WHEN TO SELL OR HOLD A STOCK: EMPIRICAL EVIDENCE FROM AN EMERGING MARKET

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Abstract:

Data from an emerging market were used to determine when to sell or hold a stock for a single model of a stock whose price is assumed to be a geometric Brownian motion in which the jump Markov process changes back and forth between positive and negative values.

Keywords: Brownian motion, Sell or hold stock, optimal time to sell

1. Introduction

We shall consider a risky asset in a financial market which is usually described as a stochastic process $S = (S_t)_{t \geq 0}$ on some probability space $(\Omega, \mathcal{A}, P)$.

A variational inequality for solving optimal stopping problems was introduced [1]. In [2], they considered a stock selling problem whose stock price is given by a geometric Brownian motion with constant coefficients.

In [3], a stock selling problem whose stock price is given by a diffusion process whose coefficients are unobservable finite state jump Markov processes was considered. He said given $X(t)$ which denote the conditional probability that the drift rate is in the increasing state, given observations of the stock price up to time t. The stock should be sold at the first time $X(t)$ hits $[0, x^*]$. This selling time has finite expected value, so the stock will always be sold after a finite time in this case. In this paper the stock selling times are the first times the stock price leaves an interval.

In [4], they presented a selling strategy paper using the information of past prices through the running maximum price; he explained that the investor with the long position in one stock decides at time $t = 0$ to close out the position before time $t = T$. The dynamics of the

price process $(S_t)_{0 < t < T}$ is assumed to be Geometric Brownian motion satisfying the stochastic

differential equation .He assumed that the investor can sell the stock at any point in time, A selling strategy τ is then the amount of time in the given period $[0, T]$ that the investor holds

the position and is based on the information accumulated to date and not on future prices. Thus, τ is restricted to be a stopping time for S_t satisfying $\tau \leq T$. The investor's objective is to

maximise, over all strategies, the probability that the selling price of the stock is greater than a given percentage of the maximum price in $[0, T]$.

In [5], it was also considered selling a stock over a certain time interval $[0, T]$, knowing that

the price is a geometric Brownian motion with certain average return over the risk free rate $a - r$ and a certain volatility σ. The answer to this question depends on the value of the a-

dimensional parameter $\alpha = (a - r)/\sigma^2$. This method used by the author allows them to prove

that whenever $\alpha > 1/2$ the optimal selling time τ^* is always at the end of the interval $\tau^* = T$,

whereas $\tau^* = 0$ in the case of $\alpha < 0$.in financial terms 'good' stocks with sufficiently large

average returns should be sold as late as possible. Whereas one should immediately get rid of 'bad' stock. These results are clearly interesting however one feels unsatisfied by the fact that the author method does not treat the case of $0 < \alpha \leq 1/2$. He concluded that one should sell

immediately as soon as $\alpha < 1/2$ and suggest that the case $\alpha < 1/2$ is in any case not interesting

financially because most stocks realize $\alpha > 1/2$ by a large margin.

In [6], the problem of finding the optimal time to sell a stock, subject to a fixed sales cost and an exponential discounting rate ρ of a continuous time with a discrete driving nose They assumed that the price of the stock fluctuates according to the equation $dY_t = Y_t \left(\mu dt + \sigma \xi(t) dt \right)$, where $\left(\xi(t) \right)$ is an alternating Markov renewal process

with values in $\{\pm 1\}$, with an exponential renewal time. They determined the critical value of ρ under which the value function is finite. They examined the validity of the "principle of smooth fit" the principle of smooth fit holds in some situation but not in others, and the difference between the others shed more light on why the principle should hold in the first

place, it turns out that this semi discrete problem admits an essentially explicit solution which exhibits a surprisingly rich structure. The corresponding result when the stock price evolves according to the Black and Scholes model is obtained as a limit case.

In [7], the optimal decision to sell or buy a stock in a given period with reference to the ultimate average of the stock price was considered. More precisely, they aim to determine an optimal selling (buying) time to maximize (minimize) the expectation of the ratio of the selling (buying) price to the ultimate average price over the period. It is an optimal stopping time problem which can be formulated as a variational inequality problem. The problem gives rise to a free boundary that corresponds to the optimal selling (buying) strategy. They provide a partial differential equation approach to characterize the free boundary (or equivalently, the optimal selling (buying) region). It turns out that the optimal selling strategy is bang-bang, which is the same as that obtained in [5] taking the ultimate maximum of the stock price as benchmark, whereas the optimal buying strategy can be a feedback one subject to the type of averaging and parameter values. Moreover, by a thorough characterization of free boundary, they reveal that the bang-bang optimal selling strategy heavily depends on the assumption that no time-vesting restrictions are imposed. If a time-vested stock is considered, then the optimal selling strategy can also be a feedback one.

In [8], the optimal timing of derivative purchases in incomplete markets was looked at. In their model, an investor attempts to maximize the spread between her model price and the offered market price through optimally timing her purchase. Both the investor and the market value the options by risk-neutral expectations but under different equivalent martingale measures representing different market views. The structure of the resulting optimal stopping problem depends on the interaction between the respective market price of risk and the option pay- off. In particular, a crucial role is played by the delayed purchase premium that is related to the stochastic bracket between the market price and the buyer's risk premia. Explicit characterization of the purchase timing is given for two representative classes of Markovian models:

(i) defaultable equity models with local intensity;

(ii) diffusion stochastic volatility models.

In [9], a simple stock price model was analyzed. The model for the stock price is given by a geometric Brownian motion whose drift rate is a two state jump Markov process. One state of the drift rate is positive. In this state the stock price should increase. The other state of the drift rate is negative. In this state the stock price should decrease. The randomness of the Brownian motion makes these periods of increase or decrease only approximate. For

this model whether a stock is "good" or not depends on the solution of a differential equation. This solution is either positive on $[0,1]$ or there is an x^* such that it is positive on $(x^*,1]$ and negative on $[0,x^*)$. When the solution is positive the stock should not be sold. When it is not, the stock should be sold as follows. Let $x(t)$, denote the conditional probability that the drift rate is in the increasing state, given observations of the stock price up to time t. The stock should be sold at the first time $x(t)$ hits $[0,x^*]$. This selling time has finite expected value, so the stock will always be sold after a finite time in this case.

In [10], it was determined in real time whether or not a given asset's price process exhibits a bubble. Due to recent progress in the characterization of asset price bubbles using the arbitrage-free martingale pricing technology, they were able to propose a new methodology for answering this question based on the asset's price volatility. They limit themselves to the special case of a risky asset's price being modeled by a Brownian driven stochastic differential equation. Such models are ubiquitous both in theory and in practice. Their methods use sophisticated volatility estimation techniques combined with the method of reproducing kernel Hilbert spaces. They illustrated these techniques using several stocks from the alleged Internet dot-com episode of 1998–2001, where price bubbles were widely thought to have existed.

In [11], a variational inequality sufficient condition for optimal stopping problems was given. The result was illustrated by computing solutions to an optimal stock selling problem. He tried to decide when to sell stocks which have rapid growth and then rapidly decline which was similar to that of Enron stock. They considered an idealized model of such a situation in which the stock price is given by a geometric Brownian motion which has an initial positive growth rate and after a random time jumps to and stays at a negative growth rate. The investor observes the stock prices but cannot observe the growth rates. Deciding when to sell is based only on past stock prices. Thus this is a partially observed optimal stopping problem which was converted to a completely observed optimal stopping. They considered the problem with two different utility functions $U(S) = \ln S$ and $U(S) = S$.

They started by stating an optimal stopping problem and giving variational inequality sufficient conditions for it.

2. Mathematical Formulation

Given that the price of a stock satisfies a stochastic differential equation and initial condition

$$dS_t = \mu_t S_t dt + \sigma S_t dW_t, \qquad S_0 = S. \qquad (2.1)$$

Where;

S_t- is a random variable-valued function of time that represents the price of the stock at time t

W_t-Brownian motion process or Weiner process with the following properties:

- Each change $W_t - W_s = (W_{s+h} - W_s) + (W_{2s+h} - W_{s+h}) + \cdots + (W_t - W_{t-h})$ is normally distributed with mean 0 and variance $t - s$.

- Each change $W_t - W_s$ is independent of the all the previous values $W_u, u \leq s$.

- Each sample path $W_t, t \geq 0$ is (a.s) continuous and as $W_0 = 0$.

μ_t -Jump Markov process or the drift, it is the deterministic rate of growth of the stock price

σ - is a positive constant that provides a measure of the degree of variation in the price, called "volatility"

The jump Markov process has two states $a > 0 and - b < 0$

The generator of the process is given by

$$\begin{pmatrix} -c & k \\ c & -k \end{pmatrix}.$$

(2.2)

The constant a gives the rate of increase, b the rate of decrease, $1/c$ is the expected time the drift a_t stays in state a and $1/k$ is the expected time the drift stays in state b.

Let $F_t = \sigma[S_r, 0 \leq r \leq t]$

be the sigma field generated by observing the price of the stock up to time t, and his selling time τ be a F_t adapted stopping time.

A solution of (2.1) can be obtained using Ito's differential rule which states that given

$$dX_t = ydt + zdW_t.$$

(2.3)

Then

$$df(X, t) = \left(y\frac{df}{dX} + \frac{df}{dt} + \frac{z^2}{2}\frac{d^2 f}{dX^2} \right) dt + z\frac{df}{dX} dW_t.$$

(2.4)

Comparing (2.3) and (2.1) and using the utility function $f(S, t) = \log S_t$ with equation (2.4)

$$y = \mu_t S_t, \qquad z = \sigma S_t,$$

$$d \log S_t = \left(\mu_t S_t \frac{df}{dS_t} + \frac{df}{dt} + \frac{\sigma^2 S_t^2}{2} \frac{d^2 f}{dS_t^2} \right) dt + \sigma S_t \frac{df}{dS_t} dW_t.$$

(2.5)

$$\frac{df}{dS_t} = \frac{1}{S_t}, \qquad \frac{df}{dt} = 0, \qquad \frac{d^2 f}{dS_t^2} = -\frac{1}{S_t^2}.$$

Then (2.5) becomes

$$d \log S_t = \left(\mu_t S_t \cdot \frac{1}{S_t} + 0 + \frac{\sigma^2 S_t^2}{2} \cdot -\frac{1}{S_t^2} \right) dt + \sigma S_t \cdot \frac{1}{S_t} dW_t.$$

(2.6)

$$d \log S_t = \left(\mu_t - \frac{\sigma^2}{2} \right) dt + \sigma dW_t.$$

(2.7)

Integrating both sides of the equation

$$\int_0^t d \log S_t = \int_0^t \left(\mu_t - \frac{\sigma^2}{2} \right) dt + \sigma dW_t.$$ (2.8)

$$\log S_t - \log S_0 = \int_0^t (\mu_t \, dt + \sigma dW_t) - \frac{\sigma^2}{2} t.$$

(2.9)

Since $S_0 = S$

(2.9) becomes

$$\log S_t = \log S + \int_0^t (\mu_t \, dt + \sigma dW_t) - \frac{\sigma^2}{2} t.$$

(2.10)

Taking the exponential both sides of equation (2.10), we have

$$S_t = S e^{\int_0^t (\mu_t \, dt + \sigma dW_t) - \frac{\sigma^2}{2} t}.$$

(2.11)

Equation (2.11) is the solution of the stock price given in (2.1).

The investor observes the stock price S_t, but cannot observe the growth rate μ_t, this is a partially observed stopping problem. In [5], they obtained a technique to reduce this type of stopping problem to a completely observed problem.

From (2.9)

$$\log S_t - \log S_0 + \frac{\sigma^2}{2} t = \int_0^t (\mu_t \, dt + \sigma dW_t).$$ (2.12)

Equation (2.12) implies observing S_t is equivalent to observing Y_t where

$$Y_t = \int_0^t (\mu_t \, dt + \sigma dW_t).$$

(2.13)

and the sigma fields

$$\sigma[S_r, 0 \le r \le t] \text{ and } \sigma[Y_r, 0 \le r \le t]$$

are equal.

Determining the stopping time of holding a stock depend on the solution of a differential equation, obtained as follows:

Let x_t denote, the conditional probability that the drift rate is in the increasing state, given observation of the stock price up to time t.

[5], gave a conditional probability of states of a jump Markov process given a measurement of the type (2.13).

If

$$x_t = Pr[\mu_t = a | Y_r, 0 \le r \le t] \text{ and } x_0 = Pr[\mu_0 = a]$$

(2.14)

then x_t is a solution of

$$dx_t = \{-cx_t + k(1 - x_t)\}dt + \left(\frac{a-b}{\sigma}\right)(1 - x_t)x_t dv_t$$

(2.15)

In (2.15), v_t is a Brownian motion process called the innovation process, it is related to the Brownian motion process W_t through the relationship

$$\int_0^t (\mu_t \, dt + \sigma dW_t) = \int_0^t \{ax_t + b(1 - x_t)\}dt + \sigma d v_t.$$

(2.16)

Applying (2.16) to (2.10)

$$\log S_t = \log S + \int_0^t \{(ax_t + b(1 - x_t))dt + \sigma dv_t\} - \frac{\sigma^2}{2}t$$

(2.17)

$$\log S_t = \log S + \int_0^t \{(ax_t + b(1 - x_t) - \frac{1}{2}\sigma^2)dt\} + \int_0^t \sigma dv_t$$

(2.18)

Re-writing (2.18) as

$$\log S_r = \log S + \int_0^r \{(ax_t + b(1 - x_t) - \frac{1}{2}\sigma^2)dt\} + \int_0^r \sigma dv_t$$

(2.19)

In (2.19) replacing r by the random time $\tau \wedge t$ and taking the expectation of both sides

$$E[\log S_{\tau \wedge t}] = \log S + E[\int_0^{\tau \wedge t} \{(ax_t + b(1 - x_t) - \frac{1}{2}\sigma^2)dt\}] + E[\int_0^{\tau \wedge t} \sigma dv_t]$$

(2.20)

The expected value of the stochastic integral

$$E[\int_0^{\tau \wedge t} \sigma dv_t]$$

(2.21)

is zero since the upper limit has finite expected value.

The expected integral of a stopping problem is given by:

$$E\left[\int_0^{\tau \wedge t}\{(ax_t + b(1 - x_t) - \tfrac{1}{2}\sigma^2)dt\}\right]$$

(2.22)

2.1.1 Preliminary Considerations

If the solution to (2.22) is non-positive, the stock is considered to be risky and should be sold off almost immediately.

If the solution to (2.22) is positive, the stock is not risky and does not necessarily need to be sold off immediately.

3. Numerical Results

We considered the stock (share) of a bank purchased at the beginning of the year 2011 for ₦18.26/unit, an investor wants to sell off some unit of the stock in the first half of the month.

The investor wants to determine the optimal time to sell off units of the stock within the first half of the month.

Table 1: Stock Price Movement Bank Quoted in NSE 1st Quarter (January-March 2011)

S/N	DATE	CLOSING PRICE	S/N	DATE	CLOSING PRICE
1	04/01/2011	18.26	33	21/02/2011	19.76
2	05/01/2011	19.17	34	22/02/2011	19.94
3	06/01/2011	20.08	35	23/02/2011	20.13
4	07/01/2011	19.08	36	24/02/2011	19.24
5	10/01/2011	18.7	37	25/02/2011	19.27
6	11/01/2011	19	38	28/02/2011	19.05
7	12/01/2011	19.95	39	01/03/2011	19.05
8	13/01/2011	19.9	40	02/03/2011	19.07
9	14/01/2011	19.99	41	03/03/2011	19.1
10	17/01/2011	19.98	42	04/03/2011	19.2
11	18/01/2011	19.9	43	07/03/2011	19.1

12	19/01/2011	20	44	08/03/2011	18.8
13	20/01/2011	20.02	45	09/03/2011	19.1
14	21/01/2011	20.5	46	10/03/2011	19.31
15	24/01/2011	20.45	47	11/03/2011	19.91
16	25/01/2011	20.49	48	14/03/2011	19.95
17	26/01/2011	20.05	49	15/03/2011	18.77
18	27/01/2011	19.7	50	16/03/2011	19
19	28/01/2011	19.55	51	17/03/2011	19.2
20	31/01/2011	19.01	52	18/03/2011	19.17
21	01/02/2011	19.85	53	21/03/2011	19
22	02/02/2011	19.9	54	22/03/2011	19.95
23	03/02/2011	19.6	55	23/03/2011	20
24	04/02/2011	19.58	56	24/03/2011	19.75
25	07/02/2011	19.6	57	25/03/2011	19.02
26	08/02/2011	19.26	58	28/03/2011	19.2
27	09/02/2011	19.81	59	29/03/2011	19.97
28	10/02/2011	19.5	60	30/03/2011	19.9
29	11/02/2011	19.46	61	31/03/2011	19.97
30	16/02/2011	19.41			
31	17/02/2011	19.6			
32	18/02/2011	19.7			

3.1. Case 1

The prices of the share analyzed for the first quarter of the year i.e. from January-March, (see table I)

From the data of the first quarter of the month, we obtain:

$$\sigma^2 \cong 0.2289$$
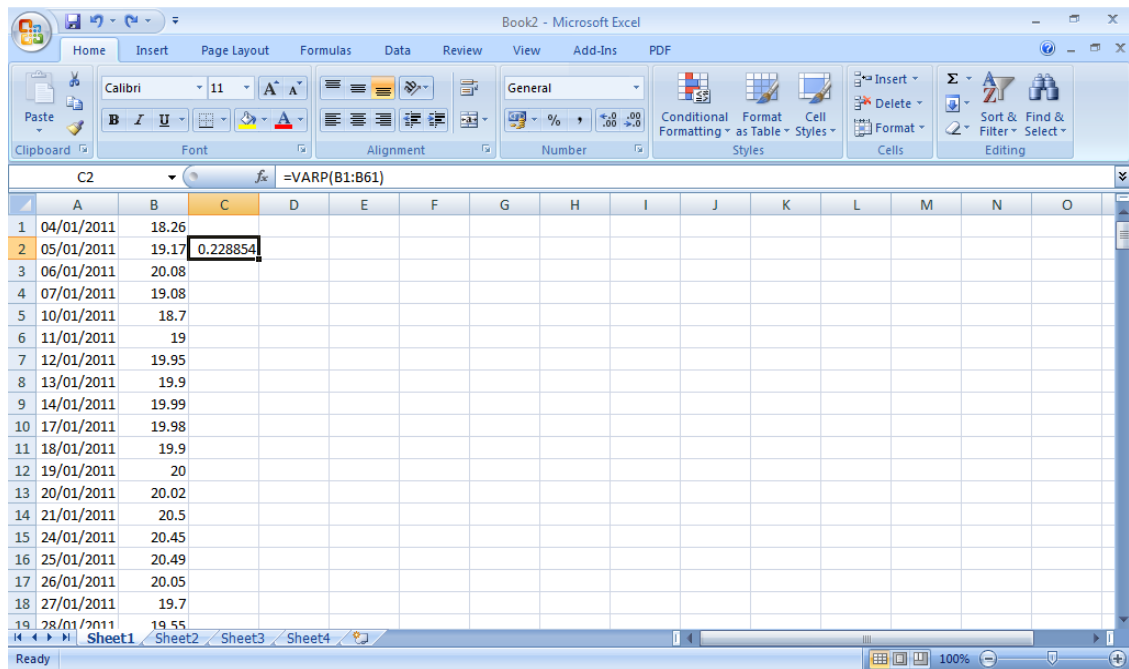
This value is obtained figure1 below

Figure 1: calculating the variance of the stock movement

Let μ_1, μ_2, μ_3 denote the mean price for the first, second and third month.
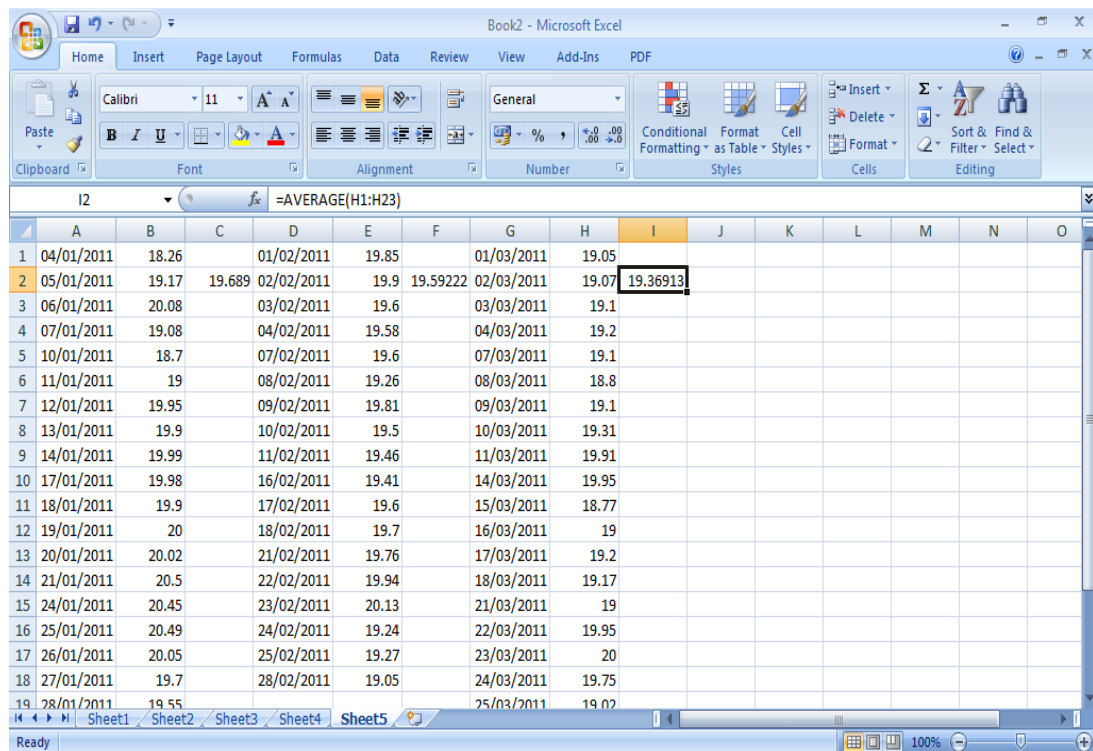
Figure 2: Calculating mean of the price movement

In figure2 above, Column C gives the mean price of the first month i.e. column B

Column F gives the mean price of the second month i.e. column E

Column I gives the mean price of the third month i.e. column H

Therefore;

$\mu_1 \cong 19.69, \ \mu_2 \cong 19.59, \ \mu_3 \cong 19.37$

Using the expected integral in (3.22),

$$E\left[\int_0^{\tau \wedge t}\{(ax_t + b(1 - x_t) - \tfrac{1}{2}\sigma^2)dt\}\right]$$

Recall that:

$x_t = Pr\ [a_t = a > 0]$, i.e. the probability that the mean is increasing positively given the

interval $0 \leq r \leq t$

a =rate of increase of μ, b =rate of decrease of $\mu.$

Since $\mu_1 > \mu_2 > \mu_3$ i.e the mean are decreasing then,

$a = 0,$

$x_t = 0,$ since the mean are not increasing positively.

Figure3: Displaying the mean values calculated.

From figure3 above;

Column C gives the rate of change of the mean given by

$$\%change = \frac{new\ value - initial\ value}{initial\ value} * 100$$

Column D gives the total rate of decrease of the mean given by;

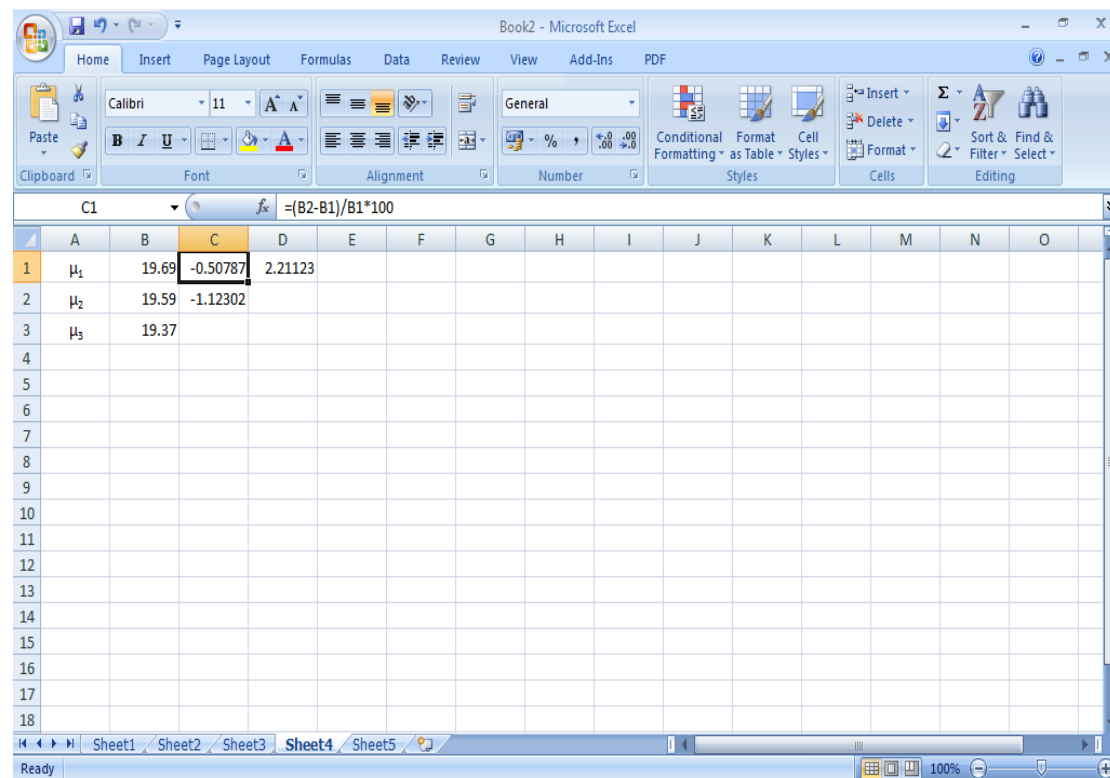$$rate\ of\ decrease = \frac{sum\ of\ rate\ of\ decrease - initial\ rate\ of\ decrease}{initial\ rate\ of\ decrease}$$

Hence

$b \cong -2.211$ (Since it is the rate of decrease)

Inserting the values of $\sigma^2 = 0.2289$, $a = 0$, $b = -2.211$, $x_t = 0$ into the expected integral of (3.22), we have,

$$E\left[\int_0^{\tau \wedge t}\{(ax_t + b(1 - x_t) - \tfrac{1}{2}\sigma^2)dt\}\right]$$

$$= E\left[\int_0^{3 \wedge 6}\{(0 + (-2.211)(1 - 0) - \tfrac{1}{2}(0.2289))dt\}\right]$$

$$= E\left[\int_0^{3 \wedge 6}\{-2.211 - 0.11445dt\}\right] = E\left[\int_0^{3 \wedge 6}\{-2.32545dt\}\right]$$

$$= E\left[-2.32545t\right]_0^3 = E[\{-2.32545(3)\} - \{-2.32545(0)\}]$$

$$= E[-6.97635]$$

Since $E[k] = k$, where k is a constant

Therefore; $E[-6.97635] = $ -6.97635.

Therefore, for the first quarter of the year the value of the expected integral equals to -6.97635.

Table 2: Stock Price Movement of a Bank Quoted in NSE 2nd Quarter (April-June 2011)

S/N	DATE	CLOSING PRICE	S/N	DATE	CLOSING PRICE
62	01/04/2011	20.4	93	20/05/2011	16.01
63	04/04/2011	20.15	94	23/05/2011	16.04
64	05/04/2011	19.82	95	24/05/2011	16.06
65	06/04/2011	19.75	96	25/05/2011	15.95
66	07/04/2011	15.2	97	26/05/2011	16.2
67	08/04/2011	15.2	98	27/05/2011	16.2
68	11/04/2011	14.86	99	31/05/2011	16.25
69	12/04/2011	14.66	100	01/06/2011	16.28

70	13/04/2011	15.2	101	02/06/2011	16.34
71	14/04/2011	15.21	102	03/06/2011	16.33
72	15/04/2011	15.46	103	06/06/2011	16.2
73	18/04/2011	15.75	104	07/06/2011	16.21
74	19/04/2011	15.75	105	08/06/2011	16.21
75	20/04/2011	15.7	106	09/06/2011	16
76	21/04/2011	16	107	10/06/2011	15.9
77	27/04/2011	16.1	108	13/06/2011	15.9
78	28/04/2011	16.07	109	14/06/2011	15.93
79	29/04/2011	16.21	110	15/06/2011	16.05
80	03/05/2011	16.59	111	16/06/2011	15.95
81	04/05/2011	16.4	112	17/06/2011	15.92
82	05/05/2011	16.4	113	20/06/2011	15.85
83	06/05/2011	16.25	114	21/06/2011	15.1
84	09/05/2011	16.06	115	22/06/2011	15.48
85	10/05/2011	16.1	116	23/06/2011	15.75
86	11/05/2011	16.25	117	24/06/2011	15.5
87	12/05/2011	16.23	118	27/06/2011	15.5
88	13/05/2011	16.1	119	28/06/2011	15.4
89	16/05/2011	16.06	120	29/06/2011	15.16
90	17/05/2011	16.42	121	30/06/2011	15.58
91	18/05/2011	16.06			
92	19/05/2011	16.25			

3.2 Case2

The prices of the share analyzed for the first half of the year i.e. from January-June, (see table I and II)

From the data of the first half of the month (using the data in table I and II), we obtain figure4 below;
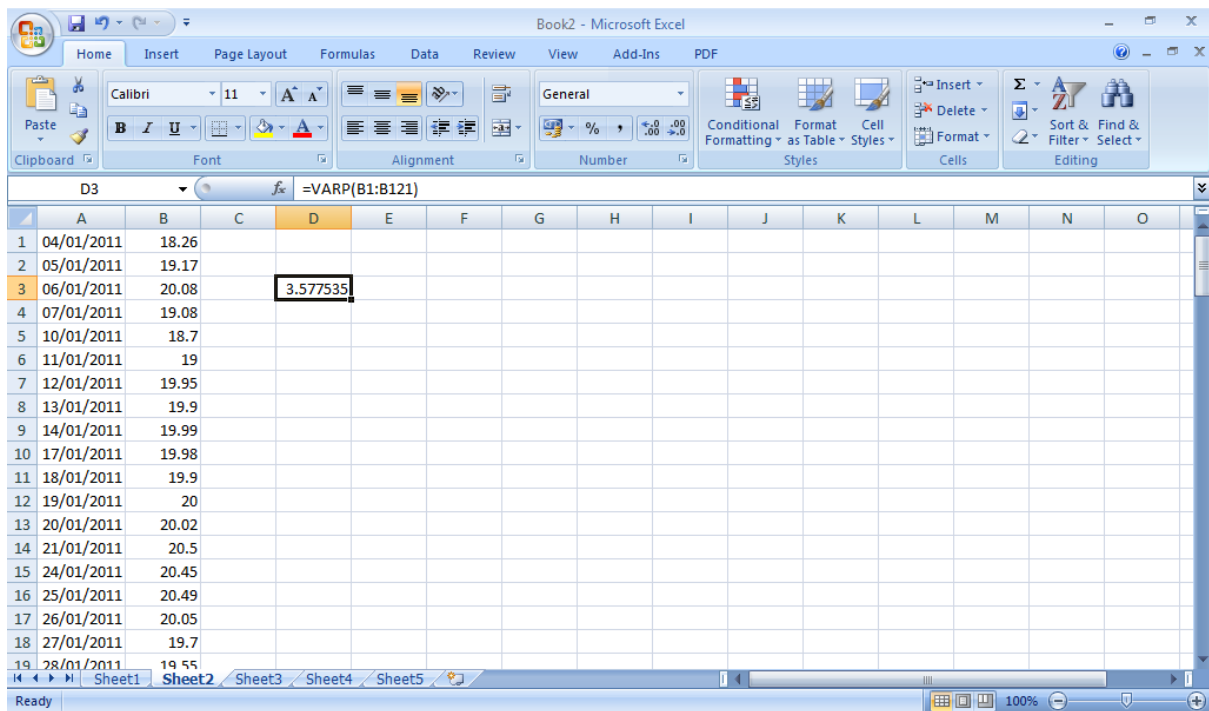
Figure4: Calculating variance of price movement in case 2.

From figure4 above, column B represent the prices of the share from January to June, we obtain:

$$\sigma^2 \cong 3.578$$

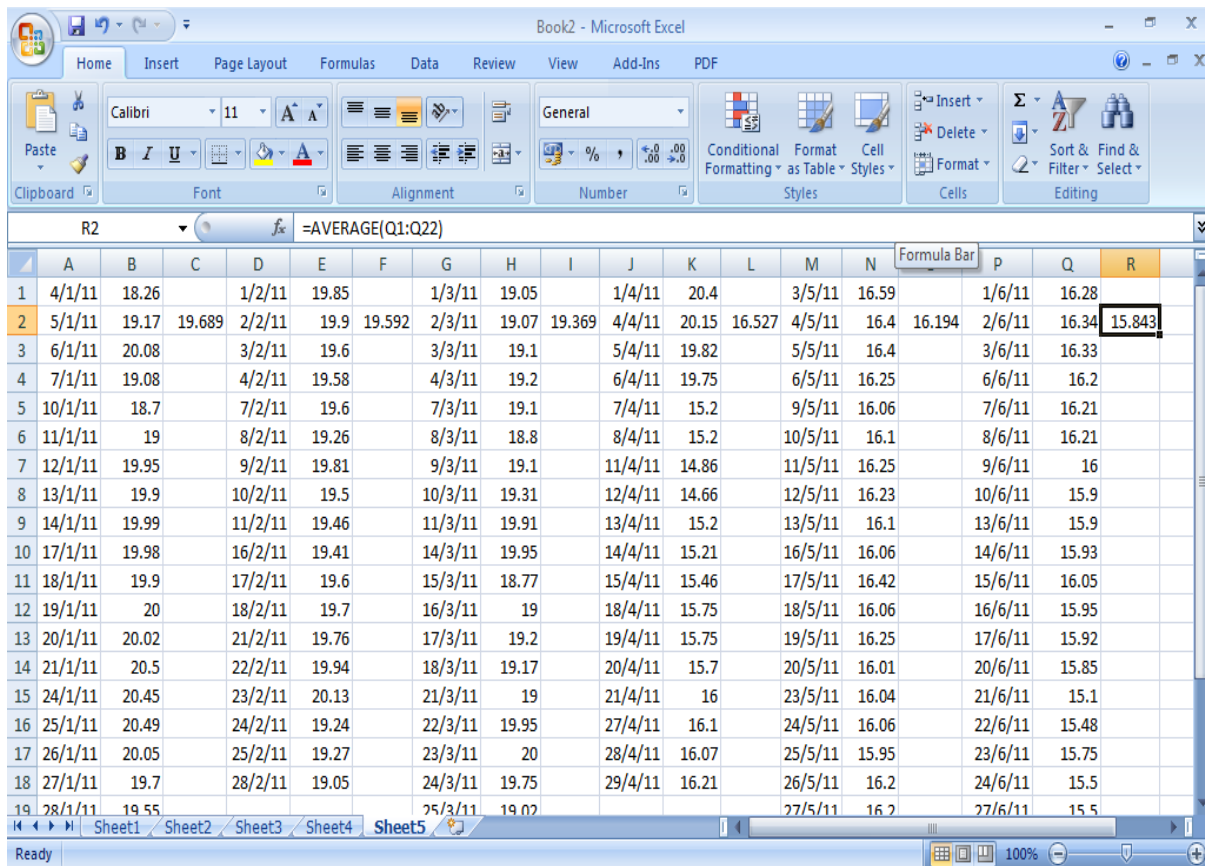| A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
4/1/11	18.26		1/2/11	19.85		1/3/11	19.05		1/4/11	20.4		3/5/11	16.59		1/6/11	16.28	
5/1/11	19.17	19.689	2/2/11	19.9	19.592	2/3/11	19.07	19.369	4/4/11	20.15	16.527	4/5/11	16.4	16.194	2/6/11	16.34	15.843
6/1/11	20.08		3/2/11	19.6		3/3/11	19.1		5/4/11	19.82		5/5/11	16.4		3/6/11	16.33	
7/1/11	19.08		4/2/11	19.58		4/3/11	19.2		6/4/11	19.75		6/5/11	16.25		6/6/11	16.2	
10/1/11	18.7		7/2/11	19.6		7/3/11	19.1		7/4/11	15.2		9/5/11	16.06		7/6/11	16.21	
11/1/11	19		8/2/11	19.26		8/3/11	18.8		8/4/11	15.2		10/5/11	16.1		8/6/11	16.21	
12/1/11	19.95		9/2/11	19.81		9/3/11	19.1		11/4/11	14.86		11/5/11	16.25		9/6/11	16	
13/1/11	19.9		10/2/11	19.5		10/3/11	19.31		12/4/11	14.66		12/5/11	16.23		10/6/11	15.9	
14/1/11	19.99		11/2/11	19.46		11/3/11	19.91		13/4/11	15.2		13/5/11	16.1		13/6/11	15.9	
17/1/11	19.98		16/2/11	19.41		14/3/11	19.95		14/4/11	15.21		16/5/11	16.06		14/6/11	15.93	
18/1/11	19.9		17/2/11	19.6		15/3/11	18.77		15/4/11	15.46		17/5/11	16.42		15/6/11	16.05	
19/1/11	20		18/2/11	19.7		16/3/11	19		18/4/11	15.75		18/5/11	16.06		16/6/11	15.95	
20/1/11	20.02		21/2/11	19.76		17/3/11	19.2		19/4/11	15.75		19/5/11	16.25		17/6/11	15.92	
21/1/11	20.5		22/2/11	19.94		18/3/11	19.17		20/4/11	15.7		20/5/11	16.01		20/6/11	15.85	
24/1/11	20.45		23/2/11	20.13		21/3/11	19		21/4/11	16		23/5/11	16.04		21/6/11	15.1	
25/1/11	20.49		24/2/11	19.24		22/3/11	19.95		27/4/11	16.1		24/5/11	16.06		22/6/11	15.48	
26/1/11	20.05		25/2/11	19.27		23/3/11	20		28/4/11	16.07		25/5/11	15.95		23/6/11	15.75	
27/1/11	19.7		28/2/11	19.05		24/3/11	19.75		29/4/11	16.21		26/5/11	16.2		24/6/11	15.5	
28/1/11	19.55					25/3/11	19.02					27/5/11	16.2		27/6/11	15.5	

Figure5: Calculating the mean of price movement in case 2

In figure5 above:

Column B represents the share price of the first month and column C represents the mean value for the first month.

Column E represents the share price of the second month and column F represents the mean value for the second month.

Column H represents the share price of the third month and column I represent the mean value for the third month.

Column K represents the share price of the fourth month and column L represents the mean value for the fourth month.

Column N represents the share price of the fifth month and column O represents the mean value for the fifth month.

Column Q represents the share price of the sixth month and column R represents the mean value for the sixth month.

Hence;

$\mu_1 \cong 19.69$, $\mu_2 \cong 19.59$, $\mu_3 \cong 19.37$, $\mu_4 \cong 16.53$, $\mu_5 \cong 16.20$, $\mu_6 \cong 15.84$.

Where μ_1, μ_2, μ_3, μ_4, μ_5, and μ_6 represent the mean of the first month, second month, third month, fourth month, fifth month and sixth month.

Since $\mu_1 > \mu_2 > \mu_3 > \mu_4 > \mu_5 > \mu_6$ i.e. the mean are decreasing, then

$a = 0,$
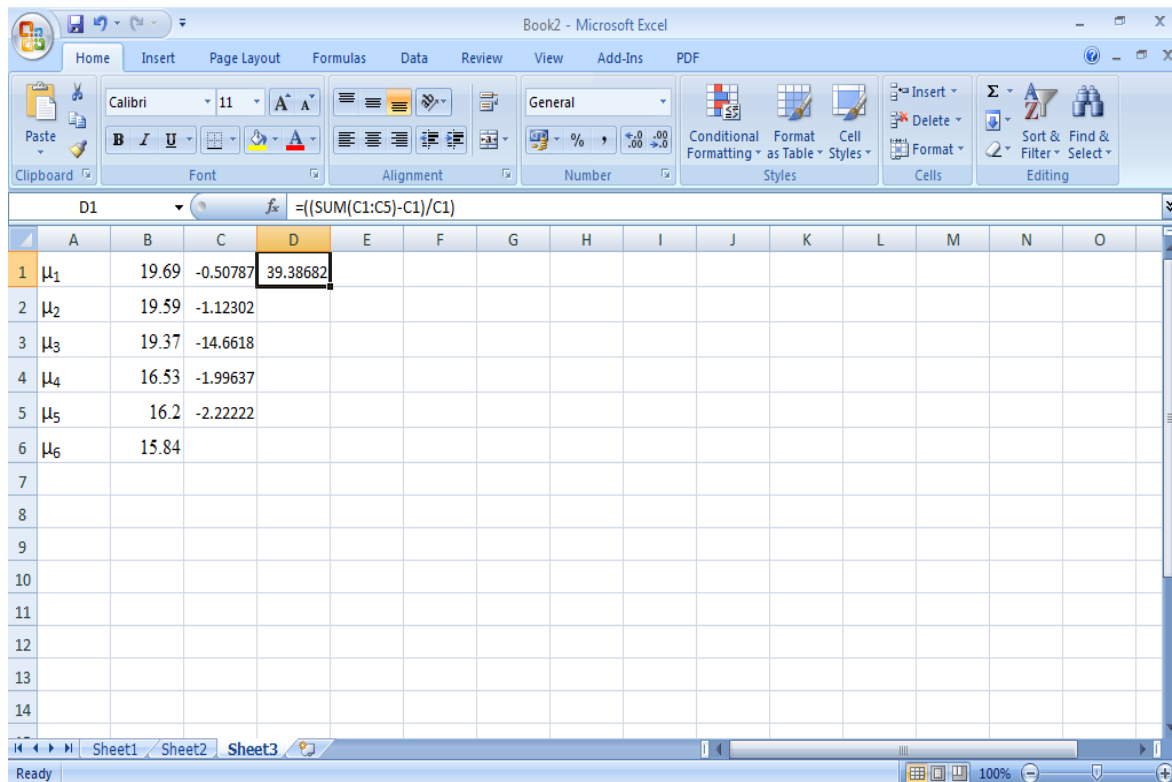
$x_t = 0,$ since the mean are not increasing positively.

Figure6: Displaying the mean of the price movements in case2

From figure 6 above;

Column C gives the rate of change of the mean given by

$$\%change = \frac{new\ value - initial\ value}{initial\ value} * 100$$

Column D gives the total rate of decrease of the mean given by;

$$rate\ of\ decrease = \frac{sum\ of\ rate\ of\ decrease - initial\ rate\ of\ decrease}{initial\ rate\ of\ decrease}$$

Hence

$b \cong -39.387$ (since it is the rate of decrease)

From (3.22)

$$E\left[\int_0^{\tau \wedge t}\{(ax_t + b(1 - x_t) - \tfrac{1}{2}\sigma^2)dt\}\right]$$

Inserting the values of $\sigma^2 = 3.578,\ a = 0,\ b = -39.387,\ x_t = 0$ into the expected integral of (3.22), we have,

$$= E\left[\int_0^6\{(0(0) + (-39.387)(1 - 0) - \tfrac{1}{2}3.578)dt\}\right]$$

$$= E\left[\int_0^6\{-39.387 - 1.789)dt\}\right] = E\left[\int_0^6\{-41.176dt\}\right]$$

$$= E\left[-41.176t\right]_0^6 = E[-247.056]$$

Since $E[k] = k,$ where k is a constant

Therefore; $E[-247.056] = -247.056.$

4. Conclusion

In the previous session of this work, an expected integral was obtained, defined as $E\left[\int_0^{\tau \wedge t}\{(ax_t + b(1 - x_t) - \tfrac{1}{2}\sigma^2)dt\}\right]$, the result of this differential equation is either positive or non-positive. If the result of the expected integral is non-positive then stock is considered to be risky and should be sold off almost immediately and if the solution is positive the stock is considered to be good and does not necessarily need to be sold off immediately.

The values of the variable of the expected integral (i.e. values for the variables σ^2, a, b and x_t) was obtained using the share prices of a Bank from January 2011- June 2011(see table I and II for the data) the investor wants to sell off some of the units of the stock within the first sixth month of the year but he wants to determine the optimal time to sell off the stock within the first six month.

Two Cases were considered; i.e. Case1 and Case2. Numerical result of the share prices for the first quarter of the year (i.e. January-March 2011) using the expected integral was obtained, while Numerical result of the share prices of the first half of the year (i.e. January- June 2011) was obtained.

For Case1, the result obtained was non-positive (i.e. -6.97635), hence the stock is regarded to be risky and it is certain that the price would further decrease, so it is optimal for the investor to sell off the stock at that period

For Case2 the result obtained was non-positive (i.e. -247.056), hence the stock is regarded to be very risky and there had been a further depreciation of the share price from that of March to June's price, hence the optimal time to sell off the stock is immediately.

In this work, we consider a case where the expected integral is non-positive, finding the optimal time to sell for a case whereby the expected integral is positive should be considered.

Reference:

Bensoussan, A. (1982), "Stochastic Control by Functional Analysis Methods", North Holland.

Min Dai and Yifei Zhong(2010), "Optimal Stock Selling/Buying Strategy with reference to the Ultimate Average", RMI Working Paper No. 10/04

Oksendal, B. (2003), 'Stochastic Differential Equations', Springer-Verlag.

Pedersen , J.L.(2007), "An Optimal Selling Strategy for Stock Trading Based on Predicting the Maximum Price"

Rishel, R. and K.Helmes (2006), "A variational inequality sufficient condition for optimal stopping with application to an optimal stock selling problem", SIAM J, control and optimization.

Rishel, R.(2006), "Whether to sell or hold a stock", Communications in Information and Systems, Vol.   6, No. 3, pp. 193-202.

Robert C. Dalang and M.O. Hongler (2004), "The right time to sell a stock whose price is driven by a Markovian noise", Ann. Appl. Probab. Volume 14, Number 4, 2176-2201.

Robert Jarrow, Younes Kchia, and Philip Protter (2007), "How to Detect an Asset Bubble", SIAM J.Financial Math.Vol.2, pp 839-865

Shiryaev, A., Q. Xu, and X. Zhou (2008), "Response to comment on :'Thou shall buy and hold'", Quantitative Finance, 8(8).

Tim Leung and Mike Ludkovski (2011), "Optimal Timing to Purchase Options", SIAM J.Financial Math.Vol.2, pp 768-793.

Zhang, Q. (2001), "Stock Trading: An Optimal Selling Rule", SIAM Journal on Control and Optimization, Vol 40.