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Research Article

Van der Waals-Like Phase Transition from Holographic Entanglement Entropy in Lorentz Breaking Massive Gravity

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Phase transition of AdS black holes in Lorentz breaking massive gravity has been studied in the framework of holography. We find that there is a first-order phase transition (FPT) and second-order phase transition (SPT) both in Bekenstein-Hawking entropy- (BHE-) temperature plane and in holographic entanglement entropy- (HEE-) temperature plane. Furthermore, for the FPT, the equal area law is checked and for the SPT, the critical exponent of the heat capacity is also computed. Our results confirm that the phase structure of HEE is similar to that of BHE in Lorentz breaking massive gravity, which implies that HEE and BHE have some potential underlying relationship.

1. Introduction

The study of HEE and quantum phase transitions of black holes has attracted a lot of interest in recent years. On one hand, HEE can be used as a perfect probe to study quantum information science [1–3], strongly correlated quantum systems [4–13], and Many-Body Systems [14, 15]. On the other hand, investigation on HEE of black holes may shed some light on understanding the nature of BHE [16, 17].

Nearly ten years ago, a holographic derivation of the HEE in conformal quantum field theories was proposed by Ryu and Takayanagi using the famous AdS/CFT correspondence [18, 19]. Recently the HEE has been used as a probe to investigate the phase structure of the Reissner-Nordstrom AdS black hole [20]. The results showed that there is a Van der Waals-like (VDW) phase transition at the same critical temperature in both the fixed charge ensemble and chemical potential ensemble in the HEE-temperature plane. They also found that the SPT occurs for the HEE at the same critical point as the BHE with nearly the same critical exponent. This work was soon generalized to the extended phase space where the cosmological constant is considered as a thermodynamical variable [21]. Very recently, the equal area law of HEE was proved to hold for the FPT in the HEE-temperature plane [22]. Based on [20], VDW phase transition of HEE in various AdS black holes has been studied in [23–32]. All of these works showed that the HEE undergoes the same VDW phase transition as that of the BHE.

Massive gravity theories have attracted considerable interest recently. One of these reasons is that these alternative theories of gravity could explain the accelerated expansion of the universe without dark energy. The graviton behaves like a lattice excitation and exhibits a Drude peak in this theory. Current experimental data from the observation of gravitational waves by advanced LIGO require the graviton mass to be smaller than the inverse period of orbital motion of the binary system; that is, $m_g < 1.2 \times 10^{-22}\,\mathrm{eV}/c^2$ [33]. Another important reason for the interest in massive gravity is that the possibility of the mass graviton could help to understand the quantum gravity effect. The first to introduce a mass to the graviton is in [34]. However this primitive linear massive gravity theory contains the so-called Boulware-Deser ghosts problem [35] that was solved by a nonlinear massive gravity theory [36, 37], where the mass terms are obtained by introducing a reference metric. Recently Vegh proposed a new reference metric to describe a class of strongly interacting quantum field theories with broken translational symmetry in the holographic framework [38]. The recent progress in massive gravity can be found in [39, 40].

Here, we consider AdS black holes in Lorentz breaking massive gravity. In the massive gravity, the graviton acquires a mass by Lorentz symmetry breaking, which is very similar to the Higgs mechanism. A review of Lorentz-violating massive gravity theory can be found in [41, 42]. In this paper, we focus on the study of the VDW phase transition of AdS black holes in Lorentz breaking massive gravity using the HEE. The main motivation of this paper is to explore whether the BHE phase transition can also be described by HEE in Lorentz breaking massive gravity. Firstly, we would like to extend proposals in [20] to study VDW phase transitions in AdS black hole with a spherical horizon in Lorentz-violating massive gravity with the HEE as a probe. What is more, we also would like to check Maxwell's equal area law and critical exponent of the heat capacity, which are two characteristic quantities in VDW phase transition.

The organization of this paper is as follows. In the next section, we shall provide a brief review of the black hole solution in Lorentz breaking massive gravity firstly. Then we will study the VDW phase transitions and critical phenomena for the AdS black hole in the BHE-temperature plane. In Section 3, we mainly concentrate on the VDW phase transition and critical phenomena in the framework of HEE. The last section is devoted to our discussions and conclusions.

2. Phase Transition and Critical Phenomena of AdS Black Holes in Lorentz Breaking Massive Gravity

2.1. Review of AdS Black Holes in Lorentz Breaking Massive Gravity. The four-dimensional Lorentz breaking massive gravity can be obtained by adding nonderivative coupling scalar fields to the standard Einstein gravity theory. As a matter field is considered, the theory can be described by the following action [41, 42]:

$$S = \int d^4x\sqrt{-g}\left[-M_{\mathrm{Pl}}^2 R + L_m + \ell^4\Psi\left(X,\Pi^{ij}\right)\right]; \tag{1}$$

here the first two terms are the curvature and ordinary matter minimally coupled to gravity, respectively, and the third term Ψ contains two functions X and Π^{ij} which relate to the four scalar fields, Ξ^0 and Ξ^i as

$$\begin{aligned} X &= \frac{\partial^\mu\Xi^0\partial_\mu\Xi^0}{\ell^4}, \\ \Pi^{ij} &= \frac{\partial^\mu\Xi^i\partial_\mu\Xi^j}{\ell^4} - \frac{\partial^\mu\Xi^i\partial_\mu\Xi^0\partial^\nu\Xi^j\partial_\nu\Xi^0}{\ell^8 X}. \end{aligned} \tag{2}$$

When the four scalar fields get a space-time depending vacuum expectation value, the system will break the Lorentz symmetry. What is more, the action can also be taken as a low-energy effective theory with the ultraviolet cutoff scale ℓ. Here the scale parameter ℓ has the dimension of mass and is in the order of $\sqrt{m_g M_{\mathrm{Pl}}}$, where m_g and M_{Pl} are the graviton mass and the Plank mass, respectively.

The AdS black hole solutions can be obtained from the above theory [43, 44]. The metric corresponding to the AdS black holes is given by

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\left(d\varphi^2 + \sin^2\varphi d\phi^2\right), \tag{3}$$

with

$$f(r) = 1 - \frac{2M}{r} - \gamma\frac{Q^2}{r^\lambda} - \frac{\Lambda r^2}{3}. \tag{4}$$

Here, the four scalar fields, Ξ^0 and Ξ^i, for this particular solution are given by

$$\begin{aligned} \Xi^0 &= \ell^2\left(t + \eta(r)\right), \\ \Xi^i &= \ell^2\alpha x^i, \end{aligned} \tag{5}$$

in which

$$\eta(r) = \pm\int\frac{dr}{f(r)} \cdot \left[1 - f(r)\left(\frac{\gamma Q^2\lambda(\lambda-1)}{12m_g^2\alpha^6}\frac{1}{r^{\lambda+1}} + 1\right)^{-1}\right]^{1/2}, \tag{6}$$

in which the scalar charge Q is related to massive gravity and the constant α which is determined by the cosmological constant Λ and the graviton mass m_g with the relation $\Lambda = 2m_g^2(1-\alpha^6)$. In this paper, we will set $\alpha > 1$ such that $\Lambda < 0$ leading to Anti-de Sitter black holes. The constant λ is a positive integration constant. When $\lambda < 1$, the ADM mass of the black hole solution diverges. For $\lambda > 1$, the metric approaches the Schwarzschild-AdS black holes with a finite mass M as $r \to \infty$. Thus we set $\lambda > 1$ in this paper. The constant $\gamma = \pm 1$. When $\gamma = 1$, the black hole only has a single horizon r_h, which is the root of the equation $f(r_h) = 0$. The function $f(r)$ for this case is given in Figure 1, which is similar to the Schwarzschild-AdS black hole. For $\gamma = -1$, the black hole is very similar to the Reissner-Nordstrom-AdS black hole. The function $f(r)$ for this case is given in Figure 2. The black hole event horizon r_h is the largest root of the equation $f(r_h) = 0$.

At the event horizon, the Hawking temperature and BHE can be written as

$$T = \frac{1}{4\pi}f'(r_h) = \frac{1}{4\pi}\left(\frac{1}{r_h} - r_h\Lambda + \frac{\gamma(\lambda-1)Q^2}{r_h^{\lambda+1}}\right), \tag{7}$$

$$S = \pi r_h^2. \tag{8}$$

The chemical potential in this black hole is

$$\Phi = -\gamma\frac{Q}{r_h^{\lambda-1}}. \tag{9}$$

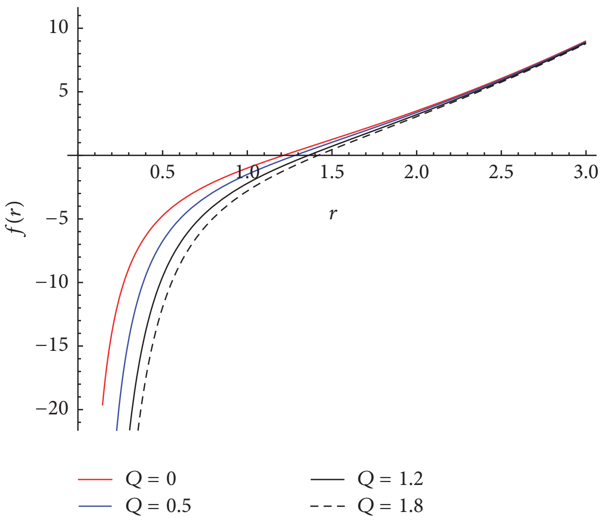

Figure 1: The figure shows $f(r)$ versus r for $\gamma = 1$ for varying Q. Here $\lambda = 2.4$, $\Lambda = -3$, and $M = 1.5$.

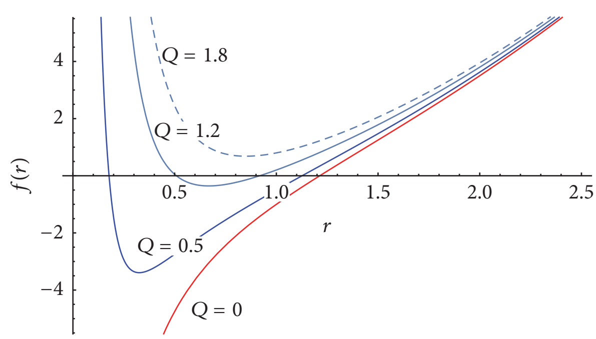

Figure 2: The figure shows $f(r)$ versus r for $\gamma = -1$ for varying Q. Here $\lambda = 2.4$, $\Lambda = -3$, and $M = 1.5$.

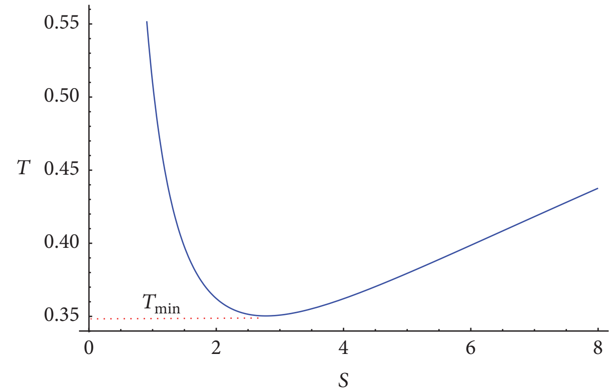

Figure 3: The figure shows T versus S for $\gamma = 1$. Here $\lambda = 2.4$, $\Lambda = -3$, and $Q = 0.3$.

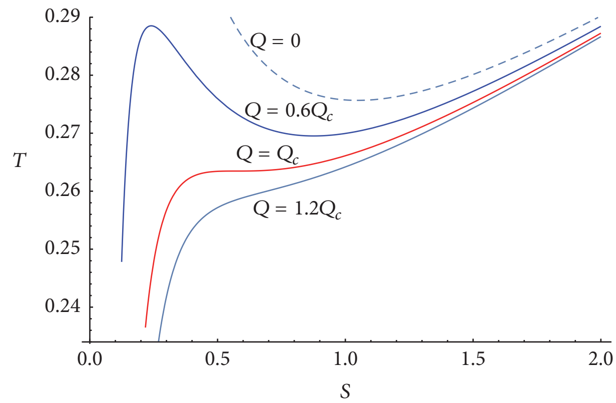

Figure 4: The figure shows T versus S for $\gamma = -1$ for varying Q. Here $\lambda = 2.4$, $\Lambda = -3$. The top dashed curve is at $Q = 0$ and the rest have $Q = 0.6Q_c$, Q_c, $1.2Q_c$, where $Q_c = 0.12346$.

We can check the first law of the black hole, which is given by

$$dM = TdS + \Phi dQ. \tag{10}$$

There have been some works to study the thermodynamics and phase transitions of black holes in Lorentz breaking massive gravity [45–48].

2.2. Van der Waals-Like Phase Transition of Bekenstein-Hawking Entropy. In this subsection, we focus on the VDW phase transition of BHE. Substituting (8) into (7) and eliminating the parameter r_h, one can get the relation between the Hawking temperature T and BHE S of the AdS black holes in massive gravity as

$$T = \frac{S^{(1/2)(-\lambda-1)}\left(\gamma\pi^{\lambda/2+1}(\lambda-1)Q - \Lambda S^{\lambda/2+1} + \pi S^{\lambda/2}\right)}{4\pi^{3/2}}. \tag{11}$$

This is the state equation of the AdS black hole thermodynamics system in massive gravity. Using (11), we investigate the phase diagram of the AdS black holes in massive gravity.

The temperature T is plotted as a function of the BHE S in Figures 3 and 4 for $\gamma = 1$ and $\gamma = -1$, respectively. In Figure 3, the temperature is plotted for $\gamma = 1$ where only one event horizon exists. This behavior of temperature is very similar to the behavior in the Schwarzschild-AdS black hole. That is to say, there is a minimum temperature $T_{\min}$ which divides the thermodynamics systems into small and large black holes. It is shown that, above the minimum temperature, small and large black holes coexist. In fact, this behavior will break for there is a first-order transition, which is similar to the Hawking-Page thermodynamic transition in [49].

In Figure 4, the temperature is plotted for $\gamma = -1$ where event horizon and inner horizon exist. Various values of Q are used to plot the relations between the temperature and horizons. The top curve corresponds to $Q = 0$. The system of this case is similar to the case $\gamma = 1$ described above. When the scalar charge Q increases, the temperature has two turning points. Further increasing of the scalar charge Q makes these two turning points merge to one. It is shown that there exists a critical point Q_c. Above the critical point, the curve does not have any turning points. Thus we find that the phase structure is very similar to that of the Van der Waals gas-fluid phase transition. It should be noted that we

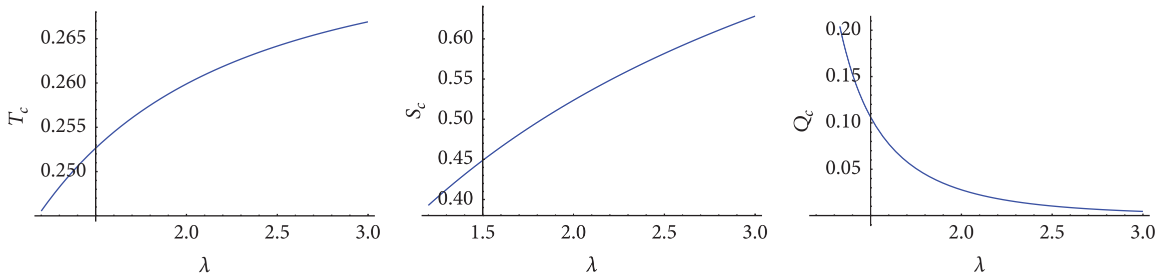

FIGURE 5: The figure shows the critical temperature T_c, the critical entropy S_c, and the critical charge Q_c versus λ for $\gamma = -1$. Here $\Lambda = -3$.

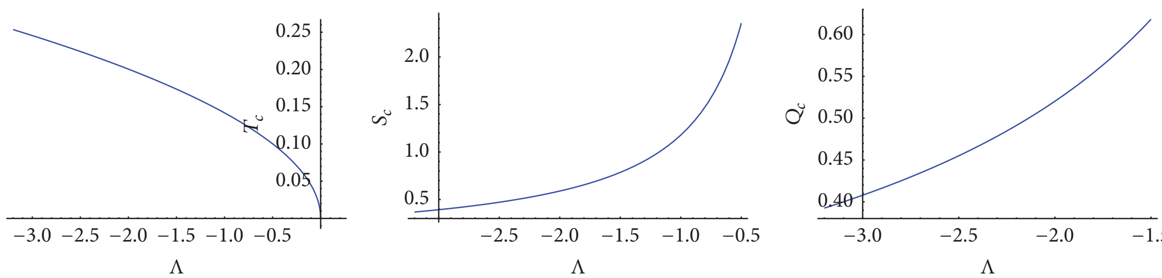

FIGURE 6: The figure shows the critical temperature T_c, the critical entropy S_c, and the critical charge Q_c versus Λ for $\gamma = -1$. Here $\lambda = 1.2$.

mainly concentrate on this type of phase structure in this paper. Furthermore using the definition of the specific heat capacity C_Q, that is,

$$C_Q = T\left(\frac{\partial S}{\partial T}\right)_Q, \tag{12}$$

one can see that the specific heat capacity is divergent at the critical point and it is obvious that this phase transition is a SPT. At this critical point, the critical charge Q_c and critical entropy S_c can be obtained by the following equations:

$$\left(\frac{\partial T}{\partial S}\right)_{Q_c,S_c} = \left(\frac{\partial^2 T}{\partial S^2}\right)_{Q_c,S_c} = 0. \tag{13}$$

After some calculation and using (11), Q_c, S_c, and the corresponding T_c can be also got as

$$Q_c = -\frac{2\left(-\lambda/\left(\lambda+2\right)\Lambda\right)^{\lambda/2}}{\gamma\left(\lambda+2\right)\left(\lambda^2-1\right)}, \tag{14}$$

$$S_c = -\frac{\pi\lambda}{\left(\lambda+2\right)\Lambda}, \tag{15}$$

$$T_c = \frac{\lambda}{2\pi\left(\lambda+1\right)\sqrt{-\lambda/\left(\lambda+2\right)\Lambda}}. \tag{16}$$

Obviously, these critical parameters depend only on the internal parameters of the systems λ and Λ. One can also see that T_c and S_c increase with λ and Q_c decreases with λ as shown in Figure 5; S_c and Q_c increase with Λ and T_c decreases with λ as shown in Figure 6. So the results show that the parameter λ promotes the thermodynamic system to reach the stable state.

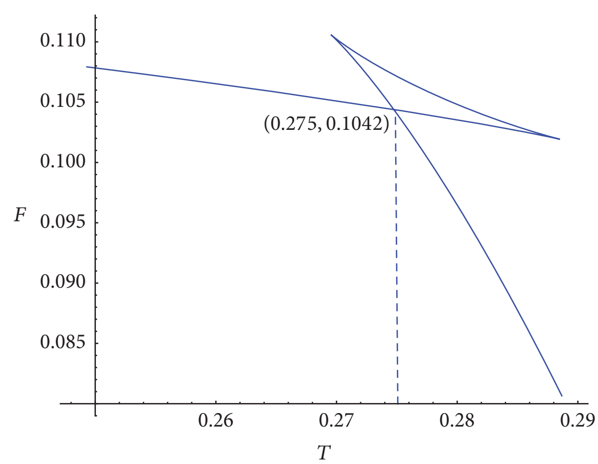

FIGURE 7: The figure shows F versus T for $\gamma = -1$. Here $\lambda = 2.4$, $\Lambda = -3$, and $Q = 0.6Q_c$.

For the FPT, we will also check whether Maxwell's equal area law holds in this thermodynamic system. As is known to all, the first-order transition temperature T^* plays a crucial role for Maxwell's equal area law. Thus in order to get T^*, we first plot the curve about the free energy with respect to the temperature T, where the free energy is defined by $F = M - TS$. The relation between F with T is plotted in Figure 7. One can see that there is a swallowtail structure, which corresponds to the unstable phase in Figure 8. The nonsmoothness of the junction implies that the phase transition is a FPT. The critical temperature T^* is apparently given by the horizontal coordinate of the junction. From Figure 7, we get $T^* = 0.2750$. Substituting this temperature T^* into (11), one can obtain three values of the entropy, $S_1 = 0.1563$,

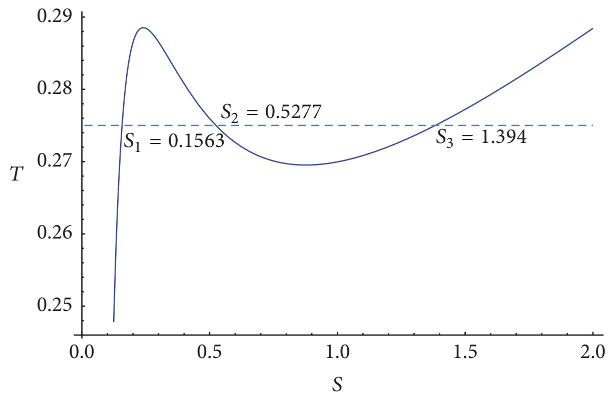

Figure 8: The figure shows T versus S for $\gamma = -1$. Here $\lambda = 2.4$, $\Lambda = -3$, and $Q = 0.6Q_c$.

$S_2 = 0.5277$, and $S_3 = 1.394$. With these values, we can now check Maxwell's equal area law

$$T^* (S_3 - S_1) = \int_{S_1}^{S_3} T(S, Q)\, dS. \quad (17)$$

After some calculation, we find that both the left and right sides of (17) are equal to 0.3404 exactly. Thus, our results show that Maxwell's equal area law is satisfied in this background.

For the SPT, we will study the critical exponent associated with the heat capacity definition in (12). Near the critical point, expanding the temperature as the very small amount $S - S_c$, we find

$$T = T_c + \left(\frac{\partial T}{\partial S}\right)_{Q_c, S_c} (S - S_c) + \left(\frac{\partial^2 T}{\partial S^2}\right)_{Q_c, S_c} (S - S_c)^2 + \left(\frac{\partial^3 T}{\partial S^3}\right)_{Q_c, S_c} (S - S_c)^3 + o(S - S_c)^4. \quad (18)$$

Using (13), the second and third terms vanish. Then using (11), (14), and (15), we get

$$T - T_c = \frac{\lambda}{16\pi^4 (-\lambda/(\lambda + 2)\Lambda)^{7/2}} (S - S_c)^3. \quad (19)$$

With the definition of the heat capacity (12), we further get $C_Q \sim (T - T_c)^{-2/3}$. So one can find that the critical exponent of the heat capacity is $-2/3$, which is the same as the one from the mean field theory in Van der Waals gas-fluid system.

3. Van der Waals-Like Phase Transition and Critical Phenomena of Holographic Entanglement Entropy

In this section, our target is to explore whether the HEE has the similar VDW phase structure and critical phenomena as those of the BHE in massive gravity. For simplicity, here we only consider the case $\gamma = -1$. Now we will investigate whether there is VDW phase transition in the HEE-temperature phase plane.

Firstly, we review some basic knowledge about HEE. For detailed introduction of HEE, one can refer to [18, 19]. For a given quantum field theory described by a density matrix ρ, HEE for a region A and its complement B are

$$S_A = -Tr_A (\rho_A \ln \rho_A), \quad (20)$$

where ρ_A is the reduced density matrix. However, it is usually not easy to get this quantity in quantum field theory. Fortunately according to AdS/CFT correspondence, [18, 19] propose a very simple geometric formula for calculating S_A for static states with the area of a bulk minimal surface as ∂A; that is,

$$S_A = \frac{\text{Area}(\gamma_A)}{4}, \quad (21)$$

where γ_A is the codimension-2 minimal surface according to boundary condition $\partial\gamma_A = \partial A$.

Subsequently using definition (21), we will calculate the HEE and study the corresponding phase transition. It is noted that the space on the boundary is spherical in the AdS black hole in massive gravity and the volume of the space is finite. Thus in order to avoid the HEE to be affected by the surface that wraps the event horizon, we will choose a small region as A. More precisely, as done in [23, 25–27], we choose region A to be a spherical cap on the boundary given by $\varphi \le \varphi_0$. Here the area can be written as

$$A = 2\pi \int_0^{\varphi_0} \Theta(r(\varphi), \varphi)\, d\varphi, \qquad \Theta = r \sin\varphi \sqrt{\frac{(r')^2}{f(r)} + r^2}, \quad (22)$$

where $r' = dr/d\varphi$. Then according to the Euler-Lagrange equation, one can get the equation of motion of $r(\varphi)$; that is,

$$\begin{aligned} 0 &= r'(\varphi)^2 \left[\sin\varphi r(\varphi)^2 f'(r) - 2\cos\varphi r'(\varphi)\right] \\ &\quad - 2r(\varphi) f(r) \left[r(\varphi)\left(\sin\varphi r''(\varphi) + \cos\varphi r'(\varphi)\right) - 3\sin\varphi r'(\varphi)^2\right] + 4\sin\varphi r(\varphi)^3 f(r)^2. \end{aligned} \quad (23)$$

After using the boundary conditions $r'(0) = 0$, $r(0) = r_0$, we can get the numeric result of $r(\varphi)$.

It is worth noting that the HEE should be regularized by subtracting off the HEE in pure AdS, because the values of the HEE in (21) are divergent at the boundary. Now let us label the regularized HEE as δS. Here we choose the size of the boundary region to be $\varphi_0 = 0.10, 0.16$ and set the UV cutoff in the dual field theory to be $r(0.099)$ and $r(0.159)$, respectively. The numeric results are shown in Figures 9 and 10. One can see that, for a given scalar charge Q, the relation between the HEE and temperature is similar to that between the BHE and temperature. That is to say, the AdS black holes thermodynamic system with the HEE undergoes the FPT and SPT one after another as the scalar charge Q increases step by step.

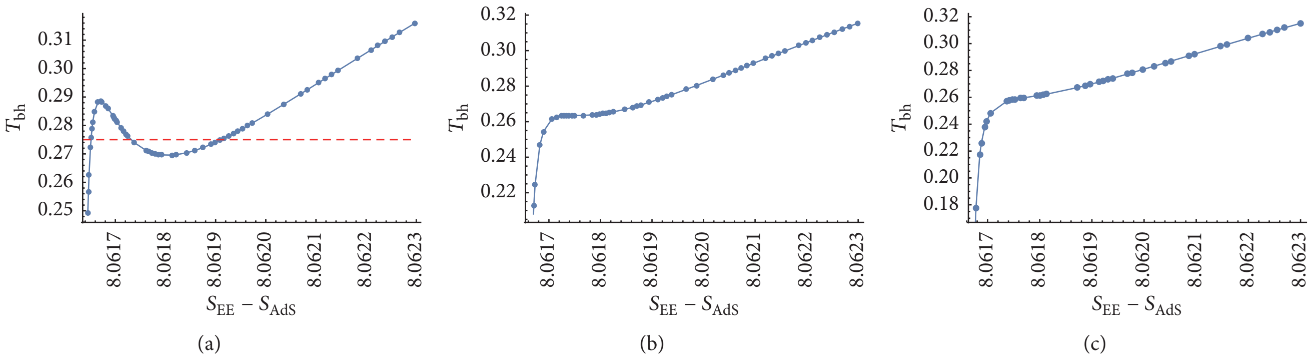

FIGURE 9: The figure shows T versus δS for $\gamma = -1$, $\varphi_0 = 0.10$. Here $\lambda = 2.4$, $Q = 0.6Q_c$, Q_c, $1.2Q_c$ from (a) to (c).

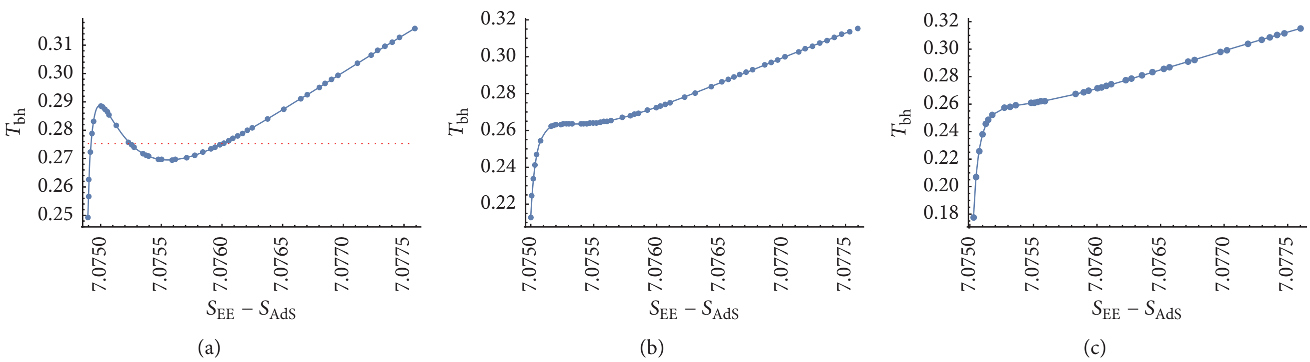

FIGURE 10: The figure shows T versus δS for $\gamma = -1$, $\varphi_0 = 0.16$. Here $\lambda = 2.4$, $Q = 0.6Q_c$, Q_c, $1.2Q_c$ from (a) to (c).

For the FPT of the HEE, we now check whether Maxwell's equal area law is satisfied. Firstly, we get the interpolating function of the temperature $T(\delta S)$ using the data obtained numerically. At the first-order phase transition point, we get the smallest and largest roots for the equation $T(\delta S) = T^*$, which are $\delta S_1 = 8.06165$, $\delta S_3 = 8.06191$ for $\varphi_0 = 0.10$ and $\delta S_1 = 7.07492$, $\delta S_3 = 7.07599$ for $\varphi_0 = 0.16$. Then using these values and the equal area law

$$T^* \left(\delta S_3 - \delta S_1\right) = \int_{\delta S_1}^{\delta S_3} T\left(\delta S\right) dS, \tag{24}$$

we find the left side equals 0.00007137, 0.00029425 and the right side equals 0.00007135, 0.00029446 for $\varphi_0 = 0.10, 0.16$, respectively. Obviously both the left and the right sides are equal within our numerical accuracy and the relative errors are less than 0.022%, 0.071 for $\varphi_0 = 0.10, 0.16$, respectively.

Now let us consider the critical exponent of the SPT in the HEE-temperature phase plane. Here comparing with the definition of specific heat capacity C_Q in (12), one can also define a specific heat capacity for the HEE as

$$C'_Q = T\left(\frac{\partial \delta S}{\partial T}\right)_Q. \tag{25}$$

Then providing a similar relation of the critical points that in (13) is also working and using (25), we can get the critical exponent of SPT of in the HEE-temperature phase. Here we employ the logarithm of the quantities $T - T_c$, $\delta S - \delta S_c$. The relation between $\log|T - T_c|$ and $\log|\delta S - \delta S_c|$ is plotted in Figure 11. The analytical results of these straight lines can also be fitted, which are for $\varphi_0 = 0.10$,

$$\log\left|T - T_c\right| = 23.469 + 3.00654 \log\left|\delta S - \delta S_c\right|, \tag{26}$$

and for $\varphi_0 = 0.16$,

$$\log\left|T - T_c\right| = 19.2089 + 3.00477 \log\left|\delta S - \delta S_c\right|. \tag{27}$$

The results show that the slopes are all around 3 and the relative errors are less than 0.218%, 0.159% for $\varphi_0 = 0.10$, 0.16, respectively, which are consistent with that of the BHE. Then one can find that the critical exponent of the specific heat capacity C'_Q is also approximately $-2/3$. That is to say, the HEE has the same SPT behavior as that of the BHE. Both of them are consistent with the result in the mean field theory of VDW gas-fluid system.

4. Conclusions

In this paper, we have investigated the VDW phase transition with the use of HEE as a probe. Firstly, we investigated the phase diagrams of the BHE in the T-S phase plane and found that the phase structure depends on the scalar charge Q and the parameter γ of the AdS black holes in this massive gravity. For the case that $\gamma = 1$ or $Q = 0$, we found that there always exists the Hawking-Page-like phase transition in this thermodynamic system, while for the case $\gamma = -1$, we found

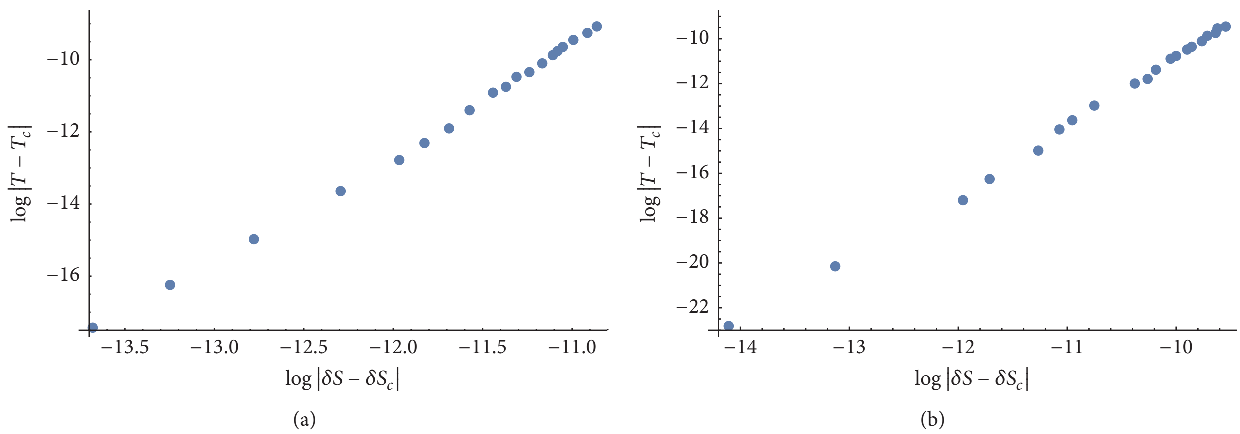

Figure 11: The figure shows $\log|T-T_c|$ versus $\log|\delta S-\delta S_c|$ for $\gamma=-1$. Here $\lambda=2.4$, $Q=Q_c$, where $\varphi_0=0.10$ in (a) and $\varphi_0=0.16$ in (b).

for the small scalar charge Q, there is always an unstable black hole thermodynamic system interpolating between the small stable black hole system and large stable black hole system. The thermodynamic transition for the small hole to the large hole is a first-order transition and Maxwell's equal area law is valid. As the scalar charge Q increases to the critical value Q_c in this space-time, the unstable black hole merges into an inflection point. We found there is a SPT at the critical point. When the scalar charge is larger than the critical charge, the black hole is stable always. That is to say, we found that there exists the VDW gas-fluid phase transition in the T-S phase plane of the AdS black hole in massive gravity.

The more interesting thing is that we found the HEE also exhibits the VDW phase structure in the T-δS plane when $\gamma=-1$ and the scalar charge $Q\neq 0$. In order to confirm this result, we further showed that Maxwell's equal area law is satisfied and the critical exponent of the specific heat capacity is consistent with that of the mean field theory of the VDW gas-fluid system for the HEE system. These results show that the phase structure of HEE is similar to that of BHE and the HEE is really a good probe to the phase transition of AdS black holes in Lorentz breaking massive gravity. This also implies that HEE and BHE exhibit some potential underlying relationship.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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