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# A discrete plant disease model with roguing and replanting

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**Abstract**

In this paper, we study a discrete plant virus disease model with roguing and replanting which is derived from the continuous case by using the well-known backward Euler method. The positivity of solutions with positive initial conditions is obtained. By applying analytic techniques and constructing a discrete Lyapunov function, we obtain the result that the disease-free equilibrium is globally attractive if $R_0 \leq 1$, and the disease is permanent if $R_0 > 1$. Numerical simulations show that the main theoretical results are true.

**Keywords:** discrete plant disease model; basic reproduction number; global attractivity; permanence

## 1 Introduction

Plants not only provide people with essential means of subsistence, but also they offer other creatures food and shelter. However, plants often suffer from multiple adverse factors in the process of growth, especially viruses, which causes the decline of plants yield and quality, even famine and social unrest.

It is well known that many serious diseases of crop plants are caused by viruses. For severe cases, plant diseases have caused large-scale damage to various crops, which resulted in a diminished output in whole regions; for instance, cocoa swollen shoot in Ghana [1, 2] and banana bunchy top in Australia [3–5]. When a crop is widely planted in a new area, plant disease prevention usually becomes important. In most cases, we rogue (remove) infected plants as a control strategy when disease breaks out. Since 1946, 190 million infected trees have been removed in Ghana [6]. In addition, we can also rogue not only visibly infected plants but also other neighboring plants which do not yet show symptoms [2, 7], but this measure may be unpopular with farmers since it involves removal of apparently healthy plants which may still be highly productive [8].

Recently, more and more attention has been paid to the discrete-time epidemic models. In [9], the authors pointed out that it is more direct, more convenient, and more accurate to describe a disease by using the discrete-time models than the continuous-time models since the statistic data about the disease situation is collected by day, week, month or year. Furthermore the discrete-time models have more wealthy dynamical behaviors, such as the discrete-time epidemic models, which have bifurcations, chaos, and other more complex dynamical behaviors. Many important and interesting results can be found in [10–22] and the references cited therein.

We know that there are usually two methods to construct discrete-time epidemic models: (i) by making use of the compartment model theory and the property of the epidemic disease, (ii) by using techniques (the backward Euler scheme, the forward Euler scheme, and Mickens' nonstandard discretization) to discretize a continuous-time epidemic model. Until now, some studies have been done on discrete-time epidemic models by using the two methods mentioned above (see [9–33]). For example, by applying Mickens' nonstandard discretization, Wang *et al.* [9] discussed dynamical behaviors for a class of discrete SIRS models with disease courses. Muroya *et al.* [22] proposed a discrete epidemic model for a disease with immunity and latency spreading in a heterogeneous host population, which was derived from the continuous case by using the well-known backward Euler method, and they obtained the result of the global stability of the disease-free equilibrium and the endemic equilibrium. According to the first method, Teng *et al.* [25] constructed a discrete SIS epidemic model with stage structure and standard incident rate and established sufficient conditions for the permanence and extinction of the disease of the model. Moreover, using the method of linearization, the local asymptotic stability of the endemic equilibrium was studied. Applying the forward Euler scheme, Hu *et al.* [30] constructed a discrete SIR epidemic model, and they studied the local stability of the disease-free equilibrium and the endemic equilibrium. In addition, numerical simulations showed plentiful and complex dynamical behaviors including bifurcations.

In this paper, we use the well-known backward Euler method to discretize a continuous-time plant virus disease model with roguing and replanting which is investigated in [8]. Our main purpose is to study dynamical behaviors of the model.

The organization of this paper is as follows. In Section 2, the model description and some preliminaries are given. Section 3 deals with the global attractivity of disease-free equilibrium of the model. In Section 4, the criterion on the permanence of the disease of the model is stated and proved. In Section 5, the numerical simulations are provided to illustrate the validity of our theoretical results. Lastly, a brief discussion is given in Section 6.

## 2 Model formulation and preliminaries

In 1994, Chan and Jeger [8] studied the following continuous-time plant virus disease model with roguing and replanting.

$$\begin{cases} S'(t) = r(K - N(t)) - \mu S(t) - k_1 S(t)\frac{I(t)}{K}, \\ E'(t) = k_1 S(t)\frac{I(t)}{K} - (\mu + k_2)E(t), \\ I'(t) = k_2 E(t) - (\mu + k_3)I(t), \\ R'(t) = k_3 I(t) - (\mu + \alpha)R(t), \end{cases} \tag{2.1}$$

where $S(t)$, $E(t)$, $I(t)$, and $R(t)$ denote the numbers of susceptible, latently infected, infectious, and post-infectious individuals at time $t$, respectively. $\mu$ is the natural mortality, which is not attributed to disease and is common to each category. $\alpha$ is an additional mortality in the post-infectious category owing to the cumulative effect of the disease. $k_i$ ($i = 1, 2, 3$) are the conversion rates of the disease's progression from susceptible to latent, from latent to infectious, and from infectious to post-infectious, respectively. $K$ is the maximum plant population size, defined in terms of agronomic considerations. It is assumed that the actual total population size is presented by $N$ ($N = S + E + I + R$), recruitment to the population is by replanting at a rate proportional $r$ to the difference between

the actual number of plants present, $N$, and the maximum population size, $K$. A qualitative analysis of this model presents the stable dynamics and threshold conditions for distinguishing the disease's extinction or permanence.

By applying a variation of backward Euler scheme discretetization, we propose the following discrete plant virus disease model, which is derived from system (2.1).

$$
\begin{cases}
S(n+1) = S(n) + r(K - N(n+1)) - \mu S(n+1) - k_1 S(n+1)\frac{I(n+1)}{K}, \\
E(n+1) = E(n) + k_1 S(n+1)\frac{I(n+1)}{K} - (\mu + k_2)E(n+1), \\
I(n+1) = I(n) + k_2 E(n+1) - (\mu + k_3)I(n+1), \\
R(n+1) = R(n) + k_3 I(n+1) - (\mu + \alpha)R(n+1), \\
N(n) = S(n) + E(n) + I(n) + R(n).
\end{cases}
\tag{2.2}
$$

Considering the biological background of model (2.2), therefore, any solution of model (2.2) satisfies the following initial condition:

$$
S(0) > 0, \qquad E(0) > 0, \qquad I(0) > 0, \qquad R(0) > 0.
\tag{2.3}
$$

**Lemma 2.1** *Any solution* $(S(n), E(n), I(n), R(n))$ *of model* (2.2) *with initial condition* (2.3), *is positive for any* $n > 0$ *and ultimately bounded.*

*Proof* Suppose that $(S(n), E(n), I(n), R(n))$ is any solution of model (2.2) with initial condition (2.3); apparently model (2.2) is equivalent to the following iteration system:

$$
\begin{cases}
E(n+1) = \frac{1}{1+\mu+k_2}(E(n) + k_1 S(n+1)\frac{I(n+1)}{K}), \\
I(n+1) = \frac{1}{1+\mu+k_3}(I(n) + k_2 E(n+1)), \\
R(n+1) = \frac{1}{1+\mu+\alpha}(R(n) + k_3 I(n+1)), \\
N(n+1) = N(n) + rK - (r+\mu)N(n+1) - \alpha R(n+1).
\end{cases}
\tag{2.4}
$$

Next, we will prove the positivity of solution by using the inductive method. When $n = 0$, we have

$$
E(1) = \frac{1}{1+\mu+k_2}\left(E(0) + \frac{k_1}{K}(N(1) - E(1) - I(1) - R(1))I(1)\right),
\tag{2.5}
$$

$$
I(1) = \frac{1}{1+\mu+k_3}(I(0) + k_2 E(1)), \qquad R(1) = \frac{1}{1+\mu+\alpha}(R(0) + k_3 I(1)),
\tag{2.6}
$$

$$
N(1) = \frac{1}{1+r+\mu}(rK + N(0) - \alpha R(1)),
\tag{2.7}
$$

and

$$
S(1) = S(0) + r(K - N(1)) - \mu S(1) - k_1 S(1)\frac{I(1)}{K}.
\tag{2.8}
$$

From (2.5)-(2.8) we see that if only $E(1)$ is confirmed, then $I(1)$, $R(1)$, $N(1)$, and $S(1)$ will be afterwards confirmed.

Firstly, we prove that if $E(1) > 0$ and then $I(1) > 0$, $R(1) > 0$, and $S(1) > 0$. From (2.6), we directly obtain $I(1) > 0$ when $E(1) > 0$. Secondly, from $I(1) > 0$ we further obtain the result that $R(1) > 0$.

Let $x = S(1)$, from (2.6) and (2.8) we obtain

$$\Phi(x) \triangleq \left(1 + \mu + \frac{k_1}{K}I(1)\right)x - S(0) - r\big(K - N(1)\big) = 0,$$

where $N(1) = x + E(1) + I(1) + R(1)$. It is obvious that $\Phi(x)$ is monotonically increasing for $x \geq 0$ when $E(1) > 0$. Because $\Phi(0) = -S(0) - r(K - I(1) - E(1) - R(1)) < 0$ and $\lim_{x \to +\infty} \Phi(x) = +\infty$, we obtain the result that $\Phi(x) = 0$ has a unique positive solution $\bar{x}$. Hence, we finally have $S(1) = \bar{x} > 0$. Moreover, we also have $N(1) = S(1) + E(1) + I(1) + R(1) > 0$.

Let $y = E(1)$, then from (2.5)-(2.7), we see that $y$ satisfies the following equation:

$$\Psi(y) \triangleq y - \frac{1}{1 + \mu + k_2}\left(E(0) + \frac{k_1}{K}I(1)\big(N(1) - I(1) - R(1) - y\big)\right) = 0,$$

where

$$N(1) = \frac{A - \alpha k_2 k_3 y}{B},$$

$$I(1) = \frac{I(0) + k_2 y}{1 + \mu + k_3},$$

and

$$R(1) = \frac{C + k_2 k_3 y}{D},$$

where

$$A = \big(rK + N(0)\big)(1 + \mu)(1 + \mu + k_3) + \alpha(1 + \mu + k_3)\big(rK + S(0) + E(0)\big) + \alpha I(0)(1 + \mu),$$

$$B = (1 + \mu + \alpha)(1 + r + \mu)(1 + \mu + k_3),$$

$$C = R(0)(1 + \mu + k_3) + k_3 I(0),$$

$$D = (1 + \mu + \alpha)(1 + \mu + k_3).$$

$\Psi(y)$ is a quadratic equation in $y$. Let us rewrite it in the form

$$\Psi(y) = ay^2 + by + c,$$

with

$$a = \frac{k_1 k_2 F}{K(1 + \mu + k_2)(1 + \mu + k_3)},$$

$$b = \left(1 - \frac{k_1}{K(1 + \mu + k_2)}\left(\frac{I(0)}{1 + \mu + k_3}F + \frac{k_2}{1 + \mu + k_3}H\right)\right),$$

$$c = -\frac{E(0)}{1 + \mu + k_2} - \frac{k_1 I(0)}{K(1 + \mu + k_2)(1 + \mu + k_3)}H.$$

Here,

$$H = \frac{A}{B} - \frac{I(0)}{1 + \mu + k_3} - \frac{C}{D},$$

$$F = \frac{\alpha k_2 k_3}{B} + \frac{k_2}{1 + \mu + k_3} + \frac{k_2 k_3}{D} + 1.$$

Obviously, $A > 0$, $B > 0$, $C > 0$, $D > 0$, $F > 0$, $a > 0$.

By the characteristic of a quadratic equation, we know that if $c < 0$, then $\Psi(y) = 0$ has a unique positive solution $\bar{y} \in (0, +\infty)$.

Now, we prove that $c < 0$. By calculating, we obtain the result that $c < 0$.

From the above discussions we see that $\Psi(y) = 0$ has a unique positive solution $\bar{y} \in (0, +\infty)$. Let $E(1) = \bar{y}$. We also have $I(1) > 0$, $R(1) > 0$, and $S(1) > 0$. Therefore, the positivity of $S(1) > 0$, $I(1) > 0$, and $R(1) > 0$ is finally obtained.

When $n = 1$, we obtain

$$E(2) = \frac{1}{1 + \mu + k_2}\left(E(1) + \frac{k_1}{K}\big(N(2) - E(2) - I(2) - R(2)\big)I(2)\right),$$

$$I(2) = \frac{1}{1 + \mu + k_3}\big(I(1) + k_2 E(2)\big), \qquad R(2) = \frac{1}{1 + \mu + \alpha}\big(R(1) + k_3 I(2)\big),$$

$$N(2) = \frac{1}{1 + r + \mu}\big(rK + N(1) - \alpha R(2)\big),$$

and

$$S(2) = S(1) + r\big(K - N(2)\big) - \mu S(2) - k_1 S(2)\frac{I(2)}{K}.$$

Besides, by arguments similar to the above, we also can obtain the result that $E(2) > 0$, $I(2) > 0$, $R(2) > 0$, and $S(2) > 0$. Finally, by using induction, we can prove that $S(n) > 0$, $E(n) > 0$, $I(n) > 0$, and $R(n) > 0$ for all $n > 0$.

In the following we will prove that the solution of model (2.2) is ultimately bounded. From the fourth equation of model (2.4), we have

$$N(n+1) = \frac{N(n) + rK - \alpha R(1)}{1 + r + \mu} \leq \frac{N(n) + rK}{1 + r + \mu}.$$

It is well known that the auxiliary equation

$$U(n+1) = \frac{U(n) + rK}{1 + r + \mu}$$

has a globally asymptotically stable equilibrium $U^* = \frac{rK}{r+\mu}$, since by the comparison principle of the difference equations, we have $\limsup_{n\to\infty} N(n) \leq \frac{rK}{r+\mu}$. In other words, $(S(n), E(n), I(n), R(n))$ is ultimately bounded. This completes the proof. □

Define $\phi(p,n) = pE(n) - I(n)$ and $\varphi(p) = \frac{pk_1 r}{r+\mu} + k_3 - (1 + \frac{1}{p})k_2$, where $p > 0$ is a constant and $n > 0$ is an integer.

**Lemma 2.2** *If there exists a constant $p > 0$ such that $\varphi(p) \leq 0$, then there exists an integer $N_1 > 0$ such that either $\phi(p,n) \geq 0$ for all $n \geq N_1$ or $\phi(p,n) \leq 0$ for all $n \geq N_1$.*

*Proof* Actually, we assume that the conclusion does not hold, then we find that $\phi(p,n)$ oscillates about 0. Hence, for any $N_1 > 0$, there exists an integer $q \geq N_1$ such that $\phi(p,q) < 0$

and $\phi(p, q + 1) \geq 0$. Thus we have

$$I(q + 1) \leq pE(q + 1) \tag{2.9}$$

and

$$
\begin{aligned}
0 &< pE(q+1) - I(q+1) - pE(q) + I(q) \\
&= p\big(E(q+1) - E(q)\big) - \big(I(q+1) - I(q)\big) \\
&= p\left[k_1 S(q+1)\frac{I(q+1)}{K} - (\mu + k_2)E(q+1)\right] - \big[k_2 E(q+1) - (\mu + k_3)I(q+1)\big] \\
&= \left[\frac{pk_1}{K}S(q+1) + (\mu + k_3)\right]I(q+1) - \left[\mu + \left(1 + \frac{1}{p}\right)k_2\right]pE(q+1) \\
&\leq \left[\frac{pk_1 r}{r + \mu} + (\mu + k_3)\right]I(q+1) - \left[\mu + \left(1 + \frac{1}{p}\right)k_2\right]pE(q+1).
\end{aligned}
$$

Substituting (2.9) into the above inequality, we further have

$$pE(q+1)\left[\frac{pk_1 r}{r + \mu} + k_3 - \left(1 + \frac{1}{p}\right)k_2\right] > 0,$$

that is, $pE(q+1)\varphi(p) > 0$. From Lemma 2.1, we have $E(q+1) > 0$, and hence $\varphi(p) > 0$, which leads to a contradiction. This completes the proof. □

## 3 Global attractivity of disease-free equilibrium

In this section, we are going to discuss the global attractivity of the disease-free equilibrium of model (2.2). In order to obtain the existence of a disease-free equilibrium and an endemic equilibrium of model (2.2), we introduce a constant

$$R_0 = \frac{rk_1 k_2}{(\mu + r)(\mu + k_2)(\mu + k_3)}.$$

It is easy to verify that model (2.2) has only a disease-free equilibrium $P_0(\frac{rK}{\mu+r}, 0, 0, 0)$ when $R_0 \leq 1$, and it has a unique endemic equilibrium $P_*(S_*, E_*, I_*, R_*)$ when $R_0 > 1$, except for $P_0$, where

$$S_* = \frac{K(\mu + k_2)(\mu + k_3)}{k_1 k_2},$$

$$E_* = \frac{\mu + k_3}{k_2}I_*,$$

$$I_* = \frac{rK - \frac{K(\mu+k_2)(\mu+k_3)(\mu+\alpha)}{k_1 k_2}}{\frac{k_1 S_*}{K} + r + r\frac{\mu+k_3}{k_2} + r\frac{k_3}{\mu+\alpha}},$$

$$R_* = \frac{k_3}{\mu + \alpha}I_*.$$

Therefore, we can claim that $R_0$ is the basic reproduction number of model (2.2).

**Theorem 3.1** *Disease-free equilibrium $P_0$ of model* (2.2) *is globally attractive iff $R_0 \leq 1$.*

*Proof* The proof of necessity is simple, we hence omit it here. Now, we prove the sufficiency. When $R_0 < 1$, we have

$$\frac{k_2}{\mu + k_3} < \frac{(\mu + r)(\mu + k_2)}{rk_1}$$

and

$$\varphi\left(\frac{k_2}{\mu + k_3}\right) = (\mu + k_2)(R_0 - 1) < 0.$$

Therefore, there exists a constant $p > \frac{k_2}{\mu+k_3}$ with $p$ sufficiently close to $\frac{k_2}{\mu+k_3}$ such that $\varphi(p) < 0$ and

$$\frac{k_2}{p} - (\mu + k_3) < 0, \tag{3.1}$$

$$\frac{rk_1 p}{\mu + r} - (\mu + k_2) < 0. \tag{3.2}$$

When $R_0 = 0$, we have

$$\frac{k_2}{\mu + k_3} = \frac{(\mu + r)(\mu + k_2)}{rk_1}$$

and

$$\varphi\left(\frac{k_2}{\mu + k_3}\right) = (\mu + k_2)(R_0 - 1) = 0.$$

Hence, for the constant $p = \frac{k_2}{\mu+k_3}$, we have $\varphi(p) = 0$, $\frac{k_2}{p} - (\mu + k_3) = 0$, and $\frac{rk_1 p}{\mu+r} - (\mu + k_2) = 0$. To sum up, we see that, if $R_0 \leq 1$, there always exists a constant $p > 0$ such that

$$\frac{k_2}{p} - (\mu + k_3) \leq 0, \tag{3.3}$$

$$\frac{rk_1 p}{\mu + r} - (\mu + k_2) \leq 0, \tag{3.4}$$

and $\varphi(p) \leq 0$. Therefore, from the point of view of Lemma 2.2, for $R_0 \leq 1$, we only need to consider the following two cases.

*Case* 1. $pE(n) \leq I(n)$ for $n \geq N_1$.

*Case* 2. $pE(n) \geq I(n)$ for $n \geq N_1$.

First of all, we consider Case 1. From the third equation of model (2.2), for all $n \geq N_1$, we have

$$I(n + 1) \leq I(n) + \left(\frac{k_2}{p} - (\mu + k_3)\right)I(n + 1). \tag{3.5}$$

Then (3.3) implies that $I(n)$ is decreasing for $n \geq N_1$. Therefore, $\lim_{n\to\infty} I(n) =: I^*$ exists and $I^* \geq 0$. Further the second equation of model (2.4) shows that $\lim_{n\to\infty} E(n) =: E^*$ exists and $E^* = \frac{\mu+k_3}{k_2}I^*$.

For any constant $\varepsilon > 0$ there exists an integer $N_\varepsilon > 0$ such that $I^* - \varepsilon \leq I(n) \leq I^* + \varepsilon$ for all $n \geq N_\varepsilon$. Then, from the third equation of model (2.4) we obtain for any $n \geq N_\varepsilon$

$$\frac{1}{1 + \mu + \alpha}\big[R(n) + k_3\big(I^* - \varepsilon\big)\big] \leq R(n+1) \leq \frac{1}{1 + \mu + \alpha}\big[R(n) + k_3\big(I^* + \varepsilon\big)\big].$$

Considering the following auxiliary equations:

$$U(n+1) = \frac{1}{1 + \mu + \alpha}\big(U(n) + k_3\big(I^* + \varepsilon\big)\big) \tag{3.6}$$

and

$$V(n+1) = \frac{1}{1 + \mu + \alpha}\big(V(n) + k_3\big(I^* - \varepsilon\big)\big), \tag{3.7}$$

the comparison theorem of difference equations implies that

$$V(n) \leq R(n) \leq U(n) \quad \text{for all } n \geq N_\varepsilon,$$

where $U(n)$ and $V(n)$ are the solutions of (3.6) and (3.7) with initial conditions $U(N_\varepsilon) = R(N_\varepsilon)$ and $V(N_\varepsilon) = R(N_\varepsilon)$, respectively. Since

$$\lim_{n\to\infty} U(n) = \frac{k_3(I^* + \varepsilon)}{\mu + \alpha}, \qquad \lim_{n\to\infty} V(n) = \frac{k_3(I^* - \varepsilon)}{\mu + \alpha},$$

we obtain

$$\frac{k_3(I^* - \varepsilon)}{\mu + \alpha} \leq \liminf_{n\to\infty} R(n) \leq \limsup_{n\to\infty} R(n) \leq \frac{k_3(I^* + \varepsilon)}{\mu + \alpha}.$$

This shows that $\lim_{n\to\infty} R(n) =: R^*$ exists and $R^* = \frac{k_3}{\mu+\alpha} I^*$. By a similar argument, we obtain the result that $\lim_{n\to\infty} N(n) =: N^*$ exists and $N^* = \frac{rK(\mu+\alpha) - \alpha k_3 I^*}{(\mu+\alpha)(r+\mu)}$.

From (3.5) we directly obtain $(\frac{k_2}{p} - (\mu + k_3))I^* \geq 0$. When $R_0 < 1$, by (3.1) it follows that $I^* = 0$. Consequently, $E^* = 0$, $R^* = 0$ and $N^* = \frac{rK}{\mu+r}$. Therefore, we finally have

$$\lim_{n\to\infty} \big(S(n), E(n), I(n), R(n)\big) = \left(\frac{rK}{\mu + r}, 0, 0, 0\right). \tag{3.8}$$

When $R_0 = 1$, if $I^* > 0$, then from the first equation of model (2.4) and $E^* = \frac{\mu + k_3}{k_2} I^*$ we obtain

$$\frac{\mu + k_3}{k_2} I^* = \frac{1}{1 + \mu + k_2}\left(\frac{\mu + k_3}{k_2} I^* + \frac{k_1}{K}\left(\frac{rK(\mu + \alpha) - \alpha k_2 I^*}{(\mu + \alpha)(r + \mu)} - \frac{\mu + k_3}{k_2} I^* - I^* - \frac{k_3 I^*}{\mu + \alpha}\right)I^*\right),$$

that is,

$$\frac{(\mu + k_2)(\mu + k_3)}{k_2} = \frac{k_1 r}{r + \mu} - \frac{k_1}{K}\left(\frac{\alpha k_2 I^*}{(\mu + \alpha)(r + \mu)} + \frac{\mu + k_3}{k_2} I^* + I^* + \frac{k_3}{\mu + \alpha} I^*\right).$$

Consequently,

$$
\begin{aligned}
0 &= \frac{(\mu + k_2)(\mu + k_3)}{k_2} - \frac{rk_1}{r + \mu} \\
&= \frac{rk_1}{r + \mu}\left(\frac{(\mu + k_2)(\mu + k_3)(r + \mu)}{rk_1k_2} - 1\right) \\
&= \frac{rk_1}{r + \mu}\left(\frac{1}{R_0} - 1\right) \\
&= -\frac{k_1}{K}I^*\left(\frac{\alpha k_2}{(\mu + \alpha)(r + \mu)} + \frac{\mu + k_3}{k_2} + 1 + \frac{k_3}{\mu + \alpha}\right) \\
&< 0,
\end{aligned}
$$

which leads to a contradiction. Hence, $I^* = 0$. Furthermore, $E^* = 0$, $R^* = 0$, and $N^* = \frac{rK}{\mu + r}$. Therefore, (3.8) holds.

Next, we consider Case 2. From the second equation of model (2.2), and according to Lemma 2.1 we have $S(n) \leq \frac{rK}{\mu + r}$. So for $n \geq N_1$, we have

$$
E(n + 1) \leq E(n) + \left(\frac{rk_1 p}{\mu + r} - (\mu + k_2)\right)E(n + 1). \tag{3.9}
$$

Then (3.4) implies that $E(n)$ is decreasing for $n \geq N_1$. Therefore, $\lim_{n\to\infty} E(n) =: E^*$ exists and $E^* \geq 0$. By a similar argument to the above, we obtain the result that $\lim_{n\to\infty} I(n) =: I^*$, $\lim_{n\to\infty} R(n) =: R^*$ and $\lim_{n\to\infty} N(n) =: N^*$ exists, respectively. Obviously,

$$
I^* = \frac{k_2}{\mu + k_3}E^*, \qquad R^* = \frac{k_3}{\mu + \alpha}I^*, \qquad N^* = \frac{rK - \alpha R^*}{r + \mu}.
$$

From (3.9) we directly obtain $(\frac{rk_1 p}{\mu + r} - (\mu + k_2))E^* \geq 0$. When $R_0 < 1$, by (3.2) it follows $E^* = 0$. Furthermore, $I^* = 0$, $R^* = 0$ and $N^* = \frac{rK}{r + \mu}$. This shows that (3.8) holds.

When $R_0 = 1$, if $E^* > 0$, then from the first equation of model (2.4), as in the above, we also obtain

$$
E^* = \frac{1}{1 + \mu + k_2}\left(E^* + \frac{k_1}{K}(N^* - E^* - I^* - R^*)I^*\right),
$$

that is,

$$
(\mu + k_2)E^* = \frac{k_1}{K}\left(\frac{rK(\mu + \alpha) - \alpha k_2 I^*}{(\mu + \alpha)(r + \mu)} - \frac{\mu + k_3}{k_2}I^* - I^* - \frac{k_3}{\mu + \alpha}I^*\right)I^*.
$$

From $E^* = \frac{\mu + k_3}{k_2}I^*$, we have

$$
(\mu + k_2)E^* = \frac{k_1}{K}\left(\frac{rK(\mu + \alpha) - \alpha k_2 I^*}{(\mu + \alpha)(r + \mu)} - \frac{\mu + k_3}{k_2}I^* - I^* - \frac{k_3}{\mu + \alpha}I^*\right)\frac{k_2}{\mu + k_3}E^*,
$$

that is,

$$
\mu + k_2 = \frac{rk_1 k_2}{(r + \mu)(\mu + k_3)} - \frac{k_1 k_2}{K(\mu + k_3)}\left(\frac{\alpha k_2 I^*}{(\mu + \alpha)(r + \mu)} + \frac{\mu + k_3}{k_2}I^* + I^* + \frac{k_3}{\mu + \alpha}I^*\right).
$$

By a similar argument to the above, we also can obtain

$$
\begin{aligned}
0 &= \mu + k_2 - \frac{rk_1k_2}{(r+\mu)(\mu+k_3)} \\
&= (\mu + k_2)\left(1 - \frac{rk_1k_2}{(\mu+k_2)(\mu+k_3)(r+\mu)}\right) \\
&= (\mu + k_2)(1 - R_0) \\
&= -\frac{k_1k_2}{K(\mu+k_3)}I^*\left(\frac{\alpha k_2}{(\mu+\alpha)(r+\mu)} + \frac{\mu+k_3}{k_2} + 1 + \frac{k_3}{\mu+\alpha}\right) \\
&< 0,
\end{aligned}
$$

which leads to a contradiction. Hence, $E^* = 0$. Furthermore, $I^* = 0$, $R^* = 0$, and $N^* = \frac{rK}{\mu+r}$. Therefore, (3.8) is true.

From the above discussions, we obtain the result that the disease-free equilibrium $P_0$ is globally attractive. This completes the proof of Theorem 3.1. □

## 4 Permanence of disease

In this section, we mainly prove the permanence of model (2.2) when $R_0 > 1$. The disease $I(n)$ in model (2.2) is to be said permanent, if there exist constants $M > m > 0$ such that for any solution $(S(n), E(n), I(n), R(n))$ of model (2.2) with initial condition (2.3) one has

$$
m \leq \liminf_{n\to\infty} I(n) \leq \limsup_{n\to\infty} I(n) \leq M.
$$

We have the following result.

**Theorem 4.1** *The disease $I(n)$ in model* (2.2) *is permanent iff $R_0 > 1$.*

*Proof* The necessity is obvious. In fact, if $R_0 \leq 1$, then from Theorem 3.1 the disease-free equilibrium is globally attractive.

Now, we prove the sufficiency. When $R_0 > 1$, we have

$$
\varphi\left(\frac{(\mu+k_2)(\mu+r)}{rk_1}\right) = (\mu+k_3)(1-R_0) < 0, \qquad \frac{k_2}{\mu+k_3} > \frac{(\mu+k_2)(\mu+r)}{rk_1}.
$$

Hence, there exists a constant $p > 0$ with $p$ sufficiently close to $\frac{(\mu+k_2)(\mu+r)}{rk_1}$ such that $\varphi(p) < 0$ and

$$
\frac{k_2}{\mu+k_3} > p > \frac{(\mu+k_2)(\mu+r)}{rk_1}.
$$

Therefore,

$$
\frac{k_2}{p} - (\mu+k_3) > 0 \tag{4.1}
$$

and

$$
rk_1 p - (\mu+k_2)(\mu+r) > 0. \tag{4.2}
$$

Let $(S(n), E(n), I(n), R(n))$ be any solution of model (2.2) with initial condition (2.3). From $\limsup_{n\to\infty} N(n) \leq \frac{rK}{r+\mu}$, we see that there exists an integer $T_0 > 0$ such that

$$0 < S(n), E(n), I(n), R(n) < \frac{rK}{r+\mu} \tag{4.3}$$

for all $n \geq T_0$. By Lemma 2.2 and $\varphi(p) < 0$, we obtain the result that there exists an integer $T_1 \geq T_0$ such that $\phi(p,n) \leq 0$ for all $n \geq T_1$ or $\phi(p,n) \geq 0$ for all $n \geq T_1$. Suppose that $\phi(p,n) \geq 0$ for all $n \geq T_1$. Then $E(n) \geq \frac{1}{p}I(n)$ for all $n \geq T_1$. From the third equation of model (2.2), we have

$$I(n+1) - I(n) = k_2 E(n+1) - (\mu + k_3)I(n+1)$$
$$\geq \left(\frac{k_2}{p} - (\mu + k_3)\right)I(n+1). \tag{4.4}$$

Hence, by (4.1) and (4.4), $I(n)$ is increasing for all $n \geq T_1$. Consequently, $\lim_{n\to\infty} I(n) =: I^*$ exists and $I^* > 0$. Further, it follows from (4.4)

$$0 \geq \left(\frac{k_2}{p} - (\mu + k_3)\right)I^* > 0,$$

which leads to a contradiction. Therefore, we only need to consider $\phi(p,n) \leq 0$ for all $n \geq T_1$. That is,

$$pE(n) \leq I(n) \quad \text{for all } n \geq T_1. \tag{4.5}$$

By (4.2), choosing a sufficiently small constant $0 < \varepsilon_0 < 1$ such that

$$0 < \frac{pk_1 r}{r+\mu} - pk_1\varepsilon_0 - (\mu + k_2). \tag{4.6}$$

Considering the following auxiliary equation:

$$W(n+1) = \frac{W(n)}{1 + \mu + \frac{k_1\rho}{K}} + \frac{1}{1 + \mu + \frac{k_1\rho}{K}} \frac{rK\mu}{r+\mu}, \tag{4.7}$$

where the parameter $0 \leq \rho \leq 1$. Equation (4.7) has a positive equilibrium $W_\rho^* = \frac{1}{\mu + \frac{k_1\rho}{K}} \frac{rK\mu}{r+\mu}$ which is globally asymptotically stable. Obviously, $\lim_{\rho\to 0} W_\rho^* = \frac{rK}{r+\mu}$. Hence, for above $\varepsilon_0$, there is a constant $\rho_0 \in (0, \varepsilon_0)$, such that $W_{\rho_0}^* > \frac{rK}{r+\mu} - \frac{\varepsilon_0 K}{2}$.

By the global asymptotic stability of equilibrium $W_{\rho_0}^*$ of (4.7), for the above $\varepsilon_0$, there exists an integer $T_2 > 0$ such that for any initial integer $n_0 > 0$ and initial value $W_0$ satisfying $0 \leq W_0 \leq \frac{rK}{r+\mu}$,

$$W_{\rho_0}(n) > W_{\rho_0}^* - \frac{\varepsilon_0 K}{2}, \quad \text{for all } n \geq n_0 + T_2,$$

where $W_{\rho_0}(n)$ is the solution of (4.7) with $\rho = \rho_0$ and initial condition $W_{\rho_0}(n_0) = W_0$.

Hence,

$$W_{\rho_0}(n) > \frac{rK}{r+\mu} - \varepsilon_0 K, \quad \text{for all } n \geq n_0 + T_2. \tag{4.8}$$

Furthermore, we consider the following auxiliary equation:

$$U(n+1) = \frac{U(n)}{1+\mu+k_2} + \frac{rk_1\eta}{1+\mu+k_2}, \tag{4.9}$$

where $0 < \eta < 1$. Obviously, (4.9) has the globally asymptotically stable equilibrium $U_\eta^* = \frac{rk_1\eta}{\mu+k_2}$. Thus, for the above $\rho_0$, there is a constant $\eta_0 \in (0, \frac{\rho_0}{2})$ such that $U_{\eta_0}^* < \frac{\rho_0}{8}$. By the global uniform asymptotic stability of equilibrium $U_{\eta_0}^*$ of (4.9), for $\frac{\rho_0}{8} > 0$, there is an integer $T_3 > 0$ such that for any initial time $n_0$ and initial value $U_0$ for which $0 \leq U_0 \leq M_0$, where $M_0$ is given above, we have

$$U_{\eta_0}(n) < U_{\eta_0}^* + \frac{\rho_0}{8}, \quad \text{for all } n \geq n_0 + T_3,$$

where $U_{\eta_0}(n)$ is the solution of (4.9) with $\eta = \eta_0$ and initial condition $U_{\eta_0}(n_0) = U_0$. Hence,

$$U_{\eta_0}(n) < \frac{\rho_0}{4}, \quad \text{for all } n \geq n_0 + T_3. \tag{4.10}$$

In order to obtain the permanence of disease $I(n)$ of model (2.2), we discuss the following three cases.

*Case* 1. $I(n) \geq \eta_0$ for all $n \geq T_1$. For this case, obviously, $I(n)$ is permanent.

*Case* 2. $I(n) < \eta_0$ for all $n \geq T_1$. From the first equation of model (2.2) and Lemma 2.1, we have

$$\begin{aligned}
S(n+1) - S(n) &= r\big(K - N(n+1)\big) - \mu S(n+1) - k_1 S(n+1)\frac{I(n+1)}{K} \\
&\geq r\left(K - \frac{rK}{r+\mu}\right) - \mu S(n+1) - k_1 S(n+1)\frac{\eta_0}{K} \\
&\geq r\left(K - \frac{rK}{r+\mu}\right) - \mu S(n+1) - k_1 S(n+1)\frac{\rho_0}{K},
\end{aligned}$$

that is,

$$S(n+1) \geq \frac{S(n)}{1+\mu+\frac{k_1\rho}{K}} + \frac{1}{1+\mu+\frac{k_1\rho}{K}}\frac{rK\mu}{r+\mu},$$

for all $n \geq T_1$. By the comparison theorem of difference equations, we have $S(n) \geq W_{\rho_0}(n)$ for all $n \geq T_1$, where $W_{\rho_0}(n)$ is the solution of (4.7) with $\rho = \rho_0$ and initial condition $W_{\rho_0}(T_1) = S(T_1)$. Since $0 \leq W_{\rho_0}(T_1) \leq \frac{rK}{r+\mu}$, by (4.8), $W_{\rho_0}(n) \geq \frac{rK}{r+\mu} - \varepsilon_0 K$ for all $n \geq T_1 + T_2$. We have

$$S(n) \geq \frac{rK}{r+\mu} - \varepsilon_0 K, \quad \text{for all } n \geq T_1 + T_2. \tag{4.11}$$

Considering the second equation of model (2.2), by (4.5), (4.6), and (4.11), for all $n \geq T_1 + T_2$

$$
\begin{aligned}
E(n+1) - E(n) &= k_1 S(n+1) \frac{I(n+1)}{K} - (\mu + k_2) E(n+1) \\
&\geq \left[ k_1 S(n+1) \frac{p}{K} - (\mu + k_2) \right] E(n+1) \\
&\geq \left[ \frac{pk_1}{K} \left( \frac{rK}{r+\mu} - \varepsilon_0 K \right) - (\mu + k_2) \right] E(n+1) \\
&= \left[ \frac{pk_1 r}{r+\mu} - pk_1 \varepsilon_0 - (\mu + k_2) \right] E(n+1).
\end{aligned}
$$

Hence, $E(n)$ is increasing for all $n \geq T_1 + T_2$. Consequently, $\lim_{n \to \infty} E(n) =: E^*$ exists and $E^* > 0$. Further, from above inequality we have

$$
0 \geq \left[ \frac{pk_1 r}{r+\mu} - pk_1 \varepsilon_0 - (\mu + k_2) \right] E^* > 0,
$$

which leads to a contradiction.

*Case* 3. There exist two integer sequences $\{m_k\}_{k=1}^{\infty}$ and $\{n_k\}_{k=1}^{\infty}$ satisfying

$$
T_1 \leq n_1 \leq m_1 < n_2 \leq m_2 < \cdots < n_k \leq m_k < \cdots
$$

and $\lim_{k \to \infty} n_k = \infty$ such that

$$
I(n) < \eta_0 \quad \text{for all } n \in \bigcup_{k=1}^{\infty} [n_k, m_k],
$$

$$
I(n) \geq \eta_0 \quad \text{for all } n \notin \bigcup_{k=1}^{\infty} [n_k, m_k].
$$

Obviously, we have $n_{k+1} - m_k \geq 2$ for all $k = 1, 2, \ldots$.

Since $R_0 = \frac{rk_1 k_2}{(\mu+r)(\mu+k_2)(\mu+k_3)} > 1$, we obtain

$$
\frac{k_2}{\mu + k_2} > \frac{(r+\mu)(\mu + k_3)}{rk_1}.
$$

Thus we can choose constants $r_1 > 0$, $r_2 > 0$, and $0 < \theta < 1$ such that

$$
r_2 k_2 - r_1 (\mu + k_2) \geq \theta, \qquad \frac{r_1 k_1 r}{r+\mu} - \varepsilon_0 r_1 k_1 - r_2 (\mu + k_3) \geq \theta. \tag{4.12}
$$

Let $T^* = \max\{T_2, T_3\}$. We can choose a positive integer $K_0$ such that

$$
\frac{r_2 \xi_0}{r_1} \frac{1}{(1-\theta)^{K_0 T^*}} - \frac{r_2 \xi}{r_1} \geq \frac{\rho_0}{4},
$$

where

$$
\xi = \frac{\eta_0}{(1 + \mu + k_3)^{(K_0+1)T^*+1}}, \qquad \xi_0 = \frac{\eta_0}{(1 + \mu + k_3)^{T^*+1}}.
$$

In the following, we will prove $I(n) \geq \xi$ for all $n \in \bigcup_{k=1}^{\infty}[n_k, m_k]$. Let $n \in \bigcup_{k=1}^{\infty}[n_k, m_k]$. If $m_k - n_k \leq (K_0 + 1)T^*$, from the third equation of model (2.2) we have

$$I(n + 1) \geq \frac{I(n)}{1 + \mu + k_3} \quad \text{for all } n \in [n_k, m_k].$$

Hence,

$$I(n + 1) \geq \frac{I(n_k - 1)}{(1 + \mu + k_3)^{n - n_k + 1}} \geq \frac{\eta_0}{(1 + \mu + k_3)^{n - n_k + 1}}$$

for all $n \in [n_k, m_k]$. Thus, we finally obtain $I(n) \geq \xi$ for all $n \in [n_k, m_k]$.

If $m_k - n_k > (K_0 + 1)T^*$, then similar to the discussion, we also have

$$I(n + 1) \geq \frac{\eta_0}{(1 + \mu + k_3)^{n - n_k + 1}} \quad \text{for all } n \in [n_k, m_k]. \tag{4.13}$$

Particularly, we obtain from (4.13) $I(n) \geq \xi$ for all $n \in [n_k, n_k + (K_0 + 1)T^*]$ and

$$I(n_k + T_2) \geq \xi_0. \tag{4.14}$$

By the second equation of model (2.2), we obtain for all $n \geq T_1$ and $n \in \bigcup_{k=1}^{\infty}[n_k, m_k]$,

$$\begin{aligned}
E(n + 1) - E(n) &= k_1 S(n + 1) \frac{I(n + 1)}{K} - (\mu + k_2)E(n + 1) \\
&\leq k_1 \frac{rK}{r + \mu} \frac{I(n)}{K} - (\mu + k_2)E(n + 1) \\
&\leq \frac{rk_1}{r + \mu} \eta_0 - (\mu + k_2)E(n + 1),
\end{aligned}$$

that is,

$$E(n + 1) < \frac{E(n)}{1 + \mu + k_2} + \frac{rk_1\eta_0}{(1 + \mu + k_2)(r + \mu)}.$$

By the comparison theorem of difference equations, we have $E(n) \leq U_{\eta_0}$ for all $n > T_1$, where $U_{\eta_0}$ is the solution of (4.9) with $\eta = \eta_0$ and initial condition $U_{\eta_0}(T_1) = E(T_1)$. Since $0 \leq U_{\eta_0}(T_1) \leq M_0$, by (4.10), $U_{\eta_0}(n) \leq \frac{\rho_0}{4}$ for all $n \geq T_1 + T_3$. Hence, $E(n) \leq \frac{\rho_0}{4}$ for all $n \geq T_1 + T_3$. So,

$$E(n) \leq \frac{\rho_0}{4} \tag{4.15}$$

for all $n \in [n_k + T_2, m_k]$, where $k = 1, 2, \ldots$.

We claim that $I(n) \geq \xi$ for all $n \in [n_k + (K_0 + 1)T^*, m_k]$. If it is not true, then there is an integer $q_0$ such that $I(n_k + (K_0 + 1)T^* + q_0) < \xi$ and $I(n) \geq \xi$ for all $n \in [n_k + (K_0 + 1)T^*, n_k + (K_0 + 1)T^* + q_0 - 1]$. Denote $t_0 = n_k + (K_0 + 1)T^* + q_0$. Let

$$v(n) = r_1 E(n) + r_2 I(n),$$

where $r_1$ and $r_2$ are constants in (4.12). Hence, as $n \in [n_k + T^*, m_k]$, from (4.12) and (4.15) we have

$$
\begin{aligned}
\Delta v(n) &= v(n+1) - v(n) \\
&= r_1 E(n+1) + r_2 I(n+1) - r_1 E(n) - r_2 I(n) \\
&= r_1 \left( k_1 \frac{S(n+1)I(n+1)}{K} - (\mu + k_2)E(n+1) \right) + r_2 \big( k_2 E(n+1) - (\mu + k_3) \big) I(n) \\
&= \left( r_1 k_1 \frac{S(n+1)}{K} - r_2(\mu + k_3) \right) I(n+1) + \big( r_2 k_2 - r_1(\mu + k_2) \big) E(n+1) \\
&\geq \left( \frac{r_1 k_1}{K} \left( \frac{rK}{r+\mu} - \varepsilon_0 K \right) - r_2(\mu + k_3) \right) I(n+1) + \big( r_2 k_2 - r_1(\mu + k_2) \big) E(n+1) \\
&\geq \theta \big( I(n+1) + E(n+1) \big) \\
&\geq \theta \big( r_2 I(n+1) + r_1 E(n+1) \big) \\
&\geq \theta v(n+1).
\end{aligned}
$$

Then

$$
v(n+1) \geq \frac{v(n)}{1-\theta} \quad \text{for all } n \in \big[ n_k + T^*, m_k \big].
$$

Consequently,

$$
v(t_0) \geq \frac{v(n_k + T^*)}{(1-\theta)^{K_0 T^* + q_0}} \geq \frac{v(n_k + T^*)}{(1-\theta)^{K_0 T^*}},
$$

that is,

$$
r_1 E(t_0) + r_2 I(t_0) \geq \frac{r_1 E(n_k + T^*) + r_2 I(n_k + T^*)}{(1-\theta)^{K_0 T^*}} \geq \frac{r_2 I(n_k + T^*)}{(1-\theta)^{K_0 T^*}}.
$$

By (4.14), we further have

$$
E(t_0) \geq \frac{r_2 \xi_0}{r_1} \frac{1}{(1-\theta)^{K_0 T^*}} - \frac{r_2 \xi}{r_1} \geq \frac{\rho_0}{4},
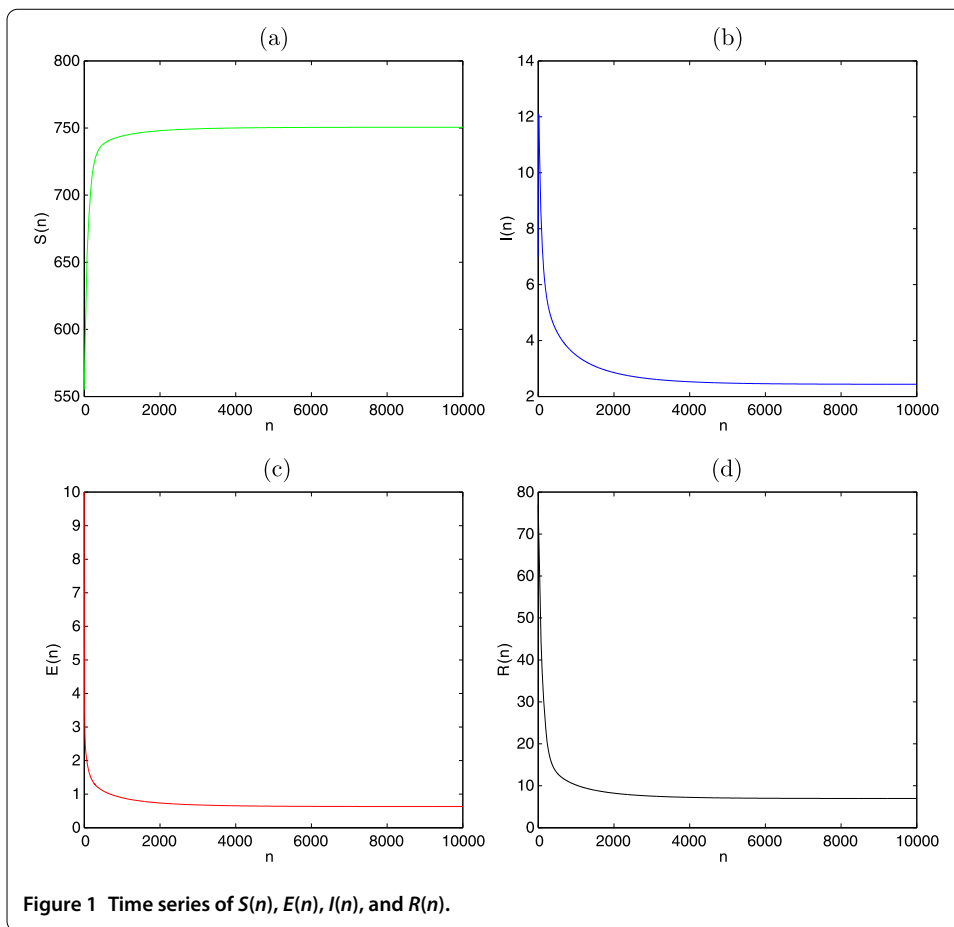$$

which leads to a contradiction with (4.15). Therefore,

$$
I(n) \geq \xi \quad \text{for all } n \in \bigcup_{k=1}^{\infty} [n_k, m_k].
$$

When $n \notin \bigcup_{k=1}^{\infty} [n_k, m_k]$, we directly have $I(n) \geq \eta_0 > \xi$. Therefore, we finally obtain the result that $I(n)$ is permanent. This completes the proof. □

## 5 Numerical simulations

In this section, we carry out numerical simulations on model (2.2) to demonstrate the results in Sections 3 and 4.

**Figure 1  Time series of *S(n)*, *E(n)*, *I(n)*, and *R(n)*.**

**Example 5.1**  The parameters of model (2.2) are chosen as follows:

$$r = 0.013, \qquad k_1 = 0.06, \qquad k_2 = 0.17, \qquad k_3 = 0.04,$$

$$\mu = 0.004, \qquad K = 1{,}000, \qquad \alpha = 0.01.$$

By calculating, we have the endemic equilibrium $P_* = (750.5882, 25.2861, 6.5446, 72.2460)$ and the basic reproduction number $R_0 = 1.0188 > 1$. Therefore, the disease is permanent from Theorem 4.1. The numerical simulations are given in Figure 1. From Figure 1, we can see that $P_*$ is globally attractive.

**Example 5.2**  The parameters are chosen as follows:

$$r = 0.011, \qquad k_1 = 0.06, \qquad k_2 = 0.17, \qquad k_3 = 0.04,$$

$$\mu = 0.004, \qquad K = 1{,}000, \qquad \alpha = 0.01.$$

By calculating, we obtain $R_0 = 0.9770 < 1$. Thus, the disease-free equilibrium of model (2.2) is globally attractive from Theorem 3.1. The numerical simulations are given in Figure 2.

**Figure 2 Time series of $S(n)$, $E(n)$, $I(n)$, and $R(n)$.**
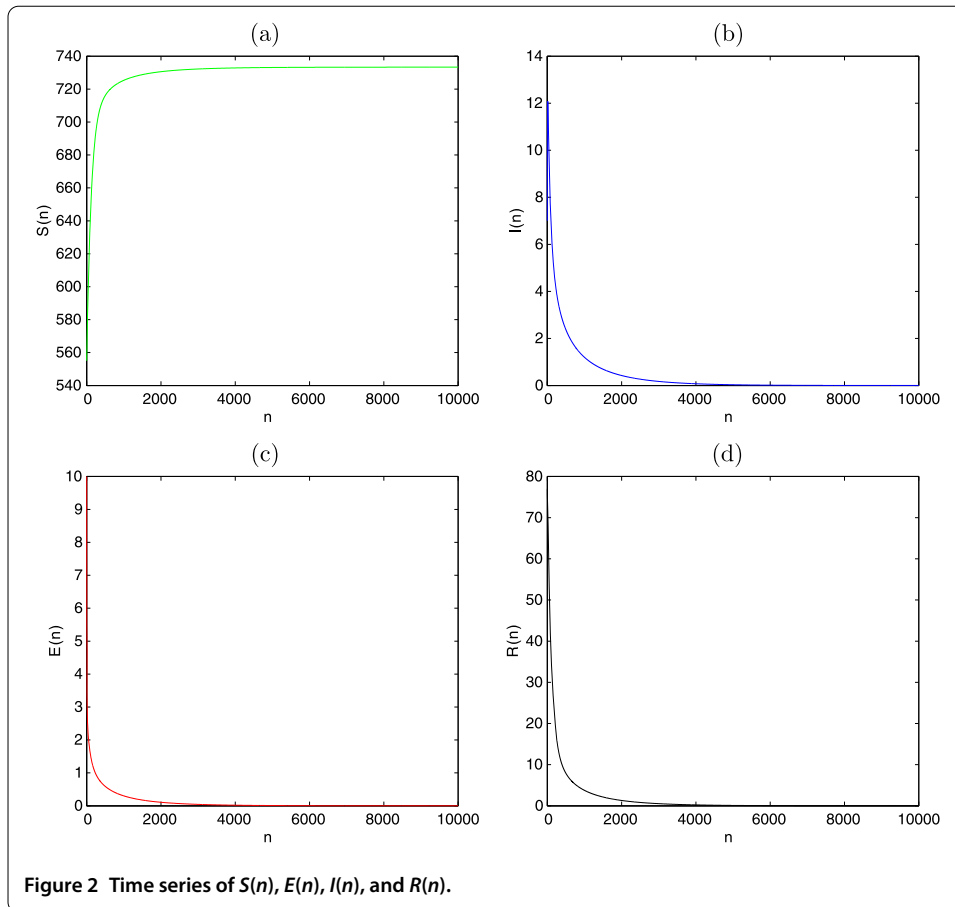
## 6 Conclusion

In this paper, we study the dynamic behaviors of a discrete plant virus disease model with roguing and replanting which is derived from the continuous case. By calculating, we obtain the basic reproduction number $R_0$. We also prove that the disease-free equilibrium of model (2.2) is globally attractive when $R_0 < 1$, in other words, the disease goes extinct. On the contrary, if $R_0 > 1$, the endemic equilibrium of model (2.2) exists and the disease will be endemic.

In Theorem 4.1, we only discuss the permanence of the disease of model (2.2). However, the numerical simulations in Figure 1 show that the endemic equilibrium of model (2.2) may be globally attractive when $R_0 > 1$.

In the real world, some plants at the beginning may show no infection, for example Calletotrichum musae. We usually use the time delay to describe this phenomenon in a mathematical model. However, the dynamic behaviors of discrete plant virus disease models with time delay are rarely considered. Therefore, an important and interesting open problem is whether we can obtain similar results on the permanence and extinction of the disease for the discrete plant virus disease models with time delay. We will discuss these problems in our future work.

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