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Research Article

Pinning Adaptive Synchronization of Delayed Coupled Dynamical Networks via Periodically Intermittent Control

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This paper investigates the exponential synchronization problem of delayed coupled dynamical networks by using adaptive pinning periodically intermittent control. Based on the Lyapunov method, by designing adaptive feedback controller, some sufficient conditions are presented to ensure the exponential synchronization of coupled dynamical networks with delayed coupling. Furthermore, a numerical example is given to demonstrate the validity of the theoretical results.

1. Introduction

Complex networks have received a great deal of attention due to their many potential practical applications [1, 2]. A family of dynamically interacting units composes a kind of complex networks which can exhibit a number of emerging phenomena. Among various dynamical behaviors of complex networks, synchronization is a significant and interesting phenomenon, such as synchronization phenomena on the Internet, synchronization transfer of digital or analog signals in communication network, and synchronization related to biological neural networks. Recently, much works have been devoted to research the synchronization problem of complex networks [3–5].

In the case where the network cannot synchronize by itself, in order to drive the network to synchronize, many effective control techniques have been reported, such as feedback control [6], sampled-data control [7], adaptive control [8, 9], pinning control [10], impulsive control [11], and intermittent control [12]. In [9], the synchronization of a class of complex network by adding an adaptive controller to all nodes has been discussed. But in practice, it is too costly and impractical to add controllers to all nodes in a large-scale network. To reduce the number of controlled nodes, pinning control is introduced [10], in which controllers are only applied to partial nodes. This case of control techniques has been earlier reported in paper [11–14]. In addition, the adaptive pinning control method, which is utilized to get the appropriate control gains effectively, has received considerable research attention. An adaptive pinning control method is proposed in [15] to synchronize for a delayed complex dynamical network with free coupling matrix. Besides these, there are many literatures to study adaptive pinning control problems of networks [16–18].

One the other hand, intermittent control has been widely used in engineering fields due to its practical and easy implementation in engineering control. In recent years, many important and interesting results on stabilization and synchronization of delayed dynamical networks by using intermittent control have been obtained. Based on ∞-norm, authors in [19] investigated a class of Cohen-Grossberg neural networks with time-varying delays by designing a periodically intermittent controller. In [20], by using periodically intermittent control, Gan studied the stochastic neural networks with leakage delay and reaction-diffusion terms; some new and less conservative synchronization conditions based on p-norm were derived. The pinning periodically intermittent control is used to achieve the synchronization of delayed complex network [21, 22]. To the best of our knowledge, the problem of adaptive pinning synchronization for delayed coupled dynamical networks has received very little research attention.

In this paper, we aim to further investigate adaptive pinning synchronization of delayed coupled dynamical network via periodically intermittent control. By using Lyapunov stability theory and designing adaptive feedback control gains, several criteria are given to guarantee synchronization of delayed coupled dynamical networks. A numerical simulation is also presented to show the effectiveness of the proposed method.

2. Model and Preliminaries

Consider the complex network consisting of N nodes and the ith node described by the following state equation:

$$\dot{x}_i(t) = Cx_i(t) + Af(x_i(t)) + Bg(x_i(t-\tau_1)) + \sum_{j=1}^{N} g_{ij}\Gamma x_j(t) + \sum_{j=1}^{N} \hat{g}_{ij}\Gamma_\tau x_j(t-\tau_2) + u_i, \quad (1)$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in R^n$ is the state variable of node i at time t; $C = (c_{ij})_{n\times n}$, $A = (a_{ij})_{n\times n}$, and $B = (b_{ij})_{n\times n}$ are system matrices, $f(x_i) = [f_1(x_{i1}), f_2(x_{i2}), \ldots, f_n(x_{in})]^T$ and $g(x_i) = [g_1(x_{i1}), g_2(x_{i2}), \ldots, g_n(x_{in})]^T$ are continuous vector functions, and τ_1 is the internal delay. $\Gamma = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ and $\Gamma_\tau = \mathrm{diag}(d_1^\tau, d_2^\tau, \ldots, d_n^\tau)$ are inner coupling matrices between the connected nodes i and j at time t and $t-\tau_2$, where τ_2 is the transmittal delay. $G = (g_{ij})_{N\times N}$ and $\hat{G} = (\hat{g}_{ij})_{N\times N}$ are the configuration matrices; if there is a link from node i to node j at time t (at time $t-\tau_2$), then $g_{ij} > 0$ ($\hat{g}_{ij} > 0$), where $j \neq i$. Otherwise, $g_{ij} = 0$ ($\hat{g}_{ij} = 0$). It is assumed that G and $\hat{G}$ satisfy the diffusive coupling connection, $\sum_{j=1}^{N} g_{ij} = 0$ and $\sum_{j=1}^{N} \hat{g}_{ij} = 0$. $u_i \in R^n$ are the control inputs. Note that the coupling configuration matrix G and matrices A, B, C, Γ and Γ_τ are not assumed to be symmetric.

The initial conditions of (1) are given by $x_i(t) = x_i^0(t)$, $t \in [t_0 - \tau, t_0]$, and $x_i^0(t) \in C([t_0-\tau, t_0], R^n)$, where $\tau = \max(\tau_1, \tau_2)$. To discuss global synchronization with one delay coupling, we define the set

$$S = \left\{ \left[x_1^T, x_2^T, \ldots, x_N^T\right]^T \mid x_i = x_j,\ i, j = 1, 2, \ldots, N \right\} \quad (2)$$

as the synchronization manifold for network (1). For all $s(t) \in S$, the dynamical equation of $s(t)$ satisfies

$$\dot{s}(t) = Cs(t) + Af(s(t)) + Bg(s(t-\tau_1)). \quad (3)$$

Define error states as $e_i(t) = x_i(t) - s(t)$ $(1 \le i \le N)$. Then, we can derive the following error dynamical system:

$$\begin{aligned}\dot{e}_i(t) &= Ce_i(t) + A(f(x_i(t)) - f(s(t))) \\ &\quad + B(g(x_i(t-\tau_1)) - g(s(t-\tau_1))) \\ &\quad + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) + \sum_{j=1}^{N} g_{ij}\Gamma_\tau e_j(t-\tau_2) + u_i, \\ &\qquad i = 1, 2, \ldots, N.\end{aligned} \quad (4)$$

For convenience of statements, one has following assumption.

H_1: There exists positive constants l_i, l_i', $i = 1, 2, \ldots, n$, for any $x, y \in R$, such that

$$\begin{aligned}|f_i(x) - f_i(y)| &< l_i |x-y|, \quad i = 1, 2, \ldots, n; \\ |g_i(x) - g_i(y)| &< l_i' |x-y|, \quad i = 1, 2, \ldots, n.\end{aligned} \quad (5)$$

In order to derive the main results, the following definitions and lemmas are needed in this paper.

Lemma 1 (see [20]). *Let $w : [\mu - \tau, +\infty) \to [0, +\infty)$ be a continuous function such that*

$$\dot{w}(t) \le -aw(t) + b \max w_t \quad (6)$$

is satisfied for $t \ge \mu$. If $a > b > 0$; then

$$w(t) \le \left[\max w_\mu\right] e^{-\varepsilon(t-\mu)}, \quad t \ge \mu, \quad (7)$$

where $\max w_t = \sup_{t-\tau\le s\le t} w(s)$ and $\varepsilon > 0$ is the smallest real root of the equation

$$\varepsilon - a + be^{\varepsilon\tau} = 0. \quad (8)$$

Lemma 2 (see [20]). *Let $w : [\mu - \tau, +\infty) \to [0, +\infty)$ be a continuous function such that*

$$\dot{w}(t) \le aw(t) + b \max w_t \quad (9)$$

is satisfied for $t \ge \mu$. If $a > 0$, $b > 0$, then

$$w(t) \le \max w_t \le \left[\max w_\mu\right] e^{(a+b)(t-\mu)}, \quad t \ge \mu, \quad (10)$$

where $\max w_t = \sup_{t-\tau\le s\le t} w(s)$.

Lemma 3 (see [6]). *Let $x, y \in R^n$. Then*

$$2x^T y \le \epsilon x^T x + \epsilon^{-1} y^T y, \quad (11)$$

for any $\epsilon > 0$.

3. Main Result

In order to realize synchronization of the couple network by pinning periodically intermittent control, some controllers are added to selected partial nodes, and the controllers u_i $(1 \le i \le N)$ can be described by

$$u_i = \begin{cases} -\bar{k}_i(t)\Gamma e_i(t), & t \in [nT, (n+\theta)T) \\ 0, & t \in [(n+\theta)T, (n+1)T), \end{cases} \quad (12)$$

where $T > 0$ denotes the control period, $0 < \theta < 1$, $n \in \mathcal{N}$:

$$\bar{k}_i(t) = \begin{cases} k_i(t), & 1 \le i \le l, \\ 0, & l+1 \le i \le N, \end{cases} \quad (13)$$

and $k_i(t)$ is the adaptive feedback strength for which the update law is to be designed. When $t \in [nT, (n+\theta)T)$ the error system (4) can be rewritten as

$$\begin{aligned}\dot{e}_i(t) = {} & Ce_i(t) + A\left(f\left(x_i(t)\right) - f\left(s(t)\right)\right) \\ & + B\left(g\left(x_i\left(t-\tau_1\right)\right) - g\left(s\left(t-\tau_1\right)\right)\right) \\ & + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) \\ & + \sum_{j=1}^{N} \widehat{g}_{ij}\Gamma_\tau e_j\left(\mathrm{t}-\tau_2\right) - \overline{k}_i(t)\Gamma e_i(t), \\ & \qquad i = 1, 2, \ldots, N.\end{aligned} \tag{14}$$

When $t \in [(n+\theta)T, (n+1)T)$, the error system (4) can be rewritten as

$$\begin{aligned}\dot{e}_i(t) = {} & Ce_i(t) + A\left(f\left(x_i(t)\right) - f\left(s(t)\right)\right) \\ & + B\left(g\left(x_i\left(t-\tau_1\right)\right) - g\left(s\left(t-\tau_1\right)\right)\right) \\ & + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) + \sum_{j=1}^{N} \widehat{g}_{ij}\Gamma_\tau e_j\left(t-\tau_2\right), \\ & \qquad i = 1, 2, \ldots, N.\end{aligned} \tag{15}$$

Our objective is to design suitable T and θ such that the delayed couple network can realize synchronization. The main results are stated as follows.

Theorem 4. *Suppose that Hypothesis H_1 holds, $\tau \le \theta T$ and $\tau \le (1-\theta)T$, where $\tau = \max\{\tau_1, \tau_2\}$. If there exist positive constants $\eta, a_i, r_i, \varepsilon_i$, $(i = 1, 2)$, such that*

(i) $\Pi_1 = \begin{bmatrix} C^s + \eta I_n & \varepsilon_1 A & L & \varepsilon_2 B \\ \varepsilon_1 A^T & -2\varepsilon_1 I_n & 0 & 0 \\ L & 0 & -2\varepsilon_1 I_n & 0 \\ \varepsilon_2 B^T & 0 & 0 & -2\varepsilon_2 I_n \end{bmatrix} \le 0,$

(ii) $\Pi_2 = \begin{bmatrix} (-\eta + a_1)I_N + d_j G^s - K^* & (1/2)d_j^\tau \widehat{G} \\ (1/2)d_j^\tau \widehat{G}^T & -r_1 I_N \end{bmatrix} \le 0,$

(iii) $\Pi_3 = \begin{bmatrix} (-\eta - a_2)I_N + d_j G^s & (1/2)d_j^\tau \widehat{G} \\ (1/2)d_j^\tau \widehat{G}^T & -r_2 I_N \end{bmatrix} \le 0,$

(iv) $a_1 > r_1 + q$,

(v) $\omega = \varepsilon - 2(a_1 + a_2 + (r_2 - r_1)e^{\varepsilon\tau})(1-\theta) > 0$,

where $C^s = (1/2)(C + C^T)$, $K = \operatorname{diag}(k_1^*, \ldots, k_l^*, 0, \ldots, 0)$, $q = \max\{l_i'^2/2\varepsilon_2,\ i = 1, \ldots, n\}$, and $\varepsilon > 0$ is the unique positive solution of the equation $\varepsilon - 2a_1 + 2(r+q)e^{\varepsilon\tau} = 0$, and choosing the adaptive law

$$\dot{k}_i(t) = \theta_i e_i^T(t)\Gamma e_i(t), \quad i = 1, 2, \ldots, l, \tag{16}$$

then the controlled couple network (1) is globally exponentially synchronized.

Proof. Construct the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{l}\frac{1}{\theta_i}\widetilde{k}_i^2, \tag{17}$$

where $\widetilde{k}_i = k_i(t) - k_i^*$ and k_i^*, $i = 1, 2, \ldots, l$, are positive constants.

Then the derivative of $V(t)$ with respect to time t along the solutions of (14) and (15) can be calculated as follows: when $nT \le t < (n+\theta)T$, for $n = 0, 1, 2, \ldots$, we get

$$\begin{aligned}\dot{V}(t) = {} & \sum_{i=1}^{N} e_i^T(t)\left\{Ce_i(t) + A\left(f\left(x_i(t)\right) - f\left(s(t)\right)\right)\right. \\ & + B\left(g\left(x_i\left(t-\tau_1\right)\right) - g\left(s\left(t-\tau_1\right)\right)\right) + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) \\ & \left.+ \sum_{j=1}^{N} \widehat{g}_{ij}\Gamma_\tau e_j\left(t-\tau_2\right) - \overline{k}_i(t)\Gamma e_i(t)\right\} + \sum_{i=1}^{l}\frac{1}{\theta_i}\left(k_i(t)\right. \\ & \left.- k_i^*\right)\dot{k}_i(t) = \sum_{i=1}^{N} e_i^T(t)\left\{Ce_i(t)\right. \\ & + A\left(f\left(x_i(t)\right) - f\left(s(t)\right)\right) \\ & + B\left(g\left(x_i\left(t-\tau_1\right)\right) - g\left(s\left(t-\tau_1\right)\right)\right) + \sum_{j=1}^{N} g_{ij}\Gamma e_j(t) \\ & \left.+ \sum_{j=1}^{N} \widehat{g}_{ij}\Gamma_\tau e_j\left(t-\tau_2\right)\right\} - \sum_{i=1}^{l} k_i(t)e_i^T(t)\Gamma e_i(t) \\ & + \sum_{i=1}^{l}\frac{1}{\theta_i}\left(k_i(t) - k_i^*\right)\dot{k}_i(t).\end{aligned} \tag{18}$$

Based on Lemma 3, we have

$$\begin{aligned} & e_i^T(t)A\left(f\left(x_i(t)\right) - f\left(s(t)\right)\right) \\ & \quad \le \frac{\varepsilon_1}{2}e_i^T(t)AA^Te_i^T(t) \\ & \qquad + \frac{1}{2\varepsilon_1}\left\|f\left(x_i(t)\right) - f\left(s(t)\right)\right\|^2 \\ & \quad \le \frac{1}{2}e_i^T(t)\left(\varepsilon_1 AA^T + \frac{1}{\varepsilon_1}LL\right)e_i(t),\end{aligned} \tag{19}$$

$$\begin{aligned} & e_i^T(t)B\left(g\left(x_i\left(t-\tau_1\right)\right) - g\left(s\left(t-\tau_1\right)\right)\right) \\ & \quad \le \frac{\varepsilon_2}{2}e_i^T(t)BB^Te_i^T(t) \\ & \qquad + \frac{1}{2\varepsilon_2}\left\|g\left(x_i\left(t-\tau_1\right)\right) - g\left(s\left(t-\tau_1\right)\right)\right\|^2 \\ & \quad \le \frac{\varepsilon_2}{2}e_i^T(t)BB^Te_i(t) \\ & \qquad + \frac{1}{2\varepsilon_2}e_i^T\left(t-\tau_1\right)L'L'e_i\left(t-\tau_1\right),\end{aligned} \tag{20}$$

where $L = \operatorname{diag}(l_1, l_2, \ldots, l_n)$, and $L' = \operatorname{diag}(l_1', l_2', \ldots, l_n')$.

Substituting (16) into the following expression, one has

$$\sum_{i=1}^{l}\frac{1}{\theta_i}\left(k_i(t)-k_i^*\right)\dot{k}_i(t)-\sum_{i=1}^{l}k_i(t)e_i^T(t)\Gamma e_i(t) = -\sum_{i=1}^{l}k_i^*e_i^T(t)\Gamma e_i(t). \tag{21}$$

Using exchange of rows and columns, it is easy to get

$$\sum_{i=1}^{N}\sum_{j=1}^{N}e_i^T(t)g_{ij}\Gamma e_j(t)=\sum_{j=1}^{n}d_j\tilde{e}_i^T(t)G\tilde{e}_j(t), \tag{22}$$

where $\tilde{e}_j(t)=[e_{1j}(t),e_{2j}(t),\dots,e_{Nj}(t)]^T$.

By following the similar steps denoted in (22), we get that

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\widehat{g}_{ij}e_i^T(t)\Gamma_\tau e_j(t-\tau_2)=\sum_{j=1}^{n}d_j^\tau\tilde{e}_j^T(t)\widehat{G}\tilde{e}_j(t-\tau_2),$$
$$\sum_{i=1}^{l}k_i^*e_i^T(t)\Gamma e_i(t)=\sum_{j=1}^{n}d_j\tilde{e}_j^T(t)K^*\tilde{e}_j(t), \tag{23}$$

where $K^*=\operatorname{diag}(k_1^*,k_2^*,\dots,k_l^*,0,\dots,0)$.

Substituting (19)–(23) into (18), we have

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{N}e_i^T(t)\left(C+\frac{1}{2}\varepsilon_1AA^T+\frac{1}{2\varepsilon_1}LL+\frac{\varepsilon_2}{2}BB^T\right)e_i(t)\\ &+\sum_{j=1}^{n}\tilde{e}_j^T(t)\left[d_j(G-K)\tilde{e}_j(t)+d_j^\tau\widehat{G}\tilde{e}_j(t-\tau_2)\right]\\ &+\frac{1}{2\varepsilon_2}\sum_{i=1}^{N}e_i^T(t-\tau_1)L'L'e_i(t-\tau_1).\end{aligned} \tag{24}$$

Define $q=\max\{l_i'^2/2\varepsilon_2,\ i=1,2,\dots,n\}$, and based on conditions (i) and (ii) of Theorem 4, one gets

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{N}\left(-a_1e_i^T(t)e_i(t)+qe_i^T(t-\tau_1)e_i(t-\tau_1)\right.\\ &\left.+r_1e_i^T(t-\tau_2)e_i(t-\tau_2)\right)\le -2a_1V(t)+2(q+r_1)\\ &\cdot\sup_{t-\tau\le s\le t}V(s).\end{aligned} \tag{25}$$

Similarly, based on condition (i) and (iii) of Theorem 4, when $(n+\theta)T\le t\le(n+1)T$, we have

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{N}e_i^T(t)\left(C+\frac{1}{2}\varepsilon_1AA^T+\frac{1}{2\varepsilon_1}LL+\frac{\varepsilon_2}{2}BB^T\right)e_i(t)\\ &+\sum_{j=1}^{n}\tilde{e}_j^T(t)\left[d_jG\tilde{e}_j(t)+d_j^\tau\widehat{G}\tilde{e}_j(t-\tau_2)\right]\\ &+\frac{1}{2\varepsilon_2}\sum_{i=1}^{N}e_i^T(t-\tau_1)L'L'e_i(t-\tau_1)\\ &\le 2a_2V(t)+2(q+r_2)\sup_{t-\tau\le s\le t}V(s).\end{aligned} \tag{26}$$

In the following, we will prove that conditions (iv) and (v) imply

$$V(t)=\sup_{-\tau\le s\le 0}V(s)e^{-\omega t},\quad t\ge 0. \tag{27}$$

Take $M_0=\max_{-\tau\le s\le 0}V(s)$ and $W(t)=e^{\varepsilon t}V(t)$. For $0\le t\le\theta T$, based on (25) and by using Lemma 1, we obtain

$$V(t)\le\max_{-\tau\le s\le 0}V(s)e^{-\varepsilon t}, \tag{28}$$

where $\varepsilon>0$ is the smallest real root of the equation

$$\varepsilon-2a_1+2(q+r_1)e^{\varepsilon\tau}=0. \tag{29}$$

Thus, we have

$$W(t)\le M_0. \tag{30}$$

For $\theta T\le t\le T$, based on (26), we have

$$\begin{aligned}\dot{W}(t) &\le \varepsilon W(t)\\ &+2e^{\varepsilon t}\left(a_2V(t)+(r_2+q)\sup_{t-\tau\le s\le t}V(s)\right)\\ &\le(\varepsilon+2a_2)W(t)+2(r_2+q)e^{\varepsilon\tau}\sup_{t-\tau\le s\le t}W(s).\end{aligned} \tag{31}$$

From Lemma 2, and noting that $\varepsilon-2a_1+2(q+r_1)e^{\varepsilon\tau}=0$, we obtain

$$\begin{aligned}W(t) &\le \sup_{\theta T-\tau\le s\le\theta T}W(s)e^{(\varepsilon+2a_2+2(r_2+q)e^{\varepsilon\tau})(t-\theta T)}\\ &\le M_0e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(t-\theta T)}\\ &\le M_0e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(1-\theta)t}.\end{aligned} \tag{32}$$

For $T\le t<(1+\theta)T$, based on (25) and by using Lemma 1, we obtain

$$V(t)\le\sup_{T-\tau\le s\le T}V(s)e^{-\varepsilon(t-T)}. \tag{33}$$

Consider that $2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})-\varepsilon>0$; then

$$\begin{aligned}W(t) &\le \sup_{T-\tau\le s\le T}W(s)e^{\varepsilon(T-s)}\\ &\le \sup_{T-\tau\le s\le T}M_0e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(s-\theta T)-\varepsilon s+\varepsilon T}\\ &\le M_0e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(1-\theta)T}\\ &\le M_0e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(1-\theta)t}.\end{aligned} \tag{34}$$

For $(1+\theta)T \le t < 2T$, one can repeat the same argument and get similar results to (31) and (32). It can be deduced from Lemma 2 that

$$W(t) \le \sup_{(1+\theta)T-\tau\le s\le(1+\theta)T} W(s)\, e^{(\varepsilon+2(r_2+q)e^{\varepsilon\tau}+2a_2)(t-(1+\theta)T)} \le M_0 e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(1-\theta)t}. \tag{35}$$

By induction, we can derive the following estimation of $W(t)$ for any integer n. For $nT \le t < (n+1)T$

$$W(t) \le M_0 e^{2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(1-\theta)t}. \tag{36}$$

Form the definition of $W(t)$ and condition (v) of Theorem 4, we obtain

$$V(t) \le M_0 e^{-(\varepsilon-2(a_1+a_2+(r_2-r_1)e^{\varepsilon\tau})(1-\theta))t} = M_0 e^{-\omega t}, \quad t \ge 0. \tag{37}$$

This implies the conclusion and the proof is complete. □

When $\Gamma = 0$, only delayed coupling exists in the networks. One has the following corollary.

Corollary 5. *Suppose that Hypothesis H_1 holds, $\tau \le \theta T$ and $\tau \le (1-\theta)T$ where $\tau = \tau_2$. If there exist positive constants η, a_i, r_i, ε_i, $(i = 1, 2)$, such that*

(i) $\Pi_1 = \begin{bmatrix} C^s+\eta I_n & \varepsilon_1 A & L & \varepsilon_2 B \\ \varepsilon_1 A^T & -2\varepsilon_1 I_n & 0 & 0 \\ L & 0 & -2\varepsilon_1 I_n & 0 \\ \varepsilon_2 B^T & 0 & 0 & -2\varepsilon_2 I_n \end{bmatrix} \le 0,$

(ii) $\Pi_2 = \begin{bmatrix} (-\eta+a_1)I_N-K^* & (1/2)d_j^\tau \widehat{G} \\ (1/2)d_j^\tau \widehat{G}^T & -r_1 I_N \end{bmatrix} \le 0,$

(iii) $\Pi_3 = \begin{bmatrix} (-\eta-a_2)I_N & (1/2)d_j^\tau \widehat{G} \\ (1/2)d_j^\tau \widehat{G}^T & -r_2 I_N \end{bmatrix} \le 0,$

(iv) $a_1 > r_1 + q$,

(v) $\omega = \varepsilon - 2(a_1 + a_2 + (r_2 - r_1)e^{\varepsilon\tau})(1-\theta) > 0$,

where $K = \operatorname{diag}(k_1^*, \ldots, k_l^*, 0, \ldots, 0)$, $q = \max\{l_i'^2/2\varepsilon_2,\ i = 1, \ldots, n\}$, and $\varepsilon > 0$ is the unique positive solution of the equation $\varepsilon - 2a_1 + 2(r+q)e^{\varepsilon\tau} = 0$, and choosing the adaptive law as (16), then the controlled couple network (1) is globally exponentially synchronized.

When $\Gamma_\tau = 0$, there is no delayed coupling in the coupled networks.

Corollary 6. *Suppose that Hypothesis H_1 holds, $\tau \le \theta T$ and $\tau \le (1-\theta)T$ where $\tau = \tau_1$. If there exist positive constants η, a_i, r_i, ε_i, $(i = 1, 2)$, such that*

(i) $\Pi_1 = \begin{bmatrix} C^s+\eta I_n & \varepsilon_1 A & L & \varepsilon_2 B \\ \varepsilon_1 A^T & -2\varepsilon_1 I_n & 0 & 0 \\ L & 0 & -2\varepsilon_1 I_n & 0 \\ \varepsilon_2 B^T & 0 & 0 & -2\varepsilon_2 I_n \end{bmatrix} \le 0,$

(ii) $\Pi_2 = (-\eta + a_1)I_N + d_j G^s - K^* \le 0,$

(iii) $\Pi_3 = (-\eta - a_2)I_N + d_j \widehat{G}^s \le 0,$

(iv) $a_1 > q = \max\{l_i'^2/2\varepsilon_2,\ i = 1, \ldots, n\}$,

(v) $\omega = \varepsilon - 2(a_1 + a_2)(1-\theta) > 0$,

where $K = \operatorname{diag}(k_1^*, \ldots, k_l^*, 0, \ldots, 0)$ and $\varepsilon > 0$ is the unique positive solution of the equation $\varepsilon - 2a_1 + 2(r+q)e^{\varepsilon\tau} = 0$, and choosing the adaptive law as (16), then the controlled couple network (1) is globally exponentially synchronized.

4. Numerical Simulation

In this section, we present a numerical simulation to illustrate the feasibility and effectiveness of our results.

Consider the coupled network (1) consisting of 6 identical Chua oscillators with time delayed nonlinearity. The dynamics of the Chua oscillator is given by

$$\dot{x}_i(t) = Cx_i(t) + Af(x_i(t)) + Bg(x_i(t-\tau_1)), \tag{38}$$

where $x_i(t) \in R^3$, $\tau_1 = 0.02$, $f(x_i(t)) = (-(1/2)\delta(\alpha-\beta)(|x_{i1}(t)+1| - |x_{i1}(t)-1|), 0, 0)^T$, and $g(x_i(t-\tau_1)) = (-\xi\gamma\sin(\nu x_{i1}(t-\tau_1)), 0, 0)^T$:

$$C = \begin{bmatrix} -\delta(1+\beta) & \delta & 0 \\ 1 & -1 & 1 \\ 0 & -\xi & -\epsilon \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$
$$\delta = 10, \quad \epsilon = 0.1636, \quad \xi = 19.53, \quad \alpha = -1.4325, \quad \beta = -0.7831, \quad \nu = 0.5, \quad \gamma = 0.2; \tag{39}$$

the other parameters of network are given as follows:

$$\Gamma = 2I_2,$$
$$\Gamma_\tau = 0.1I_2,$$
$$\tau_2 = 0.1,$$

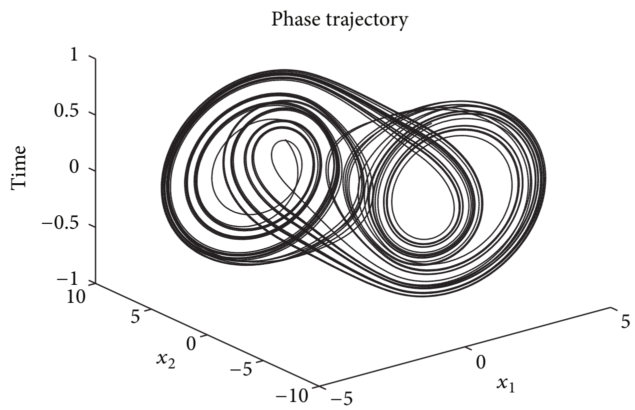

FIGURE 1: Dynamical behavior of the synchronization manifold $s(t)$.

$$G = 10\begin{bmatrix} -5 & 1 & 1 & 1 & 1 & 1 \\ 1 & -5 & 1 & 1 & 1 & 1 \\ 1 & 1 & -5 & 1 & 1 & 1 \\ 1 & 1 & 1 & -5 & 1 & 1 \\ 1 & 1 & 1 & 1 & -5 & 1 \\ 1 & 1 & 1 & 1 & 1 & -5 \end{bmatrix},$$

$$\widehat{G} = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{bmatrix}. \quad (40)$$

The dynamical behavior of the synchronization manifold $s(t)$ is shown in Figure 1.

It is easy to verify that nonlinear functions $f(\cdot)$ and $g(\cdot)$ satisfy the Lipschitz condition with $l_1 = (1/2)\delta(\beta - \alpha)$, $l_2 = \xi\gamma\nu$. By using the MATLAB LMI toolbox, a feasible solution of condition (i) of Theorem 4 is obtained as follows:

$$\begin{aligned} \varepsilon_1 &= 6.5041, \\ \varepsilon_2 &= 2.0684, \\ \eta &= -12.56. \end{aligned} \quad (41)$$

In this simulation, we add the adaptive feedback controllers to the 1st, 2nd, and 3rd nodes. Let $T = 1$, $\theta = 0.2$, $\omega = 0.1$, $a_1 = 20$, $a_2 = 13$, $r_1 = 2$, $r_2 = 0.2$, and $K^* = \mathrm{diag}(500, 500, 500, 0, 0, 0)$; thus the condition in Theorem 4 is satisfied. According to Theorem 4, the coupled network (1) can be achieved pinning adaptive synchronization.

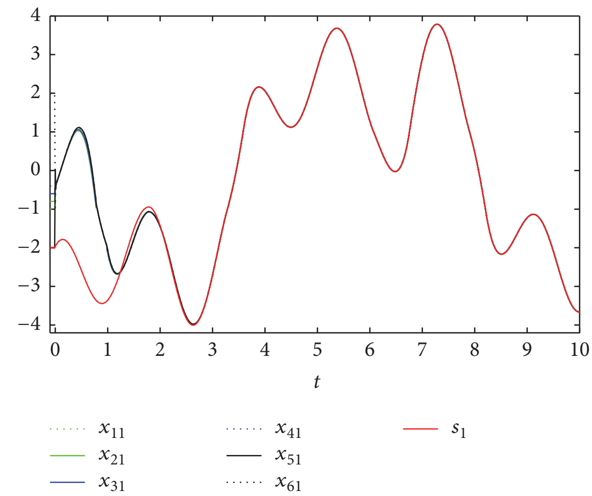

FIGURE 2: The trajectories of the state variables of $x_{i1}(t)$, $(i = 1, 2, \ldots, 6)$ in the controlled network (1) and synchronization manifold $s_1(t)$.

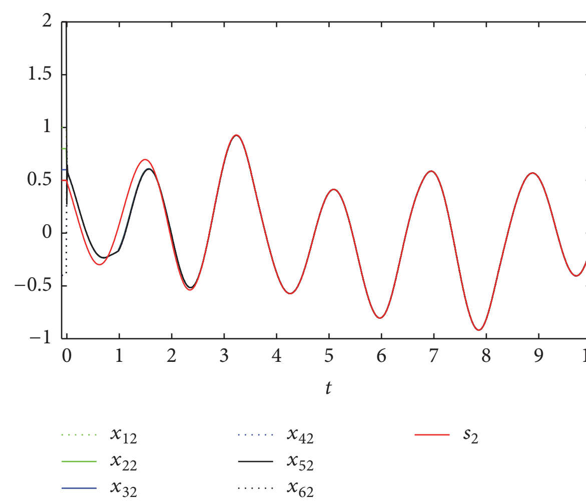

FIGURE 3: The trajectories of the state variables of $x_{i2}(t)$, $(i = 1, 2, \ldots, 6)$ in the controlled network (1) and synchronization manifold $s_2(t)$.

The initial conditions of the numerical simulation are as follows:

$$X^0 = \left[x_i^0(t) \;\; x_2^0(t) \;\; \cdots \;\; x_6^0(t) \;\; s(t)\right] = \begin{bmatrix} -1 & -0.8 & -0.6 & -0.4 & -2 & 2 & -2 \\ 1 & 0.8 & 0.6 & -0.4 & 2 & -0.4 & 0.5 \\ 0.5 & 0.3 & 1 & -2 & -0.4 & 0.4 & 1 \end{bmatrix}. \quad (42)$$

The simulation results are given in Figures 2–5. The trajectories of the state variables $x_i(t)$ and synchronization manifold are illustrated in Figures 2–4. From these figures, we can see that all nodes $x_i(t)$ tend to the synchronization manifold $s(t)$. The time evolution of the feedback control gains $\overline{k}_i(t)$ $(i = 1, 2, 3)$ is shown in Figure 5.

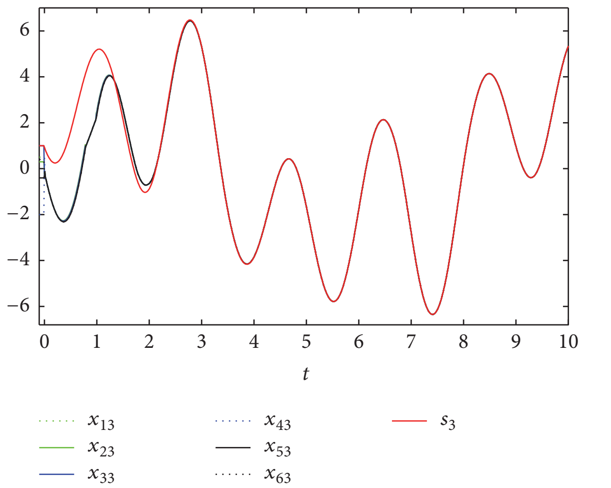

FIGURE 4: The trajectories of the state variables of $x_{i2}(t)$, $(i = 1, 2, \ldots, 6)$ in the controlled network (1) and synchronization manifold $s_3(t)$.

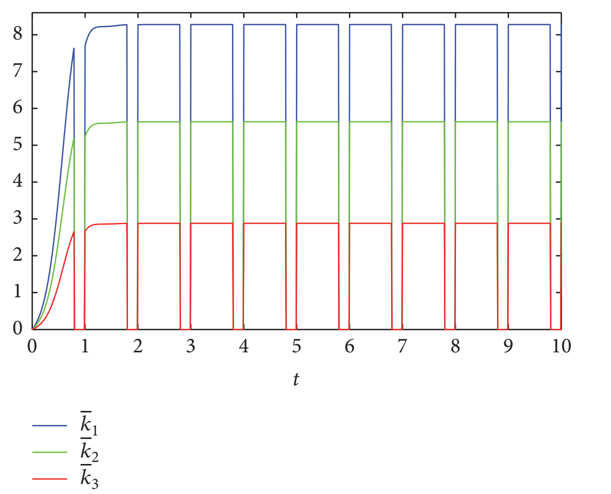

FIGURE 5: Time evolution of the control gains $\bar{k}_1, \bar{k}_2, \bar{k}_3$.

5. Conclusion

In this paper, we have investigated the exponential synchronization problem for neural networks by pinning periodically intermittent control. Based on Lyapunov stability theory and periodically intermittent control method, some novel conditions for synchronization are derived. Furthermore, numerical simulations have verified the effectiveness of the presented method.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Xueliang Liu and Shengbing Xu conceived and designed the study. Xueliang Liu carried out the main part of this manuscript. Shengbing Xu participated in the discussion and corrected the main theorem. All authors read and approved the manuscript.

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