ON THE POWER SPECTRAL DENSITY OF THE GSM SIGNALING SCHEME

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Abstract
In this paper, the Power Spectral Density of encoded Gaussian Minimum Shift Keying (GMSK) which is the Signaling Scheme of the Global System for Mobile Communication (GSM) is derived by a combined approach of the autocorrelation method and Markov Process. In the analysis, the Amplitude Modulated Pulse decomposition proposed by P. Laurent is employed to ease computation. Encoding of the message data utilizes Convolutional Code of rate1/2. Results are for both the uncoded and coded waveform comparing variation in power spread over a range of frequency.

I Introduction

The ever increasing demand for digital wireless communication system presents a serious difficulty of spectral congestion that obviously causes severe adjacent and co channel interference problems. This has led to the investigations of a wide variety of techniques for solving the endemic problems that result from spectral congestion. Among the solutions to this problem are the use of frequency–reuse techniques, efficient source encoding techniques, spectral efficient modulation schemes and/or spectral efficient multiple access scheme.

The main objective of spectral efficient modulation format is to maximize the bandwidth usage at a prescribed bit error rate with minimum expenditure of signal (minimum signal power lay off). For GSM extra constraint is placed on the modulation scheme by the fact that non-linear amplifiers which operate near saturation are incorporated in the general architecture of the system. These nonlinear devices produce extraneous signal regrowth (sidebands) when passing a signal with amplitude fluctuations through them. So a modulation scheme for this must in addition be characterized by constant amplitude to combat such signal impairments. Thus at increased data rate, for low power consumption, and under the influence of nonlinear channel, we require a modulation format that balances the respective parameter requirements. This is a typical scenario in the GSM communication system.

A modulation technique which can offer this trade-off of complexity versus spectral efficiency is GMSK. This is a type of Continuous Phase Modulation (CPM), a class of nonlinear signaling scheme that are efficient in power and bandwidth. It also generates constant envelop waveform and therefore is very useful in radio channels employing non linear amplifiers like Traveling Wave Tube (TWT). GMSK has since been adopted as the modulation scheme for the GSM digital cellular system. The performance and analysis of CPM has been reported by several researchers. However, performance of encoding GMSK has not been examined fully. The analysis of coded some classes of CPM is reported in [2], [4], [7], [10], and [16]. It is obvious that channel encoding increases the required transmission bandwidth when considered independent of modulation and thus affect the power/bandwidth trade off of system. [1], [6], and [12] presented the Power spectrum analysis of uncoded CPM with some references to GMSK. The Power spectral density analysis of some type of encoded CPM is presented in [8], and [13].

In this paper, we present the Power Spectral Density of convolutionally encoded GMSK scheme. For ease of computation, the idea of decomposition of CPM signal presented in [3] is employed where the first two Laurent pulses [3] are used as the basic GMSK signal in power spectrum computation. The method of Spectral computation of Digital FM using Autocorrelation method is presented in [9]. In this report, the combine approach of autocorrelation/Markov method as discussed in [6] and [13] is used to model the encoding process of the GMSK signal in the course of power spectrum analysis.

II GMSK Representation

GMSK modulation is a modified form of Minimum Shift Keying (MSK), and a special case of binary CPFSK in which the modulation index h is set at 0.5. In this case, the rectangular shaping pulse used in conventional MSK is replaced by a special type of non linear pulse shaping filter called Gaussian filter. This scheme ensures a narrower spectrum than that of MSK [8]. Gaussian filter has an ideal impulse response given by

$$h(t) \quad = \quad K\sqrt{\frac{2\pi}{\ln(2)}}\,(BT_b)\exp\left(\frac{(-2(BT_b)\pi\, t)^2}{\ln(2)}\right)$$

(1.0)

For the case of GMSK analysis, this pulse can be modified thus

$$g_{GMSK}(t) \quad = \quad \frac{1}{2T_b}\left\{Q\left(\frac{2\pi\, B(t-\frac{T_b}{2})}{\sqrt{\ln(2)}}\right) \;-\; Q\left(\frac{2\pi\, B(t+\frac{T_b}{2})}{\sqrt{\ln(2)}}\right)\right\}$$

(2.0)

The primary parameter here is the (BT_b) product (-3db) of the Gaussian filter. In order to reduce the sidelobes and produce a compact spectrum, the appropriate value of (BT_b) should be used. In GSM the value of the BT parameter is 0.3. If the (BT_b) product is sufficiently large, then data sequence $\{a_K\}$ which often is None-Return-to-Zero (NRZ) will pass unfiltered. Smaller values of (BT_b) product will give a good compact spectrum where B is the half power bandwidth at symbol period T_b as shown in fig. 1.0

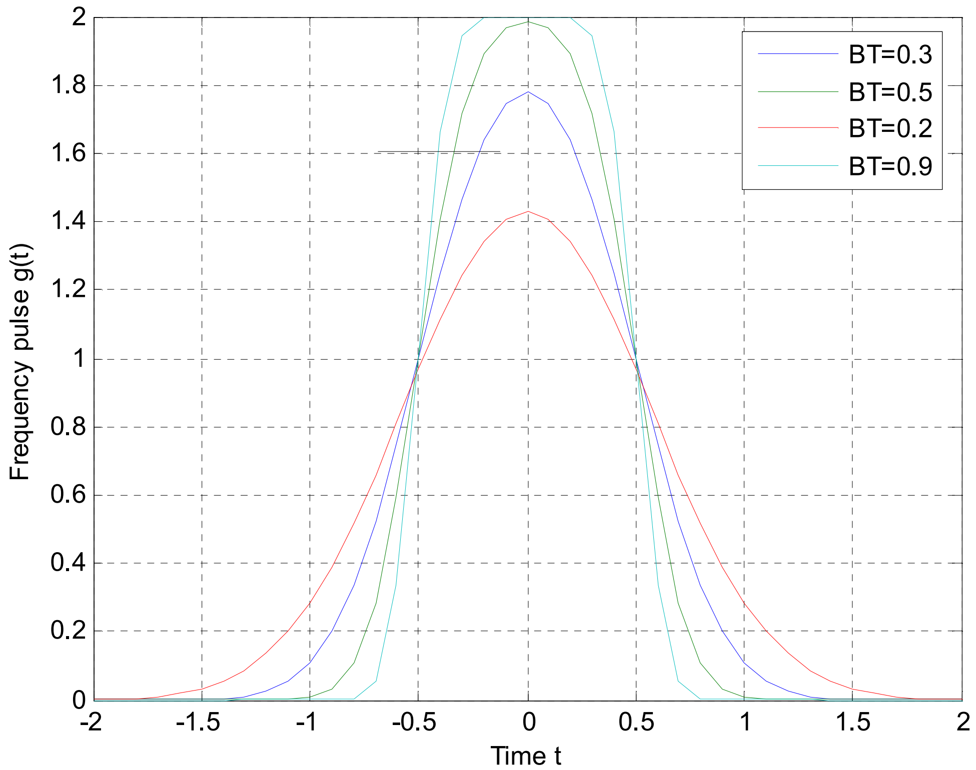

Figure 1: The frequency pulses of GMSK for BT= (0.2,0.3,0.5,0.9).

The GMSK waveform can be expressed as

$$S_{GMSK}(t) = \sqrt{\frac{2E}{T_b}}\, V\cos(2\pi f_c t \pm \pi h \sum_{k=1}^{\infty} a_K q_{GMSK}(t-kT_b))$$

$$= \sqrt{\frac{2E}{T_b}}\, V\cos(2\pi f_c t \pm \varphi(t,a)) \qquad (3.0)$$

where $\sqrt{\frac{2E}{T_b}}$ is the signal amplitude, f_c is the carrier frequency, a_k is the input data, T_b is the bit interval. The function $q_{GMSK}(t)$ is called the phase shaping pulse and is a continuous, monotonic function that determines the overall spectral characteristics of the modulated signal. It is defined as

$$q(t) = \int_{-\infty}^{t} g(\tau)\, d\tau = \frac{1}{2}\left[\frac{q_0\left(t-\frac{LT}{2}\right) - q_0\left(-\frac{LT}{2}\right)}{q_0\left(\frac{LT}{2}\right)}\right] \qquad (4.0)$$

where

$$q_0(t) = \frac{1}{4C}\left[A.erf(A) - B\,erf(B) + \frac{1}{\sqrt{\pi}}\exp(-A^2) - \frac{1}{\sqrt{\pi}}\exp(-B^2) + C\right]$$

$$C = BT\pi\sqrt{\frac{2}{\ln(2)}} \;;\; A = C\left(\frac{t}{T}+0.5\right) \;;\; B = C\left(\frac{t}{T}-0.5\right)$$

and $erf(x) \quad = \quad \frac{2}{\sqrt{\pi}}\int_{0}^{\infty}\exp(-y^{2})\,dy$.

The information carrying phase is given by

$$\varphi(t,a) \quad = \quad \pi h\sum_{k=1}^{\infty}a_{k}q_{GMSK}(t-kT_{b}) \qquad (5.0)$$

For $L=3$ and $h=\frac{1}{2}$,

$$\varphi(t,a)=\pi\{a_{k}q(t-kT_{b})+a_{k-1}q(t-(k-1)T_{b})+a_{k-2}q(t-(k-2)T_{b})\}+\frac{\pi}{2}\sum_{i=0}^{k-3}a_{i} \qquad (6.0)$$

It can be shown that the unit amplitude complex envelope of (3.0) is of the form [11]

$$S_{GMSK}(t) \quad = \quad \varphi(t,a)\exp(j2\pi f_{c}t) \qquad (7.0)$$

III Laurent Decomposition

The continuity imposed on the phase of GMSK signal depicts a kind of memory inherently built into it. The signal is best viewed as having a coded pattern directly imposed on the phase. This sought of encoding system employs an integrated approach to modulation and coding in which case, the encoding can take place in signal space as part of the modulation process. This approach offers the attractive possibility of achieving performance improvement without the bandwidth expansion which accompanies the usual concatenation of coding and modulation. The analysis of the inherent coding of CPM was shown to be achievable if one can decompose the CPM [3], [15], and [17].

In [3], P. Laurent showed that any constant amplitude binary phase modulation can be expressed as a sum of a finite number of time limited amplitude modulated pulses (AMP decomposition). This is the baseband signal that can be written as a sum of 2^{L-1} PAM signals expressed as

$$\tilde{S}_{GMSK} \quad = \quad \sum_{=0}^{2^{L-1}}\sum_{n=0}^{N-1}a_{k,n}C_{k}(t-nT_{b}) \qquad (8.0)$$

where

$$C_{k} \quad = \quad S_{0}(t)\sum_{i=0}^{L-1}S_{i+a_{K,i,L}}(t) \qquad (9.0)$$

$$S_{n}(t) \quad = \quad \frac{\sin(2h\pi\, q(t))}{\sin(\pi h)} \qquad (10.0)$$

and the parameter $a_{k,i}$ is the message bit that takes on the value 0 or 1.

For binary GMSK for GSM application, L = 3, thus we have 4 distinct pulse shapes made up of 3-fold distinct products of the $S_{n}(t)$ corresponding to 2^{L-1}. Thus [2, eqn (11)],

$$\begin{aligned}
C_{0}(t) &= S_{0}(t)S_{1}(t)S_{2}(t) \quad ; \quad 0\le t\le 4T_{b}\\
C_{1}(t) &= S_{0}(t)S_{2}(t)S_{4}(t) \quad ; \quad 0\le t\le 2T_{b}\\
C_{2}(t) &= S_{0}(t)S_{1}(t)S_{5}(t) \quad ; \quad 0\le t\le T_{b}\\
C_{3}(t) &= S_{0}(t)S_{4}(t)S_{5}(t) \quad ; \quad 0\le t\le T_{b}
\end{aligned} \qquad (11.0)$$

In [5], it was asserted that $C_{0}(t)$ and $C_{1}(t)$ are the most significant pulse durations and carry most of the signal energy. Assuming ergodic process, we can thus express (8.0) as

$$\widetilde{S}_{GMSK}(t) = \sum_{n=-\infty}^{\infty} \widetilde{a}_{0,n} C_0(t - nT_b) + \sum_{n=-\infty}^{\infty} \widetilde{a}_{1,n} C_1(t - nT_b) \quad (12.0)$$

where $a_{0,n}$ and $a_{1,n}$ are equivalent complex data symbols.

The major advantage of this decomposition approach is that it allows us to study the coding operation independent of the modulation operation. The scheme decomposes the inherent nonlinear characteristic of the CPM into finite number of linear models. Once the memory is made explicit it becomes possible to design trellis and Convolutionally encoded system for CPM. Such decomposition tends to reduce the complexity associated with calculating the Power Spectral Density.

IV Convolutional Encoding

Convolutional Coding applied to the source sequence is design to add more bits to the final bit sequence. In general, the Convolutional code is characterized as having rate $R = \frac{k}{n}$ and constraint length K, where k and n are the number of inputs and outputs respectively, in the encoder. Fig: 2.0 shows a general model for convolutional encoding of GMSK, where $R = \frac{1}{2}$, and K=5.

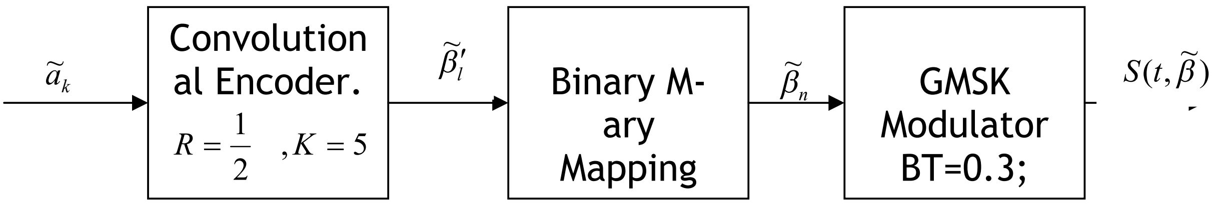

Figure 2.0: Block model of convolutionally encoded GMSK modulator.

The Convolutional polynomials for this system are respectively $G_1(X) = X^4 + X^3 + 1$ and $G_2(X) = X^4 + X^3 + X + 1$. In the figure above, a sequence of bits a_k of rate $\frac{1}{T_b}$ is the input to a rate $R = \frac{1}{2}$ convolutional encoder, producing 2 output code bits. For simplicity, we assume there are n bits mapped onto an M-ary modulation set where $M = 2^n$ or we deem it that the output of the encoder is serially fed into the modulator using an XOR gate. The basic action of encoding/mapping affects only the input into the GMSK modulator. The channel coding invariably affects both the bit interval and energy per bit. This is so since for the uncoded bit interval T_b, the coded bit interval T_c should be such that the same bandwidth is maintained, thus, $T_c = RT_b$. In the case of the energy per bit E, to maintain the same signal energy level, $E_c = RE_b$. So the energy of the uncoded bits is spread among the more numerous coded bits.

For the input complex data $\widetilde{a}_k = [\widetilde{a}_0, \widetilde{a}_1, \widetilde{a}_2 ...., \widetilde{a}_{k-1}]$, without encoding, it is expected that for $\widetilde{a}_i$ *in*, then $\widetilde{a}_i$, *out* *for* $\widetilde{a}_i = [0,1]$. There is virtually no effect on the input to the modulator. However, with the encoding process, the input to the modulator can be described thus

$$\widetilde{a}_i, in \rightarrow [\widetilde{a}_{i,\theta_m}, \widetilde{a}_{i,\theta_m}], out \quad ; \quad for \quad i, m = [0,1,2,...] \quad (13.0)$$

Hence we can express the output of the encoder as

$$\tilde{\beta}_n \quad = \quad [\tilde{a}_{i,\theta_m,0}, \tilde{a}_{i,\theta_m,1}, \tilde{a}_{i,\theta_m,2}, \ldots\ldots\ldots\ldots, \tilde{a}_{i,\theta_m,n-1}] \qquad (14.0)$$

θ_m depends on the state of the encoder memory. Thus, the state vector for the system can be expresses as

$$\Phi \quad = \quad (\tilde{\beta}_{n-1}, \tilde{\beta}_{n-2}) \qquad (15.0)$$

where $\tilde{\beta}_n$ is the transmitted bit at time *n*.

Thus eqn (12.0) becomes

$$\tilde{S}^e{}_{GMSK}(t) \quad = \quad \sum_{n=-\infty}^{\infty} \tilde{\beta}_{0,n} C_0(t - 2nT_c) + \sum_{n=-\infty}^{\infty} \tilde{\beta}_{1,n} C_1(t - 2nT_c) \qquad (16.0)$$

where $\tilde{\beta}_{0,n}$ and $\tilde{\beta}_{1,n}$ are equivalent encoded complex data symbols. The superscript e denotes encoded signal.

V Power Spectral Density

The encoded envelop signal has a generic expression of a complex signal with Real and Imaginary parts. The real part would be given as

$$\tilde{S}^e_{GMSK}(t) \quad = \quad \mathrm{Re}\{\varphi(t, \tilde{\beta}) \cos(-j2\pi f_c t)\} \qquad (17.0)$$

For simplicity, let $\varphi(t, \tilde{\beta}) \equiv \theta(t)$, then using Euler's identity,

$$S^e_{GMSK}(t) = \frac{\theta(t)\exp(j2\pi f_c t) + \theta(t)^* \exp(-j2\pi f_c t)}{2} \qquad (18.0)$$

The autocorrelation function is given by

$$R_{S_{GMSK}} R_{S_{GMSK}}(t, t+\tau) = E\left\{\frac{\theta(t)\exp(j2\pi f_c t) + \theta^*(t)\exp(-j2\pi f_c t)}{2} \times \frac{\theta(t+\tau)\exp(j2\pi f_c(t+\tau)) + \theta^*(t+\tau)\exp(-j2\pi f_c(t+\tau))}{2}\right\}$$

$$\begin{aligned} &= \tilde{R}_{\theta\theta}(t, t+\tau)\exp(j2\pi f_c(2t+\tau)) + R_{\theta\theta^*}(t, t+\tau)\exp(-j2\pi f_c \tau) \\ &+ R_{\theta\theta^*}(t, t+\tau)\exp(j2\pi f_c \tau) + \tilde{R}_{\theta\theta^*}(t, t+\tau)\exp(-j2\pi f_c(2t+\tau) \end{aligned} \qquad (19.0)$$

But

$$\varphi(t, \tilde{\beta}) = \sum_{n=-\infty}^{\infty} \tilde{\beta}_n q_{GMSK}(t - 2nT_c) \qquad (20.0)$$

Then for a random signal, the Hermitian symmetric property shows that

$$\tilde{R}_{\theta\theta}(t, t+\tau) = \sum_{n=-\infty} \sum_{n=-\infty} \tilde{\beta}_n \tilde{\beta}_m q_{GMSK}(t - 2nT_c)(t + \tau - 2mT_c) = 0 \qquad (21.0)$$

Thus

$$\varphi(t)\varphi(t+\tau) = \varphi^*(t)\varphi^*(t+\tau) = 0 \qquad (22.0)$$

Hence,

$$R_{S_{GMSK}S_{GMSK}}(t, t+\tau) = \frac{1}{4}\{\ R_{\theta^*\theta}(t, t+\tau)\exp(j2\pi f_c \tau) + R_{\theta\theta^*}(t, t+\tau)\exp(-j2\pi f_c \tau) \qquad (23.0)$$

It can easily be shown that the Fourier transform of the time average is given by

$$S^e{}_{S_{GMSK}S_{GMSK}} = \frac{1}{4}(S_{\theta^*\theta}(f - f_c) + S_{\theta\theta^*}(f + f_c)) \qquad (24.0)$$

Where $S_{\theta^*\theta}(\bullet)$ denotes the power spectrum of the equivalent encoded complex base band modulation. If we assume data symbol uncorrelation, then it can easily be deduced that the Power Spectrum is given by [13]

$$S^e{}_{\theta\theta}(f) = \frac{\sigma^2}{2T_c}\left|Q_i(f)\right|^2 + \left(\frac{\mu}{2T_c}\right)^2 \sum_{n=-\infty}^{\infty}\left|Q_i\left(\frac{n}{2T_c}\right)\right|^2 \delta\left(f - \frac{n}{2T_c}\right) - \frac{1}{T_c}\mathrm{Re}\left[Q_i(f)Q_i{}^*(f)\right]$$

(25.0)

where σ^2 and μ are the variance and mean of the stationary random process. $Q_i(t)$ is the Fourier transform of the Laurent pulses $C_i(t)$. If the effect of discrete spectrum which in itself carries no information is neglected due to the effect of differential encoding associated with the source coding, and it is assumed that no spectrum is created by the filter actions (advantage of CPM) then equation is reduced to

$$S^e{}_{\theta\theta}(f) = \frac{\sigma^2}{2T_c}\left|Q_i(f)\right|^2 = \frac{\sigma^2}{2T_c}\left|F\{\ C_i(t)\right|^2 \quad (26.0)$$

In a situation where encoding process induces memory effect on the data symbol ensuring data correlation, the Fourier transform of the correlation among data symbols is related to the variance by [13]

$$\sigma^2 \ = \sum_{K=-\infty}^{\infty} R_{\tilde{\beta}^2} \exp(-j2\pi\, f_c t) \quad (27.0)$$

Thus

$$S_{\theta\theta}(f) = \frac{1}{2T_c}\sum_{K=-\infty}^{\infty} R_{\tilde{\beta}^2} \exp(-j2\pi\, f_c t)\ \left|F\{\ C_i(t)\right|^2 \quad (28.0)$$

$$S^e_{\theta\theta}(f) = \frac{1}{2T_c}\ \left|F\{\ C_i(t)\right|^2 S_{\tilde{\beta}_i^2}(f) \quad (29.0)$$

By the nature of convolutional encoding process, the probability of observing any particular value in the sequence can be deemed to depend on the preceding values. In most practical cases, this dependency is often on the immediate preceding (Previous) sample. A procedure so described is called Markov Process. In using the Markov method, the convolutional encoder can be modeled as a Markov source characterized by a transition matrix

$$P_t^n \ = P_t = \begin{bmatrix} p_{11} & p_{12} & \cdots\cdots\cdots . & p_{1K} \\ p_{21} & p_{22} & \cdots\cdots\cdots . & p_{2K} \\ \cdot & \cdot & & \cdot \\ . & . & \cdots\cdots\cdots . & . \\ . & . & & . \\ p_{K11} & p_{K2} & \cdots\cdots\cdots . & p_{KK} \end{bmatrix} \qquad \forall\ n \geq 1 \quad (30.0)$$

Then the autocorrelation function can be expressed in terms of the transition matrix and the matrix of the correlation between the basic baseband pulses. If we assume binary sequence and let $\tilde{\beta}_{i,n} \ = \ (+1,-1); \{i = 0,1\}$, then the correlation function can be given for the first Laurent pulse as

$$R_{\tilde{\beta}_0^2}(k) = (1-2P_t)^k \quad (31.0)$$

And for the second Laurent pulse [6]

$$R_{\tilde{\beta}_1^2}(k) = \begin{cases} & l = 0 \\ -j(1-2P_t)^3 & l = 1 \\ [-j(1-2P_t)]^l & l \geq 2 \end{cases} \quad (32.0)$$

Where the transitional probability P_t is given by

$$P_t = \begin{bmatrix} 1-p_0 & p_0 \\ p_0 & 1-p_0 \end{bmatrix} \qquad (33.0)$$

From eqn (29.0)

$$S_{\tilde{\beta}_0^2}(f) = \sum_{K=-\infty}^{\infty} R_{\tilde{\beta}_i^2}(k)\exp(-j4\pi f_c k T_c) \qquad (34.0)$$

Thus for the first Laurent pulse $C_0(t)$

$$S_{\theta\theta}^{00}(f;P_t) = \frac{1}{2T_c}|Q_0(f)|^2 \left\{ \frac{4P_t(1-P_t)}{2(1-2P_t)(1+\sin 4\pi fT_c)+4P_t^2} \right\} \qquad (35.0)$$

And for the second pulse $C_1(t)$

$$S_{\theta\theta}^{11}(f;P_t) = \frac{1}{2T_c}|Q_1(f)|^2 4P_t(1-P_t) \left\{ \frac{1}{2(1-2P_t)(1+\sin 4\pi fT_c)+4P_t^2} - 2(1-2P_t)1+\sin 4\pi fT_c \right\}$$

(36.0)

The cross correlation between complex data stream induced by the memory effect that characterizes the convolutional encoding process is accounted for by the expression

$$S_{\theta\theta}^{10}(f;P_t) = \frac{1}{2T_c}|Q_0(f)Q_1{}^*(f)|^2 8P_t(1-P_t) \left\{ \frac{1}{2(1-2P_t)(1+\sin 4\pi fT_c)+4P_t^2} + 1 \right\} \cos 4\pi fT_c$$

(37.0)

Finally, the Power spectral density of the encoded GMSK is given by

$$S_{\theta\theta}^{e}(f;P_t) = S_{\theta\theta}^{00}(f;P_t) + S_{\theta\theta}^{11}(f;P_t) + S_{\theta\theta}^{10}(f;P_t) \qquad (38.0)$$

VI Result and Discussion

The power spectral densities of both the uncoded and encoded GMSK are evaluated for BT=0.3 and P_t *=0.543* and simulated using MATLAB software program. We have demonstrated that for rate ½ and constraint length K=5 the spectrum of the coded system has side lobes that are smoother but elevated than that of the uncoded system.

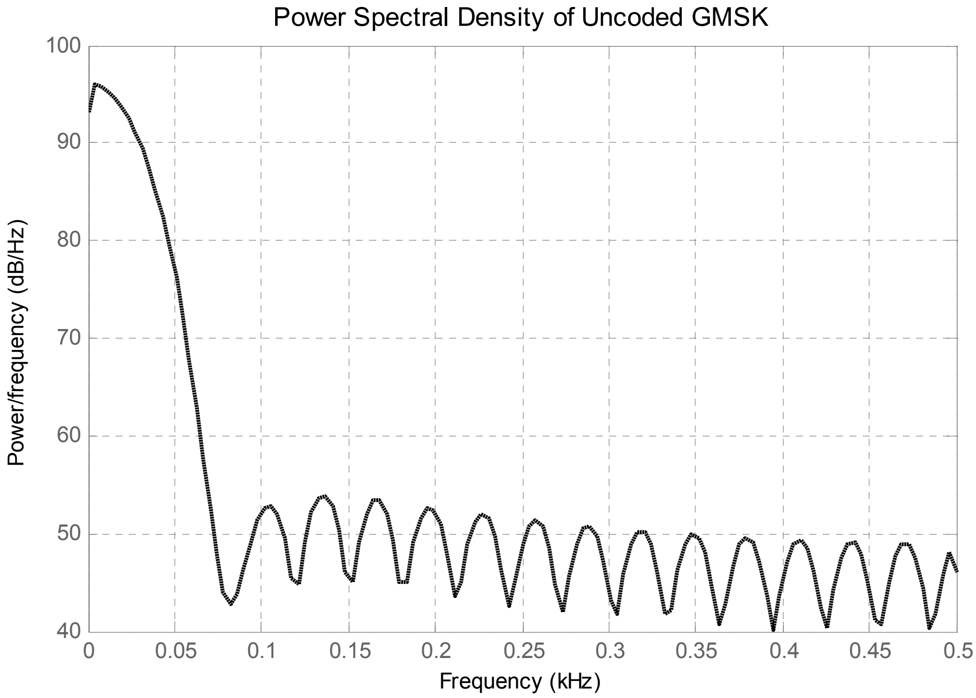

Figure 3.0: The Power Spectral Density of Uncoded GMSK

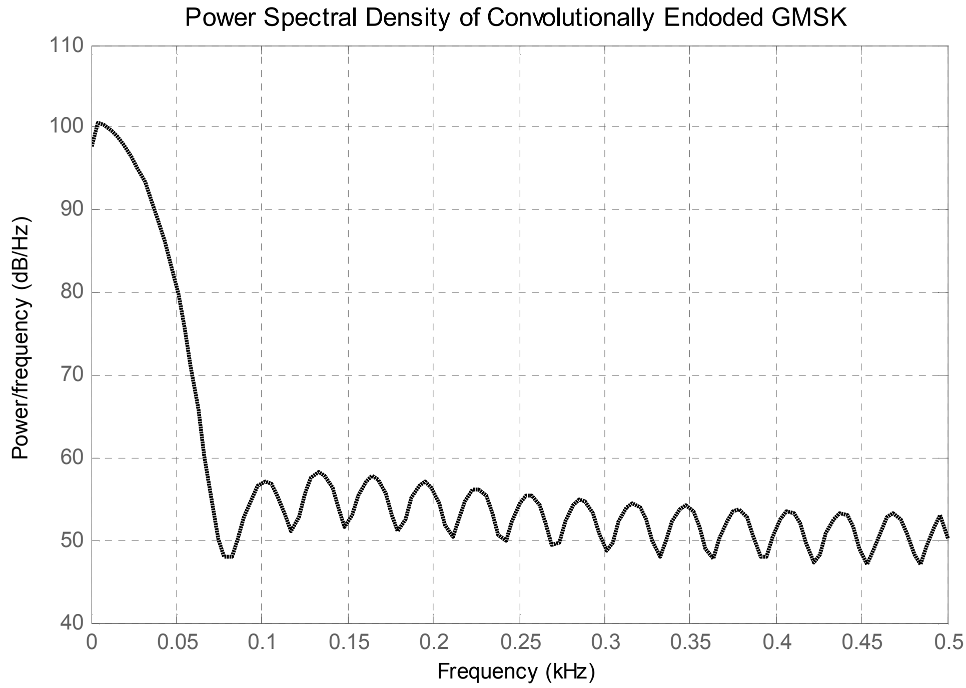

Figure 4.0: Power Spectral Density of Convolutionally Coded GMSK.

VII Conclusion

In this paper, we have derived the Power Spectral Density of Convolutionally Encoded Gaussian Minimum Shift Keying a Modulation technique adopted by the GSM Standard. The result of the MATLAB simulation of both uncoded and coded GMSK depicts variation in the Power Spectrum. . Peak-to-peak values measured from the average peak of the side-lobes to the peak of the main lobe for the uncoded and coded system are 41.91872dB and 42.39764dB respectively, showing a 0.47892dB deviation. The better performance of ½ rate encoded GMSK as compared to that of the uncoded system is due to the '*Spectral gain*' provided by the memory effect (inducing data correlation) of the convolutional encoding process employed, modeled as Markov process, which supposedly suppressed the spectral requirement from the added bit in the encoding process.

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