Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 896574, 10 pages
http://dx.doi.org/10.1155/2015/896574

Research Article

Nonfragile Robust H_∞ Filter Design for a Class of Fuzzy Stochastic Systems with Stochastic Input-to-State Stability

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Received 20 September 2014; Accepted 5 January 2015

Academic Editor: Bo Shen

The nonfragile H_∞ filtering problem for a kind of Takagi-Sugeno (T-S) fuzzy stochastic system which has a time-varying delay and parameter uncertainties has been studied in this paper. Sufficient conditions for stochastic input-to-state stability (SISS) of the fuzzy stochastic systems are obtained. Attention is focused on the design of a nonfragile H_∞ filter such that the filtering error system can tolerate some level of the gain variations in the filter and the H_∞ performance level also could be satisfied. By using the SISS result, the approach to design the nonfragile filter is proposed in terms of linear matrix inequalities. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

As the performance of a control system is affected by parameter perturbations, exogenous disturbances, measurement errors, and other uncertainties, the research of the robust control problem has had a vital status in the studies of control theory. Considering Lyapunov stability theory which is not suitable for analyzing and processing the state responses of the system with perturbations, some new methods have been developed, such as input-to-state stability (ISS). Since Sontag presented the qualitative aspect of ISS for the system response to input with bounded magnitude in 1989 [1], ISS has become an essential concept in modern controller and filter design for the nonlinear system. The ISS problem has been extensively investigated by many authors [2–6] until now. At the same time, ISS has been studied for stochastic systems. Stochastic input-to-state stability (SISS) of Lure distributed parameter control system has been investigated in [7], and sufficient conditions for SISS in Hilbert space have been presented in terms of linear operator inequalities. In [8], SISS and the H_∞ filtering problem have been considered, and the filter has been designed in LMIs. A mean-square exponential ISS problem for stochastic delay neural networks has been investigated in [9].

On the other hand, fuzzy model could turn the nonlinear models into a linear representation by partitioning the original dynamic differential equations into linear ones [10]. T-S fuzzy model [11] has been considered as an efficient technique to linearize the nonlinear systems. This model has been first put forward in the truck trailer system [12]. And another typical application is in the stirred tank reactor system which has been addressed in [13]. Until now, there have been a lot of results of T-S fuzzy system reported in literature. The stability and control problem have been investigated in [14–19] and the references therein.

Meanwhile, it is well known that state estimation can estimate the unavailable state variables or their linear combination for a given system [20, 21], and it has been found in many practical applications over decades. As a branch of state estimation theory, H_∞ filter can process the estimation problem without exact statistical data for the external noise. This problem for the T-S fuzzy system has been addressed in [22–27]; and the robust filters for stochastic systems are designed in [28, 29]. During the filter design, gain perturbations are usually unavoidable. According to [30], those gain perturbations could destabilize the filtering error system even if they are very small, which makes the filter fragile. Hence, it is reasonable to design a filter that could tolerate some level of the gain variations, which is called nonfragile filter. The nonfragile filter has received considerable attention over the past two decades; refer to [31–34] and the references therein. From what is mentioned above, it is worth noting that T-S fuzzy model can be used to divide the nonlinear stochastic systems into several subsystems. The solution to fuzzy stochastic differential equations with local martingales has been presented in [35]. The work in [36] has considered the robust fault detection problem for T-S fuzzy stochastic systems. And the stabilization for the fuzzy stochastic systems with delays has been investigated in [37–39]. The control problem has been considered in [40–45].

Motivated by the above discussion, this paper will focus on the filter design for the fuzzy stochastic system, where few results have been found. The nonfragile fuzzy delay-dependent H_∞ filter design for a T-S time-delay fuzzy stochastic system with norm-bounded parameter uncertainties is studied in this paper. The Lyapunov-Krasovskii functional technique is used and the sufficient conditions obtained are expressed in terms of linear matrix inequality (LMI) approach. This paper is organized as follows. Section 2 presents the problem formulation and preliminaries. Section 3 gives main results for the nonfragile filter design. In Section 4, a numerical example is shown to illustrate the effectiveness of the proposed methods. Section 5 concludes the paper.

Notation. The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition. Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite). $\mathbf{R}^n$ denotes the n-dimensional Euclidean space and $\mathbf{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. I stands for an identity matrix of appropriate dimension, while $I_n \in \mathbf{R}^n$ denotes a vector of ones. The notation $*$ is used as an ellipsis for terms that are induced by symmetry. diag$(\cdots)$ stands for a block-diagonal matrix. $|\cdot|$ denotes the Euclidean norm for vectors and $\|\cdot\|$ denotes the spectral norm for matrices. $\mathbf{L}_2[0,\infty)$ represents the space of square-integrable vector functions over $[0,\infty)$. $\mathsf{E}(\cdot)$ stands for the mathematical expectation operator. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Preliminaries

Consider the time-delay T-S fuzzy stochastic system with time-varying parameter uncertainties in the following form:

$$\begin{aligned}(\Sigma): dx(t) &= \sum_{i=1}^{r} h_i(s(t))[\psi_i dt + k_i d\omega(t)],\\ dy(t) &= \sum_{i=1}^{r} h_i(s(t))[C_i x(t) + C_{di}x(t-\tau(t)) + D_i v(t)]dt,\\ z(t) &= \sum_{i=1}^{r} h_i(s(t))[(L_i + \Delta L_i(t))x(t)],\\ x(t) &= \varphi(t), \quad t \in [-h_2, 0];\end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^m$ is the state; $\psi_i = (A_i + \Delta A_i(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t-\tau(t)) + (B_i + \Delta B_i(t))v(t) + E_i g(x(t))$, $k_i = (H_i + \Delta H_i(t))x(t) + (H_{di} + \Delta H_{di}(t))x(t-\tau(t))$; $\varphi(t)$ is a given real-value initial function on $[-h_2, 0]$; $\omega(t)$ is a scalar zero mean Gaussian white noise process with unit covariance; $y(t) \in \mathbf{R}^n$ is the measured output; $z(t) \in \mathbf{R}^l$ is a signal to be estimated; $v(t) \in \mathbf{R}^s$ is the input noise signal which belongs to $\mathscr{L}_2[0,\infty)$; $\tau(t)$ is a continuous differentiable function representing the time-varying delay in $x(t)$, which is assumed to satisfy $0 \leq h_1 \leq \tau(t) < h_2$ for all $t \geq 0$; and the real nonlinear function $g(x(t))$ satisfies a linear-type growth condition and local Lipschitz condition $\|g(x(t))\|^2 \leq l\|x(t)\|^2$ and $\|g(x_1(t)) - g(x_2(t))\|^2 \leq \kappa\|x_1(t) - x_2(t)\|^2$, where l and κ are two known positive constant scalars. And using the fuzzy theory, there always exists, for all t, $h_i(s(t)) \geq 0$, $i = 1, 2, \ldots, r$, $\sum_{i=1}^{r} h_i(s(t)) = 1$.

In the above nonlinear fuzzy stochastic system, A_i, A_{di}, B_i, E_i, H_i, H_{di}, C_i, C_{di}, D_i, and L_i are known constant matrices with appropriate dimensions. $\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta B_i(t)$, $\Delta H_i(t)$, $\Delta H_{di}(t)$, and $\Delta L_i(t)$ represent the unknown time-varying parameter uncertainties and are assumed to satisfy

$$\begin{bmatrix}\Delta A_i(t) & \Delta A_{di}(t) & \Delta B_i(t)\\ \Delta H_i(t) & \Delta H_{di}(t) & \Delta L_i(t)\end{bmatrix} = \begin{bmatrix}M_{1i}\\ M_{2i}\end{bmatrix} F_i(t)\begin{bmatrix}N_{1i} & N_{2i} & N_{3i}\end{bmatrix}, \tag{2}$$

where M_{1i}, M_{2i}, N_{1i}, N_{2i}, and N_{3i} are known real constant matrices and the unknown time-varying matrix function satisfying $F_i(t)^T F_i(t) \leq I$ for all t.

Now, we consider a dynamical nonfragile fuzzy filter for system (Σ):

$$\begin{aligned}d\hat{x}(t) &= \sum_{i=1}^{r} h_i(s(t))\left[\left(A_{fi} + \Delta A_{fi}(t)\right)\hat{x}(t)\,dt\right.\\ &\qquad \left. + \left(B_{fi} + \Delta B_{fi}(t)\right)dy(t) + E_{fi}g(\hat{x}(t))\right],\\ \hat{z}(t) &= \sum_{i=1}^{r} h_i(s(t))\left[\left(L_{fi} + \Delta L_{fi}(t)\right)\hat{x}(t)\right],\end{aligned} \tag{3}$$

in which the fuzzy rules have the same representations as in (1). Consider $\hat{x}(t) \in \mathbf{R}^n$ and $\hat{z}(t) \in \mathbf{R}^l$. A_{fi}, B_{fi}, L_{fi}, and E_{fi} are the filters needed to be determined. $\Delta A_{fi}(t)$, $\Delta B_{fi}(t)$, and $\Delta L_{fi}(t)$ represent the unknown time-varying parameter uncertainties and are assumed to satisfy

$$\begin{bmatrix}\Delta A_{fi}(t) & \Delta B_{fi}(t) & \Delta L_{fi}(t)\end{bmatrix} = M_{4i}F_{ai}(t)\begin{bmatrix}N_{4i} & N_{5i} & N_{6i}\end{bmatrix}, \tag{4}$$

where M_{4i}, N_{4i}, N_{5i}, and N_{6i} are known real constant matrices and the unknown time-varying matrix function satisfying $F_{ai}(t)^T F_{ai}(t) \leq I$ for all t.

Remark 1. There are two approaches to design the filter for fuzzy systems. One is dependent on the fuzzy rules when the fuzzy model is available while the other one is independent of the fuzzy rules. In this paper, we choose the first approach since the fuzzy model is known here and this approach is less conserve. So the nonfragile fuzzy rule-dependent filter is investigated in this paper as in (3).

Let $\xi(t) = [x(t)^T \;\; \hat{x}(t)^T]^T$ and $\tilde{z}(t) = z(t) - \hat{z}(t)$. The filtering error dynamic system can be written as

$$\begin{aligned}(\tilde{\Sigma}) : d\xi(t) &= \Phi(t)dt + K(t)d\omega(t),\\ \tilde{z}(t) &= \left(\tilde{L} + \Delta\tilde{L}(t)\right)\xi(t),\end{aligned} \tag{5}$$

where

$$\begin{aligned}
\Phi(t) &= \left(\tilde{A} + \Delta\tilde{A}(t)\right)\xi(t) + \left(\tilde{A}_d + \Delta\tilde{A}_d(t)\right)G\xi(t-\tau(t))\\
&\quad + \left(\tilde{B} + \Delta\tilde{B}(t)\right)v(t) + \tilde{E}g(x(t)),\\
K(t) &= \left(\tilde{H} + \Delta\tilde{H}(t)\right)\xi(t) + \left(\tilde{H}_d + \Delta\tilde{H}_d(t)\right)G\xi(t-\tau(t)),\\
\tilde{A} &= \begin{bmatrix}\overline{A} & 0\\ \overline{B}_f\overline{C} & \overline{A}_f\end{bmatrix}, \qquad \tilde{A}_d = \begin{bmatrix}\overline{A}_d\\ \overline{B}_f\overline{C}_d\end{bmatrix},\\
\tilde{H} &= \begin{bmatrix}\overline{H} & 0\\ 0 & 0\end{bmatrix}, \qquad \tilde{E} = \begin{bmatrix}\overline{E} & 0\\ 0 & \overline{E}_f\end{bmatrix},\\
\Delta\tilde{A}(t) &= \begin{bmatrix}\Delta\overline{A}(t) & 0\\ \Delta\overline{B}_f(t)\overline{C} & \Delta\overline{A}_f(t)\end{bmatrix},\\
\Delta\tilde{A}_d(t) &= \begin{bmatrix}\Delta\overline{A}_d(t)\\ \Delta\overline{B}_f(t)\overline{C}_d\end{bmatrix}, \qquad \Delta\tilde{B} = \begin{bmatrix}\Delta\overline{B}(t)\\ \Delta\overline{B}_f(t)\overline{D}\end{bmatrix},\\
\Delta\tilde{H}(t) &= \begin{bmatrix}\Delta\overline{H}(t) & 0\\ 0 & 0\end{bmatrix}, \qquad \Delta\tilde{H}_d(t) = \begin{bmatrix}\Delta\overline{H}_d(t)\\ 0\end{bmatrix},\\
\tilde{H}_d &= \begin{bmatrix}\overline{H}_d\\ 0\end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix}\overline{B}\\ \overline{B}_f\overline{D}\end{bmatrix},\\
\overline{A} &= \sum_{i=1}^{r} h_i(s(t))A_i, \qquad \overline{A}_d = \sum_{i=1}^{r} h_i(s(t))A_{di},\\
\Delta\overline{A}(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta A_i(t), \qquad \overline{H} = \sum_{i=1}^{r} h_i(s(t))H_i,\\
\overline{H}_d &= \sum_{i=1}^{r} h_i(s(t))H_{di}, \qquad \Delta\overline{A}_d(t) = \sum_{i=1}^{r} h_i(s(t))\Delta A_{di}(t),\\
\overline{B} &= \sum_{i=1}^{r} h_i(s(t))B_i, \qquad \overline{D} = \sum_{i=1}^{r} h_i(s(t))D_i,\\
\tilde{L} &= \left[\overline{L} \;\; -\overline{L}_f\right], \qquad G = [I \;\; 0],\\
\overline{E} &= \sum_{i=1}^{r} h_i(s(t))E_i, \qquad \overline{L} = \sum_{i=1}^{r} h_i(s(t))L_i,\\
\Delta\tilde{L}(t) &= \left[\Delta\overline{L}(t) \;\; -\Delta\overline{L}_f(t)\right], \qquad \overline{C} = \sum_{i=1}^{r} h_i(s(t))C_i,\\
\overline{C}_d &= \sum_{i=1}^{r} h_i(s(t))C_{di}, \qquad \Delta\overline{B}(t) = \sum_{i=1}^{r} h_i(s(t))\Delta B_i(t),\\
\overline{A}_f &= \sum_{i=1}^{r} h_i(s(t))A_{fi}, \qquad \overline{B}_f = \sum_{i=1}^{r} h_i(s(t))B_{fi},\\
\overline{L}_f &= \sum_{i=1}^{r} h_i(s(t))L_{fi}, \qquad \overline{E}_f = \sum_{i=1}^{r} h_i(s(t))E_{fi},\\
\Delta\overline{H}(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta H_i(t),\\
\Delta\overline{H}_d(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta H_{di}(t),\\
\Delta\overline{A}_f(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta A_{fi}(t),\\
\Delta\overline{B}_f(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta B_{fi}(t),\\
\Delta\overline{L}(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta L_i(t),\\
\Delta\overline{L}_f(t) &= \sum_{i=1}^{r} h_i(s(t))\Delta L_{fi}(t).
\end{aligned} \tag{6}$$

We intend to design a dynamical nonfragile fuzzy filter in the form of (3) in this paper, such that, for any scalar $0 \le h_1 < h_2$ and a prescribed level of noise attenuation $\gamma > 0$, the filtering error system $(\tilde{\Sigma})$ could be stochastic input-to-state stability and the error system $(\tilde{\Sigma})$ satisfies H_∞ performance.

Now, we present the definitions and lemmas used in this paper, which help to complete the proof of the main results.

Definition 2 (see [46]). In system (Σ), a continuously differentiable function $V(x,t) \in \vartheta^{2,1}(R^n \times R^+; R^+)$ is called a SISS Lyapunov function, if there exist functions $\alpha_1, \alpha_2, \alpha_3$, and $\alpha_4 \in K_\infty$, such that

$$\begin{aligned}\alpha_1(\|x\|) \le V(x,t) &\le \alpha_2(\|x\|),\\ LV &\le -\alpha_3(\|x\|) + \alpha_4(|v|),\end{aligned} \tag{7}$$

for any $x \in R^n$ and $u \in L_\infty$, where

$$\begin{aligned}LV(t,x) &= V_t(t,x) + V_x(t,x)f + \frac{1}{2}\operatorname{tr}\left[h^T V_{xx}(t,x)h\right],\\ f &= Ax(t) + A_d x(t-\tau) + Bv(t) + E_l g(x(t)),\\ h &= Dx(t) + E_2 x(t-\tau).\end{aligned} \tag{8}$$

Definition 3 (see [31]). The robust stochastic stable system $(\tilde{\Sigma})$ is said to satisfy the H_∞ performance; for the given scalar $\gamma > 0$ and any nonzero $v(t) \in L_2[0,\infty)$, the system $(\tilde{\Sigma})$ satisfies

$$\|\tilde{z}(t)\|_2 < \gamma\|v(t)\|_2. \tag{9}$$

Lemma 4 (see [8]). *The system (Σ) is SISS if there exists an SISS-Lyapunov function.*

3. Robust Stochastic Stabile

Theorem 5. *The filtering error system $(\widetilde{\Sigma})$ is SISS with an H_∞ attenuation level $\gamma > 0$, if there exist matrices $P = P^T > 0$, $S > 0$, $R_j = R_j^T > 0$, $j = 1,2,3$, $Q_i = Q_i^T > 0$, T_{1i}, T_{2i}, $i = 1,2$, such that the following matrix inequality holds:*

$$\Psi = \begin{bmatrix} \Omega & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} \\ * & \Psi_{22} & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{10}$$

where

$$\begin{gathered} \Omega = \begin{bmatrix} \Omega_{11} & 0 & 0 & \Omega_{14} & 0 & P\left(\widetilde{B} + \Delta\widetilde{B}(t)\right) \\ * & \Omega_{22} & 0 & \Omega_{24} & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}, \\ \Psi_{12} = \left[(h_2 - h_1)\breve{A}^T G^T R_1 \;\; (h_2 - h_1)\breve{H}^T G^T R_2 \;\; \breve{H}^T P\right], \\ \Psi_{22} = \operatorname{diag}\left\{-(h_2 - h_1) R_1, -(h_2 - h_1) R_2, -P\right\}, \\ \Psi_{13} = \left[\widetilde{T}_1 \;\; \widetilde{T}_2 \;\; \widetilde{T}_1 \;\; \widetilde{T}_2\right], \\ \Psi_{33} = \operatorname{diag}\left\{-R_2, -R_2, -(h_2 - h_1) R_1, -(h_2 - h_1) R_1\right\}, \\ \Psi_{14} = \left[\widetilde{L} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]^T, \\ \Psi_{15} = \left[\widetilde{E}^T P \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]^T, \\ \Omega_{11} = P\left(\widetilde{A} + \Delta\widetilde{A}(t)\right) + \left(\widetilde{A} + \Delta\widetilde{A}(t)\right)^T P \\ + G^T (Q_1 + Q_2 + (h_2 - h_1) R_3) G + l + S, \\ \Omega_{14} = P\left(\widetilde{A}_d + \Delta\widetilde{A}_d(t)\right), \qquad \Omega_{22} = -Q_1 + T_2 + T_2^T, \\ \Omega_{24} = -T_2 + T_2^T, \qquad \Omega_{33} = -Q_2 - T_1 - T_1^T, \\ \Omega_{34} = T_1 - T_1^T, \qquad \Omega_{44} = -T_2 - T_2^T + T_1 + T_1^T, \\ \Omega_{55} = -\frac{R_3}{(h_2 - h_1)}, \qquad \widetilde{T}_1 = \left[0 \;\; 0 \;\; T_1^T \;\; T_1^T \;\; 0 \;\; 0\right]^T, \\ \widetilde{T}_2 = \left[0 \;\; T_2^T \;\; 0 \;\; T_2^T \;\; 0 \;\; 0\right]^T, \\ \breve{A} = \left[\widetilde{A} + \Delta\widetilde{A}(t) \;\; 0 \;\; 0 \;\; \widetilde{A}_d + \Delta\widetilde{A}_d(t) \;\; 0 \;\; \widetilde{B} + \Delta\widetilde{B}(t)\right]^T, \\ \breve{H} = \left[\widetilde{H} + \Delta\widetilde{H}(t) \;\; 0 \;\; 0 \;\; \widetilde{H}_d + \Delta\widetilde{H}_d(t) \;\; 0 \;\; 0\right]^T. \end{gathered} \tag{11}$$

Proof. Choose a Lyapunov-Krasovskii candidate for system $(\widetilde{\Sigma})$ as follows:

$$\begin{aligned} V(\xi(t), t) = {} & \xi^T(t) P \xi(t) \\ & + \int_{-h_2}^{-h_1} \int_{t+\beta}^{t} \left(\Phi^T(s) G^T R_1 G \Phi(s) \right. \\ & \left. + K^T(s) G^T R_2 G K(s)\right) ds\, d\beta \\ & + \sum_{i=1}^{2} \int_{t-h_i}^{t} \xi^T(s) G^T Q_i G \xi(s)\, ds \\ & + \int_{-h_2}^{-h_1} \int_{t+\beta}^{t} \xi^T(s) G^T R_3 G \xi(s)\, ds\, d\beta. \end{aligned} \tag{12}$$

Let $\underline{\lambda}(P) = \lambda_{\min}(P)$, $\overline{\lambda}(P) = \lambda_{\max}(P)$, $\overline{\lambda}(Q_i) = \lambda_{\max}(Q_i)$, $i = 1,2$, and $\overline{\lambda}(R_j) = \lambda_{\max}(R_j)$, $j = 1,2,3$; then there exists a scalar ν_j, $j = 1,2,3$, such that

$$\begin{gathered} \underline{\lambda}(P) \|\xi(t)\|^2 \le \xi^T(t) P \xi(t) \le \overline{\lambda}(P) \|\xi(t)\|^2, \\ 0 \le \int_{t-h_i}^{t} \xi^T(s) G^T Q_i G \xi(s)\, ds \le h_i \overline{\lambda}(Q) \|\xi(t)\|^2, \quad i = 1,2, \\ 0 \le \int_{-h_2}^{-h_1} \int_{t+\beta}^{t} \Phi^T(s) G^T R_1 G \Phi(s)\, ds\, d\beta \le \nu_1 \overline{\lambda}(R) \|\xi(t)\|^2, \\ 0 \le \int_{-h_2}^{-h_1} \int_{t+\beta}^{t} K^T(s) G^T R_2 G K(s)\, ds\, d\beta \le \nu_2 \overline{\lambda}(R) \|\xi(t)\|^2, \\ 0 \le \int_{-h_2}^{-h_1} \int_{t+\beta}^{t} \xi^T(s) G^T R_3 G \xi(s)\, ds\, d\beta \le \nu_3 \overline{\lambda}(R) \|\xi(t)\|^2. \end{gathered} \tag{13}$$

It follows that

$$\begin{aligned} \underline{\lambda}(P) \|\xi(t)\|^2 \le V(\xi(t), t) \le {} & \overline{\lambda}(P) \|\xi(t)\|^2 + h_1 \overline{\lambda}(Q) \|\xi(t)\|^2 \\ & + h_2 \overline{\lambda}(Q) \|\xi(t)\|^2 + \nu_1 \overline{\lambda}(R) \|\xi(t)\|^2 \\ & + \nu_2 \overline{\lambda}(R) \|\xi(t)\|^2 + \nu_3 \overline{\lambda}(R) \|\xi(t)\|^2. \end{aligned} \tag{14}$$

From Itô formula, the stochastic differential equation can be computed as follows: $dV(\xi(t), t) = \mathbf{L}V(\xi(t), t) + 2\xi^T(t) P K(t) d\omega(t)$, where

$$\begin{aligned} & \mathbf{L}V(\xi(t), t) \\ & \quad \le 2\xi^T(t) P \Phi(t) + K^T(t) P K(t) \\ & \qquad + \xi^T(t) G^T Q_1 G \xi(t) + \xi^T(t) G^T Q_2 G \xi(t) \\ & \qquad - \xi^T(t - h_1) G^T Q_1 G \xi(t - h_1) \\ & \qquad - \xi^T(t - h_2) G^T Q_2 G \xi(t - h_2) \\ & \qquad + (h_2 - h_1) \left(\Phi^T(t) G^T R_1 G \Phi(t) + K(t)^T G^T R_2 G K(t) \right. \\ & \qquad \left. + \xi^T(t) G^T R_3 G \Phi(t)\right) \\ & \qquad - \int_{t-h_2}^{t-h_1} \left[\xi^T(s) G^T R_3 G \xi(s) + \Phi^T(s) G^T R_1 G \Phi(s) \right. \\ & \qquad \left. + K^T(s) G^T R_2 G K(s)\right] ds \end{aligned}$$

$$+ 2\eta^T(t)\overline{T}_2G\Big[\xi(t-h_1) - \xi(t-\tau(t)) - \int_{t-\tau(t)}^{t-h_1}\Phi(s)\,ds - \int_{t-\tau(t)}^{t-h_1}K(s)\,d\omega(s)\Big]$$
$$+ 2\eta^T(t)\overline{T}_1G\Big[\xi(t-\tau(t)) - \xi(t-h_2) - \int_{t-h_2}^{t-\tau(t)}\Phi(s)\,ds - \int_{t-h_2}^{t-\tau(t)}K(s)\,d\omega(s)\Big]$$
$$+ (\tau(t)-h_1)\eta^T(t)\overline{T}_2R_1^{-1}\overline{T}_2^T\eta(t) - \int_{t-\tau(t)}^{t-h_1}\eta^T(t)\overline{T}_2R_1^{-1}\overline{T}_2^T\eta(t)\,ds$$
$$+ (h_2-\tau(t))\eta^T(t)\overline{T}_1R_1^{-1}\overline{T}_1^T\eta(t) - \int_{t-h_2}^{t-\tau(t)}\eta^T(t)\overline{T}_1R_1^{-1}\overline{T}_1^T\eta(t)\,ds, \quad (15)$$

where

$$\eta^T(t) = \Big[\xi^T(t) \;\; \xi^T(t-h_1)G^T \;\; \xi^T(t-h_2)G^T \;\; \xi^T(t-\tau(t))G^T \;\; \Big(\int_{t-h_2}^{t-h_1}\xi(s)^Tds\Big)G^T \;\; v(t)\Big]. \quad (16)$$

And the final eight lines of (15) are equal to 0 from the Newton-Leibnitz formula.

Remark 6. In the proof of the theorem, we adopt Newton-Leibnitz formula to reduce the conservatism. Moreover, the results obtained in this theorem can be further extended based on fuzzy or piecewise Lyapunov-Krasovskii function.

Now, it is easy to see that

$$\begin{aligned}2\xi^T(t)P\Phi(t) &= 2\xi^T(t)P\big[(\widetilde{A}+\Delta\widetilde{A}(t))\xi(t) + (\widetilde{A}_d+\Delta\widetilde{A}_d(t))G\xi(t-\tau(t)) + (\widetilde{B}+\Delta\widetilde{B}(t))v(t)\big] \\ &\quad + \xi^T(t)P\widetilde{E}\widetilde{E}^TP\xi(t) + g(\xi(t))^Tg(\xi(t)) \\ &\le 2\xi^T(t)P\big[(\widetilde{A}+\Delta\widetilde{A}(t))\xi(t) + (\widetilde{A}_d+\Delta\widetilde{A}_d(t))G\xi(t-\tau(t)) + (\widetilde{B}+\Delta\widetilde{B}(t))v(t)\big] \\ &\quad + \xi^T(t)\big(P\widetilde{E}\widetilde{E}^TP + lI\big)\xi(t).\end{aligned} \quad (17)$$

Moreover,

$$-\int_{t-h_2}^{t-h_1}\xi^T(s)G^TR_3G\xi(s)\,ds \le -\frac{1}{h_2-h_1}\Big(\int_{t-h_2}^{t-h_1}\xi(s)ds\Big)^TG^TR_3G\Big(\int_{t-h_2}^{t-h_1}\xi(s)ds\Big). \quad (18)$$

By the above formulas (15)–(18), we can deduce that

$$\begin{aligned}\mathbf{L}V(\xi(t),t) &\le \eta^T(t)\big[\overline{\Omega} + \Psi_{12}\Psi_{22}^{-1}\Psi_{12}^T + \Psi_{13}\Psi_{33}^{-1}\Psi_{13}^T + \Psi_{15}\Psi_{15}^T\big]\eta(t) \\ &\quad - \xi^T(t)S\xi(t) + \rho v^T(t)v(t),\end{aligned} \quad (19)$$

where $\rho > 0$ is a given positive scalar and

$$\overline{\Omega} = \begin{bmatrix}\Omega_{11} & 0 & 0 & \Omega_{14} & 0 & P(\widetilde{B}+\Delta\widetilde{B}(t)) \\ * & \Omega_{22} & 0 & \Omega_{24} & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 \\ * & * & * & * & * & -\rho I\end{bmatrix}. \quad (20)$$

From (10) and (20), we can deduce

$$\eta^T(t)\big[\overline{\Omega} + \Psi_{12}\Psi_{22}^{-1}\Psi_{12}^T + \Psi_{13}\Psi_{33}^{-1}\Psi_{13}^T + \Psi_{15}\Psi_{15}^T\big]\eta(t) < 0, \quad (21)$$

which means

$$\begin{aligned}\mathbf{L}V(\xi(t),t) &\le -\xi^T(t)S\xi(t) + \rho v^T(t)v(t) \\ &\le -\lambda_{\min}(S)\|\xi(t)\|^2 + \rho\|v(t)\|^2.\end{aligned} \quad (22)$$

Together with (14), (22), and **Lemma 4**, the system $(\widetilde{\Sigma})$ is SISS.

Now, we are in the position to proof that $(\widetilde{\Sigma})$ satisfies an H_∞ attenuation level.

By Itô's formula, there is

$$\mathsf{E}(V(\xi(t),t)) = \mathsf{E}\Big(\int_0^t \mathbf{L}V(\xi(s),s)ds\Big). \quad (23)$$

Now, we consider the H_∞ performance of the filtering error system $(\widetilde{\Sigma})$. Define $J(t) = \mathsf{E}\{\int_0^t[\widetilde{z}(s)^T\widetilde{z}(s) - \gamma^2v(s)^Tv(s)]ds\}$ and consider (23). It is obvious that

$$\begin{aligned}J(t) &= \mathsf{E}\Big\{\int_0^t\big[\widetilde{z}^T(s)\widetilde{z}(s) - \gamma^2v^T(s)v(s) + \mathbf{L}V(\xi(s),s)\big]ds\Big\} - \mathsf{E}(V(\xi(t),t)) \\ &\le \mathsf{E}\Big\{\int_0^t\big[\widetilde{z}^T(s)\widetilde{z}(s) - \gamma^2v^T(s)v(s) + \mathbf{L}V(\xi(s),s)\big]ds\Big\} \\ &= \mathsf{E}\Big\{\int_0^t\Big[\xi^T(t)(\widetilde{L}+\Delta\widetilde{L}(t))^T(\widetilde{L}+\Delta\widetilde{L}(t))\xi(t) - \gamma^2v^T(t)v(t) + \mathbf{L}V(\xi(s),s)\Big]ds\Big\}.\end{aligned} \quad (24)$$

Using the same method in (15), we can deduce the following formula:

$$\begin{aligned}&\xi^T(t)\big(\widetilde{L}+\Delta\widetilde{L}(t)\big)^T\big(\widetilde{L}+\Delta\widetilde{L}(t)\big)\xi(t)\\&\quad-\gamma^2 v^T(t)\,v(t)+\mathbf{L}V(\xi(s),s)\\&\quad\le\eta^T(t)\left[\Omega+\Psi_{12}\Psi_{22}^{-1}\Psi_{12}^T+\Psi_{13}\Psi_{33}^{-1}\Psi_{13}^T\right.\\&\quad\left.+\Psi_{14}\Psi_{14}^T+\Psi_{15}\Psi_{15}^T\right]\eta(t).\end{aligned}\tag{25}$$

Then, applying the Schur complement formula to (10), there is

$$\begin{aligned}&\eta^T(t)\left[\Omega+\Psi_{12}\Psi_{22}^{-1}\Psi_{12}^T+\Psi_{13}\Psi_{33}^{-1}\Psi_{13}^T\right.\\&\quad\left.+\Psi_{14}\Psi_{14}^T+\Psi_{15}\Psi_{15}^T\right]\eta(t)<0\end{aligned}\tag{26}$$

for all $t>0$. Therefore, for all $\eta(t)\neq 0$, $J(t)<0$, which means that (9) is satisfied. This completes the proof. □

Remark 7. Since not all the delays begin at 0 moments, the delay we considered here contains both the upper bound and the lower bound, which is different from most of the existing works. Instead of the $[0,h)$ expression of the time delay, a more reliable sufficient condition is proposed in this paper.

Based on the above results, a sufficient condition for the solvability of robust H_∞ filtering problem for system $(\widetilde{\Sigma})$ is considered in the next theorem.

Theorem 8. *Consider the uncertain T-S fuzzy stochastic time-varying delay system $(\widetilde{\Sigma})$ and a constant scalar $\gamma>0$. The robust H_∞ filtering problem is solvable if there exist scalars $\varepsilon_{1i}>0$, $\varepsilon_{2i}>0$, $\varepsilon_{3i}>0$, $\varepsilon_{4i}>0$, and $\varepsilon_{5i}>0$ and matrices $P_1>0$, $P_2>0$, $S_1>0$, $S_2>0$, $R_j>0$, $j=1,2,3$, $Q_i>0$, T_{1i}, T_{2i}, $i=1,2$; W_{1i},W_{2i},W_{3i}, $1\le i\le r$, $\{\varrho_i=\varrho_i^T,1\le i\le r\}$, and $\{\pi_{ij},1\le i<j\le r\}$, such that the following LMIs hold:*

$$\begin{bmatrix}\varrho_1&\pi_{12}&\cdots&\pi_{1r}\\ *&\varrho_2&\cdots&\pi_{2r}\\ \vdots&\vdots&\ddots&\vdots\\ *&*&\cdots&\varrho_r\end{bmatrix}<0,$$

$$\begin{bmatrix}\varsigma_{ii}&\chi_{1i}&\chi_{2i}&\chi_{3i}&\chi_{4i}&\chi_{5i}\\ *&-\varepsilon_{1i}&0&0&0&0\\ *&*&-\varepsilon_{2i}&0&0&0\\ *&*&*&-\varepsilon_{3i}&0&0\\ *&*&*&*&-\varepsilon_{4i}&0\\ *&*&*&*&*&-\varepsilon_{5i}\end{bmatrix}<0,\quad(1\le i\le r),$$

$$\begin{bmatrix}\varsigma_{ij}&\chi_{1i}&\chi_{1j}&\chi_{2i}&\chi_{2j}&\chi_{3i}&\chi_{3j}&\chi_{4i}&\chi_{4j}&\chi_{5i}&\chi_{5j}\\ *&-\varepsilon_{1i}&0&0&0&0&0&0&0&0&0\\ *&*&-\varepsilon_{1j}&0&0&0&0&0&0&0&0\\ *&*&*&-\varepsilon_{2i}&0&0&0&0&0&0&0\\ *&*&*&*&-\varepsilon_{2j}&0&0&0&0&0&0\\ *&*&*&*&*&-\varepsilon_{3i}&0&0&0&0&0\\ *&*&*&*&*&*&-\varepsilon_{3j}&0&0&0&0\\ *&*&*&*&*&*&*&-\varepsilon_{4i}&0&0&0\\ *&*&*&*&*&*&*&*&-\varepsilon_{4j}&0&0\\ *&*&*&*&*&*&*&*&*&-\varepsilon_{5i}&0\\ *&*&*&*&*&*&*&*&*&*&-\varepsilon_{5j}\end{bmatrix}<0,\quad(1\le i<j\le r),\tag{27}$$

where

$$\begin{aligned}\varsigma_{ii}&=\Gamma_{ii}-\varrho_i+\varepsilon_{1i}\Xi_{1i}\Xi_{1i}^T+\varepsilon_{2i}\Xi_i\Xi_{2i}^T+\varepsilon_{3i}\Xi_{3i}\Xi_{3i}^T\\&\quad+\varepsilon_{4i}\Xi_{4i}\Xi_{4i}^T+\varepsilon_{5i}\Xi_{5i}\Xi_{5i}^T,\\ \varsigma_{ij}&=\Gamma_{ij}+\Gamma_{ji}-\pi_{ij}-\pi_{ji}+\varepsilon_{1i}\Xi_{1i}\Xi_{1i}^T+\varepsilon_{2i}\Xi_i\Xi_{2i}^T\\&\quad+\varepsilon_{3i}\Xi_{3i}\Xi_{3i}^T+\varepsilon_{4i}\Xi_{4i}\Xi_{4i}^T+\varepsilon_{5i}\Xi_{5i}\Xi_{5i}^T+\varepsilon_{1j}\Xi_{1j}\Xi_{1j}^T\\&\quad+\varepsilon_{2j}\Xi_j\Xi_{2j}^T+\varepsilon_{3j}\Xi_{3j}\Xi_{3j}^T+\varepsilon_{4j}\Xi_{4j}\Xi_{4j}^T+\varepsilon_{5j}\Xi_{5j}\Xi_{5j}^T,\end{aligned}$$

$$\chi_{1i}=\left[M_{1i}^TP_1\ \ \mathbf{0}_{1*6}\ \ (h_2-h_1)M_{1i}^TR_1\ \ \mathbf{0}_{1*10}\right]^T,$$

$$\Xi_{1i}^T=\left[N_{1i}^T\ \ \mathbf{0}_{1*3}\ \ N_{2i}^T\ \ 0\ \ N_{3i}^T\ \ \mathbf{0}_{1*11}\right],$$

$$\Xi_{5i}^T=\left[0\ \ -N_{6i}\ \ \mathbf{0}_{1*16}\right],$$

$$\chi_{2i}=\left[\mathbf{0}_{1*8}\ \ (h_2-h_1)M_{2i}^TR_2\ \ M_{2i}^TP_1\ \ \mathbf{0}_{1*8}\right]^T,$$

$$\Xi_{2i}^T=\left[N_{1i}\ \ \mathbf{0}_{1*3}\ \ N_{2i}\ \ \mathbf{0}_{1*13}\right],$$

$$\chi_{3i}=\left[0\ \ M_{4i}^TP_2\ \ \mathbf{0}_{1*16}\right]^T,$$

$$\Xi_{3i}^T=\left[N_{5i}C_j\ \ N_{4i}\ \ 0\ \ 0\ \ N_{5i}C_{dj}\ \ 0\ \ N_{5i}D_j\ \ \mathbf{0}_{1*11}\right],$$

$$\chi_{4i}=\left[\mathbf{0}_{1*17}\ \ M_{2i}^T\right]^T,\qquad\Xi_{4i}^T=\left[0\ \ N_{3i}\ \ \mathbf{0}_{1*16}\right],$$

$$\chi_{5i}=\left[\mathbf{0}_{1*17}\ \ M_{4i}^T\right]^T,$$

$$\Gamma_{ij}=\begin{bmatrix}\Gamma_{11}&\Gamma_{12}&\Gamma_{13}&\Gamma_{14}&\Gamma_{15}\\ *&\Gamma_{22}&0&0&0\\ *&*&\Gamma_{33}&0&0\\ *&*&*&-I&0\\ *&*&*&*&-I\end{bmatrix},$$

$$\Gamma_{11}=\begin{bmatrix}G_{11}&C_j^TW_{2i}^T&0&0&P_1A_{di}&0&P_1B_i\\ *&G_{22}&0&0&W_{2i}C_{dj}&0&W_{2i}D_j\\ *&*&G_{33}&0&-T_2+T_2^T&0&0\\ *&*&*&G_{44}&T_1-T_1^T&0&0\\ *&*&*&*&G_{55}&0&0\\ *&*&*&*&*&-\dfrac{R_3}{h_2-h_1}&0\\ *&*&*&*&*&*&-I\end{bmatrix},$$

$$G_{11} = P_1 A_i + A_i^T P_1 + l + Q_1 + Q_2 + (h_2 - h_1) R_3 + S_1,$$
$$G_{22} = W_{1i}^T + W_{1i} + l + S_2,$$
$$G_{33} = -Q_1 + T_2 + T_2^T,$$
$$G_{44} = -Q_2 - T_1 - T_1^T,$$
$$G_{55} = -T_2 - T_2^T + T_1 + T_1^T,$$
$$\Gamma_{12} = \begin{bmatrix} (h_2 - h_1) A_i^T R_1 & (h_2 - h_1) H_i^T R_2 & H_i^T X & H_i^T \\ (h_2 - h_1) A_i^T R_1 & (h_2 - h_1) H_i^T R_2 & H_i^T X & H_i^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (h_2 - h_1) A_{di}^T R_1 & (h_2 - h_1) H_{di}^T R_2 & H_{di}^T X & H_{di}^T \\ 0 & 0 & 0 & 0 \\ (h_2 - h_1) B_i^T R_1 & 0 & 0 & 0 \end{bmatrix},$$
$$\Gamma_{22} = \operatorname{diag}\left\{-(h_2 - h_1)R_1, -(h_2 - h_1)R_2, \begin{bmatrix} -P_1 & 0 \\ 0 & -P_2 \end{bmatrix}\right\},$$
$$\Gamma_{13} = \begin{bmatrix} \widetilde{T}_1 & \widetilde{T}_2 & (h_2 - h_1)\widetilde{T}_1 & (h_2 - h_1)\widetilde{T}_2 \end{bmatrix},$$
$$\Gamma_{33} = \operatorname{diag}\{-R_2, -R_2, -(h_2 - h_1)R_1, -(h_2 - h_1)R_1\},$$
$$\Gamma_{14}^T = \begin{bmatrix} L_i^T & -L_{fi}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Gamma_{15}^T = \begin{bmatrix} P_1 E_i^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & W_{3i}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{28}$$

When the LMIs (27) are feasible, the nonfragile filter we desired here can be chosen as

$$A_{fi} = P_2^{-1} W_{1i}, \quad B_{fi} = P_2^{-1} W_{2i}, \quad L_{fi}, \quad E_{fi} = P_2^{-1} W_{3i}, \quad i = 1, \ldots, r. \tag{29}$$

Proof. Define

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \qquad S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}. \tag{30}$$

By using the same methods in [31], it can be easily proven that the condition in Theorem 5 and the LMIs in (27) are equivalent. Then, we can conclude that the filtering error system $(\widetilde{\Sigma})$ is SISS with H_∞ performance level γ. □

Remark 9. The desired H_∞ filters can be constructed by solving the LMIs in (27), which can be implemented by using standard numerical algorithms, and no tuning of parameters will be involved.

4. Numerical Example

In this section, a numerical example is provided to show the effectiveness of the results obtained in the previous section.

Example. Consider the T-S fuzzy stochastic system $(\widetilde{\Sigma})$ with model parameters given as follows:

$$\begin{gathered} A_1 = \begin{bmatrix} -3.5 & 1 \\ -1.7 & -5.8 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -0.15 & -0.4 \\ 0 & 0.3 \end{bmatrix}, \\ A_2 = \begin{bmatrix} -2.1 & 0.6 \\ -1.7 & -2.9 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix}, \\ H_{d1} = \begin{bmatrix} -0.01 & 0.02 \\ 0.01 & -0.05 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} -0.6 & -0.1 \\ 0.2 & 0.3 \end{bmatrix}, \\ C_1 = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}, \qquad C_{d1} = \begin{bmatrix} -0.05 & -0.05 \end{bmatrix}, \\ C_2 = \begin{bmatrix} 0.2 & -0.4 \end{bmatrix}, \qquad C_{d2} = \begin{bmatrix} -0.4 & -0.5 \end{bmatrix}, \\ L_1 = \begin{bmatrix} 1.5 & -0.6 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} -0.3 & 0.2 \end{bmatrix}, \\ D_1 = 0.2, \qquad D_2 = -0.2, \\ B_1 = \begin{bmatrix} 0.9 \\ -0.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.2 \\ -0.5 \end{bmatrix}, \\ A_{d2} = \begin{bmatrix} -0.18 & 0 \\ -0.22 & -0.24 \end{bmatrix}, \qquad H_{d2} = \begin{bmatrix} -0.15 & 0.6 \\ 0.01 & 0.4 \end{bmatrix}. \end{gathered} \tag{31}$$

And the parameter uncertainties are shown as

$$\begin{gathered} M_{11} = \begin{bmatrix} 0.1 & 0.2 \\ -0.5 & 0.1 \end{bmatrix}, \qquad M_{12} = \begin{bmatrix} -0.2 & 0.1 \\ 0.3 & -0.1 \end{bmatrix}, \\ M_{21} = \begin{bmatrix} 0.8 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}, \qquad N_{11} = \begin{bmatrix} 0 & -0.3 \\ 0.1 & -0.2 \end{bmatrix}, \\ N_{21} = \begin{bmatrix} -0.2 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \qquad M_{22} = \begin{bmatrix} -0.1 & 0.2 \\ 0.4 & -0.2 \end{bmatrix}, \\ N_{12} = \begin{bmatrix} -0.5 & 0 \\ 0.2 & -0.3 \end{bmatrix}, \qquad N_{22} = \begin{bmatrix} 0 & -0.2 \\ 0 & 0.1 \end{bmatrix}. \end{gathered} \tag{32}$$

The membership functions are

$$\begin{gathered} h_1(x_1(t)) = \left(1 - \frac{x_1}{1 + \exp(x_1(t) + 1)}\right), \\ h_2(x_1(t)) = 1 - h_1(x_1(t)). \end{gathered} \tag{33}$$

By using the Matlab LMI Control Toolbox, the nonfragile robust H_∞ filtering problem is solvable to Theorem 8. It can be calculated that, for any $0 < h_1(t) \le h_2(t) \le 6$ and the nonlinear function $g(x(t)) = \sin(x_2(t))$, the robust H_∞ filtering problem can be solved with the H_∞ performance

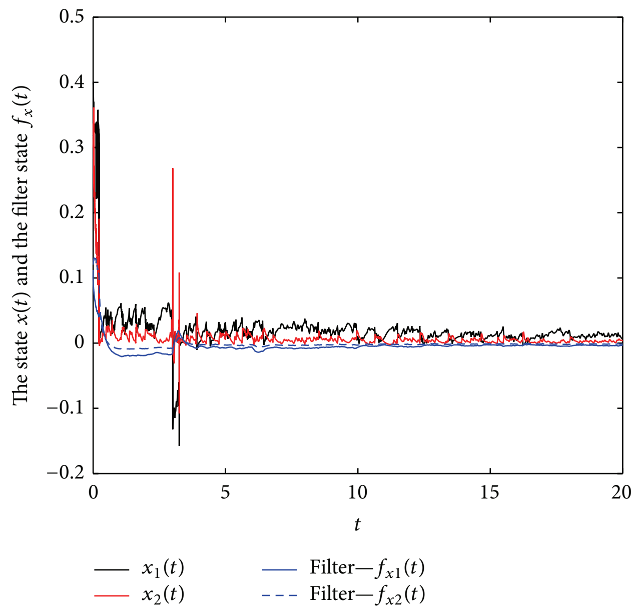

FIGURE 1: State responses of $x(t)$ and $\hat{x}(t)$.

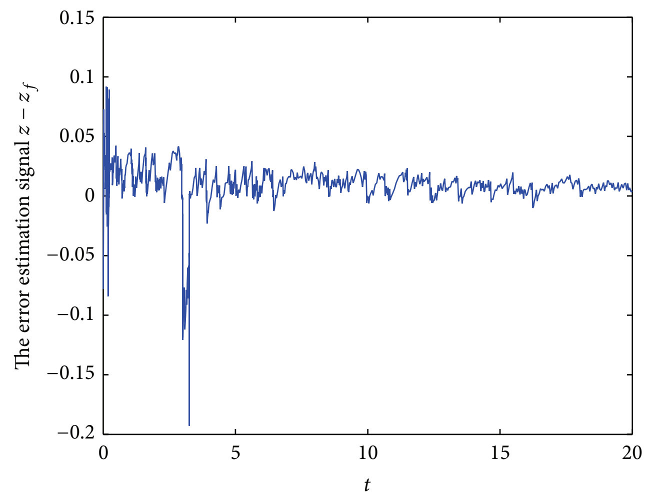

FIGURE 2: Responses of the error signal $z(t) - \tilde{z}(t)$.

level $\gamma = 0.46$. And the desired fuzzy filter can be constructed as in the form of (3) with

$$A_{f1} = \begin{bmatrix} -5.4320 & 0.4511 \\ 1.8159 & -1.5495 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -8.1142 & 3.4902 \\ 2.9687 & -5.9058 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} -1.0301 \\ 0.1040 \end{bmatrix}, \qquad B_{f2} = \begin{bmatrix} -1.0171 \\ 0.0415 \end{bmatrix},$$
$$L_{f1} = \begin{bmatrix} -0.3063 & -0.0422 \end{bmatrix}, \tag{34}$$
$$L_{f2} = \begin{bmatrix} -0.2667 & -0.0422 \end{bmatrix},$$
$$E_{f1} = \begin{bmatrix} 0.00361 & 0.0000 \\ 0.0000 & 0.00361 \end{bmatrix},$$
$$E_{f2} = \begin{bmatrix} 0.00274 & 0.00000 \\ 0.00000 & 0.00274 \end{bmatrix}.$$

The simulation results of the state responses in system (Σ) and the filter are given in Figure 1, where the initial conditions are $x_0(t) = [0.4 \quad 0.3]^T$ and $\hat{x}_0(t) = [0.1 \quad 0.1]^T$. Figure 2 shows the simulation results of the signal $\tilde{z}(t)$.

5. Conclusion

This paper considers the nonfragile H_∞ filter design problem for the uncertain time-delay T-S fuzzy stochastic system. Sufficient conditions have been addressed to guarantee that the system $(\tilde{\Sigma})$ is SISS. An LMI approach has been developed to design the fuzzy filter ensuring a prescribed H_∞ performance level of the filtering error system for all admissible uncertainties. Finally, a numerical example has been provided to show the effectiveness of the proposed filter design methods.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grants 61203048, 61304047, 11371013, and 61203047 and the foundation from Shenyang State Key Laboratory of Robotics.

References

[1] E. D. Sontag, "Smooth stabilization implies coprime factorization," *IEEE Transactions on Automatic Control*, vol. 34, no. 4, pp. 435–443, 1989.

[2] G.-D. Hu and M. Z. Liu, "Input-to-state stability of Runge-Kutta methods for nonlinear control systems," *Journal of Computational and Applied Mathematics*, vol. 205, no. 1, pp. 633–639, 2007.

[3] Q. L. Hu, Y. Y. Lv, J. Zhang, and G. F. Ma, "Input-to-state stable cooperative attitude regulation of spacecraft formation flying," *Journal of the Franklin Institute*, vol. 350, no. 1, pp. 50–71, 2013.

[4] Z.-P. Jiang and Y. Wang, "Input-to-state stability for discrete-time nonlinear systems," *Automatica*, vol. 37, no. 6, pp. 857–869, 2001.

[5] T. F. Liu, D. J. Hill, and Z.-P. Jiang, "Lyapunov formulation of ISS cyclic-small-gain in continuous-time dynamical networks," *Automatica*, vol. 47, no. 9, pp. 2088–2093, 2011.

[6] E. D. Sontag, "The ISS philosophy as a unifying framework for stability-like behavior," in *Nonlinear Control in the Year 2000 Volume 2*, vol. 259 of *Lecture Notes in Control and Information Sciences*, pp. 443–467, Springer, London, UK, 2001.

[7] Z. X. Tai, "Input-to-state stability for Lur'e stochastic distributed parameter control systems," *Applied Mathematics Letters*, vol. 25, no. 4, pp. 706–711, 2012.

[8] F. Zhao, Q. Zhang, X.-G. Yan, M. Cai, and J. Ren, "Stochastic input-to-state stability and H_∞ filtering for a class of stochastic nonlinear systems," *Applied Mathematics and Computation*, vol. 227, pp. 672–686, 2014.

[9] Q. Zhu and J. Cao, "Mean-square exponential input-to-state stability of stochastic delayed neural networks," *Neurocomputing*, vol. 131, pp. 157–163, 2014.

[10] Y. Zhang, P. Shi, S. Kiong Nguang, H. R. Karimi, and R. K. Agarwal, "Robust finite-time fuzzy H_∞ control for uncertain time-delay systems with stochastic jumps," *Journal of the Franklin Institute. Engineering and Applied Mathematics*, vol. 351, no. 8, pp. 4211–4229, 2014.

[11] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[12] K. Tanaka and M. Sano, "Robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer," *IEEE Transactions on Fuzzy Systems*, vol. 2, no. 2, pp. 119–134, 1994.

[13] Y.-Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 2, pp. 200–211, 2000.

[14] C.-W. Chen, K. Yeh, K. F. Liu, and M.-L. Lin, "Applications of fuzzy control to nonlinear time-delay systems using the linear matrix inequality fuzzy Lyapunov method," *Journal of Vibration and Control*, vol. 18, no. 10, pp. 1561–1574, 2012.

[15] J. Liu and D. Yue, "Asymptotic and robust stability of T-S fuzzy genetic regulatory networks with time-varying delays," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 8, pp. 827–840, 2012.

[16] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Reliable H_∞ control for discrete-time fuzzy systems with infinite-distributed delay," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 22–31, 2012.

[17] H. Yang, P. Shi, X. Li, and Z. Li, "Fault-tolerant control for a class of T-S fuzzy systems via delta operator approach," *Signal Processing*, vol. 98, pp. 166–173, 2014.

[18] B. Zhang, W. X. Zheng, and S. Xu, "Passivity analysis and passive control of fuzzy systems with time-varying delays," *Fuzzy Sets and Systems*, vol. 174, pp. 83–98, 2011.

[19] C. Zhang, G. Feng, J. Qiu, and W.-A. Zhang, "T—S fuzzy-model-based piecewise H_∞ output feedback controller design for networked nonlinear systems with medium access constraint," *Fuzzy Sets and Systems*, vol. 248, pp. 86–105, 2014.

[20] H. Dong, Z. Wang, and H. Gao, "On design of quantized fault detection filters with randomly occurring nonlinearities and mixed time-delays," *Signal Processing*, vol. 92, no. 4, pp. 1117–1125, 2012.

[21] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities," *Automatica*, vol. 50, no. 1, pp. 180–186, 2014.

[22] H. B. Chen, "Delay-dependent robust H_∞ filter design for uncertain neutral stochastic system with time-varying delay," *IET Signal Processing*, vol. 7, no. 5, pp. 368–381, 2013.

[23] Z. Li, S. Xu, Y. Zou, and Y. Chu, "Robust H_∞ filter design of uncertain T-S fuzzy neutral systems with time-varying delays," *International Journal of Systems Science. Principles and Applications of Systems and Integration*, vol. 42, no. 7, pp. 1231–1238, 2011.

[24] M. Liu, J. You, and X. Ma, "H_∞ filtering for sampled-data stochastic systems with limited capacity channel," *Signal Processing*, vol. 91, no. 8, pp. 1826–1837, 2011.

[25] X. Su, P. Shi, L. Wu, and Y.-D. Song, "A novel approach to filter design for T-S fuzzy discrete-time systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 6, pp. 1114–1129, 2012.

[26] J. You and S. Yin, "H_∞ filtering for time-delay TCS fuzzy systems with intermittent measurements and quantization," *Journal of the Franklin Institute*, vol. 351, pp. 3734–3751, 2014.

[27] B. Y. Zhang and W. X. Zheng, "H_∞ filter design for nonlinear networked control systems with uncertain packet-loss probability," *Signal Processing*, vol. 92, no. 6, pp. 1499–1507, 2012.

[28] M. Liu, D. W. Ho, and Y. Niu, "Robust filtering design for stochastic system with mode-dependent output quantization," *IEEE Transactions on Signal Processing*, vol. 58, no. 12, pp. 6410–6416, 2010.

[29] G. L. Wang and C. L. Su, "Delay-distribution-dependent H_∞ filtering for linear systems with stochastic time-varying delays," *IEEE Engineering and Applied Mathematics*, vol. 350, no. 2, pp. 358–377, 2013.

[30] L. H. Keel and S. P. Bhattacharyya, "Robust, fragile, or optimal?" *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1098–1105, 1997.

[31] Z. Li, Y. Chu, and S. Xu, "Delay-dependent nonfragile robust H_∞ filtering of T-S fuzzy time-delay systems," *Circuits, Systems, and Signal Processing*, vol. 29, no. 3, pp. 361–375, 2010.

[32] M. S. Mahmoud, "Resilient linear filtering of uncertain systems," *Automatica*, vol. 40, no. 10, pp. 1797–1802, 2004.

[33] G. H. Yang and J. L. Wang, "Robust nonfragile Kalman filtering for uncertain linear systems with estimator gain uncertainty," *IEEE Transactions on Automatic Control*, vol. 46, no. 2, pp. 343–348, 2001.

[34] G.-H. Yang and W.-W. Che, "Non-fragile H_∞ filter design with additive gain variations," in *Proceedings of the 45th IEEE Conference on Decision and Control (CDC '06)*, pp. 4775–4780, San Diego, Calif, USA, December 2006.

[35] W. Fei, H. Liu, and W. Zhang, "On solutions to fuzzy stochastic differential equations with local martingales," *Systems & Control Letters*, vol. 65, pp. 96–105, 2014.

[36] L. Wu and D. W. C. Ho, "Fuzzy filter design for Itô stochastic systems with application to sensor fault detection," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 1, pp. 233–242, 2009.

[37] P. Balasubramaniam and T. Senthilkumar, "Delay-dependent robust stabilization and H_∞ control for uncertain stochastic T-S fuzzy systems with discrete interval and distributed time-varying delays," *International Journal of Automation and Computing*, vol. 10, no. 1, pp. 18–31, 2013.

[38] Q. Song, Z. Zhao, and J. Yang, "Passivity and passification for stochastic Takagi-Sugeno fuzzy systems with mixed time-varying delays," *Neurocomputing*, vol. 122, pp. 330–337, 2013.

[39] J. Yang, S. Zhong, W. Luo, and G. Li, "Delay-dependent stabilization for stochastic delayed fuzzy systems with impulsive effects," *International Journal of Control, Automation and Systems*, vol. 8, no. 1, pp. 127–134, 2010.

[40] W.-H. Chen, B.-S. Chen, and W. Zhang, "Robust control design for nonlinear stochastic partial differential systems with Poisson noise: fuzzy implementation," *Fuzzy Sets and Systems*, vol. 254, pp. 83–105, 2014.

[41] Y. Li, S. Tong, T. Li, and X. Jing, "Adaptive fuzzy control of uncertain stochastic nonlinear systems with unknown dead zone using small-gain approach," *Fuzzy Sets and Systems*, vol. 235, pp. 1–24, 2014.

[42] T. Senthilkumar and P. Balasubramaniam, "Delay-dependent robust H_∞ control for uncertain stochastic T-S fuzzy systems with time-varying state and input delays," *International Journal of Systems Science: Principles and Applications of Systems and Integration*, vol. 42, no. 5, pp. 877–887, 2011.

[43] T. Wang, S. Tong, and Y. Li, "Robust adaptive fuzzy output feedback control for stochastic nonlinear systems with unknown control direction," *Neurocomputing*, vol. 106, pp. 31–41, 2013.

[44] Z. Wang, L. Huang, X. Yang, and A. Xin, "Adaptive fuzzy control for stochastic nonlinear systems via sliding mode method," *Circuits, Systems, and Signal Processing*, vol. 32, no. 6, pp. 2839–2850, 2013.

[45] Z. Xia, J. Li, and J. Li, "Delay-dependent fuzzy static output feedback control for discrete-time fuzzy stochastic systems with distributed time-varying delays," *ISA Transactions*, vol. 51, no. 6, pp. 702–712, 2012.

[46] E. D. Sontag and Y. Wang, "On characterizations of the input-to-state stability property," *Systems & Control Letters*, vol. 24, no. 5, pp. 351–359, 1995.