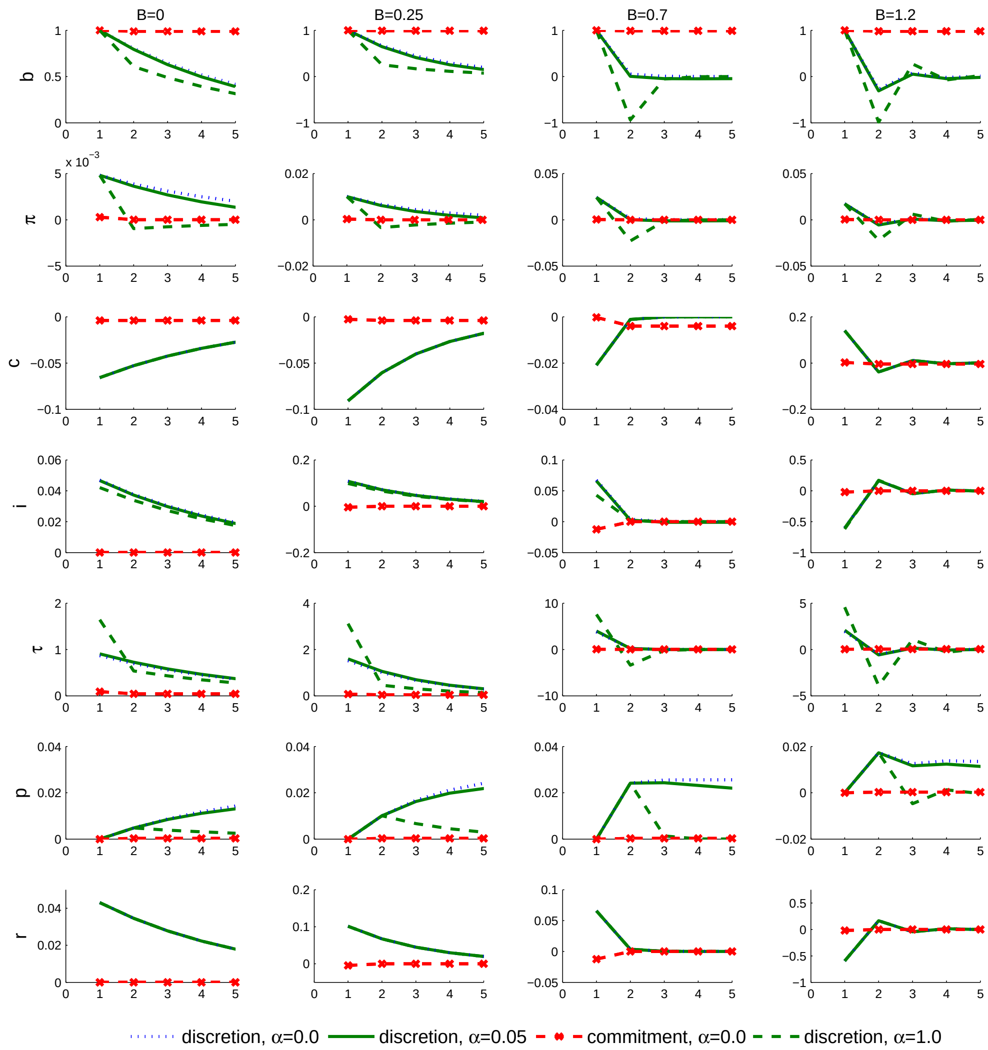

Figure A.1: Dynamic responses in cooperative regimes

Appendix B

Appendix to Chapter 3

B.1 Model Derivation

The final deterministic system for private sector is same as in the last Chapter:

$$\begin{aligned} b_{t+1} &= \frac{1}{\beta}\left(b_t - \tau\theta\hat{C} - \tau\hat{\tau}_t\right) \\ \hat{\pi}_t &= \beta\mathcal{E}_t\hat{\pi}_{t+1} + \kappa\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)\hat{C}_t \end{aligned}$$

with $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}$

B.2 Quarterly Fiscal Stabilization Discretionary Policy

The private sector observes we guess and verify later that the private sector's reaction function is a linear function of the state:

$$\pi_t = \pi_b b_t \tag{B.1}$$

Take (B.1) one step forward and use (2.9) to obtain:

$$\pi_t = \pi_b b_t + (\varkappa - \pi_b\tau\theta)\, c_t + (\nu - \pi_b\tau)\,\tau_t \tag{B.2}$$

This is a linear reaction of the private sector to government debt, consumption and tax rate.

B.2.1 Monetary Policy Maker's Reaction Function (Follower)

The monetary policy maker's optimization problem in period t can be explained by the following Bellman equation:

$$\mathcal{S}(b_t) = \min_{c_t} \left(\pi_t^2 + \lambda c_t^2 + \beta^m \mathcal{S}(b_{t+1})\right). \tag{B.3}$$

Because of the linear-quadratic nature of the problem we assume that the value function $\mathcal{S}(b_t, \eta_t)$ is quadratic in state and substitute (B.2) and (2.11) in:

$$Sb_t^2 = \min_{c_t} ((\pi_b b_t + (\varkappa - \pi_b \tau\theta) c_t \tag{B.4}$$
$$+ (\nu - \pi_b \tau) \tau_t)^2 + \lambda c_t^2 + \beta S \left(\frac{1}{\beta}(b_t - \tau\theta c_t - \tau\tau_t)\right)^2$$

from where:

$$c_t = c_b b_t + c_\tau \tau_t \tag{B.5}$$

where

$$\begin{aligned} c_b &= -\frac{\left((\varkappa - \pi_b\tau\theta)\pi_b - S\frac{\tau\theta}{\beta}\right)}{\left((\varkappa - \pi_b\tau\theta)^2 + \lambda + S\frac{\tau^2\theta^2}{\beta}\right)}, \\ c_\tau &= -\frac{\left((\varkappa - \pi_b\tau\theta)(\nu - \pi_b\tau) + S\frac{\tau^2\theta}{\beta}\right)}{\left((\varkappa - \pi_b\tau\theta)^2 + \lambda + S\frac{\tau^2\theta^2}{\beta}\right)}. \end{aligned}$$

B.2.2 Fiscal Policy Maker's Reaction Function (Leader)

The Bellman equation which describes the policy decision in period t can be written as:

$$Vb_t^2 = \min_{\tau_t} \left(\pi_t^2 + \lambda_f c_t^2 + \beta^f \left(Vb_{t+1}^2\right)\right) \tag{B.6}$$

and constraints are (B.5) and:

$$\pi_t = (\pi_b + (\varkappa - \pi_b\tau\theta) c_b) b_t + ((\varkappa - \pi_b\tau\theta) c_\tau + (\nu - \pi_b\tau)) \tau_t \tag{B.7}$$

$$b_{t+1} = \frac{1}{\beta}((1 - \tau\theta c_b) b_t - \tau (1 + \theta c_\tau) \tau_t) \tag{B.8}$$

Substitute these equation into the Bellman equation and differentiate with respect to τ_t we obtain:

$$\tau_t = \tau_b b_t \tag{B.9}$$

where

$$\tau_b = -\frac{\left(\left((\varkappa - \pi_b \tau\theta)\, c_\tau + \nu - \pi_b \tau\right)\left(\pi_b + (\varkappa - \pi_b \tau\theta)\, c_b\right) + \lambda c_\tau c_b - \tau\left(1 + \theta c_\tau\right) V \frac{1}{\beta}\left(1 - \tau\theta c_b\right)\right)}{\left(\left((\varkappa - \pi_b \tau\theta)\, c_\tau + \nu - \pi_b \tau\right)^2 + \lambda c_\tau^2 + V \frac{\tau^2}{\beta}\left(1 + \theta c_\tau\right)^2\right)}$$

B.2.3 Transition of the Economy

The transition of the economy can be now described by (B.9) and

$$\begin{aligned} c_t &= \left(c_b + c_\tau \tau_b\right) b_t = C_b b_t & \text{(B.10)} \\ \pi_t &= \left(\pi_b + (\varkappa - \pi_b \tau\theta)\, c_b + \left((\varkappa - \pi_b \tau\theta)\, c_\tau + (\nu - \pi_b \tau)\right)\tau_b\right) b_t & \\ &= \pi_b b_t & \text{(B.11)} \\ b_{t+1} &= \frac{1}{\beta}\left(1 - \tau\theta c_b - \tau\left(1 + \theta c_\tau\right)\tau_b\right) b_t & \\ &= B_b b_t & \text{(B.12)} \end{aligned}$$

B.2.4 Value Functions

Substitute (B.10)-(B.12) into (B.4) to obtain:

$$S = \pi_b^2 + \lambda C_b^2 + \beta S B_b^2$$

Similarly, substitute (B.10)-(B.12) into (B.6) to obtain:

$$V = \pi_b^2 + \lambda C_b^2 + \beta V B_b^2$$

B.2.5 Final System

For Stackelberg Equilibrium the complete system is

$$V = \pi_b^2 + \lambda\left(c_b + c_\tau \tau_b\right)^2 + \beta\left(\frac{1}{\beta}\left(1 - \tau\theta c_b - \tau\left(1 + \theta c_\tau\right)\tau_b\right)\right)^2 V \tag{B.13}$$

$$\tau_b = -\frac{\left(\left(\varkappa - \pi_b\tau\theta\right)c_\tau + \nu - \pi_b\tau\right)\left(\pi_b + \left(\varkappa - \pi_b\tau\theta\right)c_b\right) + \lambda c_\tau c_b}{\left(\left(\left(\varkappa - \pi_b\tau\theta\right)c_\tau + \nu - \pi_b\tau\right)^2 + \lambda c_\tau^2 + V\frac{\tau^2}{\beta}\left(1 + \theta c_\tau\right)^2\right)} + \frac{\tau\left(1 + \theta c_\tau\right)V\frac{1}{\beta}\left(1 - \tau\theta c_b\right)}{\left(\left(\left(\varkappa - \pi_b\tau\theta\right)c_\tau + \nu - \pi_b\tau\right)^2 + \lambda c_\tau^2 + V\frac{\tau^2}{\beta}\left(1 + \theta c_\tau\right)^2\right)} \tag{B.14}$$

$$\pi_b = \left(\pi_b + \left(\varkappa - \pi_b\tau\theta\right)c_b + \left(\left(\varkappa - \pi_b\tau\theta\right)c_\tau + \left(\nu - \pi_b\tau\right)\right)\tau_b\right) \tag{B.15}$$

$$c_b = -\frac{\left(\left(\varkappa - \pi_b\tau\theta\right)\pi_b - V\frac{\tau\theta}{\beta}\right)}{\left(\left(\varkappa - \pi_b\tau\theta\right)^2 + \lambda + V\frac{\tau^2\theta^2}{\beta}\right)} \tag{B.16}$$

$$c_\tau = -\frac{\left(\left(\varkappa - \pi_b\tau\theta\right)\left(\nu - \pi_b\tau\right) + V\frac{\tau^2\theta}{\beta}\right)}{\left(\left(\varkappa - \pi_b\tau\theta\right)^2 + \lambda + V\frac{\tau^2\theta^2}{\beta}\right)} \tag{B.17}$$

Here 5 equations for 5 unknowns $\pi_b, c_b, c_\tau, \tau_b, V$.

B.3 Biannual Fiscal Stabilization Discretionary Policy

The essential solution methods used for different length of fiscal policy cycle are similar, therefore we use $N = 2$ as an example. In this case, Equation B.11 and B.12 can be written as:

$$\begin{aligned}\pi_{p,t} &= \left(\pi_b^{-p} + \left(\varkappa - \pi_b^{-p}\tau\theta\right)c_b^p + \left(\left(\varkappa - \pi_b^{-p}\tau\theta\right)c_\tau^p + \left(\nu - \pi_b^{-p}\tau\right)\right)\tau_b^p\right)b_{p,t} \\ &= \pi_b^p b_{p,t}\end{aligned} \tag{B.18}$$

$$\begin{aligned}b_{-p,t+1} &= \frac{1}{\beta}\left(1 - \tau\theta c_b^p - \tau\left(1 + \theta c_\tau^p\right)\tau_b^p\right)b_{p,t} \\ &= B_b^p b_{p,t}\end{aligned} \tag{B.19}$$

B.3.1 Monetary Policy Maker's Optimization Problem (Follower)

The monetary policy maker's problem in any period $p \in \{e, o\}$ is identical to the one described in the previous section except that the value function depends on whether the fiscal policy maker reoptimizes or not. Assuming the quadratic form for the appropriate value function we can write the Bellman equation for the monetary policy maker in period p:

$$
\begin{aligned}
S_p b_{p,t}^2 = \min_{c_{p,t}} & \left(\pi_b^{-p} b_{p,t} + \left(\varkappa - \pi_b^{-p}\tau\theta\right) c_{p,t} + \left(\nu - \pi_b^{-p}\tau\right)\tau_{p,t}^2\right. \\
& \left. + \lambda c_{p,t}^2 + \beta S_{-p}\left(\frac{1}{\beta}\left(b_{p,t} - \tau\theta c_{p,t} - \tau\tau_{p,t}\right)\right)^2\right),
\end{aligned} \tag{B.20}
$$

where we substituted constraints (B.18) and (B.19) written for the appropriate period $p \in \{e, o\}$.

Optimization with respect $c_{p,t}$ yields:

$$
c_{p,t} = c_b^p b_{p,t} + c_\tau^p \tau_{p,t} \tag{B.21}
$$

where

$$
\begin{aligned}
c_b^p &= -\frac{\left(\left(\varkappa - \tau\theta\pi_b^{-p}\right)\pi_b^{-p} - \frac{\tau\theta}{\beta}S_{-p}\right)}{\left(\left(\varkappa - \tau\theta\pi_b^{-p}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}S_{-p}\right)}, \\
c_\tau^p &= -\frac{\left(\left(\varkappa - \tau\theta\pi_b^{-p}\right)\left(\nu - \tau\pi_b^{-p}\right) + \frac{\tau^2\theta}{\beta}S_{-p}\right)}{\left(\left(\varkappa - \tau\theta\pi_b^{-p}\right)^2 + \lambda + \frac{\tau^2\theta^2}{\beta}S_{-p}\right)}
\end{aligned}
$$

B.3.2 Fiscal Policy Maker's Optimization Problem (Leader)

The fiscal policy maker only optimizes in even time periods. The Bellman equation which describes the policy decision in period e can be written as:

$$
Vb_{e,t}^2 = \min_{\tau_{e,t}} \pi_{e,t}^2 + \lambda c_{e,t}^2 + \beta\left(\pi_{o,t+1}^2 + \lambda c_{o,t+1}^2\right) + \beta^2\left(Vb_{e,t+2}^2\right) \tag{B.22}
$$

constraints (B.21), (B.18) and (B.19) are applied at any period $p \in \{e, o\}$. Taking (B.19) two periods forward and applying (B.21), (B.18) we obtain:

$$\begin{aligned} b_{e,t+2} &= \frac{1}{\beta^2}\left(1-\tau\theta c_b^o\right)\left(1-\tau\theta c_b^e\right) b_{e,t} - \frac{1}{\beta^2}\left(1-\tau\theta c_b^o\right)\tau\left(1+\theta c_\tau^e\right)\tau_{e,t} \\ &\quad -\frac{\tau}{\beta}\left(1+\theta c_\tau^o\right)\tau_{o,t+1} \end{aligned} \tag{B.23}$$

The state in the even period $t+2$ depends on fiscal policy in both periods: $\tau_{e,t}$ and $\tau_{o,t+1}$.

We assume that the fiscal policy maker, when choosing $\tau_{e,t}$ on dates e, also sets $\tau_{e,t}$ such that

$$\tau_{o,t+1} = \tau_{e,t}. \tag{B.24}$$

We can demonstrate that such policy can have a state-space representation

$$\tau_{p,t} = \tau_b^p b_{p,t}. \tag{B.25}$$

Take (B.25) one period forward and use (B.24) to obtain

$$\begin{aligned} \tau_{o,t+1} &= \tau_b^o b_{o,t+1} = \frac{1}{\beta}\tau_b^o\left(1-\tau\theta c_b^e - \tau\left(1+\theta c_\tau^e\right)\tau_b^e\right) b_{e,t} \\ &= \tau_b^e b_{e,t}, \end{aligned}$$

from where

$$\tau_b^o = \frac{\beta\tau_b^e}{\left(1-\tau\left(\left(1+\theta c_\tau^e\right)\tau_b^e + \theta c_b^e\right)\right)} \tag{B.26}$$

Finally, we substitute (B.24) into constraint (B.23). The complete set of constraints can be written as

$$\begin{aligned}
\pi_{e,t} &= \left(\pi_b^o + (\varkappa - \pi_b^o\tau\theta)\, c_b^e\right) b_{e,t} + \left((\varkappa - \pi_b^o\tau\theta)\, c_\tau^e + (\nu - \pi_b^o\tau)\right)\tau_{e,t} && \text{(B.27)}\\
&= \Pi_b^e b_{e,t} + \Pi_\tau^e \tau_{e,t}\\
c_{e,t} &= c_b^e b_{e,t} + c_\tau^e \tau_{e,t} && \text{(B.28)}\\
c_{o,t+1} &= \frac{1}{\beta} c_b^o \left(1 - \tau\theta c_b^e\right) b_{e,t} + \left(c_\tau^o - \frac{\tau}{\beta} c_b^o \left(1 + \theta c_\tau^e\right)\right)\tau_{e,t} && \text{(B.29)}\\
&= C_b^x b_{e,t} + C_\tau^x \tau_{e,t}\\
\pi_{o,t+1} &= \frac{1}{\beta}\left(\pi_b^e + (\varkappa - \pi_b^e\tau\theta)\, c_b^o\right)\left(1 - \tau\theta c_b^e\right) b_{e,t} + \left((\varkappa - \pi_b^e\tau\theta)\, c_\tau^o + \right. && \text{(B.30)}\\
&\quad \left. \nu - \pi_b^e\tau - \frac{\tau}{\beta}\left(\pi_b^e + (\varkappa - \pi_b^e\tau\theta)\, c_b^o\right)\left(1 + \theta c_\tau^e\right)\right)\tau_{e,t}\\
&= \Pi_b^x b_{e,t} + \Pi_\tau^x \tau_{e,t}
\end{aligned}$$

$$\begin{aligned}
b_{e,t+2} &= \frac{1}{\beta^2}\left(1 - \tau\theta c_b^o\right)\left(1 - \tau\theta c_b^e\right) b_{e,t} && \text{(B.31)}\\
&\quad - \frac{\tau}{\beta}\left(\left(1 + \theta c_\tau^o\right) + \frac{1}{\beta}\left(1 - \tau\theta c_b^o\right)\left(1 + \theta c_\tau^e\right)\right)\tau_{e,t}\\
&= B_b^x b_{e,t} + B_\tau^x \tau_{e,t}
\end{aligned}$$

Substitute Equations (B.27) to (B.31) into the Bellman equation (B.22) and differentiate with respect to $\tau_{e,t}$ we obtain:

$$\tau_{e,t} = -\frac{\left(\Pi_\tau^e \Pi_b^e + \lambda c_\tau^e c_b^e + \beta \Pi_\tau^x \Pi_b^x + \beta\lambda C_\tau^x C_b^x + \beta^2 B_\tau^x B_b^x V\right)}{\left(\left(\Pi_\tau^e\right)^2 + \lambda\left(c_\tau^e\right)^2 + \beta\left(\Pi_\tau^x\right)^2 + \beta\lambda\left(C_\tau^x\right)^2 + \beta^2 V\left(B_\tau^x\right)^2\right)} b_{e,t} \tag{B.32}$$

then substitute (B.26) and (B.32) into (B.27)-(B.31) we can solve the transition of economy.

B.3.3 Value Functions

Bellman equation (B.20) yields

$$S_e = \left(\pi_b^e\right)^2 + \lambda\left(C_b^e\right)^2 + \beta S_o\left(B_b^e\right)^2 \tag{B.33}$$

$$S_o = \left(\pi_b^o\right)^2 + \lambda\left(C_b^o\right)^2 + \beta S_e\left(B_b^o\right)^2 \tag{B.34}$$

$$S_e = (\pi_b^e)^2 + \lambda (C_b^e)^2 + \beta (\pi_b^o)^2 (B_b^e)^2 + \lambda\beta (C_b^o)^2 (B_b^e)^2 + \beta^2 S_e (B_b^e)^2 (B_b^o)^2 \quad \text{(B.35)}$$

$$V = (\pi_b^e)^2 + \lambda (C_b^e)^2 + \beta (\pi_b^o)^2 (B_b^e)^2 + \lambda\beta (C_b^o)^2 (B_b^e)^2 + \beta^2 V (B_b^e)^2 (B_b^o)^2 \quad \text{(B.36)}$$

Therefore, for benevolent policy makers $V = S_e$. At the period when both benevolent policy makers reoptimize, their value functions are the same. The final system:

$$\begin{aligned}
S_o &= (\pi_b^o)^2 + \lambda (c_b^o + c_\tau^o \tau_b^o)^2 + \beta V (B_b^o)^2 \\
V &= (\pi_b^e)^2 + \lambda (c_b^e + c_\tau^e \tau_b^e)^2 + \beta \left((\pi_b^o)^2 + \lambda (c_b^o + c_\tau^o \tau_b^o)^2 + \beta (B_b^o)^2 V \right) (B_b^e)^2 \\
\tau_b^e &= -\frac{\left(\Pi_\tau^e \Pi_b^e + \lambda c_\tau^e c_b^e + \beta \Pi_\tau^x \Pi_b^x + \beta\lambda C_\tau^x C_b^x + \beta^2 B_\tau^x B_b^x V \right)}{\left((\Pi_\tau^e)^2 + \lambda (c_\tau^e)^2 + \beta (\Pi_\tau^x)^2 + \beta\lambda (C_\tau^x)^2 + \beta^2 V (B_\tau^x)^2 \right)} \\
\tau_b^o &= \frac{\beta \tau_b^e}{(1 - \tau ((1 + \theta c_\tau^e) \tau_b^e + \theta c_b^e))} \\
\pi_b^e &= \pi_b^o + (\varkappa - \pi_b^o \tau\theta)(c_b^e + c_\tau^e \tau_b^e) + (\nu - \pi_b^o \tau) \tau_b^e \\
\pi_b^o &= \pi_b^e + (\varkappa - \pi_b^e \tau\theta)(c_b^o + c_\tau^o \tau_b^o) + (\nu - \pi_b^e \tau) \tau_b^o \\
c_b^e &= -\frac{\left((\varkappa - \tau\theta\pi_b^o) \pi_b^o - \frac{\tau\theta}{\beta} S_o \right)}{\left((\varkappa - \tau\theta\pi_b^o)^2 + \lambda + \frac{\tau^2\theta^2}{\beta} S_o \right)} \\
c_b^o &= -\frac{\left((\varkappa - \tau\theta\pi_b^e) \pi_b^e - \frac{\tau\theta}{\beta} V \right)}{\left((\varkappa - \tau\theta\pi_b^e)^2 + \lambda + \frac{\tau^2\theta^2}{\beta} V \right)} \\
c_\tau^e &= -\frac{\left((\varkappa - \tau\theta\pi_b^o)(\nu - \tau\pi_b^o) + \frac{\tau^2\theta}{\beta} S_o \right)}{\left((\varkappa - \tau\theta\pi_b^o)^2 + \lambda + \frac{\tau^2\theta^2}{\beta} S_o \right)} \\
c_\tau^o &= -\frac{\left((\varkappa - \tau\theta\pi_b^e)(\nu - \tau\pi_b^e) + \frac{\tau^2\theta}{\beta} V \right)}{\left((\varkappa - \tau\theta\pi_b^e)^2 + \lambda + \frac{\tau^2\theta^2}{\beta} V \right)}
\end{aligned}$$

B.4 Biannual Fiscal Stabilization Discretionary Policy (Matrix form)

We assume a non-singular linear deterministic rational expectations model, augmented by a vector of control instruments.[1] Specifically, the evolution of the economy is explained

[1] None of the results presented here depend on the deterministic setup outlined and the consequent assumption of perfect foresight. Shocks can be included into vector y_t, see e.g. Anderson et al. (1996), Blake and Kirsanova (2012).

by the linear system

$$\begin{bmatrix} \mathsf{y}_{t+1} \\ \mathsf{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathsf{y}_t \\ \mathsf{x}_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{bmatrix} \tag{B.37}$$

where y_t is an n_1-vector of predetermined variables with initial conditions y_0 given, x_t is n_2-vector of non-predetermined (or jump) variables with $\lim_{t\to\infty} \mathsf{x}_t = 0$, u_t^F and u_t^L are the two vectors of policy instruments of two policy makers, named F and L, of size k_F and k_L respectively. For notational convenience we define the n-vector $\mathsf{z}_t = (\mathsf{y}_t', \mathsf{x}_t')'$ where $n = n_1 + n_2$, and the k-vector of control variables $\mathsf{u}_t = (\mathsf{u}_t^{L\prime}, \mathsf{u}_t^{F\prime})'$, where $k = k_F + k_L$. We assume the equations are ordered so that A_{22} is non-singular.

Typically, the second block of equations in this system represents an aggregation of the first order conditions to the optimization problem of the private sector, which has decision variables x_t. Additionally, there is a first block of equations which explains the evolution of the predetermined state variables y_t. These two blocks describe the 'evolution of the economy' as observed by policy makers.

The inter-temporal welfare criterion of policy maker i, $i \in \{L, F\}$, is defined by the quadratic loss function

$$W_t^i = \sum_{s=t}^{\infty} \beta^{s-t}(g_s^{i\prime} \mathcal{Q}^i g_s^i) = \sum_{s=t}^{\infty} \beta^{s-t}(\mathsf{z}_s' Q^i \mathsf{z}_s + 2\mathsf{z}_s' P^i \mathsf{u}_s + \mathsf{u}_s' R^i \mathsf{u}_s). \tag{B.38}$$

The elements of vector g_s^i are the goal variables of policy maker i, $g_s^i = \mathcal{C}^i(\mathsf{z}_s', \mathsf{u}_s')'$. Matrix $\mathcal{Q}^i$ is assumed to be symmetric and positive semi-definite.[2]

B.4.1 Follower's Problem

Central banker the follower re-optimizes discretionary monetary policy at every period, i.e. times t=0,1,2,3,...

[2]It is standard to assume that R is symmetric positive definite (see Anderson et al. (1996), for example). However, since many economic applications involve a loss function that places no penalty on the control variables, we note that the requirement of $\mathcal{Q}$ being positive definite can be weakened to $\mathcal{Q}$ being positive semi-definite if additional assumptions about other system matrices are met (Clements and Wimmer (2003)). The analysis in this paper is valid for $R \equiv 0$.

Even periods

Denote even periods welfare is V

$$\mathsf{y}_t' V^F \mathsf{y}_t = \min_{\mathsf{u}_t^F} \left(\mathsf{z}_t' Q^F \mathsf{z}_t + 2\mathsf{z}_t' P^F \mathsf{u}_t + \mathsf{u}_t' R^F \mathsf{u}_t + \beta^F \mathsf{y}_{t+1}' W^F \mathsf{y}_{t+1} \right) \tag{B.39}$$

$$\mathsf{y}_t' V^F \mathsf{y}_t = \min_{\mathsf{u}_t^F} \left(\begin{matrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{matrix}{}' \begin{matrix} Q_{11}^F & Q_{12}^F & P_{11}^F & P_{12}^F \\ Q_{21}^F & Q_{22}^F & P_{21}^F & P_{22}^F \\ P_{11}^{F\prime} & P_{21}^{F\prime} & R_{11}^F & R_{12}^F \\ P_{12}^{F\prime} & P_{22}^{F\prime} & R_{21}^F & R_{22}^F \end{matrix} \begin{matrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{matrix} + \beta^F \mathsf{y}_{t+1}' W^F \mathsf{y}_{t+1} \right) \tag{B.40}$$

$$\begin{aligned} \mathsf{x}_{t+1}^o &= -N^o \mathsf{y}_{t+1}^o = -N^o \left(A_{11} \mathsf{y}_t^e + A_{12} \mathsf{x}_t^e + B_{11} \mathsf{u}_t^{Le} + B_{12} \mathsf{u}_t^{Fe} \right) \\ &= A_{21} \mathsf{y}_t^e + A_{22} \mathsf{x}_t^e + B_{21} \mathsf{u}_t^{Le} + B_{22} \mathsf{u}_t^{Fe} \end{aligned}$$

from where it follows:

$$\begin{aligned} \mathsf{x}_t^e &= -(A_{22} + N^o A_{12})^{-1} \left((A_{21} + N^o A_{11}) \mathsf{y}_t + (B_{22} + N^o B_{12}) \mathsf{u}_t^F \right. \\ &\quad \left. + (B_{21} + N^o B_{11}) \mathsf{u}_t^L \right) = -J^e \mathsf{y}_t - K^e \mathsf{u}_t = -J^e \mathsf{y}_t - K^{Fe} \mathsf{u}_t^F - K^{Le} \mathsf{u}_t^L \end{aligned}$$

$$\begin{aligned} J^e &= (A_{22} + N^o A_{12})^{-1} (A_{21} + N^o A_{11}) \\ K^{Fe} &= (A_{22} + N^o A_{12})^{-1} (B_{22} + N^o B_{12}) \\ K^{Le} &= (A_{22} + N^o A_{12})^{-1} (B_{21} + N^o B_{11}) \end{aligned}$$

$$\begin{aligned} \hat{A}^e &= A_{11} - A_{12} J^e, \\ \hat{B}_1^e &= B_{11} - A_{12} K^{Le}, \\ \hat{B}_2^e &= B_{12} - A_{12} K^{Fe}. \end{aligned}$$

$$\begin{aligned} \mathsf{y}_{t+1}^o &= (A_{11} - A_{12} J^e) \mathsf{y}_t + \left(B_{11} - A_{12} K^{Le} \right) \mathsf{u}_t^L + \left(B_{12} - A_{12} K^{Fe} \right) \mathsf{u}_t^F \\ \mathsf{y}_{t+1}^o &= \hat{A}^e \mathsf{y}_t + \hat{B}_1^e \mathsf{u}_t^L + \hat{B}_2^e \mathsf{u}_t^F \end{aligned}$$

$$\begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix}' \begin{pmatrix} Q_{11}^F & Q_{12}^F & P_{11}^F & P_{12}^F \\ Q_{21}^F & Q_{22}^F & P_{21}^F & P_{22}^F \\ P_{11}^{F\prime} & P_{21}^{F\prime} & R_{11}^F & R_{12}^F \\ P_{12}^{F\prime} & P_{22}^{F\prime} & R_{21}^F & R_{22}^F \end{pmatrix} \begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix}$$

$$= \begin{pmatrix} \mathsf{y}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix}' \begin{pmatrix} I & -J' & 0 & 0 \\ 0 & -K^{L\prime} & I & 0 \\ 0 & -K^{F\prime} & 0 & I \end{pmatrix} \begin{pmatrix} Q_{11}^F & Q_{12}^F & P_{11}^F & P_{12}^F \\ Q_{21}^F & Q_{22}^F & P_{21}^F & P_{22}^F \\ P_{11}^{F\prime} & P_{21}^{F\prime} & R_{11}^F & R_{12}^F \\ P_{12}^{F\prime} & P_{22}^{F\prime} & R_{21}^F & R_{22}^F \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ -J & -K^L & -K^F \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \begin{pmatrix} \mathsf{y}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix}$$

Therefore the FOC with respect to follower's action:

$$\begin{aligned} \frac{\partial}{\partial \mathsf{u}_t^F} &= \left(P_{12}^{F\prime} - K^{F\prime}Q_{21}^F - P_{22}^{F\prime}J + K^{F\prime}Q_{22}^F J + \beta^F \hat{B}_2' W^F \hat{A}\right) \mathsf{y}_t \\ &+ \left(-P_{22}^{F\prime}K^L + K^{F\prime}Q_{22}^F K^L - K^{F\prime}P_{21}^F + R_{21}^F + \beta^F \hat{B}_2' W^F \hat{B}_1\right) \mathsf{u}_t^L \\ &+ \left(-P_{22}^{F\prime}K^F + K^{F\prime}Q_{22}^F K^F - K^{F\prime}P_{22}^F + R_{22}^F + \beta^F \hat{B}_2' W^F \hat{B}_2\right) \mathsf{u}_t^F \end{aligned}$$

$$\begin{aligned} \mathsf{u}_t^F &= -\left(\hat{R}_{22}^{Fe} + \beta^F \hat{B}_2^{e\prime} W^F \hat{B}_2^e\right)^{-1} \left(\hat{P}_2^{\prime Fe} + \beta^F \hat{B}_2^{e\prime} W^F \hat{A}^e\right) \mathsf{y}_t \\ &\quad - \left(\hat{R}_{22}^{Fe} + \beta^F \hat{B}_2^{e\prime} W^F \hat{B}_2^e\right)^{-1} \left(\hat{R}_{12}^{\prime Fe} + \beta^F \hat{B}_2^{e\prime} W^F \hat{B}_1^e\right) \mathsf{u}_t^L \end{aligned}$$

Where

$$\begin{aligned} \hat{Q}^{Fe} &= Q_{11}^F - Q_{12}^F J^e - J^{e\prime} Q_{21}^F + J^{e\prime} Q_{22}^F J^e, \\ \hat{P}_1^{Fe} &= J^{e\prime} Q_{22}^F K^{Le} - Q_{12}^F K^{Le} + P_{11}^F - J^{e\prime} P_{21}^F, \\ \hat{P}_2^{Fe} &= J^{e\prime} Q_{22}^F K^{Fe} - Q_{12}^F K^{Fe} + P_{12}^F - J^{e\prime} P_{22}^F, \\ \hat{R}_{11}^{Fe} &= K^{Le\prime} Q_{22}^F K^{Le} - K^{Le\prime} P_{21}^F - P_{21}^{F\prime} K^{Le} + R_{11}^F, \\ \hat{R}_{12}^{Fe} &= K^{Le\prime} Q_{22}^F K^{Fe} - K^{Le\prime} P_{22}^F - P_{21}^{F\prime} K^{Fe} + R_{12}^F, \\ \hat{R}_{22}^{Fe} &= K^{Fe\prime} Q_{22}^F K^{Fe} + R_{22}^F - K^{Fe\prime} P_{22}^L - P_{22}^{F\prime} K^{Fe}, \end{aligned}$$

Finally for even time t:

$$\begin{aligned} \mathsf{u}_t^{Fe} &= -F^{Fe}\mathsf{y}_t^e - L^{Fe}\mathsf{u}_t^{Le} \\ \mathsf{x}_t^e &= -\left(J^e - K^{Fe}F^{Fe}\right)\mathsf{y}_t^e - \left(K^{Le} - K^{Fe}L^{Fe}\right)\mathsf{u}_t^{Le} \\ \mathsf{y}_{t+1}^o &= \left(\hat{A}^e - \hat{B}_2^e F^{Fe}\right)\mathsf{y}_t^e + \left(\hat{B}_1^e - \hat{B}_2^e L^{Fe}\right)\mathsf{u}_t^{Le} \end{aligned}$$

Suppose we know F^{Le} in

$$\mathsf{u}_t^{Le} = -F^{Le}\mathsf{y}_t^e$$

Then

$$\begin{aligned}
\mathsf{u}_t^{Fe} &= -\left(F^{Fe}-L^{Fe}F^{Le}\right)\mathsf{y}_t^e = -G^{Fe}\mathsf{y}_t^e \\
\mathsf{x}_t^e &= -\left(\left(J^e-K^{Fe}F^{Fe}\right)-\left(K^{Le}-K^{Fe}L^{Fe}\right)F^{Le}\right)\mathsf{y}_t^e = -N^e\mathsf{y}_t^e \\
\mathsf{y}_{t+1}^o &= \left(\left(\hat{A}^e-\hat{B}_2^eF^{Fe}\right)-\left(\hat{B}_1^e-\hat{B}_2^eL^{Fe}\right)F^{Le}\right)\mathsf{y}_t^e = M^e\mathsf{y}_t^e
\end{aligned}$$

The Bellman equation characterizing discretionary policy of policy maker F, therefore, becomes

$$\begin{aligned}
V^F &= \begin{pmatrix} I \\ -F^{Le} \\ -G^{Fe} \end{pmatrix}' \begin{pmatrix} \hat{Q}^{Fe} & \hat{P}_1^{Fe} & \hat{P}_2^{Fe} \\ \hat{P}_1'^{F}e & \hat{R}_{11}^{Fe} & \hat{R}_{12}^{Fe} \\ \hat{P}_2'^{Fe} & \hat{R}_{12}'^{Fe} & \hat{R}_{22}^{Fe} \end{pmatrix} \begin{pmatrix} I \\ -F^{Le} \\ -G^{Fe} \end{pmatrix} + \beta^F M^{e\prime}W^F M^e && \text{(B.41)} \\
&= \hat{Q}^{Fe} - F^{Le\prime}\hat{P}_1'^{F} - \hat{P}_1^{Fe}F^{Le} + F^{Le\prime}\hat{R}_{11}^{Fe}F^{Le} - \hat{P}_2^{Fe}G^{Fe} - G^{Fe\prime}\hat{P}_2'^{Fe} && \text{(B.42)} \\
&\quad + F^{Le\prime}\hat{R}_{12}^{Fe}G^{Fe} + G^{Fe\prime}\hat{R}_{12}'^{Fe}F^{Le} + G^{Fe\prime}\hat{R}_{22}^{Fe}G^{Fe} + \beta^F M^{e\prime}W^F M^e && \text{(B.43)}
\end{aligned}$$

Odd periods

We denote the odd times welfare as W

$$\mathsf{y}_t'W^F\mathsf{y}_t = \min_{\mathsf{u}_t^F}\left(\mathsf{z}_t'Q^F\mathsf{z}_t + 2\mathsf{z}_t'P^F\mathsf{u}_t + \mathsf{u}_t'R^F\mathsf{u}_t + \beta^F\mathsf{y}_{t+1}'V^F\mathsf{y}_{t+1}\right) \quad \text{(B.44)}$$

$$\mathsf{y}_t'W^F\mathsf{y}_t = \min_{\mathsf{u}_t^F}\left(\begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix}' \begin{pmatrix} Q_{11}^F & Q_{12}^F & P_{11}^F & P_{12}^F \\ Q_{21}^F & Q_{22}^F & P_{21}^F & P_{22}^F \\ P_{11}^{F\prime} & P_{21}^{F\prime} & R_{11}^F & R_{12}^F \\ P_{12}^{F\prime} & P_{22}^{F\prime} & R_{21}^F & R_{22}^F \end{pmatrix} \begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix} + \beta^F\mathsf{y}_{t+1}'V^F\mathsf{y}_{t+1}\right) \quad \text{(B.45)}$$

$$\begin{aligned}
\mathsf{x}_{t+1}^e &= -N^e\mathsf{y}_{t+1}^e = -N^e\left(A_{11}\mathsf{y}_t^o + A_{12}\mathsf{x}_t^o + B_{11}\mathsf{u}_t^{Lo} + B_{12}\mathsf{u}_t^{Fo}\right) \\
&= A_{21}\mathsf{y}_t^o + A_{22}\mathsf{x}_t^o + B_{21}\mathsf{u}_t^{Lo} + B_{22}\mathsf{u}_t^{Fo}
\end{aligned}$$

from where it follows:

$$\begin{aligned}\mathsf{x}_t^o &= -(A_{22}+N^eA_{12})^{-1}\left((A_{21}+N^eA_{11})\mathsf{y}_t+(B_{22}+N^eB_{12})\mathsf{u}_t^{Fo}\right.\\ &\quad \left.+(B_{21}+N^eB_{11})\mathsf{u}_t^L\right) = -J^o\mathsf{y}_t - K^o\mathsf{u}_t = -J^e\mathsf{y}_t - K^{Fo}\mathsf{u}_t^F - K^{Lo}\mathsf{u}_t^L\end{aligned}$$

$$\begin{aligned}J^o &= (A_{22}+N^eA_{12})^{-1}(A_{21}+N^eA_{11})\\ K^{Fo} &= (A_{22}+N^eA_{12})^{-1}(B_{22}+N^eB_{12})\\ K^{Lo} &= (A_{22}+N^eA_{12})^{-1}(B_{21}+N^eB_{11})\end{aligned}$$

$$\begin{aligned}\hat{A}^o &= A_{11}-A_{12}J^o,\\ \hat{B}_1^o &= B_{11}-A_{12}K^{Lo},\\ \hat{B}_2^o &= B_{12}-A_{12}K^{Fo}.\end{aligned}$$

$$\begin{aligned}\mathsf{y}_{t+1}^e &= (A_{11}-A_{12}J^o)\,\mathsf{y}_t+\left(B_{11}-A_{12}K^{Lo}\right)\mathsf{u}_t^L+\left(B_{12}-A_{12}K^{Fo}\right)\mathsf{u}_t^F\\ \mathsf{y}_{t+1}^e &= \hat{A}^o\mathsf{y}_t+\hat{B}_1^o\mathsf{u}_t^L+\hat{B}_2^o\mathsf{u}_t^F\end{aligned}$$

$$\begin{aligned}\hat{Q}^{Fo} &= Q_{11}^F-Q_{12}^FJ^o-J^{o\prime}Q_{21}^F+J^{o\prime}Q_{22}^FJ^o,\\ \hat{P}_1^{Fo} &= J^{o\prime}Q_{22}^FK^{Lo}-Q_{12}^FK^{Lo}+P_{11}^F-J^{o\prime}P_{21}^F,\\ \hat{P}_2^{Fo} &= J^{o\prime}Q_{22}^FK^{Fo}-Q_{12}^FK^{Fo}+P_{12}^F-J^{o\prime}P_{22}^F,\\ \hat{R}_{11}^{Fo} &= K^{Lo\prime}Q_{22}^FK^{Lo}-K^{Lo\prime}P_{21}^F-P_{21}^{F\prime}K^{Lo}+R_{11}^F,\\ \hat{R}_{12}^{Fo} &= K^{Lo\prime}Q_{22}^FK^{Fo}-K^{Lo\prime}P_{22}^F-P_{21}^{F\prime}K^{Fo}+R_{12}^F,\\ \hat{R}_{22}^{Fo} &= K^{Fo\prime}Q_{22}^FK^{Fo}+R_{22}^F-K^{Fo\prime}P_{22}^L-P_{22}^{F\prime}K^{Fo},\end{aligned}$$

get

$$\begin{aligned}\mathsf{u}_t^F &= -\left(\hat{R}_{22}^{Fo}+\beta^F\hat{B}_2^{o\prime}V^F\hat{B}_2^o\right)^{-1}\left(\hat{P}_2^{\prime Fo}+\beta^F\hat{B}_2^{o\prime}V^F\hat{A}^o\right)\mathsf{y}_t\\ &\quad -\left(\hat{R}_{22}^{Fo}+\beta^F\hat{B}_2^{o\prime}V^F\hat{B}_2^o\right)^{-1}\left(\hat{R}_{12}^{\prime Fo}+\beta^F\hat{B}_2^{o\prime}V^F\hat{B}_1^o\right)\mathsf{u}_t^L\\ \mathsf{u}_t^{Fo} &= -F^{Fo}\mathsf{y}_t^o-L^{Fo}\mathsf{u}_t^{Lo}\end{aligned}$$

Finally for odd time t:

$$\begin{aligned}
\mathsf{u}_t^{Fo} &= -F^{Fo}\mathsf{y}_t^o - L^{Fo}\mathsf{u}_t^{Lo} \\
\mathsf{x}_t^o &= -\left(J^o - K^{Fo}F^{Fo}\right)\mathsf{y}_t^o - \left(K^{Lo} - K^{Fo}L^{Fo}\right)\mathsf{u}_t^{Lo} \\
\mathsf{y}_{t+1}^e &= \left(\hat{A}^o - \hat{B}_2^o F^{Fo}\right)\mathsf{y}_t^o + \left(\hat{B}_1^o - \hat{B}_2^o L^{Fo}\right)\mathsf{u}_t^{Lo}
\end{aligned}$$

Suppose we know F^{Lo} in

$$\mathsf{u}_t^{Lo} = -F^{Lo}\mathsf{y}_t^o$$

Then

$$\begin{aligned}
\mathsf{u}_t^{Fo} &= -G^{Fo}\mathsf{y}_t^o \\
\mathsf{x}_t^o &= -N^o\mathsf{y}_t^o \\
\mathsf{y}_{t+1}^e &= M^o\mathsf{y}_t^o
\end{aligned}$$

The Bellman equation characterizing discretionary policy of policy maker F, therefore, becomes

$$W^F = \begin{pmatrix} I \\ -F^{Lo} \\ -G^{Fo} \end{pmatrix}' \begin{pmatrix} \hat{Q}^{Fo} & \hat{P}_1^{Fo} & \hat{P}_2^{Fo} \\ \hat{P}_1'^{Fo} & \hat{R}_{11}^{Fo} & \hat{R}_{12}^{Fo} \\ \hat{P}_2'^{Fo} & \hat{R}_{12}'^{Fo} & \hat{R}_{22}^{Fo} \end{pmatrix} \begin{pmatrix} I \\ -F^{Lo} \\ -G^{Fo} \end{pmatrix} + \beta^F M^{o\prime} V^F M^o \tag{B.46}$$

What remains is to find F^{Lo} and F^{Le}

B.4.2 Leader's Problem

Fiscal policy is set at even periods only, i.e. t=0,2,4,6... Denote welfare S^L

$$\mathsf{y}_t^{e\prime} S^L \mathsf{y}_t^e = \min_{\mathsf{u}_t^L} \begin{pmatrix} \begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix}' \begin{pmatrix} Q_{11}^L & Q_{12}^L & P_{11}^L & P_{12}^L \\ Q_{21}^L & Q_{22}^L & P_{21}^L & P_{22}^L \\ P_{11}^{L\prime} & P_{21}^{L\prime} & R_{11}^L & R_{12}^L \\ P_{12}^{L\prime} & P_{22}^{L\prime} & R_{21}^L & R_{22}^L \end{pmatrix} \begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{pmatrix} \\ +\beta^L \begin{pmatrix} \mathsf{y}_{t+1} \\ \mathsf{x}_{t+1} \\ \mathsf{u}_{t+1}^L \\ \mathsf{u}_{t+1}^F \end{pmatrix}' \begin{pmatrix} Q_{11}^L & Q_{12}^L & P_{11}^L & P_{12}^L \\ Q_{21}^L & Q_{22}^L & P_{21}^L & P_{22}^L \\ P_{11}^{L\prime} & P_{21}^{L\prime} & R_{11}^L & R_{12}^L \\ P_{12}^{L\prime} & P_{22}^{L\prime} & R_{21}^L & R_{22}^L \end{pmatrix} \begin{pmatrix} \mathsf{y}_{t+1} \\ \mathsf{x}_{t+1} \\ \mathsf{u}_{t+1}^L \\ \mathsf{u}_{t+1}^F \end{pmatrix} \\ + \left(\beta^L\right)^2 \mathsf{y}_{t+2}^{e\prime} S^L \mathsf{y}_{t+2}^e \end{pmatrix} \tag{B.47}$$

$$\mathsf{x}^o_{t+1} = -N^o \mathsf{y}^o_{t+1} = -N^o \left(A_{11}\mathsf{y}^e_t + A_{12}\mathsf{x}^e_t + B_{11}\mathsf{u}^{Le}_t + B_{12}\mathsf{u}^{Fe}_t \right)$$
$$= A_{21}\mathsf{y}^e_t + A_{22}\mathsf{x}^e_t + B_{21}\mathsf{u}^{Le}_t + B_{22}\mathsf{u}^{Fe}_t$$

from where it follows:

$$\begin{aligned}
\mathsf{x}^e_t &= -(A_{22} + N^o A_{12})^{-1} \big((A_{21} + N^o A_{11})\mathsf{y}_t + (B_{22} + N^o B_{12})\mathsf{u}^{Fe}_t \\
&\quad + (B_{21} + N^o B_{11})\mathsf{u}^{Le}_t \big) = -J^e \mathsf{y}_t - K^{Fe}\mathsf{u}^{Fe}_t - K^{Le}\mathsf{u}^{Le}_t \\
\mathsf{u}^{Fe}_t &= -F^{Fe}\mathsf{y}^e_t - L^{Fe}\mathsf{u}^{Le}_t
\end{aligned}$$

$$\begin{aligned}
\mathsf{x}^e_t &= -\left(J^e - K^{Fe}F^{Fe}\right)\mathsf{y}^e_t - \left(K^{Le} - K^{Fe}L^{Fe}\right)\mathsf{u}^{Le}_t \\
\mathsf{u}^{Fe}_t &= -F^{Fe}\mathsf{y}^e_t - L^{Fe}\mathsf{u}^{Le}_t
\end{aligned}$$

$$\begin{aligned}
\mathsf{y}^o_{t+1} &= (A_{11} - A_{12}J^e)\,\mathsf{y}_t + \left(B_{11} - A_{12}K^{Le}\right)\mathsf{u}^L_t + \left(B_{12} - A_{12}K^{Fe}\right)\mathsf{u}^F_t \\
&= \left(A_{11} - A_{12}J^e - \left(B_{12} - A_{12}K^{Fe}\right)F^{Fe}\right)\mathsf{y}_t + \left(B_{11} - A_{12}K^{Le} - \left(B_{12} - A_{12}K^{Fe}\right)L^{Fe}\right)\mathsf{u}^L_t \\
\mathsf{y}^o_{t+1} &= \hat{A}^e\mathsf{y}_t + \hat{B}^e_1\mathsf{u}^L_t + \hat{B}^e_2\mathsf{u}^F_t
\end{aligned}$$

$$\begin{pmatrix} \mathsf{y}_t \\ \mathsf{x}_t \\ \mathsf{u}^L_t \\ \mathsf{u}^F_t \end{pmatrix} = \begin{pmatrix} I & 0 \\ -\left(J^e - K^{Fe}F^{Fe}\right) & -\left(K^{Le} - K^{Fe}L^{Fe}\right) \\ 0 & I \\ -F^{Fe} & -L^{Fe} \end{pmatrix} \begin{pmatrix} \mathsf{y}_t \\ \mathsf{u}^L_t \end{pmatrix}$$

The optimization problem becames

$$\mathsf{y}'_t S^L \mathsf{y}_t = \min_{\mathsf{u}^L_t} \left(\begin{matrix} \mathsf{y}_t \\ \mathsf{u}^L_t \end{matrix} {}' \begin{matrix} \hat{Q}^{Le} & \hat{P}^{Le} \\ \hat{P}^{Le\prime} & \hat{R}^{Le} \end{matrix} \begin{matrix} \mathsf{y}_t \\ \mathsf{u}^L_t \end{matrix} + \beta^L \begin{matrix} \mathsf{y}_{t+1} \\ \mathsf{u}^L_{t+1} \end{matrix} {}' \begin{matrix} \hat{Q}^{Lo} & \hat{P}^{Lo} \\ \hat{P}^{Lo\prime} & \hat{R}^{Lo} \end{matrix} \begin{matrix} \mathsf{y}_{t+1} \\ \mathsf{u}^L_{t+1} \end{matrix} + \left(\beta^L\right)^2 \mathsf{y}'_{t+2} S^L \mathsf{y}_{t+2} \right)$$

fiscal policy stays at the same value for two periods, which means

$$\mathsf{u}^{Lo}_{t+1} = \mathsf{u}^{Le}_t$$

Let's check that this assumption makes sense so there will be a time-consistent representation. We know that

$$\begin{aligned}
\mathsf{u}^{Lo}_{t+1} &= -F^{Lo}\mathsf{y}^o_{t+1} = -F^{Lo}\left(\left(\hat{A}^e\right)\mathsf{y}^e_t + \left(\hat{B}^e\right)\mathsf{u}^{Le}_t\right) = -F^{Lo}\left(\left(\hat{A}^e\right) - \left(\hat{B}^e\right)F^{Le}\right)\mathsf{y}^e_t \\
&= \mathsf{u}^{Le}_t = -F^{Le}\mathsf{y}^e_t
\end{aligned}$$

Therefore

$$F^{Lo} = F^{Le}\left(\hat{A}^e - \hat{B}^e F^{Le}\right)^{-1}$$

This will ensure $\mathsf{u}_{t+1}^{Lo} = \mathsf{u}_t^{Le}$.

Next

$$\begin{array}{c} \mathsf{y}_{t+1}^o \\ \mathsf{u}_{t+1}^{Lo} \end{array} = \begin{array}{cc} \left(\hat{A} - \hat{B}_2 F^{Fe}\right) & \left(\hat{B}_1 - \hat{B}_2 L^{Fe}\right) \\ 0 & I \end{array} \begin{array}{c} \mathsf{y}_t^e \\ \mathsf{u}_t^{Le} \end{array}$$

Also

$$\begin{aligned} \mathsf{y}_{t+2}^e &= \left(\hat{A}\right)\mathsf{y}_{t+1}^o + \left(\hat{B}\right)\mathsf{u}_{t+1}^{Lo} = \left(\hat{A}\right)\left(\left(\hat{A}\right)\mathsf{y}_t^e + \left(\hat{B}\right)\mathsf{u}_t^{Le}\right) + \left(\hat{B}\right)\mathsf{u}_t^{Le} \\ &= \left(\hat{A} - \hat{B}_2 F^{Fo}\right)\left(\hat{A} - \hat{B}_2 F^{Fe}\right)\mathsf{y}_t^e \\ &\quad + \left(\left(\hat{A} - \hat{B}_2 F^{Fo}\right)\left(\hat{B}_1 - \hat{B}_2 L^{Fe}\right) + \left(\hat{B}_1 - \hat{B}_2 L^{Fo}\right)\right)\mathsf{u}_t^{Le} \\ &= C\mathsf{y}_t^e + D\mathsf{u}_t^{Le} \end{aligned}$$

$$\begin{aligned} C &= \left(\hat{A}^o\right)\left(\hat{A}^e\right) \\ D &= \left(\left(\hat{A}^o\right)\left(\hat{B}^e\right) + \left(\hat{B}^o\right)\right) \end{aligned}$$

and

$$\mathsf{y}_t' S^L \mathsf{y}_t = \min_{\mathsf{u}_t^L}\left(\begin{array}{c} \mathsf{y}_t \\ \mathsf{u}_t^L \end{array}{}'\left(\begin{array}{cc} T_{11} & T_{12} \\ T_{21} & T_{22} \end{array}\right)\begin{array}{c} \mathsf{y}_t^e \\ \mathsf{u}_t^{Le} \end{array} + \left(\beta^L\right)^2\left(C\mathsf{y}_t^e + D\mathsf{u}_t^{Le}\right)' S^L\left(C\mathsf{y}_t^e + D\mathsf{u}_t^{Le}\right)\right) \tag{B.48}$$

where

$$\begin{aligned} \left(\begin{array}{cc} T_{11} & T_{12} \\ T_{21} & T_{22} \end{array}\right) &= \left(\begin{array}{c} \mathsf{y}_t \\ \mathsf{u}_t^L \end{array}\right)'\left(\begin{array}{cc} \hat{Q}^{Le} & \hat{P}^{Le} \\ \hat{P}^{Le\prime} & \hat{R}^{Le} \end{array}\right)\left(\begin{array}{c} \mathsf{y}_t \\ \mathsf{u}_t^L \end{array}\right) \\ &\quad + \beta^L\left(\begin{array}{cc} \left(\hat{A} - \hat{B}_2 F^{Fe}\right)' & 0 \\ \left(\hat{B}_1 - \hat{B}_2 L^{Fe}\right)' & I \end{array}\right)\left(\begin{array}{cc} \hat{Q}^{Lo} & \hat{P}^{Lo} \\ \hat{P}^{Lo\prime} & \hat{R}^{Lo} \end{array}\right)\left(\begin{array}{cc} \left(\hat{A} - \hat{B}_2 F^{Fe}\right) & \left(\hat{B}_1 - \hat{B}_2 L^{Fe}\right) \\ 0 & I \end{array}\right) \end{aligned}$$

FOCs:

$$\frac{\partial}{\partial \mathsf{u}_t^{Le}} = \left(T_{21} + \left(\beta^L\right)^2 D' S^L C\right)\mathsf{y}_t^e + \left(T_{22} + \left(\beta^L\right)^2 D' S^L D\right)\mathsf{u}_t^{Le} = 0$$

$$\mathsf{u}_t^{Le} = -\left(T_{22} + \left(\beta^L\right)^2 D' S^L D\right)^{-1} \left(T_{21} + \left(\beta^L\right)^2 D' S^L C\right) \mathsf{y}_t^e = -F^{Le} \mathsf{y}_t^e$$

Finally for even time t:

$$\mathsf{u}_t^{Le} = -F^{Le} \mathsf{y}_t^e$$

So the remaining Bellman equation for S is

$$S^L = \begin{matrix} I \\ -F^{Le} \end{matrix}' \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{matrix} I \\ -F^{Le} \end{matrix} + \left(\beta^L\right)^2 \left(C - DF^{Le}\right)' S^L \left(C - DF^{Le}\right) \quad \text{(B.49)}$$

Appendix C

Appendix to Chapter 4

C.1 Model Derivations

C.1.1 Households

The Lagrangian for household's utility maximisation can be written as:

$$\mathcal{E}_t \sum_{v=t}^{\infty} \left[\frac{\beta}{1+p}\right]^{v-t} [u(C_v, \xi_v) - v(N_v(z), \xi_v)]$$
$$+\lambda \left(\sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v} P_v C_v) - \mathcal{A}_t - \sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v} \int_0^1 (1-\tau_v)\left(W_v(z) N_v(z) + \Pi_v(z)\right) dz + T_{vt}) \right)$$

and the first order conditions are:

$$\frac{\partial L}{\partial N_v^s(z)} = 0 = -\left[\frac{\beta}{1+p}\right]^{v-t} v_N(N_v^s, \xi_v) + \lambda Q_{t,v}^s (1-\tau_v) W_v(z) \tag{C.1}$$

$$\frac{\partial L}{\partial C_v^s} = 0 = \left[\frac{\beta}{1+p}\right]^{v-t} u_C(C_v^s, \xi_v) - \lambda Q_{t,v}^s P_v \tag{C.2}$$

$$\frac{\partial L}{\partial \lambda} = 0 = \sum_{v=t}^{\infty} \mathcal{E}_t(Q_{t,v}^s P_v C_v^s) - \mathcal{A}_t^s \tag{C.3}$$

$$-\sum_{v=t}^{\infty} \mathcal{E}_t \left(Q_{t,v}^s \int_0^1 (1-\tau_v)\left(W_v(z) N_v^s(z) + \Pi_v(z)\right) dz \right) \tag{C.4}$$

$$\left[\frac{\beta}{1+p}\right]^{v-t} v_N(N_v^s,\xi_v) = \lambda Q_{t,v}^s(1-\tau_v)W_v(z)$$

$$W_v(z) = \frac{\varkappa}{\lambda}\left[\frac{\beta}{1+p}\right]^{v-t}\frac{\xi_v\left(N_v\xi_v\right)^{1/\psi}}{Q_{t,v}^s(1-\tau_v)}$$

$$\lambda = \left[\frac{\beta}{1+p}\right]^{v-t}\frac{\xi_v\left(C_v^s\xi_v\right)^{-1/\sigma}}{Q_{t,v}^s P_v}$$

$$W_v(z) = \frac{\varkappa\left(C_v^s\xi_v\right)^{1/\sigma}\left(N_v\xi_v\right)^{1/\psi}P_v}{(1-\tau_v)}$$

Divide $\frac{\partial L}{\partial C_v^s}$ by itself and obtain:

$$\frac{\beta}{1+p}\frac{u_C(C_{v+1}^s,\xi_{v+1})}{u_C(C_v^s,\xi_v)}\frac{P_v}{P_{v+1}} = \frac{Q_{v+1}^s}{Q_v^s} = Q_{v,v+1}^s \tag{C.5}$$

So for the generation born at time s, the household maximization leads to:

$$C_v^s = \left[\frac{1+p}{\beta}\frac{P_{v+1}}{P_v}Q_{v,v+1}^s\right]^{\sigma} C_{v+1}^s\frac{\xi_{v+1}}{\xi_v} \tag{C.6}$$

Therefore

$$C_v^s = C_t^s\frac{C_v^s}{C_t^s} = C_t^s\prod_{k=0}^{v-t-1}\left(\frac{C_{t+k+1}^s}{C_{t+k}^s}\right) = C_t^s\prod_{k=0}^{v-1-t}\left[\frac{1+p}{\beta}\frac{P_{t+k+1}}{P_{t+k}}Q_{t+k,t+k+1}^s\right]^{-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$P_v = P_t\frac{P_v}{P_t} = P_t^s\prod_{k=0}^{v-t-1}\left(\frac{P_{t+k+1}}{P_{t+k}}\right) = P_t\prod_{k=0}^{v-t-1}(1+\pi_{t+k+1})$$

We have for an *individual's* consumption and wealth from a generation born at time s :

$$P_tC_t^s + \sum_{v=t+1}^{\infty}Q_{t,v}^sP_vC_v^s = P_tC_t^s + P_tC_t^s\sum_{v=0}^{\infty}Q_{t,t+v+1}^s\frac{P_{t+1+v}}{P_t}\frac{C_{t+1+v}^s}{C_t^s}$$

$$= P_tC_t^s + P_tC_t^s\sum_{v=0}^{\infty}Q_{t,t+v+1}^s\frac{P_{t+1+v}}{P_t}\prod_{k=0}^{v}\left[\frac{1+p}{\beta}\frac{P_{t+k+1}}{P_{t+k}}Q_{t+k,t+k+1}^s\right]^{-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$= P_tC_t^s + P_tC_t^s\sum_{v=0}^{\infty}\left(\frac{\beta}{1+p}\right)^{(v+1)\sigma}\frac{P_{t+1+v}}{P_t}\prod_{k=0}^{v}\left(\frac{P_{t+k+1}}{P_{t+k}}\right)^{-\sigma}\left(Q_{t+k,t+k+1}^s\right)^{1-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}}$$

$$= P_tC_t^s + P_tC_t^s\sum_{v=1}^{\infty}\left(\frac{\beta}{1+p}\right)^{v\sigma}\prod_{k=0}^{v-1}\left(\frac{P_{t+k+1}}{P_{t+k}}\right)^{1-\sigma}\left(Q_{t+k,t+k+1}^s\right)^{1-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}} = P_tC_t^s\Phi_t$$

where

$$\Phi_t = 1+\sum_{v=1}^{\infty}\left(\frac{\beta}{1+p}\right)^{v\sigma}\prod_{k=0}^{v-1}\left(\frac{P_{t+k+1}}{P_{t+k}}Q^s_{t+k,t+k+1}\right)^{1-\sigma}\frac{\xi_{t+k}}{\xi_{t+k+1}} \tag{C.7}$$
$$= 1+\left(\frac{\beta}{1+p}\right)^{\sigma}\left(\frac{P_{t+1}}{P_t}Q^s_{t,t+1}\right)^{1-\sigma}\frac{\xi_t}{\xi_{t+1}}\Phi_{t+1}$$

and from the last FOCs,i.e. household inter-tempral budget constrain, it follows that:

$$\Phi_t P_t C^s_t = (\mathcal{A}^s_t + \mathcal{H}^s_t) \tag{C.8}$$

where nominal human capital $\mathcal{H}^s_t$ is:

$$\mathcal{H}^s_t = \mathcal{E}_t\sum_{v=t}^{\infty}Q^s_{t,v}\left(\int_0^1(1-\tau)\left(w_v(z)h^s_v(z)+\Pi_v(z)\right)dz+T^a_v\right)$$

To obtain consumption rule for all the households, we aggregate all relationships across all generations. The size of total population at time t is

$$\frac{p}{(1+p)}\sum_{s=-\infty}^{t}\left(\frac{1}{1+p}\right)^{t-s} = 1.$$

Therefore

$$C^a_t = \sum_{s=-\infty}^{t}\frac{p}{(1+p)}\left(\frac{1}{1+p}\right)^{t-s}C^s_t, \quad \mathcal{A}^a_t = \sum_{s=-\infty}^{t}\frac{p}{(1+p)}\left(\frac{1}{1+p}\right)^{t-s}\mathcal{A}^s_t$$

and we define aggregate nominal human capital as:

$$\mathcal{H}^a_t = \sum_{s=-\infty}^{t}\frac{p}{(1+p)}\left(\frac{1}{1+p}\right)^{t-s}\mathcal{E}_t\sum_{v=t}^{\infty}Q^s_{t,v}\left(\int_0^1(1-\tau)\left(w_v(z)h^s_v(z)+\Pi^s_v(z)\right)dz+T^a_v\right)$$
$$= \mathcal{E}_t\sum_{v=t}^{\infty}Q^s_{t,v}\left((1-\tau)Y_vP_v+T^a_v\right)$$

where

$$Y_vP_v = \sum_{s=-\infty}^{t}\frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}\int_0^1\left(w_v(z)h^s_v(z)+\Pi^s_v(z)\right)dz$$
$$T^a_v = \sum_{s=-\infty}^{t}\frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}T^s_v$$

Aggregating relationship (C.8) yields:

$$P_tC^a_t = \frac{1}{\Phi_t}\left(\mathcal{A}^a_t+\mathcal{H}^a_t\right).$$

We now derive a dynamic Euler equation for aggregate consumption. We note that

$$\begin{aligned}
\mathcal{A}^a_{t+1} &= \frac{1}{Q^s_{t,t+1}}\frac{1}{1+p}\sum_{s=-\infty}^{t}\frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}\left(\mathcal{A}^s_t - P_tC^s_t + \int_0^1(1-\tau)w_t(z)h^s_t(z)dz + T^s_t\right)\\
&= \frac{1}{Q^s_{t,t+1}}\frac{1}{1+p}\left(\mathcal{A}^a_t + (1-\tau)P_tY_t + T^a_t - P_tC^a_t\right)\\
\mathcal{H}^a_{t+1} &= \sum_{v=t+1}^{\infty} Q^s_{t+1,v}(1-\tau)Y_vP_v = \frac{Q^s_{t,t+1}}{Q^s_{t,t+1}}\sum_{v=t+1}^{\infty} R_{t+1,v}(1-\tau)Y_vP_v\\
&= \frac{1}{Q^s_{t,t+1}}\left(\sum_{v=t}^{\infty} Q^s_{t,v}(1-\tau)Y_vP_v - Q^s_{t,t}(1-\tau)Y_tP_t\right) = \frac{1}{Q^s_{t,t+1}}(\mathcal{H}^a_t - (1-\tau)Y_tP_t)
\end{aligned}$$

Therefore

$$\begin{aligned}
\Phi_{t+1}P_{t+1}C^a_{t+1} &= \mathcal{A}^a_{t+1} + \mathcal{H}^a_{t+1} = \mathcal{A}^a_{t+1} + \frac{1}{Q^s_{t,t+1}}(\mathcal{H}^a_t - (1-\tau)Y_tP_t)\\
&= \mathcal{A}^a_{t+1} + \frac{1}{Q^s_{t,t+1}}(\Phi_tP_tC^a_t - \mathcal{A}^a_t - (1-\tau)Y_tP_t)\\
&= \mathcal{A}^a_{t+1} + \frac{1}{Q^s_{t,t+1}}(\Phi_tP_tC^a_t - \mathcal{A}^a_{t+1} - P_tC^a_t)\\
&= -p\mathcal{A}^a_{t+1} + \frac{1}{Q^s_{t,t+1}}P_tC^a_t(\Phi_t - 1)\\
&= -p\mathcal{A}^a_{t+1} + \frac{1}{Q^s_{t,t+1}}P_tC^a_t\left(\frac{\beta}{1+p}\right)^{\sigma}\left(\frac{P_{t+1}}{P_t}Q^s_{t,t+1}\right)^{1-\sigma}\frac{\xi_t}{\xi_{t+1}}\Phi_{t+1}
\end{aligned}$$

from where, taking expectations, we obtain

$$C^a_t = \mathcal{E}_t\left(\left(\frac{1+p}{\beta}\frac{P_{t+1}}{P_t}Q^s_{t,t+1}\right)^{\sigma}\left(C^a_{t+1} + \frac{\mathcal{A}^a_{t+1}}{P_{t+1}\Phi_{t+1}}\right)\frac{\xi_{t+1}}{\xi_t}\right) \tag{C.9}$$

with

$$\mathcal{A}^a_{t+1} = \frac{1}{Q^s_{t,t+1}}\frac{1}{1+p}\left(\mathcal{A}^a_t + (1-\tau_t)P_tY_t + T^a_t - P_tC^a_t\right)$$

C.1.2 Firms

Profit Maximization A firm employs labour and produces goods z, it discount future profit with Q^f , Nominal wages W are equalized across all firms. A firm chooses employment and prices to maximize profit:

$$\max_{\{N_t(i),p^*_s(i)\}_{s=t}^{\infty}} \mathcal{E}_t\sum_{s=t}^{\infty} Q^f_{t,s}\left(y_s(z)\,p_s(z) - W_sN_s(z)\right).$$

subject to the production constraint

$$y_t(z) = Z_t N_t(z),$$

All producers of good z understand that sales depend on demand, which is a function of price. Therefore, intra-temporal consumption optimization implies

$$y_t^a(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\epsilon_t} Y_t^a$$

and each period firms are allowed to reoptimize their prices with probability $1-\gamma$, so that they remain fixed with probability γ.

$$\begin{aligned} p_t(z) &= p_t^*(z) \\ p_{t+1}(z) &= \begin{cases} p_t^*(z), & \text{with prob } \gamma \\ p_{t+1}(z), & \text{with prob} 1- \ \gamma \end{cases} \end{aligned}$$

Profit maximisation problem can be split into to separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. We can deal with each of these problems separately. As we assume that the firms live forever, the details and results for firms are the same as in Appendix to Chapter 2, see page 132 to page 136

C.1.3 Final System for private sector-Nonlinear Version:

We assume government debt is the only financial asset held by households: $B_{t+1} = \mathcal{A}_{t+1}^a$

This yields:

$$Q_{t,t+1}^s = \frac{1}{1+i_t}\frac{1}{1+p} \tag{C.10}$$

$$\Phi_t = 1 + \left(\frac{\beta}{1+p}\right)^{\sigma}\left(\Pi_{t+1}\frac{1}{(1+i_t)}\frac{1}{1+p}\right)^{1-\sigma}\Phi_{t+1}\frac{\xi_t}{\xi_{t+1}} \tag{C.11}$$

$$C_t^a = \mathcal{E}_t\left(\frac{1}{\beta^{\sigma}}\Pi_{t+1}^{\sigma}(1+i_t)^{-\sigma}\left(C_{t+1}^a + pB_{t+1}\Pi_{t+1}^{-1}\Phi_{t+1}^{-1}\right)\frac{\xi_{t+1}}{\xi_t}\right) \tag{C.12}$$

$$(F_t)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)}\left(1-\gamma\Pi_t^{\epsilon-1}\right)=(1-\gamma)\,(K_t)^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} \tag{C.13}$$

$$K_t = \frac{\varkappa\mu\,(1+1/\psi)}{(1-\tau_s)Z_s^{1+1/\psi}}\,(C_t^a+G_t)^{1+1/\psi}+\gamma\beta\mathcal{E}_t\Pi_{t+1}^{(1+1/\psi)\epsilon}K_{t+1} \tag{C.14}$$

$$F_t = (C_s^s\xi_s)^{-\frac{1}{\sigma}}\,(C_t^a+G_t)+\gamma\beta\mathcal{E}_t\Pi_{t+1}^{\epsilon-1}F_{t+1} \tag{C.15}$$

$$\Delta_t = (1-\gamma)\left(\frac{1-\gamma\Pi_t^{\epsilon-1}}{1-\gamma}\right)^{\frac{\epsilon}{\epsilon-1}}+\gamma\Pi_t^{\epsilon}\Delta_{t-1} \tag{C.16}$$

$$B_{t+1} = (1+i_t)(B_t\frac{1}{\Pi_t}+G_t-\tau_t\,(C_t^a+G_t)-T) \tag{C.17}$$

$$Y_s^a = C_t^a+G_t \tag{C.18}$$

C.1.4 The steady state

At the steady state, the system above becomes

$$\Phi = \frac{1}{\left(1-\frac{\Pi}{(1+p)(1+i)}\left(\frac{\beta(1+i)}{\Pi}\right)^{\sigma}\right)} = \frac{1}{\left(1-\frac{\beta^{\sigma}(1+i)^{\sigma-1}}{(1+p)}\right)}$$

$$C = \frac{\left(\frac{\Pi}{\beta(1+i)}\right)^{\sigma}}{1-\left(\frac{\Pi}{\beta(1+i)}\right)^{\sigma}}\frac{Bp}{\Pi\Phi}$$

$$\frac{1}{\frac{Bp}{C\Pi\Phi}+1} = \left(\frac{\Pi}{\beta(1+i)}\right)^{\sigma}$$

$$K^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} = \frac{(1-\gamma\Pi^{\epsilon-1})}{(1-\gamma)}F^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)}$$

$$K = \varkappa\mu\frac{(1+1/\psi)\,(C+G)^{1/\psi+1}}{(1-\tau)\,(1-\gamma\beta\Pi^{\epsilon(1+1/\psi)})}$$

$$F = \frac{(C)^{-\frac{1}{\sigma}}\,(C+G)}{(1-\gamma\beta\Pi^{\epsilon-1})}$$

$$\Delta = \frac{(1-\gamma)}{(1-\gamma\Pi^{\epsilon})}\left(\frac{1-\gamma\Pi^{\epsilon-1}}{1-\gamma}\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$B = (1+i)(B\frac{1}{\Pi}+G-\tau\,(C+G)-T)$$

We assume that at the steady state inflation is zero, i.e. $\Pi = 1$,the steady state values are:

$$
\begin{aligned}
\Phi &= \frac{(1+p)(1+i)}{(1+p)(1+i)-(\beta(1+i))^{\sigma}} && \text{(C.19)}\\
C &= \frac{Bp}{\Phi}\frac{1}{((\beta(1+i))^{\sigma}-1)} && \text{(C.20)}\\
K^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} &= F^{\frac{\psi}{\psi+\epsilon}(1-\epsilon)} && \text{(C.21)}\\
K &= \varkappa\mu\frac{(1+1/\psi)(C+G)^{1/\psi+1}}{(1-\tau)(1-\gamma\beta)} && \text{(C.22)}\\
F &= \frac{C^{-\frac{1}{\sigma}}(C+G)}{(1-\gamma\beta)} && \text{(C.23)}
\end{aligned}
$$

$$
\begin{aligned}
\Delta &= 1\\
B &= -\frac{(1+i)(G-\tau(C+G)-T)}{i} && \text{(C.24)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{C}{Y} &= \theta = \left(\beta^{-\sigma}(1+i)^{-\sigma}\right)\left(\frac{C}{Y}+\frac{p}{\Phi}\frac{B}{Y}\right) && \text{(C.25)}\\
&= \frac{\left(\beta^{-\sigma}(1+i)^{-\sigma}\right)p\left(\beta^{\sigma}-(1+p)(1+i)^{1-\sigma}\right)(1+i)(1-\theta-\tau)}{(1+p)(1+i)^{1-\sigma}\left(1-\left(\beta^{-\sigma}(1+i)^{-\sigma}\right)\right)i} && \text{(C.26)}\\
\frac{G}{Y} &= 1-\theta && \text{(C.27)}\\
\frac{B}{Y} &= \chi = -\frac{(1+i)(1-\theta-\tau)}{i} && \text{(C.28)}
\end{aligned}
$$

Finally

$$
\begin{aligned}
\frac{p(1-\theta-\tau)}{\theta(1+p)} &= \frac{i\left(\beta^{\sigma}-(1+i)^{-\sigma}\right)}{\left(\beta^{\sigma}-(1+p)(1+i)^{1-\sigma}\right)}\\
p(1-\theta-\tau)\beta^{\sigma}-p(1-\theta-\tau)(1+p)(1+i)^{1-\sigma} &= i\beta^{\sigma}\theta(1+p)-i(1+i)^{-\sigma}\theta(1+p)
\end{aligned}
$$

$$
i = \frac{p(1-\theta-\tau)\beta^{\sigma}-p(1-\theta-\tau)(1+p)(1+i)^{1-\sigma}}{\beta^{\sigma}\theta(1+p)-(1+i)^{-\sigma}\theta(1+p)} \tag{C.29}
$$

C.1.5 Linearization

The log-linear deviations of all variables are defined as

$$X_t = X\left(1+\hat{X}_t\right)$$

where

$$\hat{X}_t = \ln\frac{X_t}{X}$$

Therefore:

$$\begin{aligned}
(1+i_t) &= (1+i)\,(1+\hat{\imath}_t) \\
\hat{\imath}_t &= \ln\frac{(1+i_t)}{(1+i)} \\
(1+\pi_t) &= (1+\pi)\,(1+\hat{\pi}_t) \\
\hat{\pi}_t &= \ln\frac{(1+\pi_t)}{(1+\pi)} \\
(1-\tau_t) &= (1-\tau)(1-\hat{\tau}_t) \\
\hat{\tau}_t &= \ln\frac{(1-\tau_t)}{(1-\tau)}
\end{aligned}$$

and inflation:

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t} = \frac{P_{t+1}-P_t}{P_t} + 1 = 1 + \pi_{t+1}$$

Now we linearize the system we had arrived before, around zero inflation steady states (C.19) to (C.24)

Reverse of consumption propensity $\hat{\Phi}_t$

Use Equation C.11 and C.19 to linearize the reverse of consumption propensity:

$$\Phi\left(1+\hat{\Phi}_t\right) = 1 + \left(\frac{\beta}{1+p}\right)^{\sigma}\left(\frac{(1+\pi)\,(1+\hat{\pi}_{t+1})}{(1+p)\,(1+i)\,(1+\hat{\imath}_t)}\right)^{1-\sigma}\frac{\xi\left(1+\hat{\xi}_t\right)}{\xi\left(1+\hat{\xi}_{t+1}\right)}\Phi\left(1+\hat{\Phi}_{t+1}\right)$$

$$\Phi\hat{\Phi}_t = \Phi\left(\frac{\beta}{1+p}\right)^{\sigma}\left(\frac{(1+\pi)}{(1+p)\,(1+i)}\right)^{1-\sigma}\left[(1-\sigma)\,\hat{\pi}_{t+1} - (1-\sigma)\,\hat{\imath}_t + \hat{\xi}_t - \hat{\xi}_{t+1} + \hat{\Phi}_{t+1}\right]$$

$$\frac{(1+p)(1+i)}{(\beta(1+i))^{\sigma}}\hat{\Phi}_t = \hat{\Phi}_{t+1} - (1-\sigma)\left(\hat{\imath}_t - \pi_{t+1}\right) + \hat{\xi}_t - \hat{\xi}_{t+1} \tag{C.30}$$

Consumption $\hat{C}_t$

Substitute in C.12, C.20, we have

$$\begin{aligned} & C\left(1+\hat{C}_t\right) \\ = & \left(\frac{1}{\beta}\frac{(1+\pi)\left(1+\hat{\pi}_{t+1}\right)}{(1+i)\left(1+\hat{\imath}_t\right)}\right)^{\sigma}\left(C\left(1+\hat{C}_{t+1}\right)+p\frac{B}{\Phi(1+\pi)}\frac{\left(1+\hat{B}_{t+1}\right)}{\left(1+\hat{\Phi}_{t+1}\right)\left(1+\hat{\pi}_{t+1}\right)}\right)\frac{\left(1+\hat{\xi}_{t+1}\right)}{\left(1+\hat{\xi}_t\right)} \end{aligned}$$

which yields the linearized consumption:

$$\hat{C}_t = \left(\frac{1}{\beta}\frac{1}{(1+i)}\right)^{\sigma}\left(\hat{C}_{t+1} + p\frac{Y}{C\Phi}\left(\left[\frac{B}{Y}\hat{B}_{t+1}\right] - \frac{B}{Y}\hat{\pi}_{t+1} - \frac{B}{Y}\hat{\Phi}_{t+1}\right)\right) + \sigma\hat{\pi}_{t+1} - \sigma\hat{\imath}_t + \hat{\xi}_{t+1} - \hat{\xi}_t$$

assume $b_t = \chi\hat{B}_t$ where $\chi = \frac{B}{Y}$:

$$\hat{C}_t = [\beta(1+i)]^{-\sigma}\left(\mathcal{E}_t\hat{C}_{t+1} + \frac{p}{\theta\Phi}\left(b_{t+1} - \chi\pi_{t+1} - \chi\hat{\Phi}_{t+1}\right)\right) - \sigma\left(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{t+1}\right) + \hat{\xi}_{t+1} - \hat{\xi}_t \tag{C.31}$$

Philip's curve

As the non-linear system is the same as in the last Chapter, the resulted Philip's Curve will be the same as Equation(A.26)

$$\hat{\pi}_t = \beta\mathcal{E}_t\hat{\pi}_{t+1} + \kappa\left(\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)\hat{C}_t + \frac{(1-\theta)}{\psi}\hat{G}_t + \frac{\tau}{(1-\tau)}\hat{\tau}_t\right) + \hat{\xi}_t \tag{C.32}$$

where $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma}\frac{\psi}{\psi+\epsilon}$

Debt

Substitute in C.17 and C.24:

$$
\begin{aligned}
B(1+\hat{B}_{t+1}) &= (1+i)(1+\hat{\imath}_t)(B(1+\hat{B}_t)(1+\hat{\pi}_t)^{-1}+G(1+\hat{G}_t)-\tau(1+\hat{\tau}_t)Y(1+\hat{Y}_t)) \\
b_{t+1} &= (1+i)b_t-\chi(1+i)\hat{\pi}_t-(1+i)\tau\theta\hat{C} \\
&\quad +(1+i)(1-\tau)(1-\theta)\hat{G}_t+\chi i-(1+i)\tau\hat{\tau}_t
\end{aligned}
$$

Where χ is the proportion of steady state debt level to GDP, $\chi=\frac{B}{Y}=-\frac{(1+i)(1-\theta-\tau)}{i}$ and $b_t=\chi\hat{B}_t$

C.1.6 The final deterministic system for private sector

$$
\begin{aligned}
b_{t+1} &= (1+i)b_t-\chi(1+i)\hat{\pi}_t-(1+i)\tau\theta\hat{C}+(1+i)(1-\tau)(1-\theta)\hat{G}_t+\chi i-(1+i)\tau\hat{\tau}_t \\
\hat{\pi}_t &= \beta\mathcal{E}_t\hat{\pi}_{t+1}+\kappa\left(\left(\frac{1}{\sigma}+\frac{\theta}{\psi}\right)\hat{C}_t+\frac{(1-\theta)}{\psi}\hat{G}_t\right)+\hat{\xi}_t \\
\hat{C}_t &= \frac{p}{\theta\Phi[\beta(1+i)]^\sigma}\left(b_{t+1}-\chi\hat{\pi}_{t+1}-\chi\hat{\Phi}_{t+1}\right)+\frac{\hat{C}_{t+1}}{[\beta(1+i)]^\sigma}-\sigma(\hat{\imath}_t-\pi_{t+1}) \\
\hat{\Phi}_{t+1} &= \frac{(1+p)(1+i)}{(\beta(1+i))^\sigma}\hat{\Phi}_t+(1-\sigma)\hat{\imath}_t-(1-\sigma)\mathcal{E}_t\pi_{t+1}
\end{aligned}
$$

with

$$
\begin{aligned}
\kappa &= \frac{(1-\gamma)(1-\gamma\beta)}{\gamma}\frac{\psi}{\psi+\epsilon} \\
\theta &= \frac{(\beta^\sigma(1+i)^\sigma-(1+p)(1+i))(1-\tau)p}{(\beta^\sigma(1+i)^\sigma-(1+p)(1+i))p+(\beta^\sigma(1+i)^\sigma(1+p)-(1+p))i}
\end{aligned}
$$

To focus on the impact of tax and interest rate on the economy, in which the fiscal policy maker can only set the tax rate less frequent than monetary policy can decide on the interest rate, we assume the government spending is constant so the deterministic economy evolves according to system:

$$
\begin{aligned}
b_{t+1} &= (1+i)b_t-\chi(1+i)\pi_t-(1+i)\tau\theta c_t+\chi i_t-(1+i)\tau\tau_t \\
\beta\mathcal{E}_t\pi_{t+1} &= \pi_t-\kappa\left(\frac{1}{\sigma}+\frac{\theta}{\psi}\right)c_t-\kappa\frac{\tau}{(1-\tau)}\tau_t
\end{aligned}
$$

$$\frac{p}{\theta\Phi}b_{t+1} + \left(\sigma[\beta(1+i)]^{\sigma} - \chi\frac{p}{\theta\Phi}\right)\pi_{t+1} + \mathcal{E}_t c_{t+1} - \chi\frac{p}{\theta\Phi}\hat{\Phi}_{t+1} = [\beta(1+i)]^{\sigma}c_t + \sigma[\beta(1+i)]^{\sigma}i_t$$

$$(1-\sigma)\,\pi_{t+1} + \hat{\Phi}_{t+1} = \frac{(1+p)\,(1+i)}{\beta^{\sigma}\,(1+i)^{\sigma}}\hat{\Phi}_t + (1-\sigma)\,i_t$$

C.2 Impulse responses with infinite living household and positive steady state debt

Figure C.1 shows the impulse responses when $\chi = 0.1$ and $p = 0$ with different length of fiscal cycle, to compare with Figure 4.2 where $\chi = 0.1$ and $p = 0.05$, illustrates that a positive mortality rate introducing more fluctuation to the economic variables during the stabilization process.

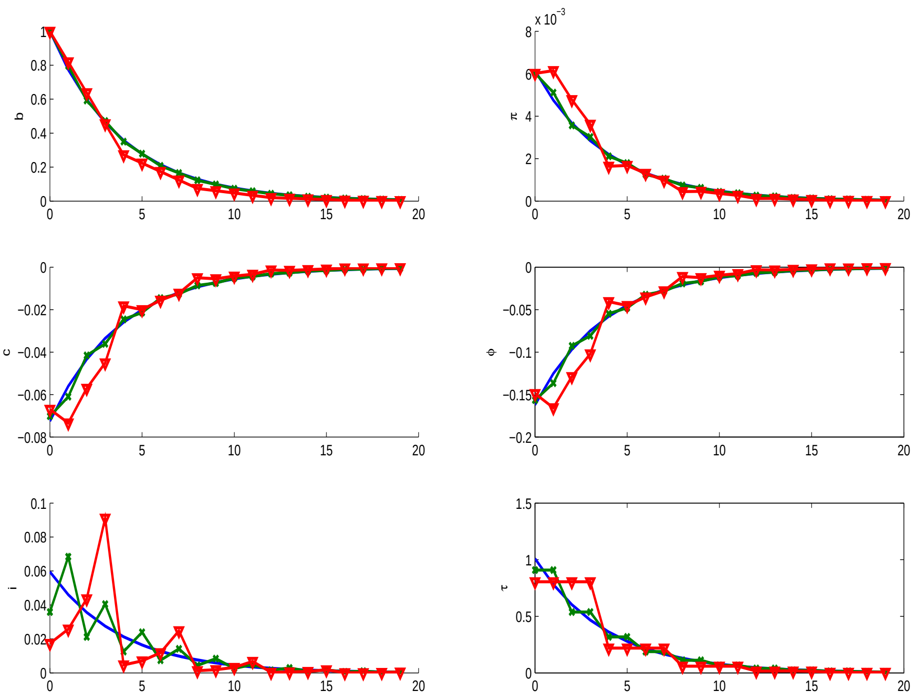

Figure C.1: Impluse responses with $\chi = 0.1$ and $p = 0$

Bibliography

Albanesi, S., V. V. Chari, and L. Christiano (2003). Expectation Traps and Monetary Policy. *Review of Economic Studies 70*, 715–741.

Ambler, S. and S. Ambler (2009). Price-level targeting and stabilization policy: A reviewŤ, bank of canada review.

Anderson, E. W., L. P. Hansen, E. R. McGrattan, and T. J. Sargent (1996). Mechanics of Forming and Estimating Dynamic Linear Economies. In H.M.Amman, D.A.Kendrick, and J.Rust (Eds.), *Handbook of Computational Economics*, pp. 171–252. Elsevier.

Atkeson, A. and M. Ogaki (1996). Wealth-varying intertemporal elasticities of substitution: Evidence from panel and aggregate data. *Journal of Monetary Economics 38*(3), 507–534.

Attanasio, O. P. and G. Weber (1993). Consumption Growth, the Interest Rate and Aggregation. *Review of Economic Stuties 60*(3), 631–649.

Attanasio, O. P. and G. Weber (1995). Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. *Journal of Political Economy 103*(6), 1121–1157.

Auerbach, A. J. (2003). Is there a Role for Discretionary Policy? In *Rethinking Stabilization Policy.* Federal Reserve Bank of Kansas City.

Backus, D. and J. Driffill (1986). The Consistency of Optimal Policy in Stochastic Rational Expectations Models. CEPR Discussion Paper 124, London.

Barro, R. and D. Gordon (1983). Rules, Discretion and Reputation in a Model of Monetary Policy. *Journal of Monetary Economics 12*, 101–121.

Batini, N. and A. Yates (2003). Hybrid Inflation and Price-Level Targeting. *Journal of Money, Credit and Banking 35*(3), 283–300.

Beetsma, R. and H. Jensen (2005). Monetary and Fiscal Policy Interactions in a Micro-founded Model of a Monetary Union. *Journal of International Economics 67*, 320–352.

Benigno, P. and M. Woodford (2003). Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach. In *NBER Macroeconomics Annual 2003*, pp. 271–333.

Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile Prices in Market Equilibrium. *Econometrica 63*(4), 841–890.

Billi, R. (2008). Price-level targeting and risk management in a low-inflation economy. Federal Reserve Bank of Kansas City.

Blake, A. P. and T. Kirsanova (2011). Inflation-Conservatism and Monetary-Fiscal Policy Interactions. *International Journal of Central Banking 7*(2), 41–83.

Blake, A. P. and T. Kirsanova (2012). Discretionary Policy and Multiple Equilibria in LQ RE Models. *Review of Economic Studies 79*, 1309–1339.

Blanchard, O. (1985). Debt, Deficits, and Finite Horizons. *Journal of Political Economy 93*(2), 223–247.

Blanchard, O. and C. Kahn (1980). The Solution of Linear Difference Models Under Rational Expectations. *Econometrica 48*, 1305–1311.

Calvo, G. (1983). Staggered Prices in a Utility-Maximising Framework. *Journal of Monetary Economics 12*, 383–398.

Calvo, G. and M. Obstfeld (1988). Optimal Time-Consistent Fiscal Policy with Finite Lifetimes. *Econometrica 56*(2), 411–432.

Campbell, J. Y. and N. G. Mankiw (1989). Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In *NBER Macroeconomics Annual*, pp. 185–246.

Chang, Y. and S.-B. Kim (2005). On the Aggregate Labor Supply. *Federal Reserve Bank of Richmond Economic Quarterly 91*(1), 21–37.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2000). Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem? *Econometrica 68*(5), 1151–1180.

Chen, X., T. Kirsanova, and C. Leith (2013). How Optimal is US Monetary Policy? Working Papers, Business School - Economics, University of Glasgow.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011). Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. *American Economic Review Papers and Proceedings 101*(2-3), 471–475.

Clarida, R. H., J. Gali, and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature 37*, 1661–1707.

Clements, D. J. and H. Wimmer (2003). Existence and uniqueness of unmixed solutions of the discrete-time algebraic Riccati equation. *Systems Control Letters 50*, 343–346.

Cooper, R. and A. John (1988). Coordination Failures in Keynesian Models. *Quarterly Journal of Economics 103*(3), 441–463.

Currie, D. and P. Levine (1993). *Rules, Reputation and Macroeconomic Policy Coordination*. Cambridge: Cambridge University Press.

Davig, T. and E. Leeper (2005). ŞAn Interpretation of Fluctuating Macro Policies. *Computing in Economics and Finance 2005 249, Society for Computational Economics* (249).

Davig, T. and E. Leeper (2006). Fluctuating Macro Policies and the Fiscal Theory. In *NBER Macroeconomics Annual*, pp. 247–298. MIT Press.

Debertoli, D. and R. Nunes (2010). Fiscal Policy under Loose Commitment. *Journal of Economic Theory 145*(3), 1005–1032.

Dennis, R. and T. Kirsanova (2012). Expectations Traps and Coordination Failureswith Discretionary Policymaking. Federal Reserve Bank of San Francisco Working paper 10-2, available at SSRN: http://ssrn.com/abstract=1664924.

Dixit, A. and L. Lambertini (2003). Interactions of Commitment and Discretion in Monetary and Fiscal Policies. *American Economic Review 93*, 1522–1542.

Dixit, A. and J. Stiglitz (1977). Monopolistic Competition and Optimum product Diversity. *American Economic Review 67*, 297–308.

Drautzburg, T. and H. Uhlig (2011). Fiscal stimulus and distortionary taxation. NBER Working Paper No. 17111.

Ellison, M. and J. Pearlman (2011). Saddlepath Learning. *Journal of Economic Theory 146*, 1500–1519.

Evans, G. (1986). Selection Criteria for Models with Non-Uniqueness. *Journal of Monetary Economics 18*, 147–157.

Evans, G. and R. Guesnerie (1993). Rationalizability, Strong Rationality, and Expectational Stability. *Games and Economic Behavior 5*, 632–646.

Evans, G. and R. Guesnerie (2003). Coordination on Saddle-Path Solutions, The Eductive Viewpoint: Linear Univariate Models. *Macroeconomic Dynamics 7*, 42–62.

Evans, G. and R. Guesnerie (2005). Coordination on Saddle-Path Solutions: The Eductive Viewpoint—Linear Multivariate Models. *Journal of Economic Theory 124*, 202–229.

Evans, G. and S. Honkapohja (2001). *Learning and Expectations in Macroeconomics.* Princeton University Press.

Favero, C. and T. Monacelli (2005). Fiscal Policy Rules and Regime (In)Stability: Evidence from the U.S. Mimeo, University of Bocconi.

Fragetta, M. and T. Kirsanova (2010). Strategic Monetary and Fiscal Policy Interactions: An Empirical Investigation. *European Economic Review 54*(7), 855–879.

Givens, G. E. (2012). Estimating Central Bank Preferences under Commitment and Discretion. *Journal of Money, Credit and Banking*. 'Forthcoming'.

Gourio, F. and P.-A. Noual (2009). The Marginal Worker and The Aggregate Elasticity of Labor Supply. Boston University, mimeo.

Guesnerie, R. and M. Woodford (1992). Endogenous Fluctuations. In J-J.Laffont (Ed.), *Advances in Economic Theory*, pp. xxx–xxx. Cambridge University Press.

Hall, R. and N. Mankiw (1994). Nominal Income Targeting. *Monetary Policy, University of Chicago Press, Chicago*, 71–93.

Hall, R. E. (1988). Intertemporal Substitution in Consumption. *Journal of Political Economy 96*(2), 339–357.

Jensen, H. (2002). Targeting Nominal Income Growth or Inflation? *American Economic Review 92*(4), 928–956.

King, M. (1997). Changes in UK Monetary Policy: Rules and Discretion in Practice. *Journal of Monetary Economics 39*, 81–97.

King, R. and A. Wolman (2004). Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria. *Quarterly Journal of Economics 199*(4), 1513–1553.

Kirsanova, T., C. Leith, and S. Wren-Lewis (2009). Monetary and fiscal policy interaction: the current consensus assignment in the light of recent developments. *Economic Journal 119*(541), F482–F496.

Kirsanova, T., M. Satchi, and D. Vines (2003). Monetary Union: Fiscal Stabilisation in the Face of Asymmetric Shocks. CEPR working paper 4433.

Kwakernaak, H. and R. Sivan (1972). *Linear Optimal Control Systems.* Wiley-Interscience.

Lambertini, L. (2006). Monetary-Fiscal Interactions with Conservative Central Bank. *Scottish Journal of Political Economy 53*, 90–128.

Lambertini, L. and R. Rovelli (2003). Monetary and fiscal policy coordination and macroeconomic stabilization: A theoretical analysis. Mimeo, University of Bologna.

Lancaster, P. and L. Rodman (1995). *Algebraic Riccati Equations.* Oxford: Oxford University Press.

Le Roux, S. and T. Kirsanova (2014). Commitment vs. Discretion in the UK:An Empirical Investigation of the Monetary and Fiscal Policy Regime. *International Journal of Central Bank*.

Leeper, E. M. (1991). Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies. *Journal of Monetary Economics 27*, 129–147.

Leith, C. and L. von Thadden (2008). Monetary and fiscal policy interactions in a New Keynesian model with capital accumulation and non-Ricardian consumers. *Journal of Economic Theory 40*(1), 279–313.

Leith, C. and S. Wren-Lewis (2000). Interactions Between Monetary and Fiscal Policy Rules. *The Economic Journal 110*, C93–C108.

Leith, C. and S. Wren-Lewis (2014). Fiscal sustainability in a new keynesian model. *Journal of Money, Credit and Banking*.

Linnemann, L. (2006). Interest rate policy, debt, and indeterminacy with distortionary taxation. *Journal of Economic Dynamics and Control 30*, 487–510.

Lucas, R. (1987). *Models of Business Cycles.* Basil Blackwell.

Nevo, A. (2001). Measuring Market Power in the Ready-to-Eat Cereal Industry. *Econometrica 69*(2), 307–342.

Oudiz, G. and J. Sachs (1985). International policy coordination in dynamic macroeconomic models. In W. H. Buiter and R. C. Marston (Eds.), *International Economic Policy Coordination*. Cambridge: Cambridge University Press.

Peterman, W. B. (2012). Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity. The Federal Reserve Board Finance and Economics Discussion Paper 2012-75.

Rogoff, K. (1985). The optimal degree of commitment to an intermediate monetary target. *The Quarterly Journal of Economics 100*(4), 1169–1189.

Roisland, O. (2006). Inflation Inertia and the Optimal Hybrid Inflation/Price-Level Target. *Journal of Money, Credit and Banking 38*(8), 2247–2251.

Rotemberg, J. J. and M. Woodford (1997). An Optimization-based Econometric Framework for the Evaluation of Monetary Policy. In *NBER Macroeconomics Annual*, pp. 297–344.

Sargent, T. and N. Wallace (1981). Some Unpleasant Monetarist Arithmetic. Federal Reserve Bank of Minneapolis Quarterly Review, 5(3).

Schabert, A. and L. von Thadden (2009). Distortionary Taxation, Debt, and the Price Level. *Journal of Money, Credit, and Banking 41*, 159–188.

Schaumburg, E. and A. Tambalotti (2007). An Investigation of the Gains from Commitment in Monetary Policy . *Journal of Monetary Economics 54*(2), 302–324.

Schmitt-Grohe, S. and M. Uribe (1997). Balanced-budget rules, distortionary taxes, and aggregate instability. *Journal of Political Economy 105*, 976–1000.

Schmitt Grohe, S. and M. Uribe (2000). Price Level Determinacy and Monetary Policy under a Balanced-Budget Requirement. *Journal of Monetary Economics 45*(1), 211–246.

Schmitt-Grohe, S. and M. Uribe (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory 114*, 198–230.

Svensson, L. (1997). Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts. *The American Economic Review 87*(1), 98–114.

Svensson, L. (2003). What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules. *Journal of Economic Literature 41*, 426–477.

Svensson, L. (2010). Inflation targeting. In *Handbook of Monetary Economics*, Volume 3. Elsevier.

Svensson, L. E. (1999). Price Level targeting versus inflation targeting: a free lunch? *Journal of Money, Credit and Banking 431*(3), 277–295.

Taylor, J. B. (1985). International Coordination in the Design of Macroeconomic Policy Rules. *European Economic Review 28*.

Vestin, D. (2006). Price-level versus inflation targeting. *Journal of Monetary Economics 53*(7), 1361–1376.

Walsh, C. (2003). Speed Limit Policies: The Output Gap and Optimal Monetary Policy. *American Economic Review 93*(1), 265–278.

Wimmer, H. (2006). A parametrization of the set of solutions of the discrete-time algebraic Riccati equation based on pairs of opposite unmixed solutions. *SIAM Journal on Control and Optimisation 44*, 1992–2005.

Woodford, M. (2001). Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking 33*, 669–728.

Woodford, M. (2003a). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ.: Princeton University Press.

Woodford, M. (2003b). Optimal interest rate smoothing. *Review of Economic Studies 70*(4), 861–886.

Yaari, M. (1965). Unvertain Lifetime, Life Insurance and the Theory of the Consumer. *Review of Economic Studies*, 137–150.