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Research Article

Nonlinear Dynamics of a PI Hydroturbine Governing System with Double Delays

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A PI hydroturbine governing system with saturation and double delays is generated in small perturbation. The nonlinear dynamic behavior of the system is investigated. More precisely, at first, we analyze the stability and Hopf bifurcation of the PI hydroturbine governing system with double delays under the four different cases. Corresponding stability theorem and Hopf bifurcation theorem of the system are obtained at equilibrium points. And then the stability of periodic solution and the direction of the Hopf bifurcation are illustrated by using the normal form method and center manifold theorem. We find out that the stability and direction of the Hopf bifurcation are determined by three parameters. The results have great realistic significance to guarantee the power system frequency stability and improve the stability of the hydropower system. At last, some numerical examples are given to verify the correctness of the theoretical results.

1. Introduction

It is well known that the hydroturbine system is the nonminimum phase system and is operated in a complex condition, constituted by controller and governor. The parameters of the hydroturbine system would change significantly under different operating conditions. According to specific goals, controlled system is researched within a given range. There exist two regulators: PID regulator (Proportion Integration Differentiation regulator) and the soft type feedback regulator (proportional-integral regulator), respectively [1]. Although the characteristics of the PID regulator are simple structure and adaptable and easily adjusting parameters, the regulation law of PID is not efficient. PI regulator is a closed-loop system with phase lag, which can reach steady state by using parameter setting. The PI regulator possesses the properties of the optimal regulator and good robustness. Furthermore, the PI regulator is easy to use in the field, similar to the parameter setting method of the PID regulator. At present, the PI regulator plays a vital role in maintaining the stability of electrical systems and is widely used in China [2, 3]. On account of lacking systematic management, it is a challenge to maintain the stability of a large hydroelectric station [4–6]. Many efforts are focused on constructing different mathematical models of the hydroturbine governing system and analyzing the stability and the bifurcation phenomena [7–16]; for example, Ling and Tao [13] analyzed the stability and the bifurcation phenomena of a proportional-integral-(PI-) (controller) type speed hydroturbine governing system with saturation.

Over the past one decade, many researchers have paid great attention to analyzing dynamic characteristic when the parameters of the hydroturbine systems are changed. For instance, using PI controller, Silva et al. [17] have revealed the problem of stabilizing of a first-order plant with time delay and obtained the stabilizing PI gain values. Shu and Pi [18] introduced a PID neural network (PIDNN) with control time delay and gave examples of analysis. Strah et al. [19] designed a speed and active power controller of hydroturbine units; some controller parameters were obtained. Li and Zhou [20] developed a gravitational search algorithm (GSA), which was applied to parameter identification of the hydraulic turbine governing system (HTGS), and analyzed the stability of the power system. Jiang et al. [5] proposed a deterministic chaotic mutation evolutionary programming (DCMEP) method to efficiently optimize the PID parameters of the hydroturbine governing systems. Utilizing a maximum peak resonance specification method, a new PID controller for automatic generation control (AGC) of hydroturbine power systems was presented by Khodabakhshian and Hooshmand [21]. On the basis of necessary and sufficient condition, Liu et al. [22] proposed a new method to analyze the stability of automatic generation control (AGC) systems with commensurate delays. Zhang et al. [23] analyzed a PID-type load frequency control (LFC) scheme by using delay-dependent robust method. Based on state space equations, Chen et al. [24] studied the nonlinear dynamical behaviors of a novel hydroturbine mode with the effect of the surge tank. Xu et al. [25] proposed a Hamiltonian model of the hydroturbine governing system, which included fractional item and time-lag, and explored the effect of the fractional item and the time-lag on the dynamic variables of the hydroturbine governing system. Wang et al. [26] studied a novel fractional-order Francis hydroturbine governing system with time delay and verified the effects of the fractional item and time delay on the system by the principle of statistical physics, respectively. The stability and Hopf bifurcation of a Goodwin model with four different delays were investigated by Zhang et al. [27]. Zhang et al. [28] analyzed a hydroturbine governing system in the process of load rejection transient and got the stable regions of the hydroturbine governing system by means of numerical simulations.

However, the existence of a Hopf bifurcation is rarely reported in proportional-integral (PI) type hydroturbine generating system with time delay. In the paper, we generate a PI hydroturbine governing system with saturation and double delays. The nonlinear dynamic behavior of the system is analyzed. The scope of some parameter values is obtained to maintain the stability of the system, which has great realistic significance in a small hydropower station.

The basic structure of the rest paper is as follows. In Section 2, we present a new PI hydroturbine governing system, which is affected by the speed control delay of the generator and the displacement-control delay of the servomotor. In Section 3, the stability of equilibrium points and Hopf bifurcation for PI hydroturbine governing system are investigated in the four different delay cases, respectively. The stability and direction of the Hopf bifurcation are illustrated in Section 4. Numerical simulations are given to support our theory by Matlab software in Section 5. Finally, a brief discussion is given in Section 6.

2. Model Description

We study a PI type hydroturbine governing system with saturation and time delay. The structure of the hydroturbine governing system is shown in Figure 1.

The transfer function of soft feedback regulator is

$$G_{\mathrm{PI}}(s) = K_P + \frac{K_I}{s}. \tag{1}$$

We use an approximate linearization approach for the hydroturbine governing system, which is a first-order mathematical model. Moreover the system is set in small perturbation. Therefore, we have

$$\begin{aligned} G_t(s) &= e_y \frac{1 - \left(e_{qy}e_h/e_y - e_{qh}\right)T_\omega s}{1 + e_{qh}T_\omega s}, \\ G_s(s) &= \frac{1}{T_a s + e_g - e_x}, \end{aligned} \tag{2}$$

where $G_t(s)$ is used in a nonelastic water column model [29, 30]. $G_s(s)$ is the transfer function from the hydroturbine moment to the speed of the generator. e_{qy} is servomotor stroke transfer coefficient of the flow rate. e_y is servomotor stroke transfer coefficient of the turbine torque. e_h is the transfer coefficient of the torque on the water head of a turbine. e_{qh} is the transfer coefficient of the flow rate on the water head of a turbine. T_ω is the water inertia time constant of a pressure guide-water system. s is the strength of the elastic water hammer effect. T_a is the sum of the machine starting time and load time constants. e_g is the load self-regulation factor. e_x is the transfer coefficient of a speed on the turbine torque. e is equal to $e_{qy}e_h/e_y - e_{qh}$.

If we neglect the load perturbations, then we obtain the state space equations as

$$\begin{aligned} \dot{x}_1 &= x_2 + -\frac{e_y e}{e_{qh}T_a} y \\ \dot{x}_2 &= -\frac{e_n}{e_{qh}T_\omega T_a}x_1 - \frac{T_a + e_n e_{qh}T_\omega}{e_{qh}T_\omega T_a}x_2 \\ &\quad + \left(-\frac{e_y}{e_{qh}T_\omega T_a} + \frac{\left(T_a + e_n e_{qh}T_\omega\right)e_y e}{e_{qh}^2 T_\omega T_a^2}\right)y, \end{aligned} \tag{3}$$

where e_n is equal to $e_g - e_x$.

When the nonlinear part can be shown by the nonlinear function $y = N(x_3 - K_P x_1)$, then we have the nonlinear state equation, which is a closed-loop system; we define x_3 as a state variable,

$$\dot{x}_3 = K_P\dot{C} + K_I\left(C - x_1\right), \tag{4}$$

where K_P is the proportional component; K_I is the integral component.

Next, when $C = 0$, by combining (3) and (4), we have the state space equations of the PI hydroturbine governing system with saturation as

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) + b_1 N\left(x_3(t) - K_P x_1(t)\right) \\ \dot{x}_2(t) &= -a_0 x_1(t) - a_1 x_2(t) \\ &\quad + \left(b_0 - a_1 b_1\right) N\left(x_3(t) - K_P x_1(t)\right) \\ \dot{x}_3(t) &= -K_I x_1(t), \end{aligned} \tag{5}$$

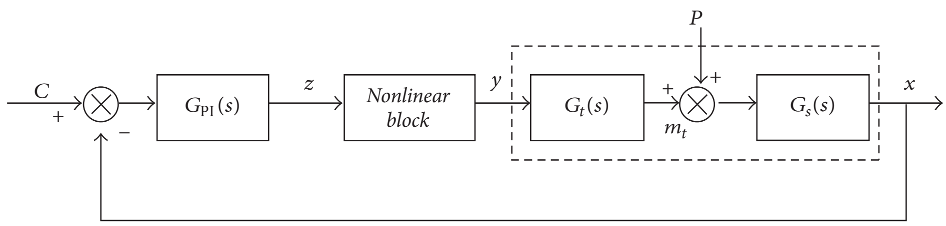

Figure 1: Structure of the hydroturbine governing system.

where $a_1 = (T_a + e_n e_{qh} T_\omega)/e_{qh} T_\omega T_a$, $a_0 = e_n/e_{qh} T_\omega T_a$, $b_1 = -e_y e/e_{qh} T_a$, $b_0 = -e_y/e_{qh} T_\omega T_a$, $e = e_{qy} e_h/e_y - e_{qh}$, and N is the constant of nonlinear part and equal to 1.

Although Ling and Tao [13] have investigated the existence and direction of the Hopf bifurcation, for the PI hydroturbine governing system, speed control delays and the displacement delays of the servomotor are never considered in the previous studies. In the paper, we consider the dynamics of the system (5) with two different delays. Therefore, we have the following PI hydroturbine governing system as

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) + b_1 N \left(x_3(t) - K_P x_1(t-\tau_1)\right) \\
\dot{x}_2(t) &= -a_0 x_1(t) - a_1 x_2(t) \\
&\quad + (b_0 - a_1 b_1) N \left(x_3(t-\tau_2) - K_P x_1(t)\right) \\
\dot{x}_3(t) &= -K_I x_1(t),
\end{aligned} \tag{6}$$

where τ_1 is the speed control delay of the generator. τ_2 is the displacement-control delay of the servomotor.

3. Stability Analysis and Hopf Bifurcation

Usually, it is not easy to find out its accurate solutions. It is sufficient to research the stability of $E^* = (x_1^*, x_2^*, x_3^*)$; we only consider $E_0 = (0, 0, 0)$. At equilibrium point E_0, the Jacobi matrix of the system (6) is

$$\begin{bmatrix} -b_1 N \cdot K_P \cdot e^{-\lambda\tau_1} & 1 & b_1 N \\ (a_1 b_1 - b_0) N \cdot K_P - a_0 & -a_1 & (b_0 - a_1 b_1) N \cdot e^{-\lambda\tau_2} \\ -K_I & 0 & 0 \end{bmatrix}, \tag{7}$$

Then the associated characteristic equation is

$$\begin{aligned}
&\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 + \left(A_4\lambda^2 + A_5\lambda\right) e^{-\lambda\tau_1} \\
&\quad + A_6 e^{-\lambda\tau_2} = 0,
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
A_1 &= a_1, \\
A_2 &= NK_P b_0 + NK_I b_1 + a_0 - NK_P a_1 b_1 \\
A_3 &= NK_I a_1 b_1, \\
A_4 &= NK_P b_1, \\
A_5 &= NK_P a_1 b_1, \\
A_6 &= -NK_I a_1 b_1 + NK_I b_0.
\end{aligned} \tag{9}$$

Next, we investigate the distribution of the roots of (8) with different delay values for τ_1 and τ_2.

It is apparent that (8) assumes the following form when $\tau_1 = 0$, $\tau_2 = 0$.

$$\lambda^3 + \lambda^2 (A_1 + A_4) + \lambda (A_2 + A_5) + A_3 + A_6 = 0. \tag{10}$$

Furthermore, we propose the Routh-Hurwitz stability criterion; a corresponding certificate shall be found [31].

Lemma 1 (see [32]). *The polynomial $L(\lambda) = \lambda^3 + p_1\lambda^2 + p_2\lambda + p_3$ with real coefficients has all roots with negative real parts if and only if the numbers p_1, p_2, p_3 are positive and the inequality $p_1 p_2 > p_3$ is satisfied.*

According to Lemma 1, all roots of (10) have negative real parts if and only if

(H1) $A_1 + A_4 > 0$, $A_2 + A_5 > 0$, $A_3 + A_6 > 0$ and $(A_1 + A_4)(A_2 + A_5) > A_3 + A_6$.

So $E^* = (x_1^*, x_2^*, x_3^*)$ is locally asymptotically stable when (H1) holds.

Case 1 ($\tau = \tau_1 = \tau_2 \neq 0$). We rewrite (8) as follows:

$$\begin{aligned}
&\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 + \left(A_4\lambda^2 + A_5\lambda + A_6\right) e^{-\lambda\tau} \\
&\quad = 0.
\end{aligned} \tag{11}$$

If $\lambda = i\omega$ is a root of (11), then we have

$$\omega^{10} + \omega^8 C_1 + \omega^6 C_2 + \omega^4 C_3 + \omega^2 C_4 + C_5 = 0, \tag{12}$$

where

$$C_1 = \frac{\left(2B_1B_2 + B_6^2 - B_1^4\right)}{B_1^2},$$
$$C_2 = \frac{\left(2B_1B_3 - 2B_1^2B_4 + B_2^2 + 2B_6B_7\right)}{B_1^2},$$
$$C_4 = \frac{\left(B_3^2 - 2B_4B_5 + 2B_7B_8\right)}{B_1^2}, \tag{13}$$
$$C_3 = \frac{\left(2B_2B_3 - 2B_1^2B_5 - B_4^2 + 2B_6B_8 + B_7^2\right)}{B_1^2},$$
$$C_5 = \frac{\left(-B_5^2 + B_8^2\right)}{B_1^2}.$$

Let $z = \omega^2$, we define

$$h(z) = z^5 + z^4C_1 + z^3C_2 + z^2C_3 + zC_4 + C_5. \tag{14}$$

From the definition of $h(z)$, we can obtain $h(+\infty) = +\infty$; thus we can get that (14) has at least one positive root if $h(0) = C_5 < 0$ holds.

We assume that (14) has five positive roots, which are defined by z_1, z_2, z_3, z_4, and z_5. Then, (12) has five positive roots $\omega_k = \sqrt{z_k}$, $k = 1, 2, 3, 4, 5$.

If we denote

$$\tau_k^{(i)} = \frac{1}{\omega_k}\left\{\arccos\left(\frac{\omega^4B_6 + \omega^2B_7 + B_8}{\omega^4B_1^2 + \omega^2B_4 + B_5}\right) + 2i\pi\right\}, \quad i = 0, 1, 2, \dots, \quad k = 1, 2, 3, 4, 5, \tag{15}$$

then $\pm i\omega_k$ is a pair of purely imaginary roots of (8) with $\tau = \tau_k^{(i)}$.

Lemma 2. *Define $\tau_0 = \tau_k^{(0)} = \min_{k\in\{1,2,3\}}\{\tau_k^{(0)}\}$ and $\omega_0 = \omega_{k_0}$. Let $\lambda(\tau) = \xi(\tau) + i\omega(\tau)$ be the root of (8) near $\tau = \tau_0$, satisfying $\xi(\tau_0) = 0$, $\omega(\tau_0) = \omega_0$; the following transversality condition holds: $(d\,\mathrm{Re}\,\lambda(\tau)/d\tau)_{\tau=\tau_k^{(i)}} \neq 0$.*

By applying Lemma 2 to (11), we can obtain the following theorem.

Theorem 3. *Suppose $H_2 = P_RQ_R + P_1Q_1 \neq 0$ holds; with the increasement of delay variable τ from zero, there is a value of τ_0, such that the positive equilibrium point E^* is locally asymptotically stable for $\tau \in [0, \tau_0)$ and unstable for $\tau > \tau_0$. Moreover, system (6) occurs with a Hopf bifurcation at E^* when $\tau = \tau_0$.*

Case 2 ($\tau_1 \neq 0, \tau_2 = 0$). We rewrite (8) as follows:

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 + A_6 + \left(A_4\lambda^2 + A_5\lambda\right)e^{-\lambda\tau_1} = 0. \tag{16}$$

Let $\lambda = i\omega$ be a root of (16), then we have

$$z^4 + z^3D_1 + z^2D_2 + zD_3 + D_4 = 0, \tag{17}$$

where

$$D_1 = \frac{\left(A_1^2A_4^2 + A_5^2 - A_4^4 - 2A_2A_4^2\right)}{A_4^2},$$
$$D_2 = \frac{\left(-2A_4^2A_5^2 + A_1^2A_5^2 + A_2^2A_4^2 - 2A_2A_5^2 - 2A_1A_3A_4^2 - 2A_1A_4^2A_6\right)}{A_4^2}, \tag{18}$$
$$D_3 = \frac{\left(-A_5^4 + A_3^2A_4^2 + A_4^2A_6^2 + A_2^2A_5^2 + 2A_3A_4^2A_6 - 2A_1A_3A_5^2 - 2A_1A_5^2A_6\right)}{A_4^2},$$
$$D_4 = \frac{\left(A_3A_5 + A_5A_6\right)^2}{A_4^2}.$$

If we denote

$$\tau_k^{(i)} = \frac{1}{\omega_k}\left\{\arccos\left(\frac{\omega^3\left(A_5 - A_1A_4\right) + \omega\left(A_3A_4 - A_2A_5 + A_4A_6\right)}{\omega^3A_4^2 + \omega A_5^2}\right) + 2i\pi\right\}, \quad i = 0, 1, 2, \dots, \quad k = 1, 2, 3, 4, \tag{19}$$

then $\pm i\omega_k$ is a pair of purely imaginary roots of (8) with $\tau_1 = \tau_k^{(i)}$.

Lemma 4. *Define* $\tau_{10} = \tau_k^{(0)} = \min_{k\in\{1,2,3\}}\{\tau_k^{(0)}\}$ *and* $\omega_0 = \omega_{k_0}$. *Let* $\lambda(\tau) = \xi(\tau) + i\omega(\tau)$ *be the root of (16) near* $\tau_1 = \tau_{10}$, *satisfying* $\xi(\tau_{10}) = 0$, $\omega(\tau_{10}) = \omega_0$, $z_k = \omega_k^2$ *and* $h'(z_k) \neq 0$; *then* $(d\,\mathrm{Re}\,\lambda(\tau_{10})/d\tau_1)_{\tau_1=\tau_k^{(i)}} \neq 0$, $(d\,\mathrm{Re}\,\lambda(\tau_{10})/d\tau_1)$, *and* $h'(z_k)$ *have the same sign.*

Next, we have to look for the conditions required for (17) to have at least one positive root.

We denote

$$h(z) = z^4 + z^3 D_1 + z^2 D_2 + zD_3 + D_4. \tag{20}$$

By applying Lemma 1 to (17), we obtain the following theorem.

Theorem 5. *For (20), the following result holds.*

(i) *If* $D_4 > 0$ *and* $\Delta = D_1D_2D_3 - D_4D_1^2 - D_3^2 \leq 0$, *then the zero solution of system (6) is asymptotically stable for* $\tau_1 > 0$.

(ii) *If* $D_4 < 0$ *and* $\Delta = D_1D_2D_3 - D_4D_1^2 - D_3^2 > 0$, *then* $E^* = (x_1^*, x_2^*, x_3^*)$ *of system (6) is asymptotically stable for* $\tau_1 \in [0, \tau_{10})$, *and it is unstable when* $\tau > \tau_{10}$.

(iii) *If all the conditions stated in (ii) and* $h'(z) \neq 0$ *are satisfied, system (6) occurs with a Hopf bifurcation at* $E^* = (x_1^*, x_2^*, x_3^*)$ *when* $\tau_1 = \tau_k^{(i)}$ $(i = 0, 1, 2, \ldots)$.

Case 3 $(\tau_1 = 0, \tau_2 \neq 0)$. We rewrite (8) as follows:

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 + A_6 e^{-\lambda\tau_2} = 0. \tag{21}$$

By letting $\lambda = i\omega$ be the root of (21), we have

$$z^3 + E_1 z^2 + E_2 z + E_3 = 0. \tag{22}$$

The roots of (22) are considered by Lemma 1. Without losing generality, we suppose that (22) has three positive roots, which are defined by z_1, z_2, and z_3.

If we denote

$$\tau_k^{(i)} = \frac{1}{\omega_k}\left\{\arccos\left(\frac{A_1\omega^2 - A_3}{A_6}\right) + 2i\pi\right\}, \qquad i = 0, 1, 2, \ldots, \quad k = 1, 2, 3, \tag{23}$$

then $\pm i\omega_k$ is a pair of purely imaginary roots of (8) with $\tau_2 = \tau_k^{(i)}$.

Define $\tau_{20} = \tau_k^{(0)} = \min_{k\in\{1,2,3\}}\{\tau_k^{(0)}\}$ and $\omega_0 = \omega_{k_0}$. Let $\lambda(\tau) = \xi(\tau) + i\omega(\tau)$ be the root of (21) near $\tau_2 = \tau_{20}$, satisfying $\xi(\tau_{10}) = 0$ and $\omega(\tau_{10}) = \omega_0$, then the following transversality condition holds.

Lemma 6. *Suppose that* $z_k = \omega_k^2$ *and* $h'(z_k) \neq 0$, *then* $(d\,\mathrm{Re}\,\lambda(\tau_{20})/d\tau_2)_{\tau_1=\tau_k^{(i)}} \neq 0$, $(d\,\mathrm{Re}\,\lambda(\tau_{20})/d\tau 2)_{\tau_1=\tau_k^{(i)}}$, *and* $h'(z_k)$ *have the same sign.*

The proof is similar to that of Lemma 4, so we ignore the proofs. By applying Lemma 6 to (21), we have the following theorem.

Theorem 7. *As* τ_2 *increases from zero, there exists a critical value* τ_{20}, *such that* E^* *is locally asymptotically stable for* $\tau_1 \in [0, \tau_{20})$ *and unstable when* $\tau_2 > \tau_{20}$. *Moreover, system (6) occurs a Hopf bifurcation at* E^* *when* $\tau_2 = \tau_{20}$.

Case 4 $(\tau_1 \neq \tau_2, \tau_1 > 0$, and $\tau_2 > 0)$. We consider (8) with τ_1 in its stability range. Regarding τ_2 as a parameter, without losing generality, we consider system (6) in Case 2. Let ωi $(\omega > 0)$ be a root of (8); then we obtain

$$F_1(\omega) + F_2(\omega)\sin(\omega\tau_1) + F_3\cos(\omega\tau_1) = 0, \tag{24}$$

where

$$\begin{aligned} F_1(\omega) &= \omega^6 + \omega^4\left(A_1^2 + A_4^2 - 2A_2\right) \\ &\quad + \omega^2\left(A_2^2 + A_5^2 - 2A_1A_3\right) + A_3^2 - A_6^2, \\ F_2(\omega) &= -2\omega^5 A_4 + \omega^3\left(2A_2A_4 - 2A_1A_5\right) \\ &\quad + 2\omega A_3 A_5, \\ F_3(\omega) &= -2\omega^5 A_4 + 2\omega^4 A_1A_4 + 2\omega^3 A_2A_4 \\ &\quad - 2\omega^2 A_3A_4. \end{aligned} \tag{25}$$

Therefore, utilizing the general Hopf bifurcation theorem for functional differential equations (FDEs) as given in Hale [33], we obtain the following results for system (6).

Theorem 8. *Suppose that (24) has at least finite positive roots,* $P_RQ_R + P_IQ_I \neq 0$ *and* $\tau_1 \in [0, \tau_{10})$, *then the positive equilibrium point* E^* *of system (6) is locally asymptotically stable for* $\tau_2 \in [0, \tau_{2*})$. *For* E^*, *system (6) undergoes a Hopf bifurcation when* $\tau_2 = \tau_{2*}$. *System (6) has a branch of periodic solutions bifurcating from the zero solution near* $\tau_2 = \tau_{2*}$.

4. Stability and Direction of the Hopf Bifurcation

In the above section, we have studied that PI hydroturbine governing model (6) with double delays undergoes a Hopf bifurcation for $\tau = \tau_k^{(i)}$ $(i = 0, 1, 2, \ldots,\ k = 1, 2, \ldots)$. In this section, we assume that system (6) undergoes a Hopf bifurcation at $\tau = \tau_0^{(i)}$ $(i = 0, 1, 2, \ldots)$. Utilizing the normal form theory and the center manifold reduction, the stability, the direction, and the bifurcation of the periodic solutions are determined.

For convenience sake, we suppose that τ_0 is less than τ_{2*}, where $\tau_{2*} \in [0, \tau_{10})$. Let $u(t) = (x_1(t), x_2(t), x_3(t))^T \in R^3$, $\tau_1 = \tau_0 + \mu$, in which $\mu \in R$, then we transform system (6) into functional differential equations (FDEs) in $C = C([-1, 0], R^3)$ as

$$\dot{x}(t) = L_\mu(x_t) + f(\mu, x_t), \tag{26}$$

where $L_\mu : C \to R^3$, $f : R \times C \to R^3$ are given, respectively. We obtain

$$L\mu(x_t) = (\tau_0+\mu)\begin{bmatrix}0 & 1 & c_1\\ c_2 & c_3 & 0\\ c_4 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi_1(0)\\ \phi_2(0)\\ \phi_3(0)\end{bmatrix} + (\tau_0+\mu)\begin{bmatrix}c_5 & 0 & 0\\ 0 & 0 & c_6\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi_1(-1)\\ \phi_2(-1)\\ \phi_3(-1)\end{bmatrix}, \tag{27}$$

$$f(u,\phi) = (\tau_0+\mu)\begin{bmatrix}c_5\phi_1(-1)\\ c_6\phi_3(-1)\\ 0\end{bmatrix},$$

where $c_1 = b_1N$, $c_2 = a_1b_1NK_P - b_0NK_P - a_0$, $c_3 = -a_1$, $c_4 = -K_I$, $c_6 = (b_0 - a_1b_1)N$, $c_5 = -b_1NK_P$ and $\phi = (\phi_1,\phi_2,\phi_3)^T \in C([-1,0],R^3)$.

Utilizing the Riesz representation theorem, there is a 3×3 matrix-valued function $\eta(\theta,\mu)$ of bounded variation for $\theta \in [-1,0]$, such that

$$L_\mu\phi = \int_{-1}^{0} d\eta(\theta,0)\,\phi(\theta), \quad \text{where } \phi \in C\left([-1,0],R^3\right). \tag{28}$$

Indeed, we may take

$$\eta(\theta,\mu) = (\tau_0+\mu)\begin{bmatrix}0 & 1 & c_1\\ c_2 & c_3 & 0\\ c_4 & 0 & 0\end{bmatrix}\delta(\theta) - (\tau_0+\mu)\begin{bmatrix}c_5 & 0 & 0\\ 0 & 0 & c_6\\ 0 & 0 & 0\end{bmatrix}\delta(\theta+1), \tag{29}$$

where δ is the Dirac delta function.

For $\phi \in C^{-1}([-1,0],R^3)$, we define

$$A^*\psi(s) = \begin{cases} -\dfrac{d\psi(s)}{ds}, & s \in (0,1] \\ \displaystyle\int_{-1}^{0} d\eta^T(t,0)\,\psi(-t), & s = 0 \end{cases} \tag{30}$$

and a bilinear inner product:

$$\langle\psi(s),\phi(\theta)\rangle = \overline{\psi}(0)\,\phi(0) - \int_{-1}^{0}\int_{\xi=0}^{\theta}\overline{\psi}(\xi-\theta)\,d\eta(\theta)\,\phi(\xi)\,d\xi, \tag{31}$$

where $\eta(\theta) = \eta(\theta,0)$; $A(0)$ and A^* are adjoint operators. From the above analysis, we obtain that $q(\theta)$ and $q^*(s)$ are eigenvectors of A and A^* corresponding to $i\omega_0\tau_k$ and $-i\omega_0\tau_k$, respectively. Suppose that $q(\theta) = (1,v_1,v_2)^T e^{i\omega_0\tau_k\theta}$ is the eigenvector of $A(0)$ corresponding to $i\omega_0\tau_k$; then $A(0)q(\theta) = i\omega_0\tau_0 q(\theta)$. It follows from the definition of $A(0)$, $L_\mu\phi$, and $\eta(\theta,\mu)$ that

$$q(\theta) = (1,v_1,v_2)^T e^{i\omega_0\tau_k\theta} = q(0)\,e^{i\omega_0\tau_k\theta}, \tag{32}$$

and similarly, by definition of A^*,

$$q^*(\theta) = D(1,v_1^*,v_2^*)^T e^{i\omega_0\tau_k\theta} = q^*(0)\,e^{i\omega_0\tau_k\theta}. \tag{33}$$

Through a simple calculation, we can obtain

$$\begin{aligned}
v_1 &= \frac{-\omega_0c_5e^{-i\omega_0\tau_k} + (\omega_0^2 + c_1c_4)\,i}{\omega_0},\\
v_2 &= \frac{c_3c_5e^{-\omega_0 i\tau_k} - c_2 - \omega_0^2 - (\omega_0c_3 + c_5e^{-\omega_0 i\tau_k})\,i}{c_6e^{-\omega_0 i\tau_k} - c_1c_3 + c_1\omega_0 i},\\
v_1^* &= \frac{c_1c_4 + \omega_0^2 - \omega_0c_5e^{-\omega_0 i\tau_k} i}{c_2\omega_0 i - c_4c_6e^{-\omega_0 i\tau_k}},\\
v_2^* &= \frac{c_2 + \omega_0^2 - c_3c_5e^{-\omega_0 i\tau_k} - (\omega_0c_3 + c_5\omega_0e^{-\omega_0 i\tau_k})\,i}{(\omega_0 i + c_3)\,c_4}.
\end{aligned} \tag{34}$$

From (31), one has

$$\begin{aligned}
\langle q^*(s),q(\theta)\rangle &= \overline{q}^*(0)\,q(0) - \int_{-1}^{0}\int_{\xi=0}^{\theta}\overline{q}^*(\xi-\theta)\,d\eta(\theta)\,q(\xi)\,d\xi\\
&= \overline{D}(1,\overline{v}_1^*,\overline{v}_2^*)(1,v_1,v_2)^T - \int_{-1}^{0}\int_{\xi=0}^{\theta}\overline{D}(1,\overline{v}_1^*,\overline{v}_2^*)\\
&\quad\cdot e^{-\omega_0 i\tau_k(\xi-\theta)}d\eta(\theta)(1,v_1,v_2)^T e^{\omega_0 i\tau_k\xi}\,d\xi = \overline{D}\{1\\
&\quad + \overline{v}_1^*v_1 + \overline{v}_2^*v_2 + \tau_k(c_5 + c_6v_2^*)\,e^{-\omega_0\tau_k i}\}.
\end{aligned} \tag{35}$$

Thus we can choose $\overline{D}$ as

$$\overline{D} = \frac{1}{1 + \overline{v}_1^*v_1 + \overline{v}_2^*v_2 + \tau_k(c_5 + c_6v_2^*)\,e^{-\omega_0\tau_k i}}, \tag{36}$$

such that $\langle q^*(s),q(\theta)\rangle = 1$ and $\langle q^*(s),\overline{q}(\theta)\rangle = 0$.

Next, through the use of the same notations in Hassard et al. [34], we can calculate the coordinates describing the center manifold C_0 at $\mu = 0$. Therefore, we have the following:

$$\begin{aligned}
g_{20} &= 2\overline{D}\tau_k\left[(c_5 + c_6v_2^*)\,e^{-i\omega_0\tau_k} + 1 + c_3\right],\\
g_{11} &= 2\overline{D}\tau_k\left[\left(c_5 + c_6\overline{v}_2^*\,|v_2|^2\right)\operatorname{Re}\{e^{i\omega_0\tau_k}\} + 1 + c_1\right],\\
g_{02} &= 2\overline{D}\tau_k\left[(c_5 + c_6v_2^*)\,e^{i\omega_0\tau_k} + 1 + c_3\right],\\
g_{21} &= \overline{D}\tau_k\left[c_5\left(W_{20}^{(1)}(-1) + W_{11}^{(1)}(-1)\right)\right.\\
&\quad\left. + c_6\left(W_{20}^{(3)}(-1) + W_{11}^{(3)}(-1)\right)\right],
\end{aligned} \tag{37}$$

where

$$
\begin{aligned}
& W_{20}(\theta) \\
& = \frac{i g_{20}}{\omega_0 \tau_k} q(0) e^{i\omega_0\tau_k\theta} + \frac{i\bar{g}_{02}}{3\omega_0\tau_k} \bar{q}(0) e^{-i\omega_0\tau_k\theta} \\
& \quad + E_1 e^{2i\omega_0\tau_k\theta}, \\
& W_{11}(\theta) \\
& = -\frac{i g_{11}}{\omega_0 \tau_k} q(0) e^{i\omega_0\tau_k\theta} + \frac{i\bar{g}_{11}}{\omega_0\tau_k} \bar{q}(0) e^{-i\omega_0\tau_k\theta} + E_2, \\
& E_1 \\
& = 2\begin{bmatrix} c_5 e^{-2i\omega_0\tau_k} & 1 & c_1 + 2i\omega_0 \\ c_2 + 2i\omega_0 & c_3 + 2i\omega_0 & c_6 e^{-2i\omega_0\tau_k} \\ c_4 + 2i\omega_0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} c_5 e^{-i\omega_0\tau_k} \\ c_6 e^{-i\omega_0\tau_k} \\ 0 \end{bmatrix}, \\
& E_2 = 2\begin{bmatrix} c_5 & 1 & c_1 \\ c_2 & c_3 & c_6 \\ c_4 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} c_5 \operatorname{Re}\left\{e^{-i\omega_0\tau_k}\right\} \\ c_6 \operatorname{Re}\left\{e^{-i\omega_0\tau_k}\right\} \\ 0 \end{bmatrix}.
\end{aligned} \tag{38}
$$

Thus, we get the following quantities:

$$
\begin{aligned}
c_1(0) &= \frac{i}{2\omega_0\tau_k}\left(g_{20}g_{11} - 2\left|g_{11}\right|^2 - \frac{1}{3}\left|g_{02}\right|^2\right) + \frac{1}{2}g_{21}, \\
\mu_2 &= -\frac{\operatorname{Re}\left\{c_1(0)\right\}}{\operatorname{Re}\left\{\lambda'(0)\right\}}, \\
T_2 &= -\frac{\operatorname{Im}\left\{c_1(0)\right\} + \mu_2 \operatorname{Im}\left\{\lambda'(0)\right\}}{\omega_0\tau_k}, \\
\beta_2 &= 2\operatorname{Re}\left\{c_1(0)\right\}.
\end{aligned} \tag{39}
$$

From the above analysis, we have the theorem as follows.

Theorem 9. *When τ_k is equal to τ_0^i, the stability and direction of the Hopf bifurcation for system (6) are confirmed by the parameters μ_2, T_2, and β_2.*

(1) *μ_2 determines the direction of the Hopf bifurcation: if $\mu_2 > 0$ and $\tau_k > \tau_0^0$, then the Hopf bifurcation is supercritical; if $\mu_2 > 0$ and $\tau_k > \tau_0^0$, then he Hopf bifurcation is subcritical. In both cases, the bifurcating periodic solutions of system (6) exist.*

(2) *T_2 determines the period of the bifurcating periodic solutions: if $T_2 > 0$, the period increases; else the period decreases.*

(3) *β_2 determines the stability of the bifurcating periodic solutions: the bifurcating periodic solutions are stable (unstable) for $\beta_2 < 0$ ($\beta_2 > 0$).*

5. Numerical Example

Suppose the hydroturbine governing system which is set works under certain operating conditions, the parameters are as follows: $N = 1$, $a_0 = 0.667$, $a_1 = 2.522$, $b_0 = 0.311$, $b_1 = -0.342$. Utilizing the function dde23( ), numerical calculations have been performed as follows.

In Case 1, system (6) has two same time delays τ, initial value $x_0 = (0.05, 0.05, 0.05)$, $K_P = 4.5$, and $K_I = -1.326$; we can obtain the Hopf bifurcation value $\tau_0 = 0.35$ s. When $\tau < \tau_0 = 0.35$ s, at the equilibrium $E^* = (x_1^*, x_2^*, x_3^*)$, system (6) is asymptotically stable, and it is unstable when $\tau > \tau_0$ are shown in Figures 2(b) and 2(d), respectively.

In Case 2, when initial value $x_0 = (0.05, 0.05, 0.05)$, $K_I < 1.7326$, $K_P = 0.5$, and $\tau_1 > 0$, the corresponding oscillation curves of $x_i(t)$ are shown in Figure 3(a). System (6) is asymptotically stable at the equilibrium point $E_0 = (0, 0, 0)$. We can compute the Hopf bifurcation value $\tau_{10} = 0.8322$ s. When $K_I \geq 1.7326$, $\tau_2 = 0$, the equilibrium point $E^* = (x_1^*, x_2^*, x_3^*)$ of system (6) is asymptotically stable for $\tau_1 \in [0, \tau_{10})$, and it is unstable when $\tau_1 > \tau_{10}$ are shown in Figures 4(b) and 4(d), respectively.

In Case 3, when initial value $x_0 = (0.05, 0.05, 0.05)$, $K_P = 5$, $K_I = -1.2$, and $\tau_1 = 0$, we obtain the Hopf bifurcation value $\tau_{20} = 0.455$ s. System (6) is asymptotically stable for $\tau_1 \in [0, \tau_{20})$, and it is unstable when τ_2 is more than τ_{20} at the equilibrium point $E^* = (x_1^*, x_2^*, x_3^*)$, which are shown in Figures 5(b) and 5(d), respectively.

In Case 4, when initial value $x_0 = (0.05, 0.05, 0.05)$, $K_I = -1.5326$, $\tau_{10} = 0.8322$ s, and $\tau_{2*} = 0.5655$ s. The phase portraits are obtained from Theorem 8 in Figures 6(b) and 6(d). When $K_P = 5$, at Hopf point $E^* = (x_1^*, x_2^*, x_3^*)$ system (6) is stable and for each $\tau_2 < \tau_{20}$, but close to τ_{20} there is a stable periodic orbit near the asymptotically stable equilibrium point $E^* = (x_1^*, x_2^*, x_3^*)$. When $\tau_2 = \tau_{2*}$ and $\tau_1 = \tau_{10}$, system (6) has a transversal Hopf point at $E^* = (x_1^*, x_2^*, x_3^*)$ (see Figure 6(d)).

6. Conclusions

In the paper, we establish a PI hydroturbine governing system with saturation and double delays. In the case of positive equilibrium point E^*, the stability of the PI hydroturbine governing system is discussed when the values of the speed control delay and the displacement delay of the servomotor is equal to zero and greater than zero, respectively. The results show that the PI hydroturbine governing system may have unexpected limit cycle oscillation when the delay parameters meet certain conditions. We obtain the scope of three parameters, which determine the stability of periodic solution, the direction of Hopf bifurcation, and the cycle of periodic solutions, respectively. Finally, the theoretical results are validated via the numerical simulation. In addition, a novel approach is proposed to analysis dynamic characteristics of the PI hydroturbine governing system with double delays.

Our work illustrates that the oscillation can be effectively controlled by decreasing speed control delay and setting

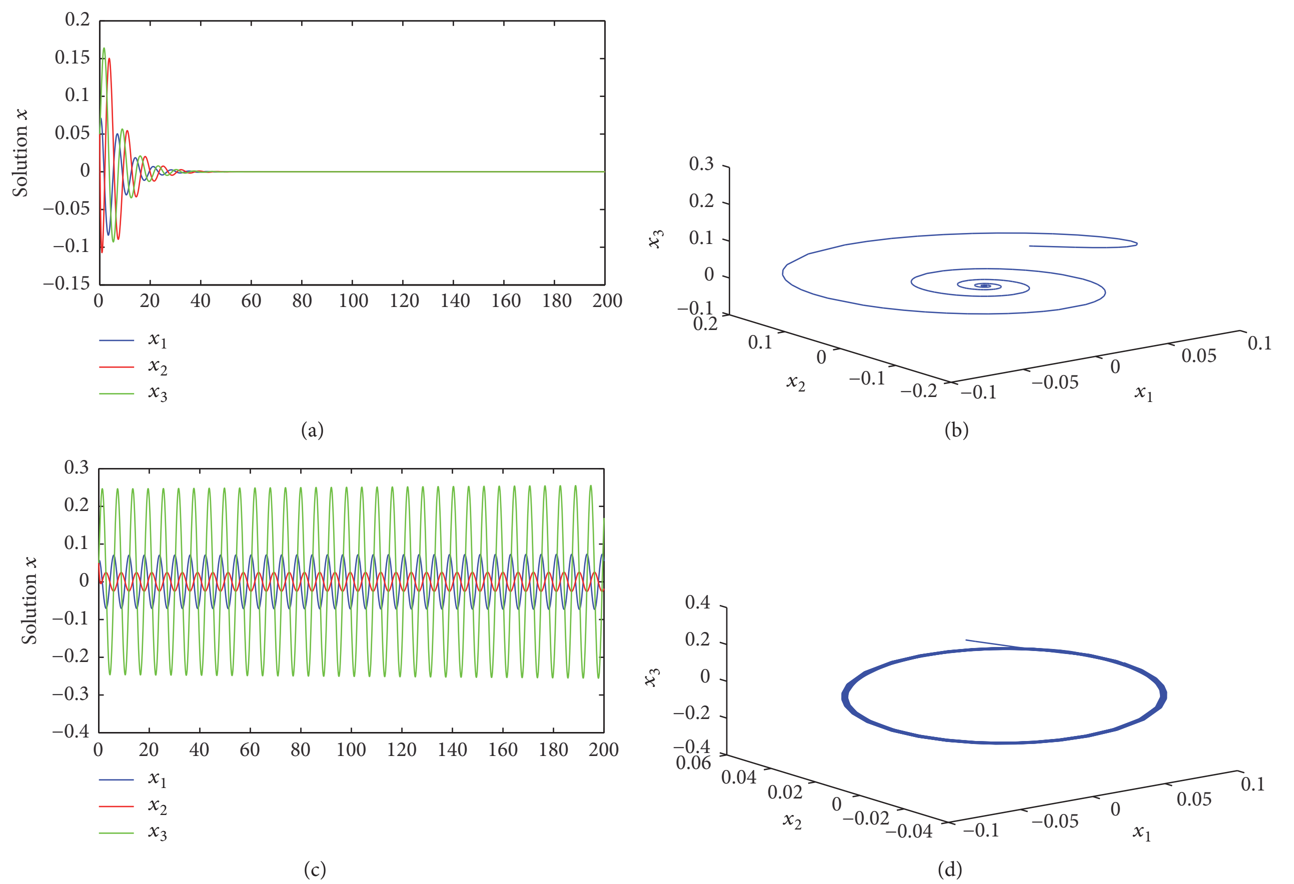

Figure 2: Time waveforms and phase portrait of the PI hydroturbine governing system. (a) Time waveforms with $K_P = 4.5$, $K_I = -1.326$, and $\tau < 0.35$ s. (b) The phase portrait with $K_P = 4.5$, $K_I = -1.326$, and $\tau = 0.15$ s. (c) Time waveforms of $x_i(t)$ with $K_P = 1.1$, $K_I = -3.6326$, and $\tau \geq 0.35$ s. (d) The phase portrait with $K_P = 1.5$ and $\tau = 0.4$ s.

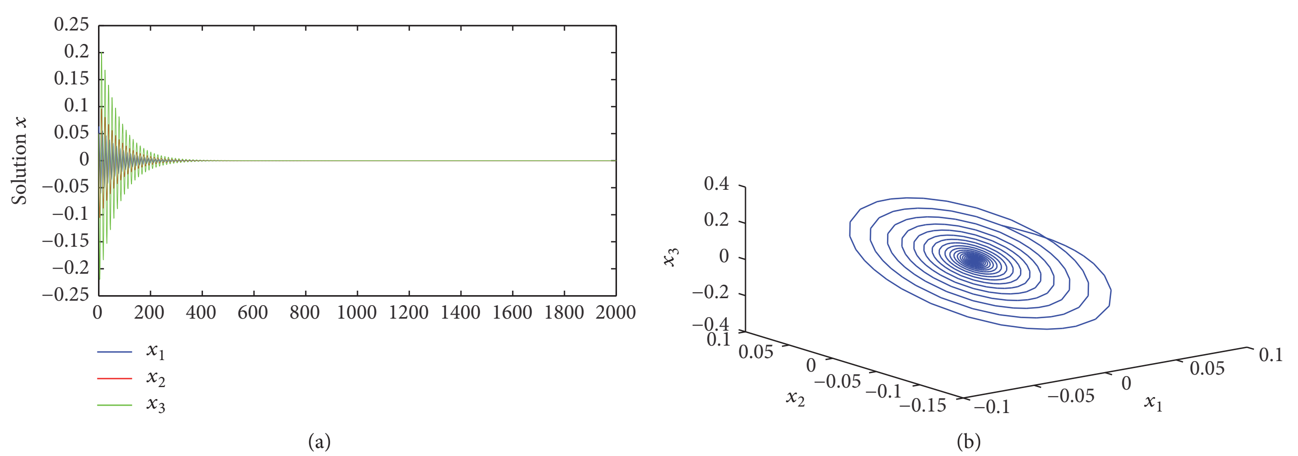

Figure 3: Time waveforms and phase portrait of the PI hydroturbine governing system with the initial value $x_0 = (0.05, 0.05, 0.05)$, $K_I < 1.7326$, and $\tau_2 = 0$. (a) Time waveforms of $x_i(t)$. (b) The phase portrait with $\tau_1 > 0$.

up the high efficiency of PI controller parameters. A time response device is designed to offset the speed control delay in the hydroturbine governing system. Utilizing data analysis method, the accuracy of servomotor displacement can be improved. The research provides theoretical guidance for hydropower station in maintaining the stability of the hydropower system. In the future work, the model of the PI hydroturbine governing system will be constituted by new materials. The rich nonlinear dynamic characteristics of the system will be analyzed accurately by the theory of fractional

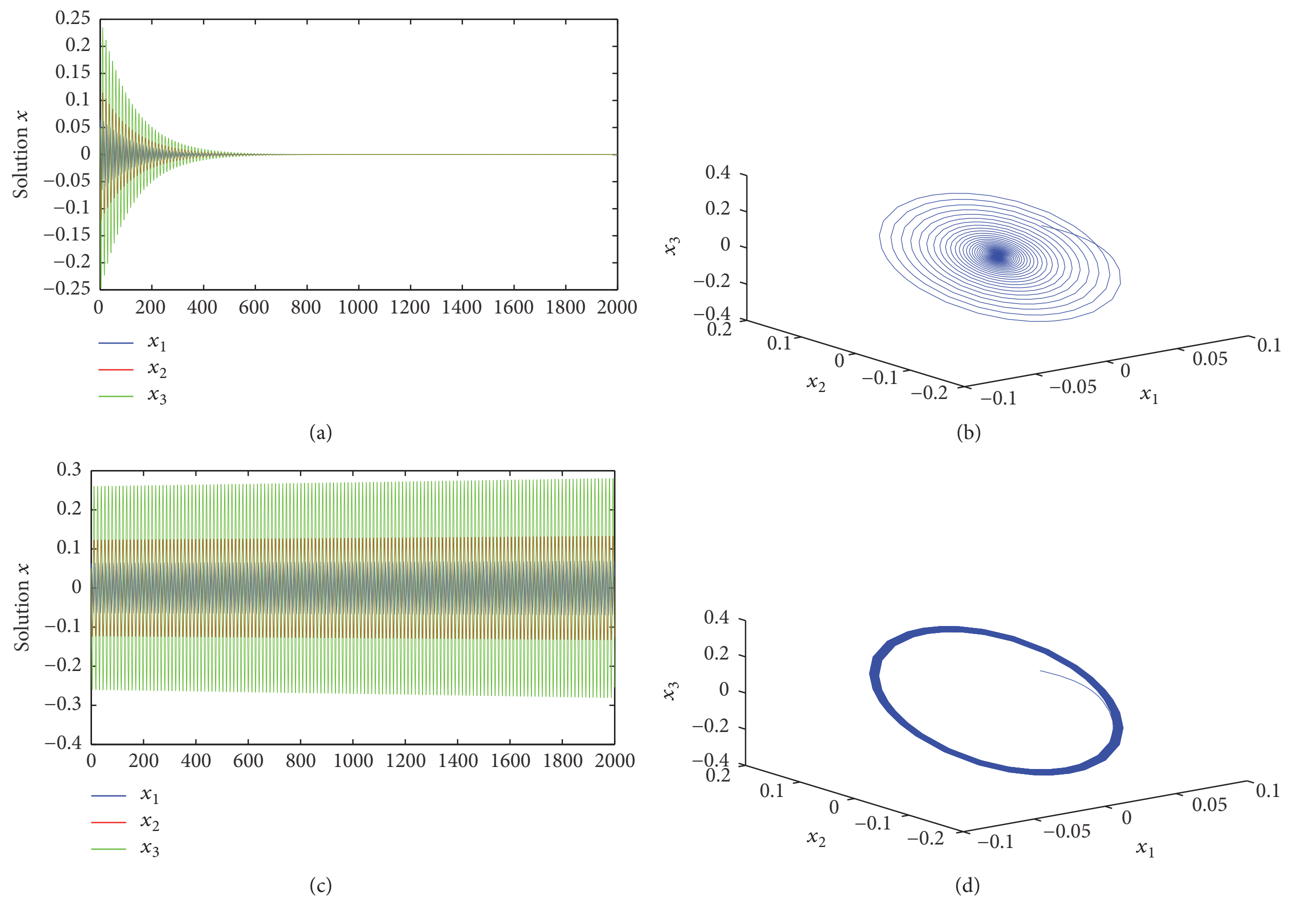

Figure 4: Time waveforms and phase portrait of the PI hydroturbine governing system. (a) Time waveforms with $0 < \tau_1 < \tau_{10}$. (b) The phase portrait with $\tau_1 = 0.75\,\mathrm{s} < \tau_{10}$. (c) Time waveforms for $\tau_1 = 0.92\,\mathrm{s} > \tau_{10}$. (d) The phase portrait with $\tau_1 = 0.84\,\mathrm{s} > \tau_{10}$.

order. These methods and results will provide new ideas to the research of the stability of the hydropower station.

Appendix

Stability Analysis and Hopf Bifurcation

According to time delays τ_1 and τ_2, we analyze the nonlinear dynamic behavior of system (6) under the four different cases.

Case A.1 ($\tau = \tau_1 = \tau_2 \neq 0$). If $\lambda = i\omega$ is a root of (11), then we have

$$\begin{gathered} \omega A_5 \sin(\omega\tau) + \left(A_6 - \omega^2 A_4\right)\cos(\omega\tau) = \omega^2 A_1 - A_3 \\ \omega A_5 \cos(\omega\tau) - \left(A_6 - \omega^2 A_4\right)\sin(\omega\tau) = \omega^3 - \omega A_1. \end{gathered} \tag{A.1}$$

From (A.1), we can obtain

$$\begin{gathered} \sin\omega\tau = \frac{\omega^5 B_1 + \omega^3 B_2 + \omega B_3}{\omega^4 B_1^2 + \omega^2 B_4 + B_5} \\ \cos\omega\tau = \frac{\omega^4 B_6 + \omega^2 B_7 + B_8}{\omega^4 B_1^2 + \omega^2 B_4 + B_5}, \end{gathered} \tag{A.2}$$

where

$$\begin{gathered} B_1 = A_4, \\ B_2 = A_1 A_5 - A_1 A_4 - A_6, \\ B_3 = A_1 A_6 - A_3 A_5, \\ B_4 = A_5^2 - 2A_4 A_6, \\ B_5 = A_6^2, \\ B_6 = A_5 - A_1 A_4, \\ B_7 = A_1 A_6 - A_1 A_5 + A_3 A_4, \\ B_8 = -A_3 A_6. \end{gathered} \tag{A.3}$$

Then we have (12). Let $z = \omega^2$; we have

$$z^5 + z^4 C_1 + z^3 C_2 + z^2 C_3 + z C_4 + C_5 = 0. \tag{A.4}$$

To analyze the existence and distribution of roots of (A.4), we have (14) and the following:

$$h'(z) = 5z^4 + 4z^3 C_1 + 3z^2 C_2 + 2z C_3 + C_4. \tag{A.5}$$

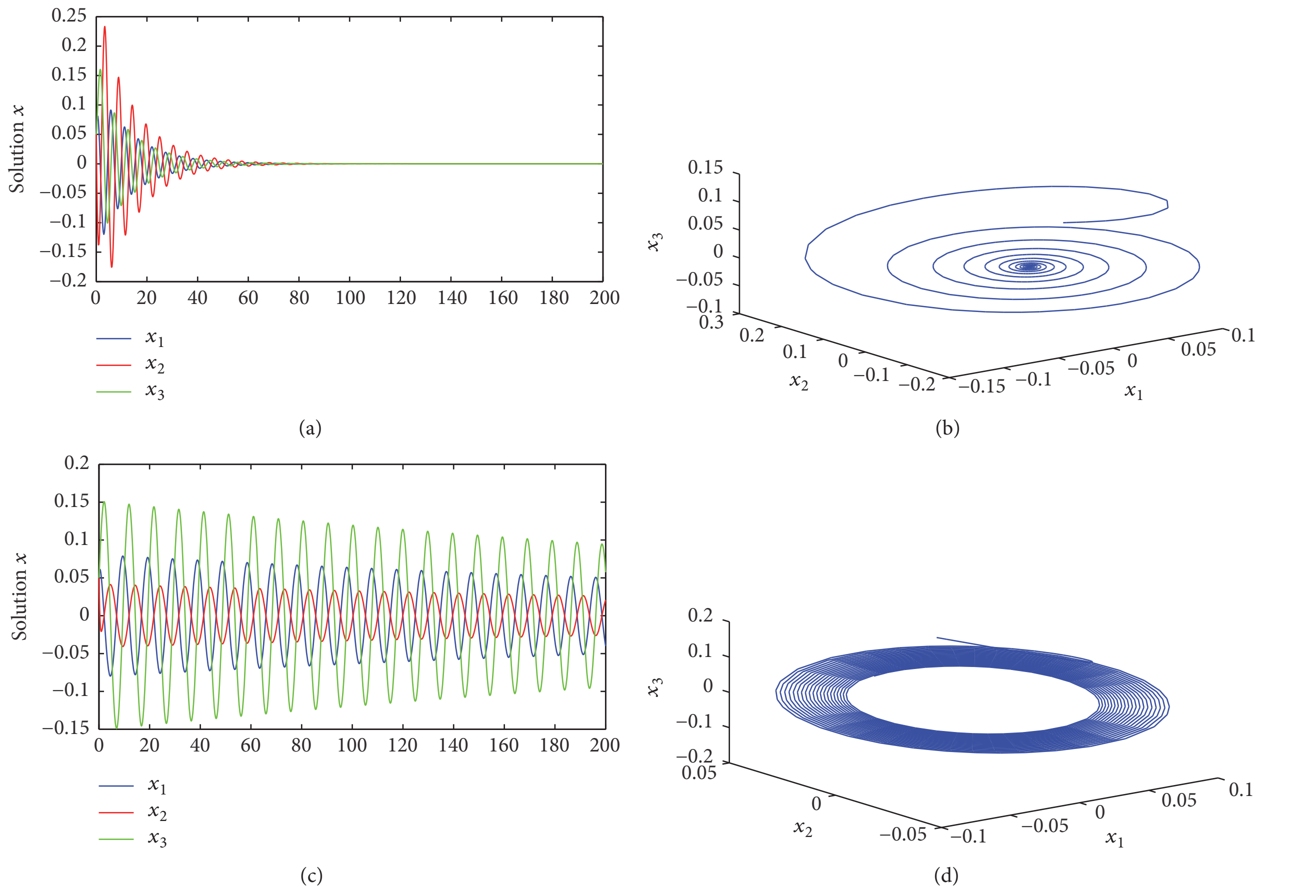

FIGURE 5: The corresponding oscillation curves and phase portrait of the PI hydroturbine governing system. (a) The corresponding oscillation curves for $K_P = 5$ and $\tau_2 \in [0, \tau_{20})$. (b) The phase portrait with $K_P = 5$ and $\tau_2 = 0.35\,\text{s} \in [0, \tau_{20})$. (c) The corresponding oscillation curves with $K_P = 1.5$ and $\tau_2 \geq \tau_{20}$. (d) The phase portrait with $K_P = 1.5$ and $\tau_2 = 0.46\,\text{s} \geq \tau_{20}$.

From the above analysis, we have Lemma 2. The proof is as follows.

Proof. We denote

$$\begin{aligned}
P &= \left[3\lambda^2 + 2A_1\lambda + A_2 + \left(2A_4\lambda + A_5\right)e^{-\lambda\tau}\right]_{\lambda=\omega i} \\
&= -3\omega^2 + A_2 + A_5\cos(\omega\tau) + 2A_4\omega\sin(\omega\tau) \\
&\quad + \left(2A_4\omega\cos(\omega\tau) + 2\omega A_1 - A_5\sin(\omega\tau)\right)i \coloneqq P_R \\
&\quad + P_I i, \\
Q &= \left[-\lambda\left(A_4\lambda^2 + A_5\lambda + A_6\right)e^{-\lambda\tau}\right]_{\lambda=\omega i} = \omega^3 A_4 \\
&\quad \cdot \sin(\omega\tau) + \omega^2 A_5\cos(\omega\tau) - \omega A_6\sin(\omega\tau) \\
&\quad + \left(\omega^3 A_4\cos(\omega\tau) - \omega^2 A_5\sin(\omega\tau)\right. \\
&\quad \left. - \omega A_6\cos(\omega\tau)\right)i \coloneqq Q_R + Q_I i.
\end{aligned} \tag{A.6}$$

Taking the derivative of λ with respect to τ in (11), one can get

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{3\lambda^2 + 2A_1\lambda + A_2 + \left(2A_4\lambda + A_5\right)e^{-\lambda\tau}}{-\lambda\left(A_4\lambda^2 + A_5\lambda + A_6\right)e^{-\lambda\tau}} - \frac{\tau}{\lambda}. \tag{A.7}$$

By substituting $\lambda = \omega i$ into (A.7) we have

$$\begin{aligned}
&\left(\frac{d\,\mathrm{Re}\,\lambda(\tau)}{d\tau}\right)^{-1}_{\tau=\tau_k^{(i)}} \\
&= \mathrm{Re}\left[\frac{3\lambda^2 + 2A_1\lambda + A_2 + \left(2A_4\lambda + A_5\right)e^{-\lambda\tau}}{-\lambda\left(A_4\lambda^2 + A_5\lambda + A_6\right)e^{-\lambda\tau}}\right]_{\tau=\tau_k^{(i)}} \\
&= \frac{P_R Q_R + P_I Q_I}{P_R^2 + Q_I^2}.
\end{aligned} \tag{A.8}$$

If $H_2 = P_R Q_R + P_I Q_I \neq 0$, we can obtain

$$\left(\frac{d\,\mathrm{Re}\,\lambda(\tau)}{d\tau}\right)_{\tau=\tau_k^{(i)}} \neq 0. \tag{A.9}$$

This proves Lemma 2. □

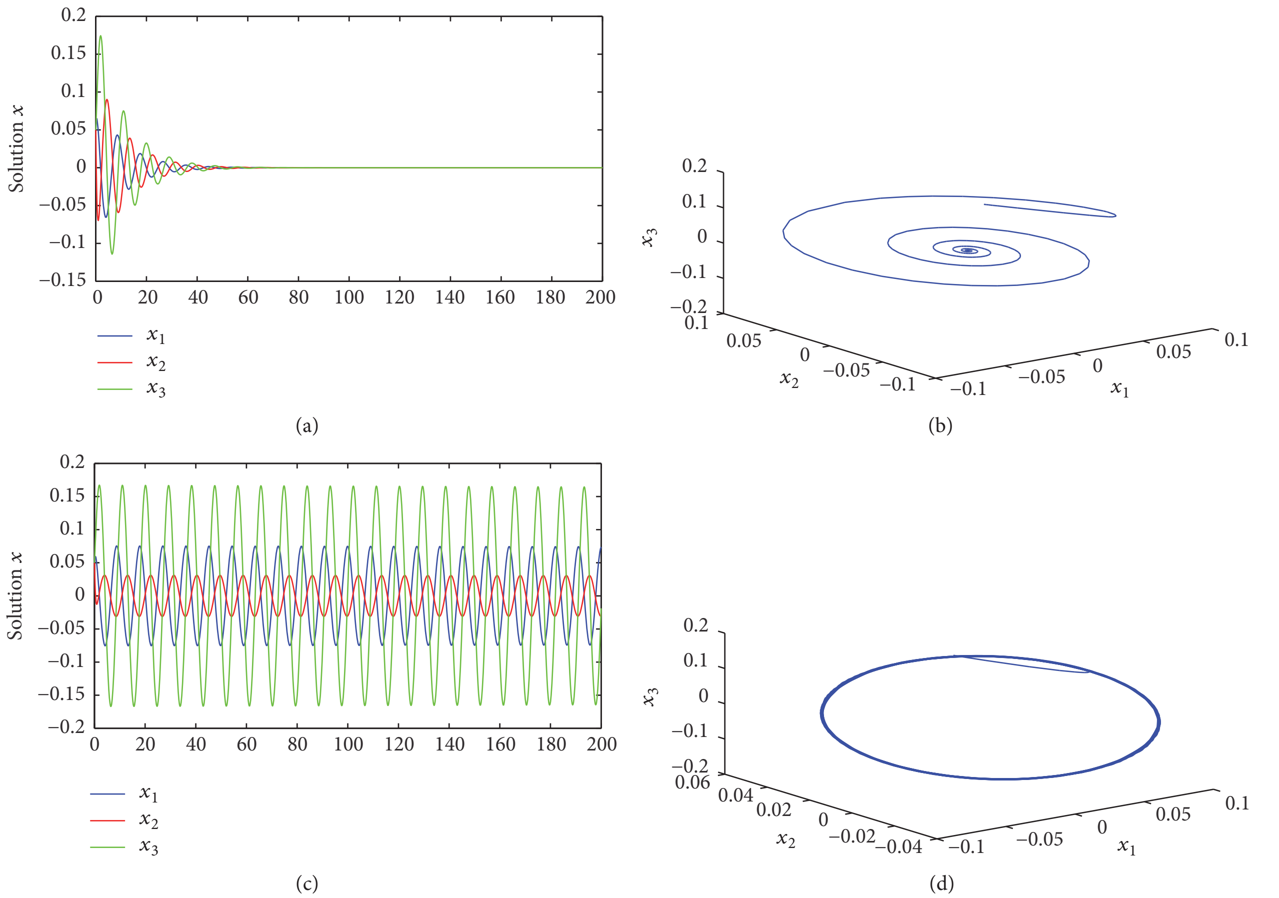

FIGURE 6: The corresponding oscillation curves and phase portrait of the PI hydroturbine governing system. (a) The corresponding oscillation curves with $K_P = 5$ and $\tau_2 \in [0, \tau_{20})$. (b) The phase portrait with $K_P = 5$ and $\tau_2 \in [0, \tau_{20})$. (c) The corresponding oscillation curves with $K_P = 1.2$, $\tau_1 = 0.01$ s, and $\tau_2 \geq \tau_{2*}$. (d) Hopf bifurcation occurs for $K_P = 1.2$, $\tau_1 = 0.01$ s, and $\tau_2 = \tau_{2*}$.

Case A.2 ($\tau_1 \neq 0, \tau_2 = 0$). Let $\lambda = i\omega$ be a root of (16), then we have

$$\begin{aligned}&A_3 + A_6 - \omega^2 A_4 \cos(\omega\tau) - \omega^2 A_1 + \omega A_5 \sin(\omega\tau)\\&\quad + \left(\omega A_5 \cos(\omega\tau) + \omega A_2 - \omega^3 + \omega^2 A_4 \sin(\omega\tau)\right) i\\&= 0.\end{aligned} \tag{A.10}$$

The real and imaginary parts are separated; we obtain

$$\begin{aligned}-\omega^2 A_4 \cos(\omega\tau) + \omega A_5 \sin(\omega\tau) &= \omega^2 A_1 - A_3 - A_6\\ \omega A_5 \cos(\omega\tau) + \omega^2 A_4 \sin(\omega\tau) &= \omega^3 - \omega A_2.\end{aligned} \tag{A.11}$$

From (A.11), we can obtain

$$\begin{aligned}&\sin(\omega\tau)\\&\quad = \frac{\omega^4 A_4 + \omega^2 (A_1 A_5 - A_2 A_4) - (A_3 A_5 + A_5 A_6)}{\omega^3 A_4^2 + \omega A_5^2}\\&\cos(\omega\tau)\\&\quad = \frac{\omega^3 (A_5 - A_1 A_4) + \omega (A_3 A_4 - A_2 A_5 + A_4 A_6)}{\omega^3 A_4^2 + \omega A_5^2}.\end{aligned} \tag{A.12}$$

It follows that

$$\omega^8 + \omega^6 D_1 + \omega^4 D_2 + \omega^2 D_3 + D_4 = 0, \tag{A.13}$$

where

$$\begin{aligned}D_1 &= \frac{\left(A_1^2 A_4^2 + A_5^2 - A_4^4 - 2A_2 A_4^2\right)}{A_4^2},\\ D_2 &= \frac{\left(-2A_4^2 A_5^2 + A_1^2 A_5^2 + A_2^2 A_4^2 - 2A_2 A_5^2 - 2A_1 A_3 A_4^2 - 2A_1 A_4^2 A_6\right)}{A_4^2},\end{aligned}$$

$$D_3 = \frac{\left(-A_5^4 + A_3^2A_4^2 + A_4^2A_6^2 + A_2^2A_5^2 + 2A_3A_4^2A_6 - 2A_1A_3A_5^2 - 2A_1A_5^2A_6\right)}{A_4^2},$$

$$D_4 = \frac{\left(A_3A_5 + A_5A_6\right)^2}{A_4^2}. \tag{A.14}$$

Then we obtain (17).

Assume that (17) has positive roots. Without losing generality, we suppose that (17) has four positive roots, which are defined by z_1, z_2, z_3, and z_4. Then, (17) has four positive roots $\omega_k = \sqrt{z_k}$, $k = 1, 2, 3, 4$.

From the above analysis, we have Lemma 4. The proof is as follows.

Proof. Taking the derivative of λ with respect to τ_1 in (16), we have

$$\left(\frac{d\lambda}{d\tau_1}\right)^{-1} = \frac{\left(3\lambda^2 + 2A_1\lambda + A_2\right)e^{\lambda\tau_1}}{\lambda\left(A_4\lambda^2 + A_5\lambda\right)} + \frac{2A_4\lambda + A_5}{\lambda\left(A_4\lambda^2 + A_5\lambda\right)} - \frac{\tau_1}{\lambda}. \tag{A.15}$$

In addition, we have

$$\begin{aligned} \left[\lambda\left(A_4\lambda^2 + A_5\lambda\right)\right]_{\lambda=\omega_k i} &= -\omega_k{}^2A_5 - \omega_k{}^3A_4 i, \\ \left[3\lambda^2 + 2A_1\lambda + A_2\right]_{\lambda=\omega_k i} &= A_2 - 3\omega_k{}^2 + 2A_1\omega_k i, \\ \left[2A_4\lambda + A_5\right]_{\lambda=\omega_k i} &= A_5 + 2A_4\omega_k i. \end{aligned} \tag{A.16}$$

For simplification of analysis, we define ω_k as ω and $\tau_k^{(i)}$ as τ_1. From (A.15)-(A.16) and (A.12), we can obtain

$$\begin{aligned} &\left(\frac{d\operatorname{Re}\lambda\left(\tau_{10}\right)}{d\tau_1}\right)^{-1}_{\tau_1=\tau_k^{(i)}} \\ &= \operatorname{Re}\left[\frac{\left(3\lambda^2 + 2A_1\lambda + A_2\right)e^{\lambda\tau_1}}{\lambda\left(A_4\lambda^2 + A_5\lambda\right)}\right]_{\tau_1=\tau_k^{(i)}} \\ &+ \operatorname{Re}\left[\frac{2A_4\lambda + A_5}{\lambda\left(A_4\lambda^2 + A_5\lambda\right)}\right]_{\tau_1=\tau_k^{(i)}} = \frac{1}{\Lambda}\left\{-\omega\left(A_2\right.\right. \\ &\left.- 3\omega^2\right)\left[\omega A_5\cos\left(\omega\tau\right) + \omega^2A_4\sin\left(\omega\tau\right)\right] \\ &+ 2A_1\omega^2\left[\omega A_5\sin\left(\omega\tau\right) - \omega^2A_4\cos\left(\omega\tau\right)\right] \\ &\left.- 2\omega^4A_4^2 - \omega^2A_5^2\right\} = \frac{1}{\Lambda}\left\{4A_4^2\omega^6 + 3\omega^4\left(A_1^2A_4^2\right.\right. \\ &\left.+ A_5^2 - A_4^4 - 2A_2A_4^2\right) + 2\omega^2\left(-2A_4^2A_5^2 + A_1^2A_5^2\right. \\ &\left.+ A_2^2A_4^2 - 2A_2A_5^2 - 2A_1A_3A_4^2 - 2A_1A_4^2A_6\right) \\ &+ A_3^2A_4^2 - A_5^4 + A_4^2A_6^2 + A_2^2A_5^2 + 2A_3A_4^2A_6 \\ &\left.- 2A_1A_3A_5^2 - 2A_1A_5^2A_6\right\} = \frac{1}{\Lambda}\cdot h'\left(z_k\right), \end{aligned} \tag{A.17}$$

where $\Lambda = \omega^6A_4^4 + \omega^4A_5^2A_4^2$. Thus we have

$$\begin{aligned} &\operatorname{sign}\left[\frac{d\operatorname{Re}\lambda\left(\tau_{10}\right)}{d\tau_1}\right]_{\tau_1=\tau_k^{(i)}} \\ &= \operatorname{sign}\left[\frac{d\operatorname{Re}\lambda\left(\tau_{10}\right)}{d\tau_1}\right]^{-1}_{\tau_1=\tau_k^{(i)}} = \operatorname{sign}\left[\frac{h'\left(z_k\right)}{\Lambda}\right] \\ &\neq 0. \end{aligned} \tag{A.18}$$

Furthermore, since $z_k > 0$, we conclude that $[d\operatorname{Re}\lambda(\tau_{10})/d\tau_1]_{\tau_1=\tau_k^{(i)}}$ and $h'(z_k)$ have the same sign. □

Case A.3 ($\tau_1 = 0, \tau_2 \neq 0$). By letting $\lambda = i\omega$ be the root of (21), we have

$$\begin{aligned} \cos\left(\omega\tau_2\right) &= \frac{\left(A_1\omega^2 - A_3\right)}{A_6} \\ \sin\left(\omega\tau_2\right) &= \frac{\left(A_2\omega - \omega^3\right)}{A_6}. \end{aligned} \tag{A.19}$$

It follows that

$$\omega^6 + E_1\omega^4 + E_2\omega^2 + E_3 = 0, \tag{A.20}$$

where $E_1 = -2A_2$, $E_2 = A_1^2 + A_2^2 - 2A_1A_3$, $E_3 = A_3^2 - A_6^2$.

If we define $z = \omega^2$, we have (22); then we also obtain Lemma 6 and Theorem 7.

Case A.4 ($\tau_1 \neq \tau_2, \tau_1 > 0$, and $\tau_2 > 0$). Without losing generality, we consider system (6) in Case 2. Let ωi ($\omega > 0$) be a root of (8), then we obtain

$$\begin{aligned} &-\omega^2A_4\cos\left(\omega\tau_1\right) + \omega A_5\sin\left(\omega\tau_1\right) \\ &\quad= \omega^2A_1 - A_3 - A_6\cos\left(\omega\tau_2\right) \\ &\omega A_5\cos\left(\omega\tau_1\right) + \omega^2A_4\sin\left(\omega\tau_1\right) \\ &\quad= \omega^3 - A_2\omega + A_6\sin\left(\omega\tau_2\right). \end{aligned} \tag{A.21}$$

Eliminating τ_2, we obtain (24).

If (24) has finite positive roots, we define the roots of (24) as $\omega_1, \omega_2, \ldots, \omega_k$, such that there is a sequence $\{\tau_{2_i}^{(j)} \mid j = 1, 2, \ldots\}$ for every fixed ω_i $(i = 1, 2, \ldots, k)$.

From (A.21), if we denote

$$\tau_{2_i}^{(j)} = \frac{1}{\omega_i}\left\{\arccos\left(\frac{\omega A_5 \sin(\omega\tau_1) - \omega^2 A_4 \cos(\omega\tau_1) + A_3 - A_1\omega^2}{A_6}\right) + 2j\pi\right\}, \quad i = 1, 2, \ldots, k, \; j = 0, 1, 2, \ldots \tag{A.22}$$

then $\pm i\omega_i$ is a pair of purely imaginary roots of (24) with $\tau_2 = \tau_{2_i}^{(j)}$.

Let $\tau_{2*} = \{\min(\tau_{2_i}^{(j)}) \mid i = 1, 2, \ldots, k, \; j = 1, 2, \ldots\}$, when $\tau_2 = \tau_{2*}$, (24) has a pair of purely imaginary roots $\pm i\omega_*$ for $\tau_1 \in [0, \tau_{10})$.

Next, in order to verify the transversality condition of Hopf bifurcation, we differentiate (8) with respect to τ_2 and substitute $\tau_2 = \tau_{2*}$; then we have

$$\left(\frac{d\,\mathrm{Re}\,\lambda(\tau_2)}{d\tau_2}\right)^{-1}_{\tau_2=\tau_{2*}} = -\frac{P_R Q_R + P_I Q_I}{P_R^2 + Q_I^2}, \tag{A.23}$$

where

$$\begin{aligned}
P_R &= A_6 \cos(\omega\tau_2), \\
P_I &= -A_6 \sin(\omega\tau_2), \\
Q_R &= -3\omega^2 + \cos(\omega\tau_1)\left(A_5 + A_4\omega^2\right) \\
&\quad + \sin(\omega\tau_1)\left(2A_2A_4\omega - A_5\omega\right), \\
Q_I &= 2A_1\omega + \cos(\omega\tau_1)\left(2A_2A_4 - A_5\omega\right) \\
&\quad - \sin(\omega\tau_1)\left(A_5 + A_4\omega^2\right),
\end{aligned} \tag{A.24}$$

$$P_R Q_R + P_I Q_I \neq 0.$$

Therefore, Theorem 8 is true.

Nomenclature

$G_t(s)$: The transfer function of the guide vane to the hydroturbine moment
$G_s(s)$: The transfer function from the Hydroturbine moment to the speed of the generator
e_y: Servomotor stroke transfer coefficient of the turbine torque (p.u.)
e_{qy}: Servomotor stroke transfer coefficient of the flow rate (p.u.)
e_{qh}: The transfer coefficient of the flow rate on the water head of a turbine (p.u.)
e_h: The transfer coefficient of the torque on the water head of a turbine (p.u.)
T_a: The sum of the machine starting time and load time constant (s)
K_P: The proportional component
P: The load perturbation
m_t: Turbine output torque (N·m)
C: The reference input
x_1, x_2, x_3: The intermediate variables
s: The strength of the elastic water hammer effect
T_ω: The water inertia time constant of a pressure guide-water system (s)
e_x: The transfer coefficient of a speed on the turbine torque (p.u.)
y: The displacement of the servomotor (m)
e_g: The load self-regulation factor (p.u.)
τ_1: The speed control delay of the generator (s)
τ_2: The displacement-control delay of the servomotor (s)
K_I: The integral component
x: The output of the system
z: The nonlinear part
N: The constant of nonlinear part
e, e_n: The intermediate variables.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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