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ON QUADRUPLE INTEGRAL EQUATIONS INVOLVING TRIGONOMETRIC KERNELS

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ABSTRACT. A general technique is developed for the solution of quadruple integral equations involving trigonometric kernels. Four such sets are solved explicitly. Application is made to the problem of three-collinear cracks in linear plane elasticity.

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1. INTRODUCTION.

Dual, Triple, Quadruple, and higher order integral equations arise in punch and crack problems in the linear theory of Elasticity and the solution of dual integral equations goes back to Busbridge in 1938 [1]. Because of their importance in applicatons, dual integral equations have been considered by a large number of investigators, and a thorough account is given by Sneddon [2]. Sneddon [2] also considers a particular case of Triple integral equations and those equations have recently been considered by Lowndes and Srivastava [3], Chakrabarti [4], Singh [5,6], and others. A set of quadruple integral equations has been considered by Jain and Singh [7], and so on.

In this paper we outline a general method for solving quadruple integral equations involving trigonometric kernels and apply this method to a number of such sets. We later on discuss their application to the problem of three collinear cracks, not all of same length, situated symmetrically about the origin on the x-axis and evaluate a number of stress concentration factors.

2. We consider the set of equations

$$\int^{\infty} tA(t)\cos xt\,dt = f_1(x), \quad 0 < x < a \tag{2.1a}$$

$$\int_0^{\infty} A(t)\cos xt\,dt = g_2(x), \quad a < x < b \tag{2.1b}$$

$$\int_0^{\infty} tA(t)\cos xt\,dt = f_3(x), \quad b < x < 1 \tag{2.1c}$$

and
$$\int_0^{\infty} A(t)\cos xt\,dt = g_4(x), \quad x > 1. \tag{2.1d}$$

These equations are to be solved simultaneously. $f_1(x)$, $f_3(x)$, $g_2(x)$ and $g_4(x)$ are given and $A(t)$, $0 < t < \infty$ is to be determined so as to satisfy all these equations.

We notice that if we write

$$\int_0^\infty tA(t)\cos xt\, dt = f_2(x), \quad a < x < b \tag{2.2}$$

and if we are able to determine $f_2(x)$, then the equations (2.1) reduce to a pair of dual integral equations whose solution is known. We proceed to determine $f_2(x)$.

We write

$$F(x) = f_1(x) \oplus f_2(x) \oplus f_3(x), \quad 0 < x < 1, \tag{2.3}$$

where (2.3) stands for

$$F(x) = f_1(x), \quad 0 < x < a \tag{2.4a}$$

$$= f_2(x), \quad a < x < b \tag{2.4b}$$

$$\text{and} \quad = f_3(x), \quad b < x < 1 \tag{2.4c}$$

In terms of $F(x)$ and $g_4(x)$, the solution of equations (2.1) is given by [8]

$$A(t) = \frac{2}{\pi}\int_0^1 uJ_0(ut)F_1(u)du + \frac{2}{\pi}\int_1^\infty uJ_0(ut)G_1(u)du \tag{2.5a}$$

where

$$F_1(u) = \int_0^u \frac{F(x)dx}{\sqrt{u^2 - x^2}} \tag{2.5b}$$

and

$$G_1(u) = -\int_u^\infty \frac{g_4'(x)dx}{\sqrt{x^2 - u^2}} \tag{2.5c}$$

where $'$ stands for differentiation w.r.t. x.

We now notice that

$$F_1(u) = \int_0^u \frac{f_1(x)dx}{\sqrt{u^2 - x^2}}, \quad 0 < u < a \tag{2.6a}$$

$$= \int_0^a \frac{f_1(x)dx}{\sqrt{u^2 - x^2}} + \int_a^u \frac{f_2(x)dx}{\sqrt{u^2 - x^2}}, \quad a < u < b \tag{2.6b}$$

$$\text{and} \quad = \int_0^a \frac{f_1(x)dx}{\sqrt{u^2 - x^2}} + \int_a^b \frac{f_2(x)dx}{\sqrt{u^2 - x^2}} + \int_b^u \frac{f_3(x)dx}{\sqrt{u^2 - x^2}}, \quad b < u < 1. \tag{2.6c}$$

Substituting from (2.5) into (2.1b) and simplifying, we get

$$\frac{\pi}{2}\, g_2(x) = \int_0^a f_1(\xi)K(x,\xi)d\xi + \int_a^b f_2(\xi)K(x,\xi)d\xi$$

$$+ \int_b^1 f_3(\xi)K(x,\xi)d\xi + \int_1^\infty \frac{uG_1(u)du}{\sqrt{u^2 - x^2}}, \quad a < x < b \tag{2.7a}$$

where

$$K(x,\xi) = \int_{Max(x,\xi)}^1 \frac{udu}{\sqrt{u^2 - x^2}\,\sqrt{u^2 - \xi^2}} \tag{2.7b}$$

Equation (2.7) is our equation to determine $f_2(\xi)$, $a < \xi < b$. We proceed to solve this equation.

Differentiating (2.7) w.r.t. x, we obtain

$$\begin{aligned} P\int_a^b \frac{f_2(\xi)\sqrt{1-\xi^2}}{\xi^2-x^2}\,d\xi &= \frac{\pi}{2}\,g_2'(x)\frac{\sqrt{1-x^2}}{x} - \int_0^a \frac{f_1(\xi)\sqrt{1-\xi^2}}{\xi^2-x^2}\,d\xi \\ &\quad - \int_b^1 \frac{f_3(\xi)\sqrt{1-\xi^2}}{\xi^2-x^2}\,d\xi - \sqrt{1-x^2}\int_1^\infty \frac{uG_1(u)du}{(u^2-x^2)^{\frac{3}{2}}}, \\ &= R(x), \text{ say} \qquad a<x<b, \end{aligned} \tag{2.8}$$

where P denotes the principal value of an improper integral.

Equation (2.5) can be solved easily [9]. The solution is

$$\begin{aligned} f_2(\xi) &= \frac{-2\xi}{\pi^2\sqrt{\xi^2-a^2}\,\sqrt{b^2-\xi^2}\,\sqrt{1-\xi^2}}P\int_a^b \frac{\sqrt{x^2-a^2}\,\sqrt{b^2-x^2}\,R(x)2x\,dx}{x^2-\xi^2} \\ &\quad + \frac{D\xi}{\sqrt{\xi^2-a^2}\,\sqrt{b^2-\xi^2}\,\sqrt{1-\xi^2}}, \quad a<\xi<b, \end{aligned} \tag{2.9}$$

where D is a constant of integration, which must be determined by requiring that $f_2(\xi)$ satisfy (2.7).

This solves the equations (2.1) completely. With $f_2(\xi)$ known from (2.9), the solution of (2.1) is given by (2.5).

3. We now consider the set of equations

$$\int_0^\infty tA(t)\sin xt\,dt = f_1(x), \quad 0<x<a \tag{3.1a}$$

$$\int_0^\infty A(t)\sin xt\,dt = g_2(x), \quad a<x<b \tag{3.1b}$$

$$\int_0^\infty tA(t)\sin xt\,dt = f_3(x), \quad b<x<1 \tag{3.1c}$$

and $$\int_0^\infty A(t)\sin xt\,dt = g_4(x), \quad x>1. \tag{3.1d}$$

This time we write

$$\int_0^\infty tA(t)\sin xtdt\,dt = f_2(x), \quad a<x<b \tag{3.2}$$

$A(t)$ is now given by

$$A(t) = \frac{2}{\pi}\int_0^1 J_1(ut)h_1(u)du - \frac{2}{\pi}\int_1^\infty uJ_1(ut)h_2(u)du \tag{3.3a}$$

where

$$h_1(u) = \int_0^u \frac{xF(x)dx}{\sqrt{u^2-x^2}}, \tag{3.3b}$$

$$h_2(u) = \frac{d}{du}\int_u^\infty \frac{g_4(x)dx}{\sqrt{x^2-u^2}} \tag{3.3c}$$

and $$F(x) = f_1(x)\oplus f_2(x)\oplus f_3(x) \tag{3.3d}$$

as before. This time the equation to determine $f_2(x)$ is found to be

$$\begin{aligned} \int_a^b \xi f_2(\xi)K(x,\xi)d\xi &= \frac{\pi}{2}g_2(x) + \int_1^\infty \frac{xh_2(u)du}{\sqrt{u^2-x^2}} \\ &\quad - \int_0^a \xi f_1(\xi)K(x,\xi)d\xi - \int_b^1 \xi f_3(\xi)K(x,\xi)d\xi = \\ &= R(x), \text{ say} \qquad a<x<b \end{aligned} \tag{3.4a}$$

where

$$K(x,\xi) = \int_{Max(x,\xi)}^{1} \frac{x}{u} \frac{1}{\sqrt{u^2-x^2}\sqrt{u^2-\xi^2}}\, du. \tag{3.4b}$$

Differentiating equation (3.4) w.r.t x and solving for $f_2(x)$, we get

$$f_2(\xi) = \frac{-2}{\pi^2\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}} P\int_a^b \frac{\sqrt{x^2-a^2}\,\sqrt{b^2-x^2}\,\sqrt{1-x^2}\,R'(x)2x\,dx}{x^2-\xi^2}$$
$$+\frac{D}{\sqrt{\xi^2-a^2}\,\sqrt{b^2-\xi^2}\,\sqrt{1-\xi^2}}, \quad a<\xi<b \tag{3.5}$$

The constant D must again be determined by substitution into (3.4).

4. We consider the set of equations

$$\int_0^\infty A(t)\sin xt\, dt = f_1(x), \quad 0<x<a \tag{4.1a}$$
$$\int_0^\infty tA(t)\sin xt\, dt = g_2(x), \quad a<x<b \tag{4.1b}$$
$$\int_0^\infty A(t)\sin xt\, dt = f_3(x), \quad b<x<1 \tag{4.1c}$$
$$\text{and} \quad \int_0^\infty tA(t)\sin xt\, dt = g_4(x), \quad x>1. \tag{4.1d}$$

Once again we write

$$\int_0^\infty A(t)\sin xt dt = f_2(x), \quad a<x<b \tag{4.2}$$

and proceed to determine $f_2(x)$.

This time, $A(t)$ is given by

$$A(t) = \int_0^1 uJ_0(ut)F_2(u)du + \int_1^\infty uJ_0(ut)G_2(u)du \tag{4.3a}$$

where

$$F_2(u) = \frac{2}{\pi}\int_0^u \frac{F'(x)dx}{\sqrt{u^2-x^2}}, \tag{4.3b}$$
$$G_2(u) = \frac{2}{\pi}\int_u^\infty \frac{g_4(x)dx}{\sqrt{x^2-u^2}}, \tag{4.3c}$$
$$\text{and} \quad F(x) = f_1(x)\oplus f_2(x)\oplus f_3(x) \tag{4.3d}$$

as before. The equation to determine $f_2(x)$ this time is found to be

$$\int_0^a f_1'(\xi)K(x,\xi)d\xi + \int_a^b f_2'(\xi)K(x,\xi)d\xi + \int_b^1 f_3'(\xi)K(x,\xi)d\xi$$
$$= \frac{\pi C}{2} - \frac{\pi}{2}\int_a^x g_2(\xi)d\xi - \int_1^\infty \frac{uG_2(u)du}{\sqrt{u^2-x^2}} \tag{4.4a}$$

where

$$K(x,\xi) = \int_{Max(x,\xi)}^1 \frac{udu}{\sqrt{u^2-x^2}\,\sqrt{u^2-\xi^2}} \tag{4.4b}$$

and where C is a constant of integration.

Differentiating (4.4) w.r.t. x, we obtain

$$P\int_a^b \frac{f_2'(\xi)\sqrt{1-\xi^2}}{\xi^2-x^2}\,d\xi = -\frac{\pi}{2}\frac{\sqrt{1-x^2}}{x}g_2(x) - \int_0^a \frac{f_1'(\xi)\sqrt{1-\xi^2}}{\xi^2-x^2}\,d\xi$$
$$-\int_b^1 f_3'(\xi)\frac{\sqrt{1-\xi^2}}{\xi^2-x^2}\,d\xi - \sqrt{1-x^2}\int_1^\infty \frac{uG_2(u)du}{(u^2-x^2)^{\frac{3}{2}}}\,du$$
$$= R(x), \text{ say}, \quad a<x<b. \tag{4.5}$$

and the solution this time is given by

$$f_2'(\xi) = \frac{-2\xi}{\pi^2\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}}P\int_a^b \frac{\sqrt{x^2-a^2}\sqrt{b^2-x^2}R(x)2x}{x^2-\xi^2}\,dx$$
$$+\frac{D\xi}{\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}}, \quad a<\xi<b \tag{4.6}$$

so that

$$f_2(\xi) = \int_a^\xi f_2'(\xi)d\xi + f_1(a^-) \tag{4.7}$$

D must again be determined by substitution into (4.4) or this time, by the requirement that

$$f_2(b^-) = \int_a^b f_2'(\xi)d\xi + f_1(a^-) = f_3(b^+),$$

i.e. by requiring $F(x)$ to be continuous at the points $x=a$ and at $x=b$.

5. We consider this time the same equations as in Article 4 with sin function there replaced by cos.
However, this time, the proper question to ask is [8];
.Find the constant C and the function $A(t)$ such that

$$C+\int_0^\infty A(t)\cos xt\,dt = f_1(x), \quad 0<x<a \tag{5.1a}$$
$$\int_0^\infty tA(t)\cos xt\,dt = g_2(x), \quad a<x<b \tag{5.1b}$$
$$C+\int_0^\infty A(t)\cos xt\,dt = f_3(x), \quad b<x<1 \tag{5.1c}$$
$$\text{and} \quad \int_0^\infty tA(t)\cos xt\,dt = g_4(x), \quad x>1. \tag{5.1d}$$

We again write

$$C+\int_0^\infty A(t)\cos xt\,dt = f_2(x), \quad a<x<b \tag{5.2}$$

and this time, we get

$$A(t) = \frac{2t}{\pi}\int_0^1 uJ_0(ut)F_1(u)du + \frac{2t}{\pi}\int_1^\infty uJ_0(ut)G_1(u)du$$
$$-Ct\int_0^1 uJ_0(ut)du \tag{5.3}$$

where

$$F_1(u) = \int_0^u \frac{F(x)dx}{\sqrt{u^2-x^2}}, \tag{5.4a}$$
$$G_1(u) = \int_u^\infty \frac{g(x)dx}{\sqrt{x^2-u^2}}, \tag{5.4b}$$
$$g(x) = -\int_x^\infty g_4(x)dx, \tag{5.4c}$$
$$\text{and} \quad F(x) = f_1(x)\oplus f_2(x)\oplus f_3(x) \tag{5.4d}$$

as before. C is now given by [8]

$$C = \frac{2}{\pi}[F_1(1) - G_1(1)] \tag{5.5}$$

and the equation to determine $f_2(x)$ is now found to be

$$\int_a^b f_2(\xi)\left[K_1(x,\xi) + \frac{1}{\xi}\frac{\partial K_2}{\partial \xi}\right] d\xi = -R(x), \tag{5.6a}$$

where

$$R(x) = -\frac{\pi}{2x}\left[D_1 + \int_a^x g_2(\xi)d\xi\right] - \int_1^\infty \frac{G_1'(u)}{\sqrt{u^2-x^2}}\,du + \int_0^a f_1(\xi)\left[\int_x^1 \frac{udu}{\sqrt{u^2-x^2}(u^2-\xi^2)^{\frac{3}{2}}}\right] d\xi$$

$$+\int_b^1 \frac{f_3(\xi)}{\xi}\frac{\partial K_3}{\partial \xi}\, d\xi \tag{5.6b}$$

where

$$K_1(x,\xi) = \int_b^1 \frac{udu}{\sqrt{u^2-x^2}(u^2-\xi^2)^{\frac{3}{2}}}, \tag{5.6c}$$

$$K_2(x,\xi) = \int_{Max(x,\xi)}^b \frac{udu}{\sqrt{u^2-x^2}\sqrt{u^2-\xi^2}} \tag{5.6d}$$

and $$K_3(x,\xi) = \int_{Max(x,\xi)}^1 \frac{udu}{\sqrt{u^2-x^2}\sqrt{u^2-\xi^2}} \tag{5.6e}$$

Simplification of equation (5.6) gives

$$P\int_a^b \frac{f_2(\xi)}{\sqrt{1-\xi^2}}\frac{1}{\xi^2-x^2}\,d\xi = \frac{R(x)}{\sqrt{1-x^2}}$$

and this equation may be solved as before. The constants of integration may be found by requiring that $F(x)$ (which denotes temperatures in a temperature problem, see ([8]) is continuous at $x = a$ and at $x = b$.

6. AN APPLICATION.

We apply the equations in section 2 to the problem of finding stresses in an elastic body in plane elasticity, where the body has three colinear cracks lying along $(-1,-b)$, $(-a,a)$ and $(b,1)$. While the problems of one crack, that of two symmetrical cracks, and of an infinite row of cracks opened by the same (equal) pressure on the surface of each crack have been solved by Sneddon and Lowengrub [10], the problem of three symmetrical cracks, not all of them equal, does not seem to have been solved.

If we denote the displacements (u,v) in the xy-plane by the expressions [10],

$$u = -\frac{1}{2}\int_0^\infty (1-2\eta-ty)A(t)e^{-ty}\sin xt\; dt \tag{6.1a}$$

and

$$v = \frac{1}{2}\int_0^\infty (2-2\eta+ty)A(t)e^{-ty}\cos xt\; dt \tag{6.1b}$$

then the stresses $\sigma_{xx}(x,y)$, $\sigma_{yy}(x,y)$ and $\sigma_{xy}(x,y)$ are given by

$$\sigma_{xx} = -\frac{d}{dx}\int_0^\infty (1-yt)A(t)e^{-ty}\sin xt\, dt \tag{6.2a}$$

$$\sigma_{yy} = -\frac{d}{dx}\int_0^\infty (1+yt)A(t)e^{-ty}\sin xt\, dt \tag{6.2b}$$

$$\text{and}\quad \sigma_{xy} = -y\int_0^\infty t^2A(t)e^{-ty}\sin xt\, dt \tag{6.2c}$$

so that if $\sigma_{yy}(x,0) = -\alpha$ on $(-1,-b)$, $(-a,a)$, and $(b,1)$ and $v = 0$ on the rest of the x-axis, the problem in $A(t)$ is given by equations (2.1) with $f_1(x) = \alpha$ in $0 < x < a$, $f_3(x) = \alpha$ in $b < x < 1$, $g_2(x) = 0$ in $a < x < b$ and $g_4(x) = 0$ in $x > 1$. Because the problem is linear, we shall take $\alpha = 1$. $f_2(\xi)$, which is equal to $-\sigma_{yy}$ in $a < \xi < b$, in this case, is found to be

$$f_2(\xi) = 1 - \frac{\xi(a^2+b^2-2\xi^2)}{2\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}} + \frac{D\xi}{\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}}, \quad a<\xi<b \tag{6.3}$$

where D must be found from

$$D\int_a^b \frac{\xi}{\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}}\ln\frac{\sqrt{1-x^2}+\sqrt{1-\xi^2}}{\sqrt{|\xi^2-x^2|}}\,d\xi = -\int_0^1 \ln\frac{\sqrt{1-x^2}+\sqrt{1-\xi^2}}{\sqrt{|\xi^2-x^2|}}\,d\xi$$
$$+\int_a^b \frac{\xi(a^2+b^2-2\xi^2)}{2\sqrt{\xi^2-a^2}\sqrt{b^2-\xi^2}\sqrt{1-\xi^2}}\ln\frac{\sqrt{1-x^2}+\sqrt{1-\xi^2}}{\sqrt{|\xi^2-x^2|}}\,d\xi \tag{6.4}$$

It is easy to double-check (by differentiating (6.4), for example) that D as given by (6.4) is independent of x in $a < x < b$.

Since $f_2(\xi)$ is $-\sigma_{yy}(\xi,0)$ in $a < \xi < b$, the stress intensity factors k_1 and k_2 at the points $(a,0)$ and $(b,0)$ are given by

$$k_1 = -\lim_{x\to a^+}\sqrt{2(x-a)}\; f_2(x) \tag{6.5a}$$

and

$$k_2 = -\lim_{x\to b^-}\sqrt{2(b-x)}\; f_2(x). \tag{6.5b}$$

Also the stress intensity factor k_3 at $(1,0)$ is easily found to be $\frac{2}{\pi}F_1(1)$, where after a substitution, $F_1(1)$ is given by

$$F_1(1) = \frac{\pi}{2} - \frac{b^2-a^2}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 2\theta}{1-a^2\cos^2\theta-b^2\sin^2\theta}\,d\theta + D\int_0^{\frac{\pi}{2}}\frac{d\theta}{1-a^2\cos^2\theta-b^2\sin^2\theta} \tag{6.6}$$

NUMERICAL RESULTS.

Values of stress concentration factors k_1, k_2, and k_3 for various values of a and b are given in Table 1. It is to be seen that for a given value of b, i.e. for a given position of the crack $b < x < 1$, as a increases, the concentration factor k_1 which is less than k_2 in the beginning overtakes k_2. This happens for $b \geq \cdot 3$. If $b << \cdot 3$, i.e. if the outlying crack in $b < x < 1$ is quite large, then the stress concentration factor k_2 in it can be quite large indeed. The stress concentration factor k_3 at $(1,0)$ in the crack $b < x < 1$ is relatively less affected by the presence of the crack in $0 < x < a$. The stress concentration factor for two equal cracks ($a = 0$) and for three equal cracks ($2a = 1 - b$) are given in Tables 2 and 3 respectively. If $2l$ is the length of a crack, then in the case of a single crack, the stress concentration factor in it is proportional to $\sqrt{l}$. To account for this effect, the factors in Table 2 and Table 3 have been divided by $\sqrt{l}$ where $2l$ is the length of the crack. It is to be seen from Table 2 that, as $r = \frac{2b}{(1-b)}$, which is the distance between the cracks divided by the length of the crack, gets larger, the effect on stress concentration factors, due to the presence of other cracks, decreases. At $r = 2$, this effect is less than 2% while at $r = \frac{6}{7}$, the effect on k_2 is about 6%, and at $r = \frac{1}{9.5}$, the effect on k_2 is about 47%. The effect on k_3 is relatively smaller. $k_1 = 0$ in this case. These observations are in accordance with the remarks made by Sneddon and Lowengrub [10, p. 44]. In the case of three equal cracks as $r = (3b-1)/(2-2b)$ = distance between cracks/length of each crack, gets smaller, the effect on the stress concentration factors is more pronounced. At $r = \frac{1}{6}$, k_1 is about 45% higher than what it would be without the two neighbouring cracks, k_2 is about 41% higher and k_3 is only about 14% higher. In the case of three equal cracks, $k_1 > k_2$ in every case, so that the middle crack has a higher stress concentration factor than its neighbour on either side. In each case, the effect on k_3, the stress concentration factor on the outer edge of the outlying crack, is relatively small.

Table 4 gives the values of $k/\sqrt{\ell}$, where k is the largest stress-concentration factor, for 2, 3 and an infinite number of equal length cracks for various values of r, the ratio of the distance between cracks to the length of each crack. It is to be noted that $k \to 1$ as $r \to \infty$ and $k \to \infty$ as $r \to 0$, so that the relationship $k = k(r)$ is hyperbolic in nature. On a log-log graph, however, this relationship becomes extremely close to linear (see Fig. 1) and we get the simple relationship

$$k/\sqrt{\ell} \simeq \alpha r^{-\beta}$$

where

$$\alpha = .6070,\ \beta = .3591 \quad \text{for the case of 2 cracks;}$$

$$\alpha = .6515,\ \beta = .3776 \quad \text{for the case of 3 cracks;}$$

$$\text{and} \quad \alpha = .6939,\ \beta = .4885 \quad \text{for an infinite number of cracks.}$$

These formulae give results very close to the computed values for $0 < r < .1$. For large values of r, $k/\sqrt{\ell} \to 1$.

b \ a/b	0	.01	.05	.1	.3	.5	.7	.9	.95	.99
	0	.1670	.3737	.5301	.9510	1.3311	1.8516	3.1905	4.3624	8.8717
.01	1.6685	1.6686	1.6703	1.6756	1.7355	1.8816	2.2124	3.3626	4.4754	8.9163
	.8331	.8331	.8331	.8332	.8344	.8370	.8420	.8534	.8602	.8742
	0	.1025	.2294	.3254	.5820	.8094	1.1125	1.8677	2.5162	4.9704
.05	1.0151	1.0152	1.0161	1.0191	1.0532	1.1360	1.3226	1.9643	2.5784	4.9940
	.7718	.7718	.7718	.7720	.7742	.7790	.7880	.8081	.8197	.8423
	0	.0869	.1946	.2759	.4922	.6805	.9260	1.5234	2.0307	3.9354
.1	.8417	.8420	.8428	.8452	.8724	.9386	1.0870	1.5935	2.0746	3.9511
	.7287	.7287	.7288	.7291	.7321	.7389	.7516	.7785	.7937	.8228
	0	.0762	.1704	.2415	.4265	.5771	.7561	1.1575	1.4917	2.7406
.3	.6264	.6264	.6269	.6287	.6487	.6968	.8032	1.1564	1.4840	2.7329
	.6109	.6109	.6111	.6116	.6175	.6306	.6544	.7040	.7304	.7779
	0	.0794	.1776	.2515	.4402	.5837	.7360	1.0410	1.2917	2.2460
.5	.5088	.5088	.5093	.5108	.5281	.5694	.6600	.9535	1.2191	2.2072
	.5062	.5062	.5065	.5073	.5157	.5343	.5687	.6407	.6783	.7433
	0	.0863	.1931	.2732	.4750	.6199	.7550	.9750	1.1485	1.8439
.7	.3890	.3890	.3894	.3907	.4051	.4397	.5158	.7601	.9762	1.7551
	.3887	.3887	.3890	.3899	.3998	.4222	.4657	.5626	.6147	.7032
	0	.0951	.2127	.3008	.5213	.6739	.8008	.9367	1.0132	1.3610
.9	.2237	.2237	.2240	.2248	.2339	.2563	.3065	.4745	.6252	1.1484
	.2237	.2237	.2239	.2246	.2328	.2523	.2939	.4087	.4834	.6216
	0	.0973	.2181	.3084	.5343	.6900	8174	.9364	.9833	1.1894
.95	.1581	.1581	.1583	.1589	.1656	.1819	.2189	.3469	.4655	.8825
	.1581	.1581	.1583	.1589	.1652	.1804	.2141	.3179	.3961	.5619
	0	.0995	.2225	.3147	.5450	.7036	.8325	.9445	.9720	1.0208
.99	.0707	.0707	.0708	.0711	.0741	.0816	.0788	.1606	.2217	.4586
	.0707	.0707	. 0708	.0711	.0741	.0814	.0983	.1573	2125	.3864

Table 1

Values of k_1, k_2 and k_3 at $(a,0)$, $(b,0)$ and $(1,0)$ respectively for three cracks situated along $(-1,-b)$, $(-a,a)$ and $(b,1)$. k_1, k_2 and k_3 are listed in the first, second and third row respectively.

b	.01	.05	.1	.3	.5	.7	.9	.95	.99
$k_2/\sqrt{\ell}$	2.3716	1.4729	1.2551	1.0587	1.0176	1.0043	1.0004	1.0001	1.0000
$k_3/\sqrt{\ell}$	1.1841	1.1200	1.0863	1.0326	1.0125	1.0036	1.0003	1.0001	1.0000

Table 2

Value of $k_2/\sqrt{\ell}$ and $k_3/\sqrt{\ell}$ for two equal cracks $2\ell = 1 - b$ is the length of each crack and $s = 2b$ is the distance between the cracks. $k_1 = 0$ in this case.

b	$k_1/\sqrt{\ell}$	$k_2/\sqrt{\ell}$	$k_3/\sqrt{\ell}$
.99	1.00003	1.00002	1.00001
.95	1.0007	1.0004	1.0004
.9	1.0028	1.0018	1.0017
.7	1.0343	1.0249	1.0179
.5	1.1675	1.1388	1.0686
.4	1.4481	1.4070	1.1371
.34	3.0435	3.0126	1.2862
.334	7.3369	7.3232	1.3852

Table 3

Values of $k_1/\sqrt{\ell}$, $k_2/\sqrt{\ell}$ and $k_3/\sqrt{\ell}$ for three equal cracks. $2\ell = 1 - b$ is the length of each crack. $s = (3b - 1)/2$ is the distance between the cracks. The ratio of the length of each crack divided by the distance between the cracks goes up from .0101 to 666 as we go down the table.

r \ no. of cracks	2	3	∞
5.0	1.0038	1.0071	1.0117
1.0	1.0480	1.0766	1.1284
.5	1.1125	1.1675	1.2861
.1	1.4914	1.6539	2.2069
.05	1.7950	2.0323	2.9866
.01	3.0044	3.5387	6.4296
.005	3.8419	4.5884	9.0481
.001	7.0404	8.6332	20.1518
.0005	9.2417	11.4377	28.4847
.0001	17.7159	22.3166	63.6683

Table 4

Values of $k/\sqrt{\ell}$ for two, three and an infinite number of cracks for various values of r.

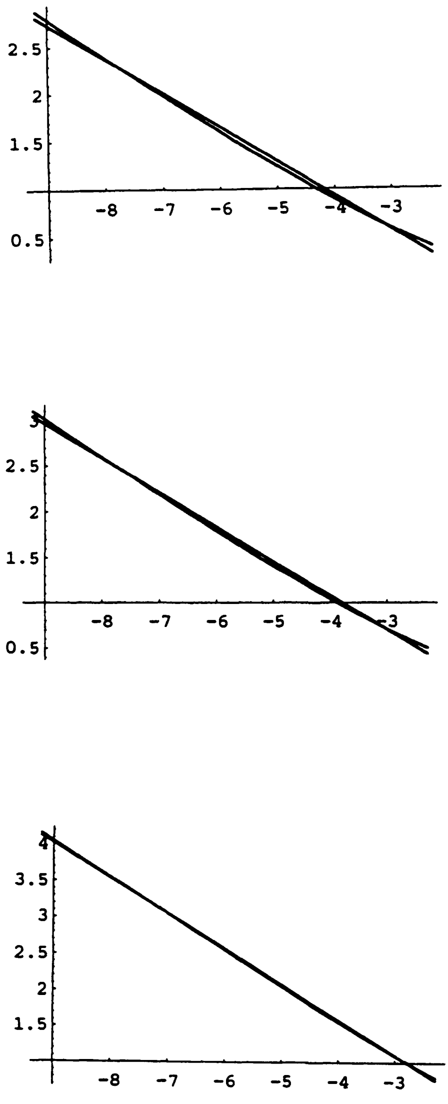

Figure 1. Linear regression of F in the equation $\ln(k/\sqrt{\ell}) = F(\ln r)$ for the case of two cracks (top), 3 cracks (middle), and an infinite number of cracks (bottom), for $r \leq .1$. k is the largest stress concentration factor.

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