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Research Article

Exact Solutions for $(4 + 1)$-Dimensional Nonlinear Fokas Equation Using Extended F-Expansion Method and Its Variant

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The construction of exact solution for higher-dimensional nonlinear equation plays an important role in knowing some facts that are not simply understood through common observations. In our work, $(4+1)$-dimensional nonlinear Fokas equation, which is an important physical model, is discussed by using the extended F-expansion method and its variant. And some new exact solutions expressed by Jacobi elliptic function, Weierstrass elliptic function, hyperbolic function, and trigonometric function are obtained. The related results are enriched.

1. Introduction

It has recently become more interesting to obtain exact solutions of nonlinear partial differential equations. These equations are mathematical models of complex physical phenomena that arise in engineering, applied mathematics, chemistry, biology, mechanics, physics, and so forth. Thus, the investigation of the traveling wave solutions to nonlinear evolution equations (NLEEs) plays an important role in mathematical physics. A lot of physical models have supported a wide variety of solitary wave solutions.

In the recent years, much efforts have been spent on this task and many significant methods have been established such as inverse scattering transform [1], Backlund and Darboux transform [2], Hirota [3], homogeneous balance method [4], symmetry reductions method [5–8], Jacobi elliptic function method [9], tanh-function method [10], exp-function method [11–13], simple equation method [14], the meshless methods [15–20], G'/G-expansion method [21–23], F-expansion method [24, 25], improved F-expansion method [26, 27], and extended F-expansion method [28].

Wang and Li [25] developed a new algebraic method, belonging to the simplest equation method [29–32], to seek more new solutions of NLEEs that can be expressed as polynomial in an elementary function which satisfies a more general subequation than other subequations like Riccati equation, auxiliary ordinary equation, elliptic equation, and generalized Riccati equation. The Fans method not only gives a unified formation to construct various traveling wave solutions but also provides a guideline to classify the various types of traveling wave solutions according to five parameters. An extended F-expansion method is proposed by Yomba in 2005 by giving more solutions of the general subequation. Using the new method, exact solutions of many NLEEs are successfully obtained [28].

The higher-dimensional integrable model is one of the important problems in mathematical physics which can be obtained from several lower-dimensional integrable equations by extending Lax pairs to higher dimensions. Fokas [33] extended the integrable Kadomtsev-Petviashvili and Davey-Stewartson equation to present a new $(4 + 1)$-dimensional (4-dimensional space and one-dimensional time) nonlinear wave equation which is given by

$$4u_{tx} - u_{xxxy} + u_{xyyy} + 12u_x u_y + 12uu_{xy} - 6u_{zw} = 0. \quad (1)$$

Due to important applications of higher-dimensional equations in real world problems, it is necessary to investigate its analytic solutions. Therefore, (1) is studied by many authors. In [34], Yang and Yan investigated symmetries of (1) including point symmetries and the potential symmetries. In [35], Lee et al. discussed the exact solutions of (1) by modified tanh-coth method, extended Jacobi elliptic function method, and the exp-function method. In [36], Kim obtained a group of exact solutions using G'/G-expansion method.

In [37], we have successfully applied the extended F-expansion method on a higher-order wave equation of KdV type. In this work, we apply this method and its variant on (4 + 1)-dimensional nonlinear Fokas equation for obtaining new exact traveling solutions. Compared with [37], the subequation is discussed more in detail. Besides the four cases discussed in [37], the other two cases of the subequation are also considered, that is, $h_0 = h_4 = 0$ and $h_2 = h_4 = 0$, so we can obtain richer results. In addition, in present paper two forms of solution are adopted. They are (4) and (5). Equation (4) is the same with the form in [37]. However, (5) is a new form, by which some new solutions can be obtained. The details can be found in Section 3.

The organization of the paper is as follows: in Section 2, a brief description of the extended F-expansion for finding traveling wave solutions of nonlinear equations is given. In Section 3, we will study (1) by the extended F-expansion methods. Finally conclusions are given in Section 4.

2. Description of the Extended F-Expansion Methods

Based on F-expansion method, the main procedures of the extended F-expansion method are as follows [22].

Step 1. Consider a general nonlinear PDE in the form

$$F\left(u, u_x, u_t, u_{xx}, u_{xt}, \ldots\right) = 0. \tag{2}$$

Using $u(x,t) = U(\xi)$, $\xi = x - ct$, we can rewrite (2) as the following nonlinear ODE:

$$F\left(U, U', U'', \ldots\right) = 0, \tag{3}$$

where the prime denotes differentiation with respect to ξ.

Step 2. Suppose that the solution of ODE (3) can be written as follows:

$$U(\xi) = A_0 + \sum_{i=1}^{n}\left(A_i F^i(\xi) + B_i F^{-i}(\xi)\right) \tag{4}$$

or

$$U(\xi) = A_0 + \sum_{i=1}^{n}\left(A_i F^i(\xi) + B_i F^{i-1} F'(\xi)\right), \tag{5}$$

where A_i, B_i $(i = 1, 2, \ldots n)$ are constants to be determined later, n is a positive integer that is given by the homogeneous balance principle, and $F(\xi)$ satisfies the following equation:

$$\left(F'(\xi)\right)^2 = h_0 + h_1 F(\xi) + h_2 F^2(\xi) + h_3 F^3(\xi) + h_4 F^4(\xi), \tag{6}$$

where h_0, h_1, h_2, h_3, h_4 are constant.

Step 3. Substituting (4) or (5) along with (6) into (3) and then setting all the coefficients of $F^j(\xi) F'^k(\xi)$ $(j = 1, 2, \ldots, k = 0, 1)$ of the resulting system to zero yields a set of over-determined nonlinear algebraic equations for A_0, A_i, B_i $(i = 1, 2, \ldots n)$.

Step 4. Assuming that the constants A_0, A_i, B_i $(i = 1, 2, \ldots n)$ can be obtained by solving the algebraic equations in Step 3 and then substituting these constants and the solutions of (6), depending on the special conditions chosen for the h_0, h_1, h_2, h_3, and h_4 into (4) (or (5)), we can obtain the explicit solutions of (2) immediately.

3. Exact Solutions of (1)

Making a transformation $u(x, y, z, t, w) = \phi(\xi)$ with $\xi = \alpha x + \beta y + \gamma z + \delta w + \epsilon t$, (1) can be reduced to the following ODE:

$$\begin{aligned}&\left(\alpha\beta^3 - \alpha^3\beta\right)\phi^{(4)} + \left(4\alpha\epsilon - 6\gamma\delta\right)\phi'' \\ &\quad + 12\alpha\beta\left(\phi'\right)^2 + 12\alpha\beta\phi\phi'' = 0,\end{aligned} \tag{7}$$

where α, β, γ, δ, and ε are nonzero constants. Balancing $\phi^{(4)}$ and $(\phi')^2$ in (7), we obtain $n + 4 = 2n + 2$ which gives $n = 2$.

Suppose that (7) owns the solutions in the form

$$\phi(\xi) = A_0 + A_1 F(\xi) + A_2 F^2(\xi) + \frac{B_1}{F(\xi)} + \frac{B_2}{F^2(\xi)}. \tag{8}$$

Substituting (8) and (6) into (7) and then setting all the coefficients of $F^k (k = -6, \ldots, 6)$ of the resulting system to zero, we can obtain the following results.

3.1. $h_3 = h_4 = 0$. In this situation, we obtain the following set of nontrivial solutions:

$$\begin{aligned}&\left\{A_1 = A_2 = B_1 = 0, B_2 = h_0\left(\alpha^2 - \beta^2\right),\right. \\ &\left.\quad \epsilon = \frac{-2\alpha\beta^3 h_2 + 3\gamma\delta - 6\alpha\beta A_0 + 2\alpha^3\beta h_2}{2\alpha}\right\}, \\ &\left\{A_1 = B_1 = B_2 = 0, A_2 = h_4\left(\alpha^2 - \beta^2\right),\right. \\ &\left.\quad \epsilon = \frac{-2\alpha\beta^3 h_2 + 3\gamma\delta - 6\alpha\beta A_0 + 2\alpha^3\beta h_2}{2\alpha}\right\}, \\ &\left\{A_1 = B_1 = 0, A_2 = h_4\left(\alpha^2 - \beta^2\right), B_2 = h_0\left(\alpha^2 - \beta^2\right),\right. \\ &\left.\quad \epsilon = \frac{-2\alpha\beta^3 h_2 + 3\gamma\delta - 6\alpha\beta A_0 + 2\alpha^3\beta h_2}{2\alpha}\right\},\end{aligned} \tag{9}$$

where A_0, h_0, h_2, and h_4 are arbitrary constants and α, β, γ, and δ are nonzero constants.

Table 1: Solutions of $F(\xi)$ in $F'^2 = h_0 + h_2\ F^2 + h_4\ F^4$.

Case	h_0	h_2	h_4	$F(\xi)$
1	1	$-(m^2+1)$	m^2	$\text{sn}(\xi), \text{cd}(\xi)$
2	$1-m^2$	m^2-1	$-m^2$	$\text{cn}(\xi)$
3	m^2-1	$2-m^2$	-1	$\text{dn}(\xi)$
4	m^2	$-(m^2+1)$	1	$\text{ns}(\xi), \text{dc}(\xi)$
5	$-m^2$	$2m^2-1$	$1-m^2$	$\text{nc}(\xi)$
6	-1	$2-m^2$	m^2-1	$\text{nd}(\xi)$
7	1	$2-m^2$	$1-m^2$	$\text{sc}(\xi)$
8	1	$2m^2-1$	$-m^2(1-m^2)$	$\text{sd}(\xi)$
9	$1-m^2$	$2-m^2$	1	$\text{cs}(\xi)$
10	$-m^2(1-m^2)$	$2m^2-1$	1	$\text{sd}(\xi)$
11	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$\text{ns}(\xi) \pm \text{cs}(\xi)$
12	$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{4}$	$\text{nc}(\xi) \pm \text{sc}(\xi)$
13	$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$\text{ns}(\xi) \pm \text{ds}(\xi)$
14	$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\text{sn}(\xi) \pm i\,\text{cn}(\xi)$

Substituting (9) into (8), we obtain, respectively, the following formal solution of (1):

$$u(x,y,z,w,t) = A_0 + \frac{h_0(\alpha^2-\beta^2)}{F^2(\xi)}, \tag{10}$$

$$u(x,y,z,w,t) = A_0 + h_4(\alpha^2-\beta^2)F^2(\xi), \tag{11}$$

$$\begin{aligned} u(x,y,z,w,t) = {} & A_0 + h_4(\alpha^2-\beta^2)F^2(\xi) \\ & + \frac{h_0(\alpha^2-\beta^2)}{F^2(\xi)}, \end{aligned} \tag{12}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + \epsilon t$ and α, β, γ, δ, and ϵ are determined in (9).

When $h_1 = h_3 = 0$, the general elliptic equation (6) is reduced to the auxiliary ordinary equation

$$F'(\xi)^2 = h_0 + h_2F^2(\xi) + h_4F^4(\xi). \tag{13}$$

The solutions of (13) are given in Table 1. Combining (10)–(12) with Table 1, many exact solutions of (1) can be obtained. For simplicity, we just give out one case in Table 1; the other cases can be discussed similarly.

When $h_0 = 1$, $h_2 = -(m^2+1)$, $h_4 = m^2$, the solution of (13) is $F(\xi) = \text{sn}(\xi,m)$ or $F(\xi) = \text{cd}(\xi,m)$. Substituting them into (10)–(12), we can obtain the following Jacobi Elliptic function solutions of (1).

Form (10), one has

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{(\alpha^2-\beta^2)}{\text{sn}^2(\xi,m)} \\ &= A_0 + (\alpha^2-\beta^2)\,\text{ns}^2(\xi,m), \end{aligned} \tag{14}$$

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{(\alpha^2-\beta^2)}{\text{cd}^2(\xi,m)} \\ &= A_0 + (\alpha^2-\beta^2)\,\text{dc}^2(\xi,m), \end{aligned} \tag{15}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((-2\alpha\beta^3(-m^2-1) + 3\gamma\delta - 6\alpha\beta A_0 + 2\alpha^3\beta(-m^2-1))/2\alpha)t$.

When $m \to 1$, $\text{sn}(\xi,m) \to \tanh(\xi)$, solution (14) becomes

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{(\alpha^2-\beta^2)}{\tanh^2(\xi)} \\ &= A_0 + (\alpha^2-\beta^2)\coth^2(\xi), \end{aligned} \tag{16}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((4\alpha\beta^3 + 3\gamma\delta - 6\alpha\beta A_0 - 4\alpha^3\beta)/2\alpha)t$.

When $m \to 0$, $\text{dc}(\xi,m) \to \sec(\xi)$, solution (15) becomes

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{(\alpha^2-\beta^2)}{\cos^2(\xi)} \\ &= A_0 + (\alpha^2-\beta^2)\sec^2(\xi), \end{aligned} \tag{17}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((2\alpha\beta^3 + 3\gamma\delta - 6\alpha\beta A_0 - 2\alpha^3\beta)/2\alpha)t$.

From (11), we have

$$u(x,y,z,w,t) = A_0 + m^2\left(\alpha^2-\beta^2\right)\operatorname{sn}^2(\xi,m), \quad (18)$$

$$u(x,y,z,w,t) = A_0 + m^2\left(\alpha^2-\beta^2\right)\operatorname{cd}^2(\xi,m), \quad (19)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((-2\alpha\beta^3(-m^2-1) + 3\gamma\delta - 6\alpha\beta A_0 + 2\alpha^3\beta(-m^2-1))/2\alpha)t$.

When $m \to 1$, $\operatorname{sn}(\xi,m) \to \tanh(\xi)$, solution (18) becomes

$$u(x,y,z,w,t) = A_0 + \left(\alpha^2-\beta^2\right)\tanh^2(\xi), \quad (20)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((4\alpha\beta^3 + 3\gamma\delta - 6\alpha\beta A_0 - 4\alpha^3\beta)/2\alpha)t$.

From (12), we have

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{\left(\alpha^2-\beta^2\right)}{\operatorname{sn}^2(\xi,m)} \\ &\quad + m^2\left(\alpha^2-\beta^2\right)\operatorname{sn}^2(\xi,m), \end{aligned} \quad (21)$$

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{\left(\alpha^2-\beta^2\right)}{\operatorname{cd}^2(\xi,m)} \\ &\quad + m^2\left(\alpha^2-\beta^2\right)\operatorname{cd}^2(\xi,m). \end{aligned} \quad (22)$$

When $m \to 1$, $\operatorname{sn}(\xi,m) \to \tanh(\xi)$, solution (21) becomes

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \left(\alpha^2-\beta^2\right)\coth^2(\xi) \\ &\quad + \left(\alpha^2-\beta^2\right)\tanh^2(\xi), \end{aligned} \quad (23)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((4\alpha\beta^3 + 3\gamma\delta - 6\alpha\beta A_0 - 4\alpha^3\beta)/2\alpha)t$.

When $m \to 0$, $\operatorname{cd}(\xi,m) \to \cos(\xi)$, solution (22) becomes (17).

Remark 1. Considering the form of (5), we can suppose that (7) owns the solutions in the form

$$\begin{aligned} \phi(\xi) &= A_0 + A_1F(\xi) + A_2F^2(\xi) \\ &\quad + B_1F'(\xi) + B_2F(\xi)F'(\xi). \end{aligned} \quad (24)$$

Substituting (24) and (13) into (7) and then setting all the coefficients of $F^j(\xi)F'^k(\xi)$ $(j = 0,1,\ldots,5,\ k = 0,1)$ of the resulting system to zero, we can obtain the following result:

$$\left\{A_1 = B_2 = 0, A_2 = \frac{h_4}{2}\left(\alpha^2-\beta^2\right), B_1 = \pm\frac{\sqrt{h_4}}{2}\left(\alpha^2-\beta^2\right), \right.$$
$$\left. \epsilon = \frac{\alpha^3\beta h_2 - \alpha\beta^3 h_2 - 12\alpha\beta A_0 + 6\gamma\delta}{4\alpha}\right\}, \quad (25)$$

where h_0, h_2, and A_0 are arbitrary constants, $h_4 > 0$, and α, β, γ, and δ are nonzero constants.

Substituting (25) into (24), we obtain the following formal solution of (1):

$$\begin{aligned} u(x,y,z,w,t) &= A_0 + \frac{h_4}{2}\left(\alpha^2-\beta^2\right)F^2(\xi) \\ &\quad \pm \frac{\sqrt{h_4}}{2}\left(\alpha^2-\beta^2\right)F'(\xi). \end{aligned} \quad (26)$$

Combining (26) with Table 1, many exact solutions of (1) can be obtained. For simplicity, we just give out one case in Table 1.

When $h_0 = 1$, $h_2 = -(m^2+1)$, $h_4 = m^2$, the solution of (13) is $F(\xi) = \operatorname{sn}(\xi,m)$ or $F(\xi) = \operatorname{cd}(\xi,m)$. Substituting them into (10)–(12), we can obtain the following Jacobi Elliptic function solutions of (1):

$$\begin{aligned} &u(x,y,z,w,t) \\ &\quad = A_0 + \frac{m^2}{2}\left(\alpha^2-\beta^2\right)\operatorname{sn}^2(\xi,m) \\ &\quad\quad \pm \frac{m}{2}\left(\alpha^2-\beta^2\right)\operatorname{cn}(\xi,m)\operatorname{dn}(\xi,m), \end{aligned} \quad (27)$$

$$\begin{aligned} &u(x,y,z,w,t) \\ &\quad = A_0 + \frac{m^2}{2}\left(\alpha^2-\beta^2\right)\operatorname{cd}^2(\xi,m) \\ &\quad\quad \pm \frac{m}{2}\left(1-m^2\right)\left(\alpha^2-\beta^2\right)\operatorname{sn}(\xi,m)\operatorname{nd}^2(\xi,m), \end{aligned} \quad (28)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((\alpha^3\beta(-m^2-1) - \alpha\beta^3(-m^2-1) - 12\alpha\beta A_0 + 6\delta\gamma)/4\alpha)t$.

When $m \to 1$, $\operatorname{sn}(\xi,m) \to \tanh(\xi)$, $\operatorname{cn}(\xi,m) \to \operatorname{sech}(\xi)$, and $\operatorname{dn}(\xi,m) \to \operatorname{sech}(\xi)$, solution (27) becomes

$$\begin{aligned} &u(x,y,z,w,t) \\ &\quad = A_0 + \frac{1}{2}\left(\alpha^2-\beta^2\right)\tanh^2(\xi) \\ &\quad\quad \pm \frac{1}{2}\left(\alpha^2-\beta^2\right)\operatorname{sech}^2(\xi) \\ &\quad = A_0 + \frac{\alpha^2-\beta^2}{2}\left(1-2\operatorname{sech}^2(\xi)\right), \end{aligned} \quad (29)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((-2\alpha^3\beta + 2\alpha\beta^3 - 12\alpha\beta A_0 + 6\delta\gamma)/4\alpha)t$.

Some typical wave figures are given as follows. (14) is an unbounded periodic wave solution, as shown in Figure 1(a). As m increases, the period also increases gradually, as shown in Figure 1(b). When $m \to 1$, it becomes unbounded solitary wave solution, as shown in Figure 1(c). (18) is a smooth periodic wave solution, as shown in Figure 2(a). As m increases, the period also increases gradually, as shown in Figure 2(b). When $m \to 1$, it becomes a solitary wave solution, as shown in Figure 2(c). The figures of other solutions are similar, so we do not give them out.

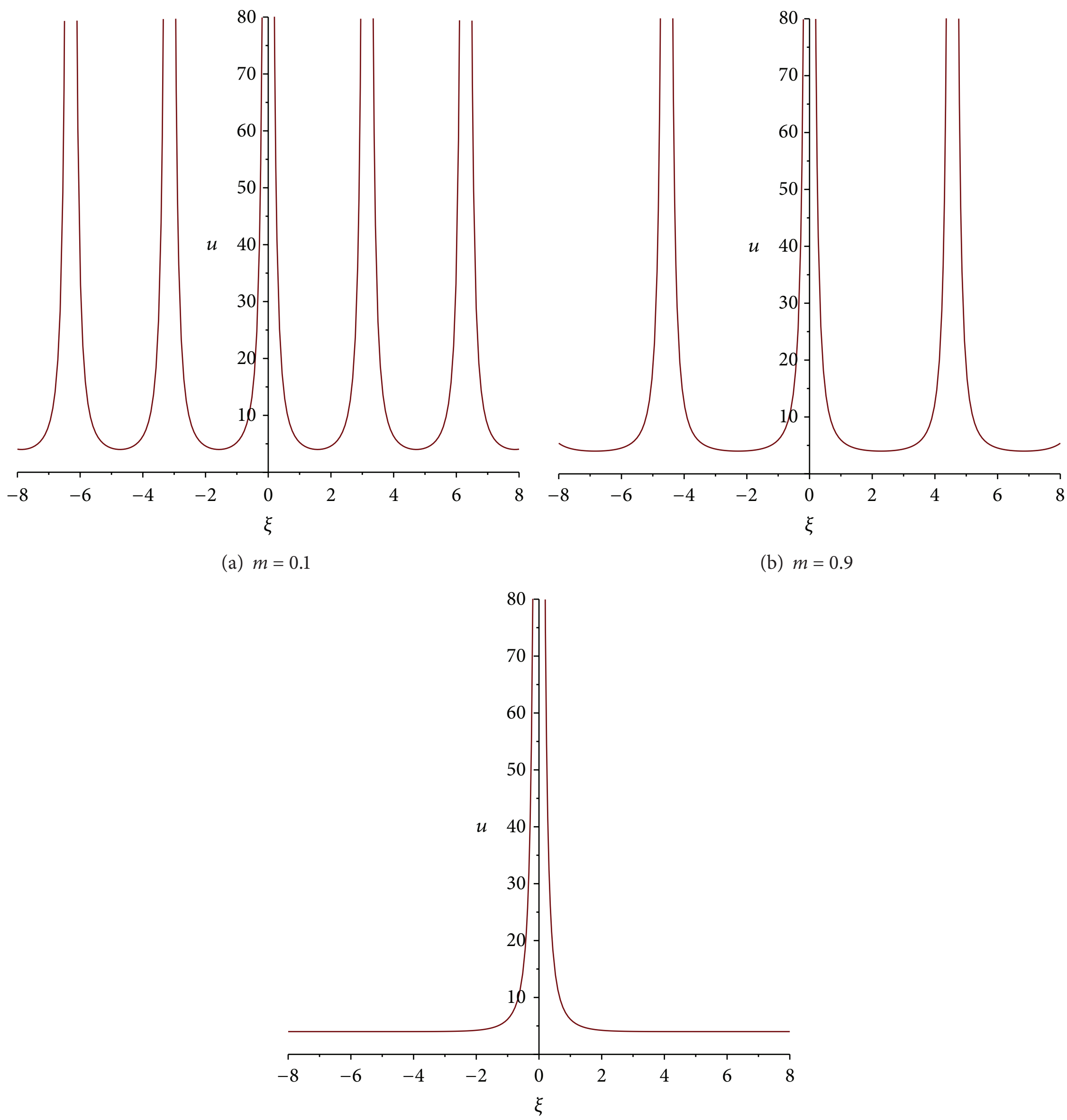


Figure 1: 2-D figures of solution (14) with $\alpha = 2$, $\beta = 1$, $A_0 = 1$ in the intervals $\xi \in [-6, 6]$.

3.2. $h_0 = h_4 = 0$. In this situation, we have the following result:

$$\left\{A_2 = B_1 = B_2 = 0, A_1 = \frac{h_3}{4}\left(\alpha^2 - \beta^2\right), \quad \epsilon = \frac{-\alpha\beta^3 h_2 + \alpha^3\beta h_2 + 6\gamma\delta - 12\alpha\beta A_0}{4\alpha}\right\}, \tag{30}$$

where A_0, h_1, h_2, and h_3 are arbitrary constants.

Substituting (30) into (8), we obtain the following formal solution of (1):

$$u(x, y, z, w, t) = A_0 + \frac{h_3}{4}\left(\alpha^2 - \beta^2\right) F(\xi), \tag{31}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + \epsilon t$ and α, β, γ, δ, and ϵ are determined in (30).

When $h_0 = h_4 = 0$, the general elliptic equation (6) is reduced to the auxiliary ordinary equation

$$F'(\xi)^2 = h_1 F(\xi) + h_2 F^2(\xi) + h_3 F^3(\xi). \tag{32}$$

Combining (31) with solutions of (32), many exact solutions of (1) can be obtained. The process is similar to the case of $h_1 = h_3 = 0$; we omit it.

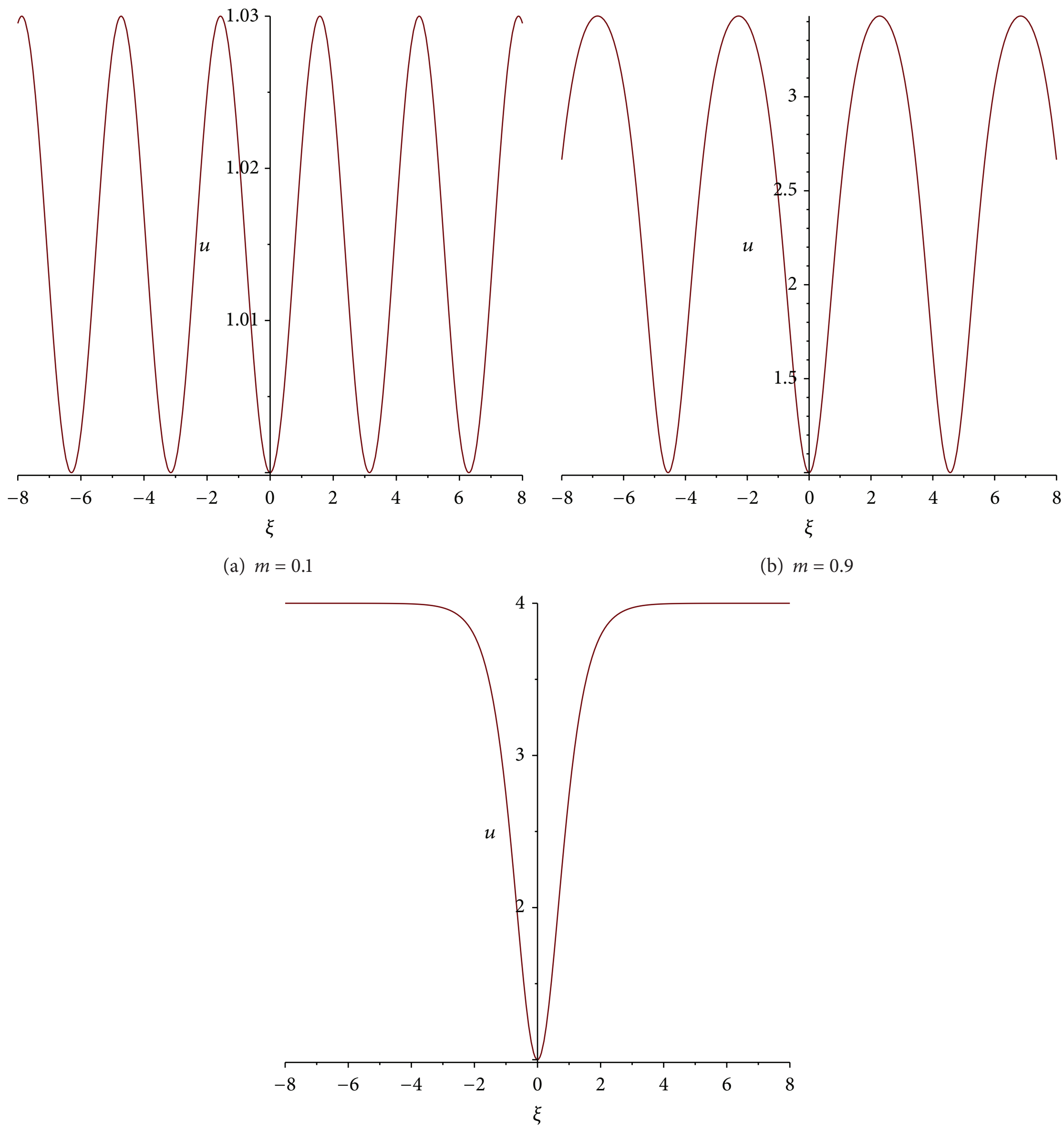


FIGURE 2: 2-D figures of solution (18) with $\alpha = 2$, $\beta = 1$, $A_0 = 1$ in the intervals $\xi \in [-6, 6]$.

3.3. $h_0 = h_1 = 0$. In this situation, we have the following result:

$$\left\{ B_1 = B_2 = 0, A_1 = \frac{h_3}{2}\left(\alpha^2 - \beta^2\right), \right.$$
$$A_2 = \frac{h_3^2}{4h_2}\left(\alpha^2 - \beta^2\right), h_4 = \frac{h_3^2}{4h_2}, \tag{33}$$
$$\left. \epsilon = \frac{-h_2\beta^3\alpha + 6\gamma\delta - 12A_0\beta\alpha + h_2\beta\alpha^3}{4\alpha} \right\},$$

where A_0, h_2, and h_3 are arbitrary constants.

Substituting (33) into (8), we obtain the following formal solution of (1):

$$u(x, y, z, w, t) = A_0 + \frac{h_3}{2}\left(\alpha^2 - \beta^2\right) F(\xi) + \frac{h_3^2}{4h_2}\left(\alpha^2 - \beta^2\right) F(\xi)^2, \tag{34}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + \epsilon t$ and α, β, γ, δ, and ϵ are determined in (33).

When $h_0 = h_1 = 0$, the general elliptic equation (6) is reduced to the auxiliary ordinary equation

$$F'(\xi)^2 = h_2 F(\xi)^2 + h_3 F^3(\xi) + h_4 F^4(\xi). \tag{35}$$

Table 2: Solutions of (33) with $\Delta = h_3^2 - 4h_2h_4$, $\varepsilon = \pm 1$.

Case	h_2, Δ	$F(\xi)$	Case	h_2, Δ	$F(\xi)$
1	$h_2 > 0$	$\dfrac{-h_2h_3 \operatorname{sech}^2((\sqrt{h_2}/2)\xi)}{h_3^2 - h_2h_4(1+\varepsilon\tanh((\sqrt{h_2}/2)\xi)^2}$	2	$h_2 > 0$	$\dfrac{-h_2h_3 \operatorname{csch}^2((\sqrt{h_2}/2)\xi)}{h_3^2 - h_2h_4(1+\varepsilon\coth((\sqrt{h_2}/2)\xi)^2}$
3	$h_2 > 0,\ \Delta > 0$	$\dfrac{2h_2 \operatorname{sech}(\sqrt{h_2}\xi)}{\varepsilon\sqrt{\Delta} - h_3 \operatorname{sech}(\sqrt{h_2}\xi)}$	4	$h_2 < 0,\ \Delta > 0$	$\dfrac{2h_2 \sec(\sqrt{-h_2}\xi)}{\varepsilon\sqrt{\Delta} - h_3 \sec(\sqrt{-h_2}\xi)}$
5	$h_2 > 0,\ \Delta < 0$	$\dfrac{2h_2 \operatorname{csch}(\sqrt{h_2}\xi)}{\varepsilon\sqrt{-\Delta} - h_3 \operatorname{csch}(\sqrt{h_2}\xi)}$	6	$h_2 < 0,\ \Delta > 0$	$\dfrac{2h_2 \csc(\sqrt{-h_2}\xi)}{\varepsilon\sqrt{\Delta} - h_3 \csc(\sqrt{-h_2}\xi)}$
7	$h_2 > 0,\ h_4 > 0$	$\dfrac{-h_2 \operatorname{sech}^2((\sqrt{h_2}/2)\xi)}{h_3 + 2\varepsilon\sqrt{h_2h_4}\tanh((\sqrt{h_2}/2)\xi)}$	8	$h_2 < 0,\ h_4 > 0$	$\dfrac{-h_2 \sec^2((\sqrt{-h_2}/2)\xi)}{h_3 + 2\varepsilon\sqrt{-h_2h_4}\tan((\sqrt{-h_2}/2)\xi)}$
9	$h_2 > 0,\ h_4 > 0$	$\dfrac{h_2 \operatorname{csch}^2((\sqrt{h_2}/2)\xi)}{h_3 + 2\varepsilon\sqrt{h_2h_4}\coth((\sqrt{h_2}/2)\xi)}$	10	$h_2 < 0,\ h_4 > 0$	$\dfrac{-h_2 \csc^2((\sqrt{-h_2}/2)\xi)}{h_3 + 2\varepsilon\sqrt{-h_2h_4}\cot((\sqrt{-h_2}/2)\xi)}$
11	$h_2 > 0,\ \Delta = 0$	$\dfrac{-h_2}{h_3}\left(1+\varepsilon\tanh\left(\dfrac{\sqrt{h_2}}{2}\right)\xi\right)$	12	$h_2 > 0,\ \Delta = 0$	$\dfrac{-h_2}{h_3}\left(1+\varepsilon\coth\left(\dfrac{\sqrt{h_2}}{2}\right)\xi\right)$
13	$h_2 > 0$	$\dfrac{4h_2\exp\left(\varepsilon\sqrt{h_2}\xi\right)}{\left(\exp\left(\varepsilon\sqrt{h_2}\xi\right) - h_3\right)^2 - 4h_2h_4}$	14	$h_2 > 0,\ h_3 = 0$	$\dfrac{4h_2\exp\left(\varepsilon\sqrt{h_2}\xi\right)}{1 - 4h_2h_4\exp\left(2\varepsilon\sqrt{h_2}\xi\right)}$

If $h_2 > 0$, $h_3^2 - 4h_2h_4 = 0$, the solutions of (35) are given by the following:

$$F_1(\xi) = \frac{-h_2}{h_3}\left(1 \pm \coth\left(\frac{\sqrt{h_2}}{2}\xi + \xi_0\right)\right),$$
$$F_2(\xi) = \frac{-h_2}{h_3}\left(1 \pm \tanh\left(\frac{\sqrt{h_2}}{2}\xi + \xi_0\right)\right), \quad (36)$$

where ξ_0 is arbitrary constant.

Substituting (36) into (34), we obtain the following hyperbolic function solutions of (1):

$$u(x,y,z,w,t) = A_0 - \frac{h_2}{2}\left(\alpha^2 - \beta^2\right)\left(1 \pm \coth(\xi + \xi_0)\right) + \frac{h_2}{4}\left(\alpha^2 - \beta^2\right)\left(1 \pm \coth(\xi + \xi_0)\right)^2,$$
$$u(x,y,z,w,t) = A_0 - \frac{h_2}{2}\left(\alpha^2 - \beta^2\right)\left(1 \pm \tanh(\xi + \xi_0)\right) + \frac{h_2}{4}\left(\alpha^2 - \beta^2\right)\left(1 \pm \tanh(\xi + \xi_0)\right)^2, \quad (37)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((-h_2\beta^3\alpha + 6\gamma\delta - 12A_0\beta\alpha + h_2\beta\alpha^3)/4\alpha)t$.

Remark 2. Considering the form of (5), we can suppose that (7) owns the solutions in the form

$$\phi(\xi) = A_0 + A_1F(\xi) + A_2F^2(\xi) + B_1F'(\xi) + B_2F(\xi)F'(\xi). \quad (38)$$

Substituting (38) and (35) into (7) and then setting all the coefficients of $F^j(\xi)F'^k(\xi)$ $(j = 0, 1, \ldots, 5,\ k = 0, 1)$ of the resulting system to zero, we can obtain the following result:

$$\left\{A_0 = A_0, A_1 = \frac{h_3}{4}\left(\alpha^2 - \beta^2\right), A_2 = \frac{h_4}{2}\left(\alpha^2 - \beta^2\right),\right.$$
$$B_1 = \pm\frac{\sqrt{h_4}}{2}\left(\alpha^2 - \beta^2\right), B_2 = 0, \quad (39)$$
$$\left.\epsilon = \frac{\alpha^3\beta h_2 - \alpha\beta^3h_2 - 12\alpha\beta A_0 + 6\delta\gamma}{4\alpha}\right\},$$

where h_0, h_2, and A_0 are arbitrary constants, $h_4 \geqslant 0$, and α, β, γ, and δ are nonzero constants.

The solutions of (35) are given in Table 2.

Substituting (39) into (38), we obtain the following formal solution of (1):

$$u(x,y,z,w,t) = A_0 + \frac{h_3}{4}\left(\alpha^2 - \beta^2\right)F(\xi) + \frac{h_4}{2}\left(\alpha^2 - \beta^2\right)F^2(\xi) \pm \frac{\sqrt{h_4}}{2}\left(\alpha^2 - \beta^2\right)F'(\xi), \quad (40)$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + \epsilon t$ and α, β, γ, δ, and ϵ are determined in (39).

Combining (40) with Table 2, many exact solutions of (1) can be obtained. For simplicity, we do not list them carefully. Using case 2 in Table 2, we can obtain the following type of wave figures. Figures 3(a) and 3(b) are kink waves and Figure 3(c) is new type of singular wave. It is interesting that they are dependent on $\Delta = h_3^2 - 4h_2h_4$.

3.4. $h_0 \neq 0$, $h_1 \neq 0$, $h_2 \neq 0$, $h_3 \neq 0$, $h_4 \neq 0$. In this case, there exists three parameters r, p, and q such that

$$\begin{aligned}\left(F'(\xi)\right)^2 &= h_0 + h_1F(\xi) + h_2F^2(\xi) \\ &\quad + h_3F^3(\xi) + h_4F^4(\xi) \\ &= \left(r + pF(\xi) + qF^2(\xi)\right)^2.\end{aligned} \tag{41}$$

Equation (41) is satisfied only if the following relations hold:

$$\begin{gathered} h_0 = r^2, \qquad h_1 = 2rp, \qquad h_2 = 2rq + p^2, \\ h_3 = 2pq, \qquad h_4 = q^2. \end{gathered} \tag{42}$$

Equation (33) is the general Riccati equation. The solutions of (41) are listed in [12]. There are 24 group solutions named ϕ_i^I, $(i = 1, 2, \ldots 24)$, which we do not list for simplicity.

Substituting (41) and (8) into (7) and then setting all the coefficients of F^k $(k = -5, \ldots, 5)$ of the resulting system to zero, we can obtain the following results:

$$\begin{aligned} \Big\{ & A_0 = A_0, B_1 = B_2 = 0, A_1 = pq\left(\alpha^2 - \beta^2\right), \\ & A_2 = q^2\left(\alpha^2 - \beta^2\right), \\ & \epsilon = \left(8\alpha^3\beta qr + \alpha^3\beta p^2 - \alpha\beta^3p^2\right. \\ & \qquad \left. -12\alpha\beta A_0 - 8\alpha\beta^3qr + 6\gamma\delta\right) \\ & \quad \times (4\alpha)^{-1} \Big\} \end{aligned} \tag{43}$$

$$\begin{aligned} \Big\{ & A_0 = A_0, A_1 = A_2 = 0, B_1 = rp\left(\alpha^2 - \beta^2\right), \\ & B_2 = r^2\left(\alpha^2 - \beta^2\right), \\ & \epsilon = \left(8\alpha^3\beta qr + \alpha^3\beta p^2 - \alpha\beta^3p^2\right. \\ & \qquad \left. -12\alpha\beta A_0 - 8\alpha\beta^3qr + 6\gamma\delta\right) \\ & \quad \times (4\alpha)^{-1} \Big\}, \end{aligned} \tag{44}$$

where r, p, and q are arbitrary constants, and

$$\begin{aligned} \Big\{ & A_0 = A_0, A_1 = B_1 = p = 0, \\ & A_2 = q^2\left(\alpha^2 - \beta^2\right), B_2 = r^2\left(\alpha^2 - \beta^2\right), \\ & \epsilon = \frac{4\alpha^3\beta qr - 6\alpha\beta A_0 - 4\alpha\beta^3qr + 3\gamma\delta}{2\alpha} \Big\}, \end{aligned} \tag{45}$$

where r and q are arbitrary constants.

Substituting (43)–(45) into (8), we obtain, respectively, the following formal solutions of (1):

$$\begin{aligned} u(x, y, z, w, t) &= A_0 + pq\left(\alpha^2 - \beta^2\right)F(\xi) \\ &\quad + q^2\left(\alpha^2 - \beta^2\right)F(\xi)^2, \\ u(x, y, z, w, t) &= A_0 + rp\left(\alpha^2 - \beta^2\right)F(\xi)^{-1} \\ &\quad + r^2\left(\alpha^2 - \beta^2\right)F(\xi)^{-2}, \end{aligned} \tag{46}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((8\alpha^3\beta qr + \alpha^3\beta p^2 - \alpha\beta^3p^2 - 12\alpha\beta A_0 - 8\alpha\beta^3qr + 6\gamma\delta)/4\alpha)t$, and

$$\begin{aligned} u(x, y, z, w, t) &= A_0 + q^2\left(\alpha^2 - \beta^2\right)F(\xi)^2 \\ &\quad + r^2\left(\alpha^2 - \beta^2\right)F(\xi)^{-2}, \end{aligned} \tag{47}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((4\alpha^3\beta qr - 6\alpha\beta A_0 - 4\alpha\beta^3qr + 3\gamma\delta)/2\alpha)t$.

Substituting solutions of (41) $F(\xi) = \phi_i^I$, $(i = 1, 2, \ldots 24)$ into (46)–(47), we can obtain a lot of solutions of (1). We just give one example.

When $p^2 - 4pq > 0$ and $pq \neq 0$, $\phi_1^I = -(1/2q)(p + \sqrt{p^2 - 4qr}\tanh((\sqrt{p^2 - 4qr}/2)\xi))$. Substituting ϕ_1^I into (46) and (51), we have

$$\begin{aligned} u(x, y, z, w, t) &= A_0 + pq\left(\alpha^2 - \beta^2\right)\phi_1^I \\ &\quad + q^2\left(\alpha^2 - \beta^2\right)\phi_1^{I2}, \\ u(x, y, z, w, t) &= A_0 + rp\left(\alpha^2 - \beta^2\right)\phi_1^I - 1 \\ &\quad + r^2\left(\alpha^2 - \beta^2\right)\phi_1^{I-2}, \end{aligned} \tag{48}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((8\alpha^3\beta qr + \alpha^3\beta p^2 - \alpha\beta^3p^2 - 12\alpha\beta A_0 - 8\alpha\beta^3qr + 6\gamma\delta)/4\alpha)t$.

When $p = 0$, $qr < 0$, $\phi = r\tanh(\sqrt{-qr}\xi)$. Substituting ϕ into (47), we have

$$\begin{aligned} u(x, y, z, w, t) &= A_0 + q^2\left(\alpha^2 - \beta^2\right)\phi^2 \\ &\quad + r^2\left(\alpha^2 - \beta^2\right)\phi^{-2}, \end{aligned} \tag{49}$$

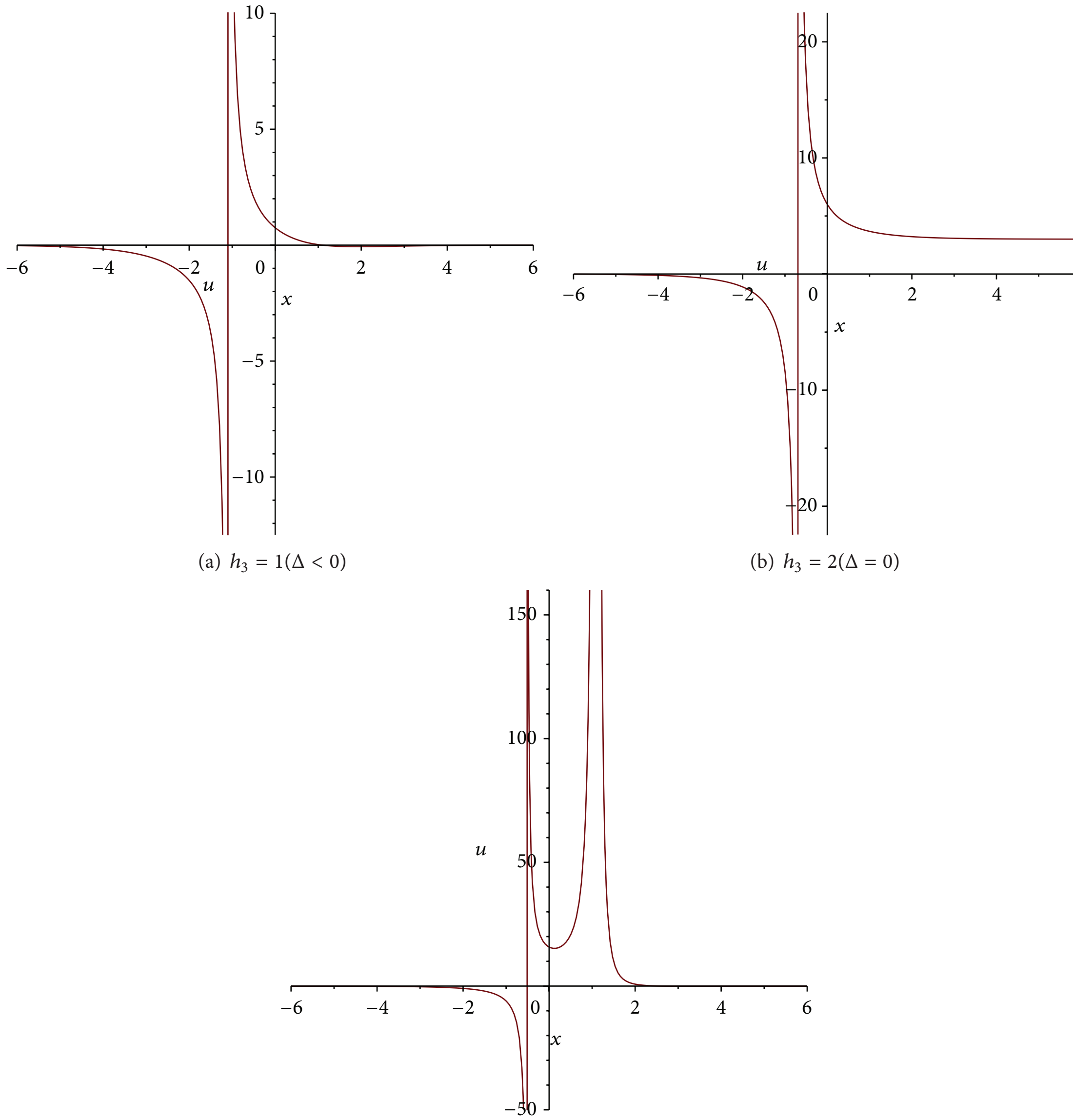
where $\xi = \alpha x + \beta y + \gamma z + \delta w + ((4\alpha^3\beta qr - 6\alpha\beta A_0 - 4\alpha\beta^3qr + 3\gamma\delta)/2\alpha)t$.

3.5. $h_0 \neq 0$, $h_1 \neq 0$, $h_2 = 0$, $h_3 \neq 0$, $h_4 \neq 0$. In this case, there exists three parameters r, p, and q such that

$$\begin{aligned}\left(F'(\xi)\right)^2 &= h_0 + h_1F(\xi) + h_3F^3(\xi) + h_4F^4(\xi) \\ &= \left(r + pF(\xi) + qF^2(\xi)\right)^2.\end{aligned} \tag{50}$$

Equation (50) is satisfied only if the following relations hold:

$$h_0 = r^2, \qquad h_1 = 2rp, \qquad h_3 = 2pq, \qquad h_4 = q^2. \tag{51}$$



(a) $h_3 = 1(\Delta < 0)$

(b) $h_3 = 2(\Delta = 0)$

(c) $h_3 = 3(\Delta > 0)$

FIGURE 3: 2-D figures of solution (40) with $\alpha = 2$, $\beta = 1$, $A_0 = 0$, $h_2 = h_4 = 1$ in the intervals $\xi \in [-6, 6]$.

The following constraint should exist between r, p, and q parameters:

$$p^2 = -2qr, \quad qr < 0. \tag{52}$$

Therefore, we can discuss the solution of (1) similarly as 3.4 under the condition (52). Here, we omit it.

3.6. $h_2 = 0$, $h_4 = 0$. In this situation, we have the following result:

$$\begin{gathered} A_2 = B_1 = B_2 = 0, \qquad A_1 = \frac{h_3}{4}\left(\alpha^2 - \beta^2\right), \\ \epsilon = -\frac{3}{2}\frac{-\gamma\delta + 2A_0\beta\alpha}{\alpha}. \end{gathered} \tag{53}$$

Substituting (53) into (8), we obtain the following formal solution of (1):

$$u(x, y, z, w, t) = A_0 + \frac{h_3}{4}\left(\alpha^2 - \beta^2\right)F(\xi), \tag{54}$$

where $\xi = \alpha x + \beta y + \gamma z + \delta w + \epsilon t$ and α, β, γ, δ, and ϵ are determined in (53).

When $h_2 = h_3 = 0$, the general elliptic equation (6) is reduced to the auxiliary ordinary equation

$$F'(\xi)^2 = h_0 + h_1 F(\xi) + h_3 F^3(\xi). \tag{55}$$

The solution of (55) is the Weierstrass elliptic doubly periodic type solution is as follows:

$$F(\xi) = \wp\left(\frac{\sqrt{h_3}}{2}\xi, g_2, g_3\right), \quad h_3 > 0. \tag{56}$$

Substituting (56) into (54), the solution of (1) is

$$u(x, y, z, w, t) = A_0 + \frac{h_3}{4}\left(\alpha^2 - \beta^2\right)\wp\left(\frac{\sqrt{h_3}}{2}\xi, g_2, g_3\right), \quad h_3 > 0, \tag{57}$$

where $g_2 = -4h_1/h_3$ and $g_3 = -4h_0/h_3$.

4. Conclusions

The investigation of the exact solutions of higher-dimensional integrable models is one of the important problems in mathematical physics. In our work, (4 + 1)-dimensional nonlinear Fokas equation (1), which is an important physical model, is discussed by using the extended F-expansion method and its variant. Some new exact solutions expressed by Jacobi elliptic function, Weierstrass elliptic function, hyperbolic function, and trigonometric function are obtained and some typical wave figures are given including periodic wave, solitary wave, kink wave, and some new types. The correctness of all the solutions is verified by substituting them into original equation (1). Comparing with [34–36], it is easy to see that our method is more straightforward and the form of the solutions obtained in our paper is also more simple and many solutions are new. The related results are enriched.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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