Karapınar and Agarwal *Fixed Point Theory and Applications* 2013, **2013**:216
http://www.fixedpointtheoryandapplications.com/content/2013/1/216

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

A note on 'Coupled fixed point theorems for α-ψ-contractive-type mappings in partially ordered metric spaces'

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Abstract
In this paper, we show that some examples in (Mursaleen *et al.* in Fixed Point Theory Appl. 2012:124, 2012) are not correct. Then, we extend, improve and generalize their results. Finally, we state some examples to illustrate our obtained results.
MSC: 47H10; 54H25

Keywords: coupled fixed point; fixed point; ordered set; metric space; α-ψ contractive mapping; α-admissible; (c)-comparison function; partial order

1 Introduction and preliminaries

In the sequel, let X be a non-empty set. Throughout the text, we use indifferently the notation Y or X^2 to denote the product space $X \times X$. Let $T: X \to X$ and $F: X^2 \to X$ be two mappings. From now on, $\preceq$ will denote a partial order on X, and d will be a metric on X. To determine sufficient conditions in order to ensure the existence of the following kind of points is the main aim of the present manuscript.

Definition 1.1 (See [1]) An element $(x, y) \in X^2$ is called a *coupled fixed point* of a mapping $F: X^2 \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Definition 1.2 (See [1]) Let $(X, \preceq)$ be a partially ordered set, and let $T: X \to X$ be a mapping. Then T is said to be *non-decreasing with respect to* $\preceq$ if $x \preceq y$ implies $Tx \preceq Ty$, and it is *non-increasing* (*w.r.t.* $\preceq$) if $x \preceq y$ implies $Tx \succeq Ty$ for every $x, y \in X$.

Definition 1.3 (See, *e.g.*, [2]) Let $(X, \preceq)$ be a partially ordered set. A sequence $\{x_n\} \subset X$ is said to be non-decreasing with respect to $\preceq$ if $x_n \preceq x_{n+1}$ for all n.

Definition 1.4 (See [2]) Let $(X, \preceq)$ be a partially ordered set, and let d be a metric on X. We say that $(X, \preceq, d)$ is regular if for every non-decreasing sequence $\{x_n\} \subset X$ such that $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \preceq x$ for all k.

Definition 1.5 (See [1]) Let $(X, \preceq)$ be a partially ordered set, and let $F: X^2 \to X$ be a mapping. The mapping F is said to have the *mixed monotone property* if $F(x, y)$ is monotone non-decreasing in x and monotone non-increasing in y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad x_1 \preceq x_2 \quad \Longrightarrow \quad F(x_1, y) \preceq F(x_2, y)$$

and

$$y_1, y_2 \in X, \quad y_1 \preceq y_2 \quad \Longrightarrow \quad F(x, y_1) \succeq F(x, y_2).$$

The partial order $\preceq$ on X can be induced on X^2 in the following way

$$(x,y),(u,v) \in X^2, \quad (x,y) \preceq_2 (u,v) \quad \Longleftrightarrow \quad x \preceq u \quad \text{and} \quad y \succeq v. \tag{1}$$

We say that (x,y) is comparable to (u,v) if either $(x,y) \preceq_2 (u,v)$ or $(x,y) \succeq_2 (u,v)$. According to the definitions above, a sequence $\{(x_n,y_n)\} \subset X^2$ is non-decreasing with respect to $\preceq_2$ if $(x_n,y_n) \preceq_2 (x_{n+1},y_{n+1})$ for all n.

If d is a metric on X, we will consider the metrics $d_2, d_{\max} : Y \times Y \to [0,\infty)$ defined, for all $(x,y),(u,v) \in Y$, by

$$d_2\big((x,y),(u,v)\big) = d(x,u) + d(y,v), \qquad d_{\max}\big((x,y),(u,v)\big) = \max\big(d(x,u), d(y,v)\big).$$

Notice that $(X^2, \preceq_2, d_2)$ is regular if for every non-decreasing sequence $\{(x_n,y_n)\} \subset X^2$ such that $(x_n,y_n) \xrightarrow{d_2} (x,y) \in Y$ as $n \to \infty$, there exists a subsequence $\{(x_{n(k)},y_{n(k)})\}$ of $\{(x_n,y_n)\}$ such that $(x_{n(k)},y_{n(k)}) \preceq_2 (x,y)$ for all k.

Given a mapping $F : X^2 \to X$, the mapping $T_F : Y \to Y$ will be defined as follows:

$$T_F(x,y) = \big(F(x,y), F(y,x)\big) \quad \text{for all } (x,y) \in Y.$$

The following result can be easily shown.

Lemma 1.1 (See, *e.g.*, [3]) *The following properties hold*:
(a) *if* (X,d) *is complete, then* (Y,d_2) *and* $(Y,d_{\max})$ *are complete*;
(b) F *has the mixed monotone property on* $(X,\preceq)$ *if, and only if,* T_F *is monotone non-decreasing with respect to* $\preceq_2$;
(c) $(x,y) \in X \times X$ *is a coupled fixed point of* F *if and only if* (x,y) *is a fixed point of* T_F.

Let Ψ be the family of functions $\psi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

(Ψ_1) ψ is non-decreasing;
(Ψ_2) $\sum_{n=1}^{+\infty} \psi^n(t) < \infty$ for all $t > 0$, where ψ^n is the nth iterate of ψ.

These functions are known in the literature as Bianchini-Grandolfi gauge functions in some sources and as (c)-comparison functions in others (see, *e.g.*, [4]). They have a crucial role in fixed point theory (see, *e.g.*, [5–7]). It is easily proved that if ψ is a (c)-comparison function, then $\psi(t) < t$ for any $t > 0$.

Very recently, Samet *et al.* [8] introduced the following concepts.

Definition 1.6 Let (X,d) be a metric space, and let $T : X \to X$ be a given mapping. We say that T is an α-ψ-contractive mapping if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that

$$\alpha(x,y) d(Tx,Ty) \le \psi\big(d(x,y)\big) \quad \text{for all } x,y \in X. \tag{2}$$

Clearly, any contractive mapping (that is, a mapping satisfying the Banach contraction property associated to $k \in (0,1)$) is an α-ψ-contractive mapping with $\alpha(x,y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$ for all $t \geq 0$.

Definition 1.7 Let $T : X \to X$, and let $\alpha : X \times X \to [0,\infty)$. We say that T is α-admissible if for all $x, y \in X$, we have

$$\alpha(x,y) \geq 1 \quad \Longrightarrow \quad \alpha(Tx,Ty) \geq 1.$$

Various examples of such mappings are presented in [8]. The characterization of this notion for the setting of G-metric spaces was considered in [9, 10]. The main results in [8] are the following fixed point theorems.

Theorem 1.1 *Let (X,d) be a complete metric space, and let $T : X \to X$ be an α-ψ-contractive mapping. Suppose that*
(i) *T is α-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *T is continuous*.
Then there exists $u \in X$ such that $Tu = u$.

Theorem 1.2 *Let (X,d) be a complete metric space, and let $T : X \to X$ be an α-ψ-contractive mapping. Suppose that*
(i) *T is α-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.*
Then there exists $u \in X$ such that $Tu = u$.

Theorem 1.3 *Adding to the hypotheses of Theorem* 1.1 (*resp. Theorem* 1.2) *the condition* '*For all $x, y \in X$, there exists $z \in X$ such that $\alpha(x,z) \geq 1$ and $\alpha(y,z) \geq 1$*,' *we obtain the uniqueness of the fixed point*.

In [8], the authors also mentioned some existing results can be considered as a particular case of their main results, see *e.g.* [11–19]. Later, Karapınar and Samet [2] extended and generalized the result of Samet *et al.* [8] by stating the following definitions.

Definition 1.8 Let (X,d) be a metric space, and let $T : X \to X$ be a given mapping. We say that T is a generalized α-ψ-contractive mapping of type I if there exist two functions $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have

$$\alpha(x,y)d(Tx,Ty) \leq \psi\bigl(M(x,y)\bigr), \tag{3}$$

where $M(x,y) = \max\{d(x,y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\}$.

Definition 1.9 Let (X,d) be a metric space, and let $T : X \to X$ be a given mapping. We say that T is a generalized α-ψ-contractive mapping of type II if there exist two functions

$\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have

$$\alpha(x,y)d(Tx,Ty) \leq \psi\big(N(x,y)\big), \tag{4}$$

where $N(x,y) = \max\{d(x,y), \frac{d(x,Tx)+d(y,Ty)}{2}\}$.

Remark 1.1 Clearly, since ψ is non-decreasing, every α-ψ-contractive mapping is a generalized α-ψ-contractive mapping of types I and II. Notice also that every generalized α-ψ-contractive mapping of type II is also a generalized α-ψ-contractive mapping of type I.

Karapınar and Samet [2] proved the following theorems.

Theorem 1.4 *Let (X,d) be a complete metric space. Suppose that $T : X \to X$ is a generalized α-ψ-contractive mapping of type I (respectively, of type II) and satisfies the following conditions*:

(i) *T is α-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *T is continuous.*

Then there exists $u \in X$ such that $Tu = u$.

Theorem 1.5 *Let (X,d) be a complete metric space. Suppose that $T : X \to X$ is a generalized α-ψ-contractive mapping of type I (respectively, of type II), and the following conditions hold*:

(i) *T is α-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.*

Then there exists $u \in X$ such that $Tu = u$.

For the uniqueness of a fixed point of a generalized α-ψ-contractive mapping, we will consider the following hypothesis, in which $\mathrm{Fix}(T)$ denotes the set of all fixed points of T.

(H) For all $x, y \in \mathrm{Fix}(T)$, there exists $z \in X$ such that $\alpha(x,z) \geq 1$ and $\alpha(y,z) \geq 1$.

Theorem 1.6 *Adding condition* (H) *to the hypotheses of Theorem* 1.4 (*resp. Theorem* 1.5), *we obtain that u is the unique fixed point of T.*

Inspired by the above mentioned results, Mursaleen *et al.* [20] characterized the idea to prove the existence and uniqueness of a coupled fixed point. Before starting the main theorem of Mursaleen *et al.*, we recall the basic definition and fundamental results in coupled fixed point theory.

The first result in the existence and uniqueness of fixed point of contraction mapping in partially ordered complete metric spaces was given by Ran and Reurings [21]. Following this initial work, a number of authors have investigated the fixed points of various mappings and their applications in the theory of differential equations. A notion of *coupled fixed point* was defined by Guo and Laksmikantham [22]. After that, Bhaskar and Lakshmikantham [1] proved the existence and uniqueness of a coupled fixed point in the context of partially ordered metric spaces by introducing the notion of mixed monotone property. In that paper, the authors proved the existence and uniqueness of a solution of periodic boundary value problems. After this remarkable paper, the notion of coupled fixed point have attracted attention of a number of authors (see, *e.g.*, [23–47]).

Mursaleen *et al.* [20] reconsidered the notion of an α-admissible mapping, introduced by Samet *et al.* [8], in the following way:

Definition 1.10 (See [20]) Let $F : X^2 \to X$ and $\alpha : X^2 \times X^2 \to [0,\infty)$ be two mappings. We say that F is α-admissible if for all $x, y, u, v \in X$, we have

$$\alpha\bigl((x,y),(u,v)\bigr) \geq 1 \quad \Longrightarrow \quad \alpha\bigl(\bigl(F(x,y),\bigl(F(y,x)\bigr)\bigr),\bigl(F(u,v),\bigl(F(v,u)\bigr)\bigr)\bigr) \geq 1.$$

Remark 1.2 Notice that Definition 1.10 is exactly the same with Definition 1.7 by choosing X^2.

The main results in [20] are the following ones.

Theorem 1.7 *Let $(X,\preceq)$ be a partially ordered set, and suppose that there is a metric d on X such that (X,d) is a complete metric space. Let $F : X \times X \to X$ be a mapping having the mixed monotone property of X. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0,\infty)$ such that for all $x, y, u, v \in X$, the following holds*

$$\alpha\bigl((x,y),(u,v)\bigr)d\bigl(F(x,y),F(u,v)\bigr) \leq \psi\left(\frac{d(x,u)+d(y,v)}{2}\right) \tag{5}$$

for all $x \succeq u$ and $y \preceq v$. Suppose also that

(i) *F is α-admissible;*

(ii) *there exists $x_0, y_0 \in X$ such that*

$$\alpha\bigl((x_0,y_0),\bigl(F(x_0,y_0),F(y_0,x_0)\bigr)\bigr) \geq 1 \quad \textit{and} \quad \alpha\bigl((y_0,x_0),\bigl(F(y_0,x_0),F(x_0,y_0)\bigr)\bigr) \geq 1;$$

(iii) *F is continuous.*

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0,y_0)$ and $y_0 \succeq F(y_0,x_0)$, then F has a coupled fixed point, that is, there exist $x, y \in X$ such that

$$F(x,y) = x \quad \textit{and} \quad F(y,x) = y.$$

Theorem 1.8 *Let $(X,\preceq)$ be a partially ordered set, and suppose that there is a metric d on X such that (X,d) is a complete metric space. Let $F : X \times X \to X$ be a mapping having the mixed monotone property of X. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0,\infty)$ such that for all $x, y, u, v \in X$, the following holds*

$$\alpha\bigl((x,y),(u,v)\bigr)d\bigl(F(x,y),F(u,v)\bigr) \leq \psi\left(\frac{d(x,u)+d(y,v)}{2}\right) \tag{6}$$

for all $x \succeq u$ and $y \preceq v$. Suppose also that

(i) *F is α-admissible;*

(ii) *there exists* $x_0, y_0 \in X$ *such that*

$$\alpha\big((x_0,y_0),\big(F(x_0,y_0),F(y_0,x_0)\big)\big) \geq 1 \quad \text{and} \quad \alpha\big((y_0,x_0),\big(F(y_0,x_0),F(x_0,y_0)\big)\big) \geq 1;$$

(iii) *if* $\{x_n\}$ *and* $\{y_n\}$ *are sequences in* X *such that* $\alpha((x_n,y_n),(x_{n+1},y_{n+1})) \geq 1$ *and* $\alpha((y_n,x_n),(y_{n+1},x_{n+1})) \geq 1$ *for all* n, *and* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then* $\alpha((x_n,y_n),(x,y)) \geq 1$ *and* $\alpha((y_n,x_n),(y,x)) \geq 1$ *for all* n.

If there exist $x_0, y_0 \in X$ *such that* $x_0 \preceq F(x_0,y_0)$ *and* $y_0 \succeq F(y_0,x_0)$, *then* F *has a coupled fixed point, that is, there exist* $x, y \in X$ *such that*

$$F(x,y) = x \quad \text{and} \quad F(y,x) = y.$$

Theorem 1.9 *In addition to the hypothesis of Theorem* 1.7, *suppose that for every* $(x,y),(s,t) \in X \times X$, *there exists* $(u,v) \in X \times X$, *such that*

$$\alpha\big((x,y),(u,v)\big) \geq 1 \quad \text{and} \quad \alpha\big((s,t),(u,v)\big) \geq 1,$$

and also assume that (u,v) *is comparable to* (x,y) *and* (s,t). *Then* F *has a unique fixed point*.

In this paper, we show that coupled fixed point results of Mursaleen *et al.* [20] can be obtained by usual fixed point theorems. Moreover, by giving an example, we conclude that the main result of Mursaleen *et al.* [20] is not strong enough to be applied to their own examples. The object of this paper is to extend, improve and generalize their results in a more simple set up. Finally, we also note that the remarks and comments of this paper are also valid for [48].

2 Main results

We start this section by giving an example to show the weakness of Theorem 1.7. First, we notice that the function $F(x,y) = \frac{1}{4}xy$ in Example 3.7 in [20, 48] and the function $F(x,y) = \frac{1}{4}\ln(1+|x|) + \frac{1}{4}\ln(1+|y|)$ in Example 3.8 in [20, 48] do not satisfy the mixed monotone property.

Now, we state the following example.

Example 2.1 Let $X = \mathbb{R}$ and $d : X \times X \to [0,\infty)$ be the Euclidean metric. Consider a mapping $\alpha : X^2 \times X^2 \to [0,\infty)$ defined by

$$\alpha\big((x,y),(u,v)\big) = \begin{cases} 1, & \text{if } x \geq u, y \leq v, \\ 0, & \text{otherwise.} \end{cases}$$

Define a mapping $F : X \times X \to X$ as

$$F(x,y) = \frac{3x-y}{5} \quad \text{for all } x, y \in X.$$

It is clear that F is mixed monotone, but we claim that it does not satisfy condition (5). Indeed, assume that there exists $\psi \in \Psi$ such that

$$\alpha\big((x,y),(u,v)\big)d\big(F(x,y),F(u,v)\big) \leq \psi\left(\frac{d(x,u)+d(y,v)}{2}\right) \tag{7}$$

holds for all $x \geq u$ and $v \geq y$. Let us take $x \neq u$, $y = v$ in the previous inequality. Hence, $t = |x - u| > 0$ and inequality (7) turns into

$$\frac{3t}{5} = \frac{3|x-u|}{5} = d\bigl(F(x,y), F(u,v)\bigr) \leq \psi\left(\frac{|x-u|}{2}\right) = \psi\left(\frac{t}{2}\right). \tag{8}$$

Recall that $\psi(t) < t$ for any $t > 0$. Hence, inequality (8) turns into

$$\frac{3t}{5} \leq \psi\left(\frac{t}{2}\right) < \frac{t}{2},$$

which is a contradiction. Hence, Theorem 1.7 is not applicable to the operator F in order to prove that $(0,0)$ is the unique coupled fixed point of F.

We notice that Theorem 1.7 is not strong enough to conclude that F has a coupled fixed point. Inspired by Example 2.1, we suggest the following statement instead of Theorem 1.7.

Theorem 2.1 *Let $(X, \preceq)$ be a partially ordered set, and suppose there is a metric d on X such that (X,d) is a complete metric space. Let $F : X^2 \to X$ be a mapping having the mixed monotone property on X. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0,\infty)$ such that for all $x,y,u,v \in X$ the following holds*

$$\alpha\bigl((x,y),(u,v)\bigr)\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \leq \psi\left(\frac{d(x,u) + d(y,v)}{2}\right) \tag{9}$$

for which $x \succeq u$ and $y \preceq v$. Suppose also that

(i) *F is α-admissible*;

(ii) *there exists $x_0, y_0 \in X$ such that*

$$\alpha\bigl((x_0,y_0),\bigl(F(x_0,y_0),F(y_0,x_0)\bigr)\bigr) \geq 1 \quad \textit{and} \quad \alpha\bigl((y_0,x_0),\bigl(F(y_0,x_0),F(x_0,y_0)\bigr)\bigr) \geq 1;$$

(iii) *F is continuous.*

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0,y_0)$ and $y_0 \succeq F(y_0,x_0)$, then F has a coupled fixed point, that is, there exist $x,y \in X$ such that

$$F(x,y) = x \quad \textit{and} \quad F(y,x) = y.$$

Proof Following the lines of the proof of Theorem 1.7, we conclude the result. To avoid the repetition, we omit the details. □

Analogously, instead of Theorem 1.8, we state the following.

Theorem 2.2 *Let $(X, \preceq)$ be a partially ordered set, and suppose that there is a metric d on X such that (X,d) is a complete metric space. Let $F : X^2 \to X$ be a mapping having the mixed monotone property on X. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0,\infty)$ such that for all $x,y,u,v \in X$, the following holds*

$$\alpha\bigl((x,y),(u,v)\bigr)d\bigl(F(x,y),F(u,v)\bigr) \leq \psi\left(\frac{d(x,u) + d(y,v)}{2}\right) \tag{10}$$

for which $x \succeq u$ and $y \preceq v$. Suppose also that

(i) *F is α-admissible*;
(ii) *there exists* $x_0, y_0 \in X$ *such that*

$$\alpha\big((x_0,y_0),\big(F(x_0,y_0),F(y_0,x_0)\big)\big) \geq 1 \quad \textit{and} \quad \alpha\big((y_0,x_0),\big(F(y_0,x_0),F(x_0,y_0)\big)\big) \geq 1;$$

(iii) *if* $\{x_n\}$ *and* $\{y_n\}$ *are sequences in X such that* $\alpha((x_n,y_n),(x_{n+1},y_{n+1})) \geq 1$ *and* $\alpha((y_n,x_n),(y_{n+1},x_{n+1})) \geq 1$ *for all n, and* $\lim_{n\to\infty} x_n = x$ *and* $\lim_{n\to\infty} y_n = y$, *then* $\alpha((x_n,y_n),(x,y)) \geq 1$ *and* $\alpha((y_n,x_n),(y,x)) \geq 1$ *for all n*.

If there exist $x_0, y_0 \in X$ *such that* $x_0 \preceq F(x_0,y_0)$ *and* $y_0 \succeq F(y_0,x_0)$, *then F has a coupled fixed point, that is, there exist* $x, y \in X$ *such that*

$$F(x,y) = x \quad \textit{and} \quad F(y,x) = y.$$

Proof Following the lines of the proof of Theorem 1.8, we easily conclude the result. We omit the details. □

Let us reconsider Example 2.1.

Example 2.2 Let $X = \mathbb{R}$, and let $d : X \times X \to [0,\infty)$ be the Euclidean metric. Consider the mapping $\alpha : X^2 \times X^2 \to [0,\infty)$ defined as

$$\alpha\big((x,y),(u,v)\big) = \begin{cases} 1, & \text{if } x \geq u, y \leq v \text{ or } x \leq u, y \geq v, \\ 0, & \text{otherwise.} \end{cases}$$

Consider the mapping $F : X \times X \to X$ defined by

$$F(x,y) = \frac{3x - y}{5} \quad \text{for all } x, y \in X.$$

Clearly, F has a mixed monotone property, and we claim that it also satisfies condition (9). Indeed, if $\alpha((x,y),(u,v)) = 0$, then the result is straightforward. Suppose $\alpha((x,y),(u,v)) = 1$. Without loss of generality, assume that $x \geq u$ and $y \leq v$. Then we have that

$$\begin{aligned} &\alpha\big((x,y),(u,v)\big)\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \\ &\quad = \frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \\ &\quad = \frac{1}{10}\big|3(x-u) - (y-v)\big| + \frac{1}{10}\big|3(y-v) - (x-u)\big| \\ &\quad \leq \frac{2}{5}\big[|x-u| + |y-v|\big] \end{aligned} \tag{11}$$

holds for all $x \geq u$ and $v \geq y$. On the other hand,

$$\frac{d((x,y) + d(u,v))}{2} = \frac{1}{2}\big[|x-u| + |y-v|\big]. \tag{12}$$

Hence, it is sufficient to choose $\psi(t) = \frac{4t}{5}$ to provide all conditions of Theorem 2.1. Notice that the point $(0,0)$ is the unique coupled fixed point of F.

Example 2.3 Let $X = [-1,1]$, and let $d : X \times X \to [0,\infty)$ be the Euclidean metric. Consider the mapping $\alpha : X^2 \times X^2 \to [0,\infty)$ defined as

$$\alpha\bigl((x,y),(u,v)\bigr) = \begin{cases} 1, & \text{if } x \geq u, y \leq v \text{ or } x \leq u, y \geq v, \\ 0, & \text{otherwise.} \end{cases}$$

Define a mapping $F : X \times X \to X$ as

$$F(x,y) = \frac{5x^3 - y^3}{24} \quad \text{for all } x, y \in X.$$

Then F is mixed monotone and satisfies all conditions of Theorem 2.1. Indeed, if $\alpha((x,y),(u,v)) = 0$, the result is trivial. Suppose $\alpha((x,y),(u,v)) = 1$. Without loss of generality, assume that $x \geq y$, $u \geq v$. Then we have that

$$\begin{aligned} &\alpha\bigl((x,y),(u,v)\bigr)\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \\ &\quad= \frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \\ &\quad= \frac{1}{48}\bigl|5\bigl(x^3 - u^3\bigr) - \bigl(y^3 - v^3\bigr)\bigr| + \frac{1}{48}\bigl|5\bigl(y^3 - v^3\bigr) - \bigl(x^3 - u^3\bigr)\bigr| \\ &\quad\leq \frac{1}{8}\bigl[\bigl|x^3 - u^3\bigr| + \bigl|y^3 - v^3\bigr|\bigr] \leq \frac{3}{8}\bigl[|x-u| + |y-v|\bigr] \end{aligned}$$

holds for all $x \geq u$ and $y \leq v$. On the other hand,

$$\frac{d((x,y) + d(u,v))}{2} = \frac{1}{2}\bigl[|x-u| + |y-v|\bigr].$$

Hence, it is sufficient to choose $\psi(t) = \frac{3t}{4}$ to provide all conditions of Theorem 2.1. Notice that the point $(0,0)$ is the coupled fixed point of F.

Now, we improve Example 3.8 in [20] in the following way.

Example 2.4 Let $X = [0,\infty)$, and let $d : X \times X \to [0,\infty)$ be a Euclidean metric. Consider the mapping $\alpha : X^2 \times X^2 \to [0,\infty)$ defined by

$$\alpha\bigl((x,y),(u,v)\bigr) = \begin{cases} 1, & \text{if } x \geq u, y \leq v \text{ or } x \leq u, y \geq v, \\ 0, & \text{otherwise.} \end{cases}$$

Define the mapping $F : X \times X \to X$ as

$$F(x,y) = \begin{cases} \frac{1}{4}[\ln(1+x) - \ln(1+y)], & \text{if } x \geq y, \\ 0, & \text{otherwise.} \end{cases}$$

Then F is mixed monotone and satisfies all conditions of Theorem 2.1. Indeed, if $\alpha((x,y),(u,v)) = 0$, the result follows trivially. Suppose $\alpha((x,y),(u,v)) = 1$. Without loss of generality, assume that $x \geq u$ and $y \leq v$. Then we have

$$\begin{aligned}
&\alpha\big((x,y),(u,v)\big)\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \\
&\quad= \frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \\
&\quad= \frac{1}{8}\big|\big(\ln(1+x) - \ln(1+u)\big) - \big(\ln(1+y) - \ln(1+v)\big)\big| \\
&\qquad+ \frac{1}{8}\big|\big(\ln(1+y) - \ln(1+v)\big) - \big(\ln(1+x) - \ln(1+u)\big)\big| \\
&\quad\leq \frac{1}{8}\left[\left|\ln\left(\frac{(1+x)}{1+u}\right)\right|\right] + \frac{1}{8}\left[\left|\ln\left(\frac{(1+v)}{1+y}\right)\right|\right] \\
&\qquad+ \frac{1}{8}\left[\left|\ln\left(\frac{(1+y)}{1+v}\right)\right|\right] + \frac{1}{8}\left[\left|\ln\left(\frac{(1+u)}{1+x}\right)\right|\right] \\
&\quad\leq \frac{1}{4}\big[\ln\big(1+|x-u|\big) + \ln\big(1+|y-v|\big)\big], \\
&\quad\leq \frac{1}{2}\ln\left(1 + \frac{[|x-u| + |y-v|]}{2}\right)
\end{aligned} \tag{13}$$

holds for all $x \geq u$ and $v \geq y$. On the other hand,

$$\frac{d((x,y) + d(u,v))}{2} = \frac{1}{2}\big[|x-u| + |y-v|\big]. \tag{14}$$

Hence, it is sufficient to choose $\psi(t) = \frac{1}{2}\ln(1+t)$, $t > 0$ to provide all conditions of Theorem 2.1. Notice that the point $(0,0)$ is the coupled fixed point of F.

For the uniqueness of the coupled fixed point, we state the following theorem.

Theorem 2.3 *In addition to the hypothesis of Theorem* 2.1, *suppose that for every* $(x,y),(s,t) \in X \times X$, *there exists* $(u,v) \in X \times X$ *such that*

$$\alpha\big((x,y),(u,v)\big) \geq 1 \quad \textit{and} \quad \alpha\big((s,t),(u,v)\big) \geq 1,$$

and also assume that (u,v) *is comparable to* (x,y) *and* (s,t). *Then* F *has a unique fixed point.*

To avoid the repetition, we omit the proof, since the result is easily obtained by following the lines of the proof of Theorem 1.9.

3 From coupled fixed point theorem to usual fixed point theorem

According to the definitions above, we reconsider Definition 1.10 in the following way.

Definition 3.1 Let $F : X^2 \to X$, and let $\alpha^* : X^2 \times X^2 \to [0,\infty)$. The operator F is α^*-admissible if for all $(x,y),(u,v) \in X^2$, we have

$$\alpha^*\big((x,y),(u,v)\big) = \alpha^*\big((y,x),(v,u)\big) \quad \text{and} \tag{15}$$

$$\alpha^*\big((x,y),(u,v)\big) \geq 1 \quad \Longrightarrow \quad \alpha^*\big(\big(F(x,y),\big(F(y,x)\big)\big),\big(F(u,v),\big(F(v,u)\big)\big)\big) \geq 1. \tag{16}$$

Lemma 3.1 *Let $F : X^2 \to X$, and let $\alpha^* : X^2 \times X^2 \to [0,\infty)$. If F is α^*-admissible, then the operator $T_F : X^2 \to X^2$, defined by $T_F(x,y) = (F(x,y), F(y,x))$ for all $(x,y) \in X^2$, is α-admissible in the sense of Definition* 1.7, *that is,*

$$\alpha(z,w) \geq 1 \quad \Longrightarrow \quad \alpha\big(T_F(z), T_F(w)\big) \geq 1$$

for all $z = (x,y), w = (u,v) \in Y = X^2$, where $\alpha^* = \alpha : Y \times Y \to [0,\infty)$.

We omit the proof, since it is straightforward.

Remark 3.1 Regarding the definition of $d_2 : X \times X \to [0,\infty)$, that is,

$$d_2\big((x,y),(u,v)\big) = d(x,u) + d(y,v) = d(y,v) + d(x,u) = d_2\big((y,x),(v,u)\big),$$

one can conclude the assumption (15) is very natural but not necessary. For example, consider a partially ordered set $(X,\preceq)$, then, we set $(X \times X, \preceq_2)$ as in (1). Now, one can define

$$\alpha\big((x,y)(u,v)\big) = \begin{cases} 1, & \text{if } (x,y) \preceq_2 (u,v), \\ 2, & \text{if } (x,y) \succeq_2 (u,v), \\ 0, & \text{otherwise}, \end{cases}$$

which is clearly not equal to $\alpha^*((x,y)(u,v))$.

Theorem 3.1 *Let (X,d) be a complete metric space, and let $F : X \times X \to X$ be a mapping. Suppose that there exist $\psi \in \Psi$ and $\alpha^* : X^2 \times X^2 \to [0,\infty)$ such that the following holds*

$$\alpha^*\big((x,y),(u,v)\big)\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \leq \psi\left(\frac{d(x,u) + d(y,v)}{2}\right) \tag{17}$$

for all $x,y,u,v \in X$. Suppose also that

(i) *F is α^*-admissible*;
(ii) *there exists $(x_0,y_0) \in X$ such that*

$$\alpha^*\big((x_0,y_0),\big(F(x_0,y_0),F(y_0,x_0)\big)\big) \geq 1;$$

(iii) *F is continuous.*

Then, there exist $x,y \in X$ such that

$$F(x,y) = x \quad \text{and} \quad F(y,x) = y.$$

Remark 3.2 Theorem 3.1 coincides with Theorem 1.7 if we replace α with α^* in the statement of Theorem 1.7.

Theorem 3.2 *Theorem* 3.1 *follows from Theorem* 1.1.

Proof From (17), for all $(x,y),(u,v) \in X \times X$, we have

$$\alpha^*\big((x,y),(u,v)\big)\frac{d(F(x,y),F(u,v)) + d(F(v,u),F(y,x))}{2} \leq \psi\left(\frac{d(x,u) + d(y,v)}{2}\right),$$

that is,

$$\alpha^*\big((x,y),(u,v)\big)\delta\big(T_F(x,y),T_F(u,v)\big) \leq \varphi\big(\delta\big((x,u),(y,v)\big)\big)$$

for all $(x,y),(u,v) \in Y$, where $\delta : Y \times Y \to [0,\infty)$ is the metric on Y given by

$$\delta\big((x,y),(u,v)\big) = \frac{d_2((x,y),(u,v))}{2} \quad \text{for all } (x,y),(u,v) \in Y,$$

and $T_F(x,y) = (F(x,y),F(y,x))$ for all $(x,y) \in X^2$. Thus, we proved that the mapping T_F satisfies condition (2) and hence all conditions of Theorem 1.1 are satisfied. Then T_F has a fixed point, which implies that F has a coupled fixed point. □

Theorem 3.3 *Let (X,d) be a complete metric space, and let $F : X \times X \to X$ be a mapping. Suppose that there exist $\psi \in \Psi$ and $\alpha^* : X^2 \times X^2 \to [0,\infty)$ such that the following holds*

$$\alpha^*\big((x,y),(u,v)\big)\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} \leq \psi\left(\frac{d(x,u)+d(y,v)}{2}\right) \tag{18}$$

for all $x,y,u,v \in X$. Suppose also that

(i) *F is α-admissible;*

(ii) *there exists $(x_0,y_0) \in X$ such that*

$$\alpha\big((x_0,y_0),\big(F(x_0,y_0),F(y_0,x_0)\big)\big) \geq 1;$$

(iii) *if $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $\alpha^*((x_n,y_n),(x_{n+1},y_{n+1})) \geq 1$ for all n and $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then $\alpha^*((x_n,y_n),(x,y)) \geq 1$.*

Then, there exist $x,y \in X$ such that

$$F(x,y) = x \quad \textit{and} \quad F(y,x) = y.$$

Remark 3.3 Theorem 3.3 coincides with Theorem 1.8 if we replace α with α^* in the statement of Theorem 1.8.

Theorem 3.4 *Theorem* 3.3 *follows from Theorem* 1.2.

Proof Following the lines of the proof of Theorem 3.2, we observe that T satisfies condition (2) and hence all conditions of Theorem 1.2 are satisfied. Then T_F has a fixed point, which implies that F has a coupled fixed point. □

Theorem 3.5 *In addition to the hypothesis of Theorem* 3.2, *suppose that for every $(x,y),(s,t) \in X \times X$, there exists $(u,v) \in X \times X$ such that*

$$\alpha\big((x,y),(u,v)\big) \geq 1 \quad \textit{and} \quad \alpha\big((s,t),(u,v)\big) \geq 1,$$

and also assume that (u,v) is comparable to (x,y) and (s,t). Then F has a unique fixed point.

Proof Following the lines of the proof of Theorem 3.2, we observe that T satisfies condition (2) and hence all conditions of Theorem 1.3 are satisfied. Then T_F has a unique fixed point, which implies that F has a unique coupled fixed point. □

Fixed point theorems on metric spaces endowed with a partial order

In the last decades, one of the most attractive research topics in fixed point theory was to prove the existence of fixed point on metric spaces endowed with partial orders. The first result in this direction was reported by Turinici [49] in 1986. Following this interesting paper, Ran and Reurings in [21] characterized the Banach contraction principle in partially ordered sets with some applications to matrix equations. Later, the results in [21, 49] were further extended and improved by many authors (see, for example, [50–54] and the references cited therein). In this section, we will deduce that more general form of Theorem 1.7 (respectively, Theorem 1.8) can be obtained from our Theorem 3.1 (respectively, Theorem 3.2).

We obtain the following result whose analog can be found in [41].

Proposition 3.1 *Let $(X,\preceq)$ be a partially ordered set, and let d be a metric on X such that (X,d) is complete. Let $F: X\times X\to X$ have a mixed monotone property. Suppose that there exists a function $\psi\in\Psi$ such that*

$$\frac{d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}{2}\le\psi\left(\frac{d(x,u)+d(y,v)}{2}\right)$$

for all $(x,y),(u,v)\in Y$ with $(x,y)\succeq_2(u,v)$. Suppose also that the following conditions hold:

(i) *there exists $(x_0,y_0)\in Y$ such that $(x_0,y_0)\preceq_2(F(x_0,y_0),F(y_0,x_0))$;*

(ii) *F is continuous or $(X,\preceq_2,d)$ is regular.*

Then F has a coupled fixed point. Moreover, if for all $(x,y),(u,v)\in Y$ there exists $(z,w)\in Y$ such that $(x,y)\preceq_2(z,w)$ and $(u,v)\preceq_2(z,w)$, we have uniqueness of the fixed point.

Proof Define the mapping $\alpha^*: Y\times Y\to[0,\infty)$ by

$$\alpha^*\big((x,y)(u,v)\big)=\begin{cases}1 & \text{if } (x,y)\preceq_2(u,v) \text{ or } (x,y)\succeq_2(u,v),\\ 0, & \text{otherwise.}\end{cases}$$

Clearly, T_F is an α-ψ-contractive mapping, that is,

$$\alpha^*\big((x,y),(u,v)\big)d_2\big(T_F(x,y),T_F(u,v)\big)\le\psi\big(d_2\big((x,y),(u,v)\big)\big)$$

for all $(x,y),(u,v)\in Y$. From condition (i), we have $\alpha^*((x_0,y_0),(T_F(x_0,y_0),T_F(y_0,x_0)))\ge 1$. Due to Lemma 1.1, T_F is non-decreasing mapping with respect to $\preceq_2$. Moreover, for all $(x,y),(u,v)\in Y$, from the monotone property of T_F, we have

$$\begin{aligned}&\alpha^*\big((x,y),(u,v)\big)\ge 1 \quad\Longrightarrow\quad (x,y)\succeq_2(u,v)\quad\text{or}\\ &(x,y)\preceq_2(u,v)\quad\Longrightarrow\quad T_F(x,y)\succeq T_F(u,v)\quad\text{or}\\ &T_F(x,y)\preceq T_F(u,v)\quad\Longrightarrow\quad \alpha^*\big(T_F(x,y),T_F(u,v)\big)\ge 1.\end{aligned}$$

Thus F is α^*-admissible. Hence, due to Lemma 3.1 T_F is α-admissible. Now, if F is continuous, the existence of a fixed point follows from Theorem 3.1. Suppose now that $(X, \preceq_2, d)$ is regular. Let $\{x_n\}$ be a sequence in X such that $\alpha^*((x_n, y_n), (x_{n+1}, y_{n+1})) \geq 1$ for all n and $(x_n, y_n) \to (x, y) \in X^2$ as $n \to \infty$. From the regularity hypothesis, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \preceq x$ and $y_{n(k)} \succeq y$ for all k. This implies from the definition of α^* that $\alpha^*((x_{n(k)}, y_{n(k)}), (x, y)) \geq 1$ for all k. In this case, the existence of a fixed point follows from Theorem 3.3. To show the uniqueness, let $(x, y), (u, v) \in X^2$. By hypothesis, there exists $(z, w) \in X^2$ such that $(x, y) \preceq_2 (z, w)$ and $(u, v) \preceq_2 (z, w)$, which implies from the definition of α^* that $\alpha^*((x, y), (z, w)) \geq 1$ and $\alpha^*((u, v), (z, w)) \geq 1$. Thus, we deduce the uniqueness of the fixed point by Theorem 3.3. □

Now, we state the result of [29] as an easy consequence of Proposition 3.1.

Corollary 3.1 *Let $(X, \preceq)$ be a partially ordered set, and d be a metric on X such that (X, d) is complete. Let $F : X \times X \to X$ have a mixed monotone property. Suppose that there exists a function $k \in [0, 1)$ such that*

$$\bigl[d\bigl(F(x,y), F(u,v)\bigr) + d\bigl(F(y,x), F(v,u)\bigr)\bigr] \leq k\bigl[d(x,u) + d(y,v)\bigr]$$

for all $(x, y), (u, v) \in Y$ with $(x, y) \succeq_2 (u, v)$, where $\succeq_2$ is defined as in (1). Suppose also that the following conditions hold:

(i) *there exists $(x_0, y_0) \in Y$ such that $(x_0, y_0) \preceq_2 (F(x_0, y_0), F(y_0, x_0))$;*

(ii) *F is continuous or $(X, \preceq_2, d)$ is regular.*

Then F has a coupled fixed point. Moreover, if for all $(x, y), (u, v) \in Y$ there exists $(z, w) \in Y$ such that $(x, y) \preceq_2 (z, w)$ and $(u, v) \preceq_2 (z, w)$, we have uniqueness of the fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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Received: 6 June 2013 Accepted: 30 July 2013 Published: 14 August 2013

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doi:10.1186/1687-1812-2013-216
Cite this article as: Karapınar and Agarwal: **A note on 'Coupled fixed point theorems for α-ψ-contractive-type mappings in partially ordered metric spaces'.** *Fixed Point Theory and Applications* 2013 **2013**:216.