Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 828979, 10 pages
http://dx.doi.org/10.1155/2015/828979

Research Article

Limit Theorems for Local Cumulative Shock Models with Cluster Shock Structure

Jianming Bai, Yun Chen, Chun Yuan, and Xiaoling Yin

School of Management, Lanzhou University, Lanzhou 730000, China

Correspondence should be addressed to Chun Yuan; ldycc123@163.com

Received 29 November 2014; Revised 12 February 2015; Accepted 27 February 2015

Academic Editor: Daniela Boso

This paper considers a more general shock model with insurance and financial risk background, in which the system is subject to two types of shocks called primary shocks and secondary shocks. Each primary shock causes a series of secondary shocks according to some cluster pattern. In reliability applications, a primary shock can represent an issue of insurance policies of an insurer company, and the secondary shocks then denote the relevant insurance claims generated by the policy. We focus on the local cumulative shock process where only a certain number of the most recent primary and secondary shocks are accumulated. This process is a very new topic in the available literature which is more flexible and realistic in modeling some more complex reliability situations such as bankrupt behavior of an insurance company. Based on the theory of infinite divisibility and stable distributions, we establish a central limit theorem for the local cumulative shock process and obtain the conditions for the process to converge to an infinitely divisible distribution or to an α-stable law. Also, by choosing the proper scale parameters, the process converges to a normal distribution.

1. Introduction

A shock model in reliability is an operating system subject to successive shocks of random magnitudes and random arrival times. In applications, such a system may appear in engineering, economics, and natural objects. The shocks can be overload and abrupt changes of temperature or voltage for a mechanical device and electronic equipment and be natural disasters for an ecological system or financial crises for an economic system. These shocks have impacts on the system and cause the final breakdown of the system.

In the literatures, the standard mathematical setup for the reaching law of shocks is a stochastic point process $\{N(t);\ t \geq 0\}$ with the shock instants $0 < S_1 < S_2 < \cdots$, and the shock magnitudes come from a family of nonnegative random variables $\{A_i\}$, $i = 1, 2, \ldots$. The main object in focus is the lifetime (or the failure time) of the system. Let τ denote the lifetime and $z > 0$ be the prefixed threshold of the system, then the two classical cases, cumulative shock model and extreme shock model, are defined, respectively, as

$$\{\tau \leq t\} \Longleftrightarrow \left\{\sum_{i=1}^{N(t)} A_i > z\right\}, \quad (1)$$

$$\{\tau \leq t\} \Longleftrightarrow \{\max\{A_1, \ldots, A_{N(t)}\} > z\} \quad (2)$$

for arbitrary $t > 0$. Hence, shock effects are accumulated in the cumulative shock case, and the system fails just when the cumulative magnitudes exceeds the threshold level. While in the extreme shock case, shock effects are memoryless and the system breaks down as soon as the magnitude of an individual shock is larger than the threshold. With relations (1) and (2), the so-called cumulative shock process $\{X(t);\ t \geq 0\}$ and maximum shock process $\{M(t);\ t \geq 0\}$ can be defined by

$$X(t) = \sum_{i=1}^{N(t)} A_i, \quad (3)$$

$$M(t) = \max\{A_1, \ldots, A_{N(t)}\}, \quad (4)$$

respectively. It is clear that the lifetime properties of the system are determined entirely by the processes $\{X(t)\}$ in the cumulative case and $\{M(t)\}$ in the extreme case.

Due to the important theory value and the broad application areas, shock models remain an academic focus in reliability researches during the last three decades. The main literatures on the two types of shock models include

Agrafiotis and Tsoukalas [1], Bai et al. [2], Gut [3, 4], Gut and Hüsler [5], Igaki et al. [6], Skoulakis [7], Finkelstein and Marais [8], Mercier and Pham [9], Omey and Vesilo [10], Sumita and Zuo [11], Wang et al. [12], and others. In these works, various reliability backgrounds are provided, the distributed characteristics of the system lifetime are discussed, and the asymptotic properties of the cumulative and maximum shock processes are investigated. In general, the study develops along the two directions. One is gradually profound exploration of the reliability properties of models. We can find that, with an evolvement of research objects from the early simple systems to some complex systems presently, the key problems also turn to distribution properties and limiting behaviors of the system lifetime and estimations for the failure probability from the lifetime distribution classes. The other is realistic extensions of models. In the past several years, the model structure and the relevant failure mechanism are rechanged or readjusted based on classical models to meet the various features of the real reliability systems, which is regarded as a main trend in current research.

In this paper, we setup a new shock model (called local cumulative shock model) based on a cluster point process and discuss the limit properties of the relevant shock process (called local cumulative shock process). Under the cluster structure, a primary shock, whenever it occurs, can trigger a series of secondary shocks, and the system fails when the superposed effect of primary and secondary shocks just over a certain local period exceeds the threshold. This model is an extension of the classical extreme shock model. With a more practical structure and failure mechanism than classical models, it is suitable to describe some complex reliability systems relating to earthquake disaster, network failure, and insurance risk.

The rest of this paper is organized as follows. In Section 2, we present a local cumulative shock model with a cluster structure and give the basic assumptions. The fundamental properties and weak limit theorems (based on the infinite divisibility and Lévy-Khintchine representation) of the local cumulative shock process are discussed in Sections 3 and 4, respectively. Finally, Section 5 concludes the paper.

2. Local Cumulative Shock Model with Cluster Structure

At the beginning, we list the main notations which are used in this paper.

$\{N(t);\ t \geq 0\}$ primary shock process with shock instants $0 < S_1 < S_2 < \cdots$, which is assumed to be a nonhomogeneous Poisson process.

$\lambda(t)$ intensity function of $\{N(t)\}$, and then cumulative intensity function is $\Lambda(t) = \int_0^t \lambda(s)\mathrm{d}s$.

X_i magnitude of the ith primary shock.

$\{M_i(t);\ t \geq 0\}$ arrival process of secondary shocks triggered by the ith primary shock. For $i = 1, 2, \ldots$, $\{M_i(t)\}$'s are assumed to be i.i.d. stochastic point processes.

Y_{ij} magnitude of the jth secondary shock caused by the ith primary shock.

$\{X(t);\ t \geq 0\}$ cumulative shock process which is a superposition of a primary process and a group of secondary processes.

$\{\Delta X_h(t);\ t \geq h\}$ local cumulative shock process defined as $\Delta X_h(t) = X(t) - X(t-h)$.

$G_\Delta(\theta)$ moment generating function of $\Delta X_h(t)$.

$C_\Delta(\theta)$ characteristic function of $\Delta X_h(t)$.

$(\delta^2(t), \nu(t,\cdot), b(t))$ characteristic triplet of $\Delta X_h(t)$, where $\nu(t,\cdot)$ is a Lévy measure.

$Z_h(t)$ regularized process of $\{\Delta X_h(t)\}$ with centering function $\mu(t)$ and regularizing function $\sigma(t) > 0$.

$C_Z(\theta)$ characteristic function of $Z_h(t)$.

$(\delta'^2(t), \nu'(t,\cdot), b'(t))$ characteristic triplet of $Z_h(t)$ with Lévy measure $\nu'(t,\cdot)$.

$U_1, \ldots, U_n$ i.i.d. random variables defined on $[0, t]$ with common distribution function $\Lambda(s)/\Lambda(t)$, where $s \in (0, t)$.

$\mathbf{1}_{\{\cdot\}}$ indicator function of a random event $\{\cdot\}$.

A generalized cumulative shock model and its lifetime properties are already discussed in our latest work [13], where the system considered is subject to two types of shocks, called primary shocks and secondary shocks, respectively, and each primary shock causes a series of secondary shocks according to a "cluster" mechanism. Then, the shock process has a cluster structure and is a superposition of a primary (shock) process and a group of adjunct (shock) processes, and the system fails once the totally superposed effect of the primary and secondary shocks exceeds the threshold level. Let $\{N(t);\ t \geq 0\}$ be the primary process with shock points $0 < S_1 < S_2 < \cdots$ and let $\{M_i(t);\ t \geq 0\}$ be the adjunct process caused by the ith primary shock. The relevant cumulative shock process $\{X(t);\ t \geq 0\}$ can be defined, through the totally superposed shock effect by time t, as

$$X(t) = \sum_{i=1}^{N(t)} \left(cX_i + \sum_{j=1}^{M_i(t-S_i)} Y_{ij} \right) = c\sum_{i=1}^{N(t)} X_i + \sum_{i=1}^{N(t)} \sum_{j=1}^{\infty} Y_{ij} \mathbf{1}_{\{S_i + D_{ij} \leq t\}}, \quad t \geq 0, \tag{5}$$

where S_i and X_i represent the occurrence instant and magnitude of the ith primary shock, $i = 1, 2, \ldots$; and for each i, $\{D_{ij}\}$ is a point sequence satisfying $0 = D_{i0} < D_{i1} < \cdots$; then, $S_i + D_{ij}$ and Y_{ij} represent the occurrence instant and magnitude of the jth secondary shock caused by the ith primary shock; $j = 1, 2, \ldots$; $\mathbf{1}_{\{\cdot\}}$ is the indicator function of event $\{\cdot\}$.

In particular, the coefficient "c" in Model (5) can be set differently to describe the different superposition patterns of primary shocks and secondary shocks for different applications. For example, for a seismic hazard, the total damage is the accumulated effect of both the main-quake (the primary shock) and all associated after-quakes (the secondary shocks). In this case we set $c = 1$. This is a very natural situation and the details for earthquake cluster background can be found in Ogata [14], Daley and Vere-Jones [15], and relevant references therein. Also, an upgrade of a computer software does not affect the computer system, but it may induce some software consistency issues which affect the system operation for a period of time. This is just the situation of $c = 0$, where only secondary shocks damage the system and all primary shocks are ineffective. The relevant cases are discussed by Hohn et al. [16] and Fäy et al. [17]. Moreover, we consider the repair on some system as the primary shock, each repair is imperfect and causes a cluster of redundant faults which can be considered as the secondary shocks. Thus, the two types of shocks have the opposite effects to the system and $c = -1$. Another interesting interpretation of this case is an insurance risk issue, whenever the insurer company issues an insurance policy and charges a corresponding premium, it has to burden a series of potential claim risks induced by this policy. Where a policy premium means a primary shock and the insurance claims play the secondary shocks. For the corresponding details of insurance and finance applications, please see mainly Rolski et al. [18], Denuit et al. [19], and Lindskog and McNeil [20].

With the above cluster structure, the new cumulative shock model associating with (5) is more appropriate for modeling some real and complex reliability situations. However, is the system's failure behaviour dependent purely on the total accumulation of shocks over $[0, t]$, the entire history of the system operation? An observation on the fate of Lehman Brothers Holdings implies a negative answer. Indeed, it is difficult to convince that a century-old insurance or financial firm will be immortal as its long successful experience. In fact, for many realistic reliability systems such as an earthquake-prone region and an insurance or financial company, an unpredictable misfortune may be initiated by some momentous events only recently rather than very long ago. This suggests that we concentrate on the impact events over some latest period and to regard a new reliability issue.

By this background, we consider a further case based on Model (5). For some $h > 0$, let

$$\begin{aligned}\Delta X_h(t) &= X(t) - X(t-h)\\ &= c\sum_{i=1}^{N(t)} X_i + \sum_{i=1}^{N(t)}\sum_{j=1}^{\infty} Y_{ij}\mathbf{1}_{\{S_i+D_{ij}\le t\}}\\ &\quad - c\sum_{i=1}^{N(t-h)} X_i - \sum_{i=1}^{N(t-h)}\sum_{j=1}^{\infty} Y_{ij}\mathbf{1}_{\{S_i+D_{ij}\le t-h\}}\\ &= c\sum_{i=1}^{N(t-h,t)} X_i + \sum_{i=1}^{N(t)}\sum_{j=1}^{M_i(t-S_i-h,t-S_i)} Y_{ij}, \quad t \ge h,\end{aligned} \tag{6}$$

where $\eta(t-h,t) := \eta(t) - \eta(t-h)$ denotes the arrival counts of a counting process $\{\eta(t)\}$ in the interval $(t-h, t]$. Thus, $\Delta X_h(t)$ measures the cumulative effect of both primary and secondary shocks over a recent period of length h and induces a stochastic process $\{\Delta X_h(t);\ t \ge h\}$, according to which we can define a new lifetime of the system as

$$\tau = \inf\{t : \Delta X_h(t) > z\}, \tag{7}$$

and the relevant failure event can be expressed as

$$\{\tau \le t\} \Longleftrightarrow \{\Delta X_h(s) > z,\ \text{for } h < s \le t\}. \tag{8}$$

It establishes a novel failure mechanism: the system fails if and only if the superposed effect of the primary and secondary shocks accumulated in the recent duration $(t-h, t]$, rather than the entire history $[0, t]$ as in Model (5), exceeds the threshold level.

Associating with the failure mechanism (8) is a new shock model; we call the local cumulative shock model with cluster structure, whose lifetime property is completely determined by $\{\Delta X_h(t);\ t \ge h\}$, the local cumulative shock process. Note that when $h \to 0$, the failure occurs only when the magnitude of a single shock exceeds z, resulting in the classical extreme shock model defined by (2) and (4). Also, as $h \to t$ for each given t, $(t-h, t] \to (0, t]$ and $\Delta X_h(t) \to X(t)$ of (5) and the new model becomes the generalized cumulative shock model of cluster structure.

To illustrate the differences between the two shock processes $\{X(t)\}$ and $\{\Delta X_h(t)\}$, we give a group of MATLAB numerical simulations below. Where the primary process $\{N(t)\}$ follows a homogeneous Poisson process and the adjunct processes $\{M_i(t)\}$ are an independent and identically distributed (i.i.d.) family of homogeneous Poisson processes. When the secondary shocks Y_{ij} have an i.i.d. light-tailed distribution (an exponential distribution is used here), the sample paths of the cumulative shock process $\{X(t)\}$ and the corresponding local cumulative shock process $\{\Delta X_h(t)\}$ with the case of $c = 1$ are shown in Figure 1, and the sample paths with the case of $c = -1$ in Figure 2, respectively. Meanwhile, the sample paths of $\{X(t)\}$ and $\{\Delta X_h(t)\}$ with the secondary shocks Y_{ij} of an i.i.d. heavy-tailed distribution (heavy-tailed Weibull distribution here) display as Figure 3 ($c = 1$) and Figure 4 ($c = -1$), respectively.

These illustrations show some distinct characteristics between the local cumulative shock process $\{\Delta X_h(t)\}$ and the cumulative shock process $\{X(t)\}$. For example, (i) taking different values (1 or -1) of the parameter c (which means different superposition patterns of primary shocks and secondary shocks) changes the trend of $\{X(t)\}$, but it seems to have no obvious influence on $\{\Delta X_h(t)\}$; (ii) when the secondary shocks Y_{ij} follow a heavy-tailed distribution, the sample paths of $\{X(t)\}$ have more drastic upward jumps than those in the case of Y_{ij} has a light-tailed distribution. The similar differences between the two cases can also be found from the sample paths of $\{\Delta X_h(t)\}$; (iii) it can be observed that $\{\Delta X_h(t)\}$ fluctuates more intensely than $\{X(t)\}$ in the same time span, whether the secondary shocks Y_{ij} have light-tailed or heavy-tailed distributions, also whether the coefficient $c = 1$ or -1. In summary, these characteristics show stronger randomness (or lesser predictability) for the local cumulative shock process $\{\Delta X_h(t)\}$ than for the cumulative shock process $\{X(t)\}$ and provide a more reasonable explanation for the bankruptcy behavior in insurance and financial industry

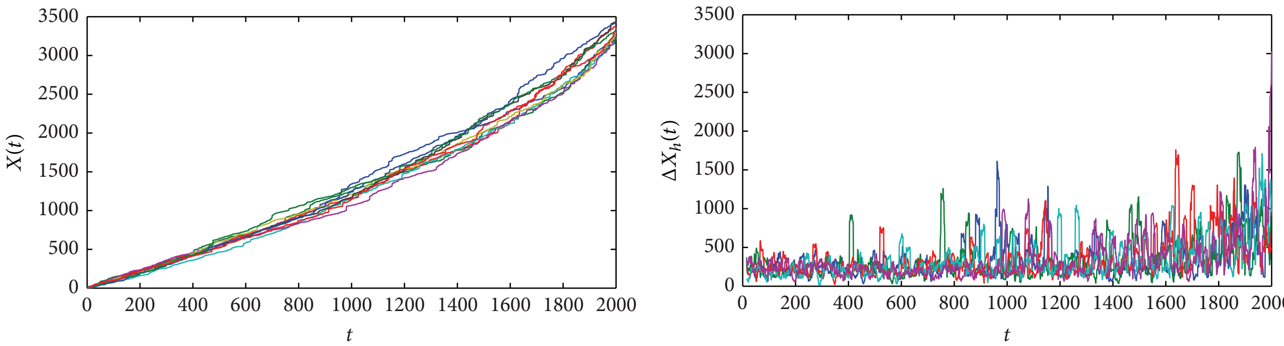

Figure 1: Sample paths with exponentially distributed secondary shocks and $c = 1$.

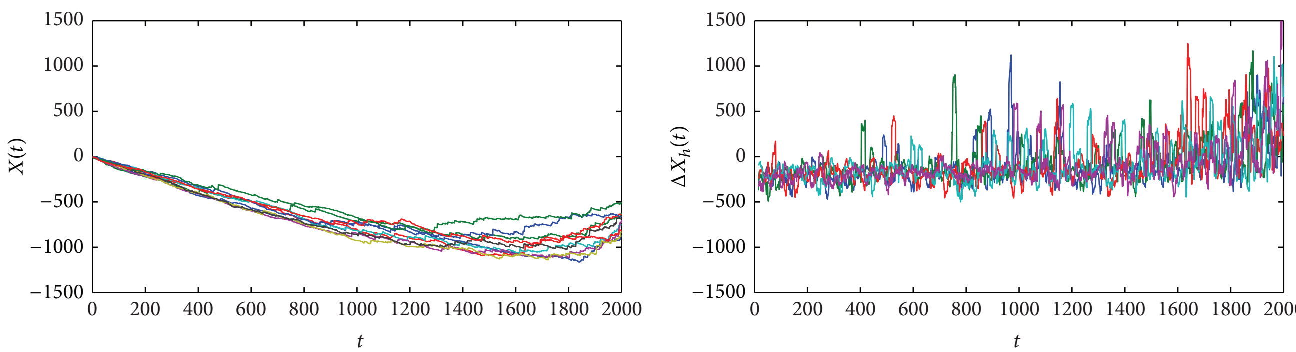

Figure 2: Sample paths with exponentially distributed secondary shocks and $c = -1$.

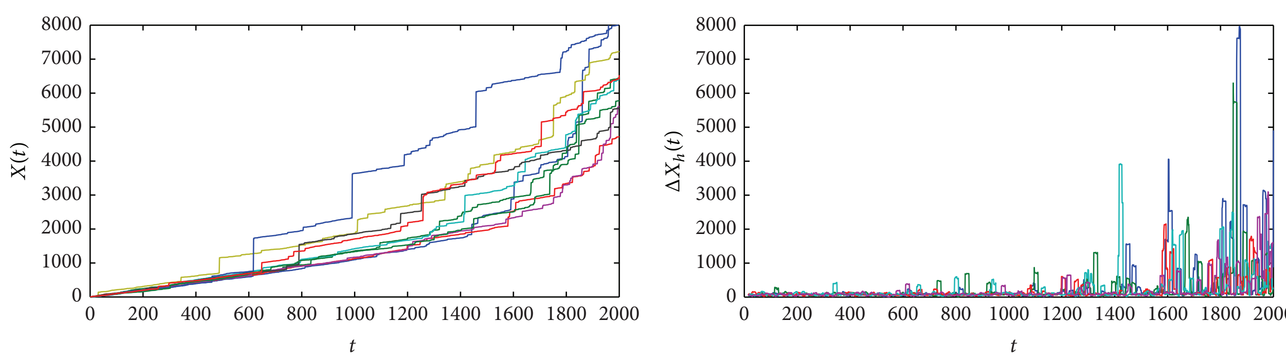

Figure 3: Sample paths with Weibull distributed secondary shocks and $c = 1$.

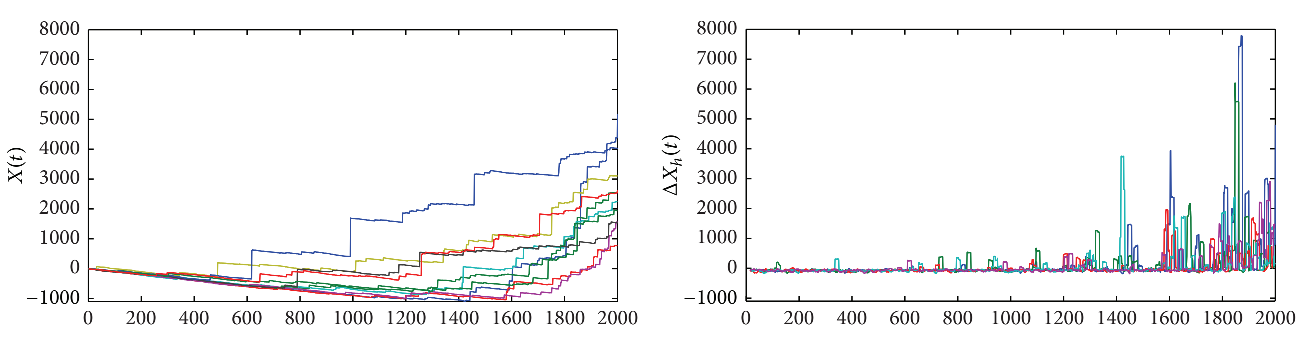

Figure 4: Sample paths with Weibull distributed secondary shocks and $c = -1$.

such as Lehman Brothers Holdings: the fateful risk is more like short-term determined and not necessarily long-history dependent.

In this paper, we consider the local cumulative shock model with cluster shock structure and mainly focus on the weak convergence of $\{\Delta X_h(t)\}$, the local cumulative shock process defined by (6). $\{\Delta X_h(t)\}$ is a very novel stochastic process, it defines a new reliability mechanism for some applications and has not been found in the current research literature. Our discussion is based on the following basic assumptions.

(A_1) The primary process $\{N(t);\ t \geq 0\}$ is a nonhomogeneous Poisson process with intensity function $\lambda(t) < \infty$ and cumulative intensity function $\Lambda(t) = \int_0^t \lambda(s)\mathrm{d}s$ satisfying $\Lambda(t) \to \infty$ as $t \to \infty$.

(A_2) The adjunct processes $\{M_i(t);\ t \geq 0\}$, $i = 1, 2, \ldots$, are an i.i.d. family of counting processes of stationary increments and with arrival instants of $0 = D_{i0} < D_{i1} < \cdots$. For any $t < \infty$, we have $M_1(t) < \infty$.

(A_3) The primary shock magnitudes X_i's are nonnegative and i.i.d. random variables with finite variance $\mathrm{Var}(X_1) > 0$.

(A_4) The secondary shock magnitudes Y_{ij}'s are nonnegative and i.i.d. random variables, for all i and j.

(A_5) The families $\{X_i\}$, $\{Y_{ij}\}$, $\{N(t)\}$, and $\{M_i(t)\}$ are mutually independent.

3. Fundamental Properties

For describing the properties of $\{\Delta X_h(t);\ t \geq h\}$, we use the notations

$$\gamma_i = (X_i; D_{i1}, D_{i2}, \ldots; Y_{i1}, Y_{i2}, \ldots),$$
$$\psi(t, S_i, \gamma_i) = cX_i \mathbf{1}_{\{S_i \leq t\}} + \sum_{j \geq 1} Y_{ij} \mathbf{1}_{\{S_i + D_{ij} \leq t\}}. \tag{9}$$

Then, the local cumulative shock process $\{\Delta X_h(t)\}$ has a new representation

$$\begin{aligned}\Delta X_h(t) &= \sum_{S_i \leq t} \psi(t, S_i, \gamma_i) - \sum_{S_i \leq t-h} \psi(t-h, S_i, \gamma_i)\\ &:= \sum_{S_i \leq t} \psi((t-h, t), S_i, \gamma_i)\\ &= \sum_{i=1}^{N(t)} \psi((t-h, t), S_i, \gamma_i), \quad t \geq h.\end{aligned} \tag{10}$$

By assumptions (A_2)–(A_5), $\{\gamma_i\}$ is a family of i.i.d. random vectors and independent of the primary process $\{N(t)\}$, and the components X, D, and Y in γ are mutually independent. We present several basic properties of $\{\Delta X_h(t)\}$ based on the moment generating function.

Proposition 1. *Under assumptions (A_1)–(A_5), given $t > h$, the moment generating function of $\Delta X_h(t)$ can be expressed as*

$$G_\Delta(\theta) = \exp\left\{\Lambda(t)\left(E\left[\mathrm{e}^{\theta\psi((t-h,t),U_1,\gamma_1)}\right] - 1\right)\right\}, \tag{11}$$

where U_1 is a random variable defined on $[0, t]$ with the distribution function $\Lambda(\cdot)/\Lambda(t)$.

Proof. By the definition of the moment generating function, we have

$$\begin{aligned}G_\Delta(\theta) &= E\left[\mathrm{e}^{\theta\Delta X_h(t)}\right]\\ &= E\left[\mathrm{e}^{\theta\sum_{i=1}^{N(t)}\psi((t-h,t),S_i,\gamma_i)}\right]\\ &= \sum_{n=0}^{\infty} P\{N(t) = n\} E\left[\mathrm{e}^{\theta\sum_{i=1}^{n}\psi((t-h,t),U_i,\gamma_i)}\right]\\ &= \sum_{n=0}^{\infty}\left(E\left[\mathrm{e}^{\theta\psi((t-h,t),U_1,\gamma_1)}\right]\right)^n \frac{\Lambda^n(t)}{n!}\mathrm{e}^{-\Lambda(t)}\\ &= \exp\left\{\Lambda(t)\left(E\left[\mathrm{e}^{\theta\psi((t-h,t),U_1,\gamma_1)}\right] - 1\right)\right\},\end{aligned} \tag{12}$$

where $U_1, \ldots, U_n$ are i.i.d. random variables on $[0, t]$ with the common distribution function $\Lambda(s)/\Lambda(t)$. The last step based on the fact that

$$\psi((t-h, t), U_i, \gamma_i) = cX_i \mathbf{1}_{\{t-h<U_i\leq t\}} + \sum_{j\geq 1} Y_{ij}\mathbf{1}_{\{t-h<U_i+D_{ij}\leq t\}}, \quad i = 1, 2, \ldots, \tag{13}$$

are i.i.d. random variables. □

From (11), we obtain

$$\begin{aligned}E[\Delta X_h(t)] &= \Lambda(t) E[\psi((t-h, t), U_1, \gamma_1)]\\ &= \int_0^t E[\psi((t-h, t), s, \gamma_1)]\lambda(s)\,\mathrm{d}s,\\ \mathrm{Var}(\Delta X_h(t)) &= \Lambda(t) E\left[\psi^2((t-h, t), U_1, \gamma_1)\right]\\ &= \int_0^t E\left[\psi^2((t-h, t), s, \gamma_1)\right]\lambda(s)\,\mathrm{d}s.\end{aligned} \tag{14}$$

Note that (11) and (14) contain the moment generating function and the moments of $\psi((t-h, t), U_1, \gamma_1)$. Thus, we need to present the properties of $\psi((t-h, t), U_1, \gamma_1)$.

Proposition 2. *Under assumptions (A_1)–(A_5), given $t > h$, the moment generating function of $\psi_1 = \psi((t-h, t), U_1, \gamma_1)$ is*

$$G_{\psi_1}(\theta) = \sum_{m=0}^{\infty} p_m E^m\left[e^{\theta Y_{11}}\right] \cdot\left(\frac{\Lambda(t) - \Lambda(t-h)}{\Lambda(t)} E\left[\mathrm{e}^{\theta cX_1}\right] + \frac{\Lambda(t-h)}{\Lambda(t)}\right), \tag{15}$$

where $p_m = P\{M_1(h) = m\}$, $m = 0, 1, \ldots$.

Proof. Letting $M_1 = M_1(t-U_1-h, t-U_1)$, then

$$\psi\left((t-h,t),U_1,\gamma_1\right) = cX_1\mathbf{1}_{\{t-h<U_1\le t\}} + \sum_{j\ge 1} Y_{1j}\mathbf{1}_{\{t-h<U_1+D_{1j}\le t\}} = cX_1\mathbf{1}_{\{t-h<U_1\le t\}} + \sum_{j=1}^{M_1} Y_{1j}. \tag{16}$$

Note that M_1 has stationary increments (by assumption (A_2)); we have

$$\begin{aligned}
G_{\psi_1}(\theta) &= E\left[e^{\theta\psi((t-h,t),U_1,\gamma_1)}\right] \\
&= E\left[E\left[e^{\theta cX_1\mathbf{1}_{\{t-h<U_1\le t\}}}e^{\theta\sum_{j=1}^{M_1}Y_{1j}} \mid U_1\right]\right] \\
&= E\left[\sum_{m=0}^{\infty} P\{M_1 = m \mid U_1\} \cdot E\left[e^{\theta cX_1\mathbf{1}_{\{t-h<U_1\le t\}}}e^{\theta\sum_{j=1}^{M_1}Y_{1j}} \mid M_1 = m, U_1\right]\right] \\
&= \sum_{m=0}^{\infty} p_m E\left[e^{\theta cX_1\mathbf{1}_{\{t-h<U_1\le t\}}}e^{\theta\sum_{j=1}^{m}Y_{1j}}\right] \\
&= \sum_{m=0}^{\infty} p_m \left(E\left[e^{\theta cX_1}e^{\theta\sum_{j=1}^{m}Y_{1j}}\right] P\{t-h<U_1\le t\} + E\left[e^{\theta\sum_{j=1}^{m}Y_{1j}}\right] P\{U_1\le t-h\}\right) \\
&= \frac{1}{\Lambda(t)}\sum_{m=0}^{\infty} p_m \left(E\left[e^{\theta cX_1}\right]E\left[e^{\theta\sum_{j=1}^{m}Y_{1j}}\right] \cdot (\Lambda(t)-\Lambda(t-h)) + E\left[e^{\theta\sum_{j=1}^{m}Y_{1j}}\right]\Lambda(t-h)\right) \\
&= \frac{1}{\Lambda(t)}\sum_{m=0}^{\infty} p_m E^m\left[e^{\theta Y_{11}}\right]\left((\Lambda(t)-\Lambda(t-h))E\left[e^{\theta cX_1}\right] + \Lambda(t-h)\right).
\end{aligned} \tag{17}$$

The result has been proved. □

For a given t, we can compute first two moments (if exists) of $\psi((t-h,t),U_1,\gamma_1)$ and of $\Delta X_h(t)$ and get the following conclusion.

Proposition 3. *Under assumptions (A_1)–(A_5), for a given $t > h$, the moment generating function of $\Delta X_h(t)$ is*

$$G_\Delta(\theta) = e^{\Lambda(t)(G_{\psi_1}(\theta)-1)}, \tag{18}$$

where $G_{\psi_1}(\theta)$ is given by (15), and the mean and variance are

$$\begin{aligned}
E\left[\Delta X_h(t)\right] &= E\left[N(t-h,t)\right]cE\left[X_1\right] + E\left[N(t)\right]E\left[M_1\right]E\left[Y_{11}\right], \\
\operatorname{Var}\left(\Delta X_h(t)\right) &= E\left[N(t-h,t)\right] \cdot \left(c^2E\left[X_1^2\right] + 2cE\left[M_1\right]E\left[X_1\right]E\left[Y_{11}\right]\right) \\
&\quad + E\left[N(t)\right]\left(E\left[M_1\right]\operatorname{Var}\left(Y_{11}\right) + E\left[M_1^2\right]E^2\left[Y_{11}\right]\right).
\end{aligned} \tag{19}$$

Also define the distribution function of $\psi((t-h,t),s,\gamma_1)$ as

$$H(y;t,s) = P\{\psi((t-h,t),s,\gamma_1)\le y\}, \quad 0\le s\le t, \ y\in R. \tag{20}$$

We have another result.

Proposition 4. *Under assumptions (A_1)–(A_5), for a given $t > h$, the logarithmically characteristic function of $\Delta X_h(t)$ is*

$$i\theta b(t) - \frac{1}{2}\theta^2\delta^2(t) + \int_{R\setminus\{0\}}\left(e^{i\theta y} - 1 - \frac{i\theta y}{1+y^2}\right)\nu(t,dy), \tag{21}$$

where

$$\begin{aligned}
b(t) &= \int_R \frac{y}{1+y^2}\nu(t,dy), \\
\delta^2(t) &= \lim_{\varepsilon\to 0}\int_{|y|<\varepsilon} y^2\nu(t,dy),
\end{aligned} \tag{22}$$

$$\begin{aligned}
\nu(t,dy) &= \int_0^t H(y+dy;t,s)\lambda(s)\,ds - \int_0^t H(y;t,s)\lambda(s)\,ds \\
&:= \int_0^t H(dy;t,s)\lambda(s)\,ds
\end{aligned} \tag{23}$$

is a measure on R.

Proof. Denote the characteristic function of $\Delta X_h(t)$ by $C_\Delta(\theta)$. It follows from (11) that

$$C_\Delta(\theta) = G_\Delta(i\theta) = \exp\left\{\Lambda(t)\left(G_{\psi_1}(i\theta)-1\right)\right\}, \tag{24}$$

where $G_{\psi_1}(\mathrm{i}\theta)$ is the characteristic function of $\psi_1 = \psi((t-h,t),U_1,\gamma_1)$. Conditioning on U_1, we have

$$\begin{aligned}\log C_\Delta(\theta) &= \Lambda(t)\left(E\left[\mathrm{e}^{\mathrm{i}\theta\psi((t-h,t),U_1,\gamma_1)}\right]-1\right)\\ &= \int_0^t \left(E\left[\mathrm{e}^{\mathrm{i}\theta\psi((t-h,t),s,\gamma_1)}\right]-1\right)\lambda(s)\,\mathrm{d}s\\ &= \int_0^t\int_R \left(\mathrm{e}^{\mathrm{i}\theta y}-1\right)H(\mathrm{d}y;t,s)\,\lambda(s)\,\mathrm{d}s\\ &= \int_R \left(\mathrm{e}^{\mathrm{i}\theta y}-1\right)\nu(t,\mathrm{d}y)\\ &= \mathrm{i}\theta\int_R \frac{y}{1+y^2}\nu(t,\mathrm{d}y)\\ &\quad + \lim_{\varepsilon\to 0}\int_{|y|<\varepsilon}\left(\mathrm{e}^{\mathrm{i}\theta y}-1-\frac{\mathrm{i}\theta y}{1+y^2}\right)\nu(t,\mathrm{d}y)\\ &\quad + \int_{R\setminus\{0\}}\left(\mathrm{e}^{\mathrm{i}\theta y}-1-\frac{\mathrm{i}\theta y}{1+y^2}\right)\nu(t,\mathrm{d}y)\\ &= \mathrm{i}\theta b(t)-\frac{1}{2}\theta^2\delta^2(t)\\ &\quad + \int_{R\setminus\{0\}}\left(\mathrm{e}^{\mathrm{i}\theta y}-1-\frac{\mathrm{i}\theta y}{1+y^2}\right)\nu(t,\mathrm{d}y).\end{aligned} \tag{25}$$

The last equality is due to the definition of (22) and the fact that

$$\mathrm{e}^{\mathrm{i}\theta y}-1-\frac{\mathrm{i}\theta y}{1+y^2}\sim-\frac{1}{2}\theta^2y^2,\quad |y|\longrightarrow 0, \tag{26}$$

where "~" means limit equivalence. □

It can be shown that the measure $\nu(t,\cdot)$ defined by (23) satisfies

$$\int_R \left(y^2\wedge 1\right)\nu(t,\mathrm{d}y)<\infty \tag{27}$$

and also guarantees

$$b(t)<\infty,\qquad 0\le\delta^2(t)<\infty. \tag{28}$$

Thus, $\nu(t,\cdot)$ is the Lévy measure and $(\delta^2(t),\nu(t,\cdot),b(t))$ is the characteristic triplet of $\Delta X_h(t)$ for given $t>h$, respectively.

4. Central Limit Theorems

In this section, we discuss the limit theorems of $\{\Delta X_h(t)\}$, that is, the weak convergence of the regularized process

$$Z_h(t) := \frac{\Delta X_h(t)-\mu(t)}{\sigma(t)},\quad t\ge h, \tag{29}$$

as $t\to\infty$, where $\mu(t)$ and $\sigma(t)>0$ are the appropriately selected centering and regularizing functions. To this end, we need to examine the convergence property of the corresponding characteristic function. The discussion of this section is based on the standard theories of infinitely divisible distributions and stable distributions (please see Cont and Tankov [21], Sato [22], Lin et al. [23], and Embrechts et al. [24] for the details).

4.1. Lévy-Khintchine Representation and Infinite Divisibility. By (21) and the Lévy-Khintchine representation of an infinitely divisible distribution, we know that $\Delta X_h(t)$ is infinitely divisible for given $t>h$, and its characteristic function is fully determined by the Lévy triplet $(\delta^2(t),\nu(t,\cdot),b(t))$ defined in (22) and (23). Such a one-to-one correspondence also holds for their convergence results as $t\to\infty$. Thus, we use the notation

$$\Delta X_h(t)\approx\left(\delta^2(t),\nu(t,\cdot),b(t)\right) \tag{30}$$

to represent this relation and only discuss the weak convergence of triplet $(\delta^2(t),\nu(t,\cdot),b(t))$. Note that the infinite divisibility of $\Delta X_h(t)$ is due to the nature of the Poisson primary shock process.

Then, by (29) and Proposition 4, the regularized process $\{Z_h(t)\}$ is also infinitely divisible. For appropriately selected $\mu(t)$ and $\sigma(t)$, we have the following theorem.

Theorem 5. *Under assumptions (A_1)–(A_5), for a given $t>h$, $Z_h(t)$ is infinitely divisible and*

$$Z_h(t)\approx\left(\delta'^2(t),\nu'(t,\cdot),b'(t)\right), \tag{31}$$

where

$$\nu'(t,\mathrm{d}y)=\nu(t,\sigma(t)\,\mathrm{d}y)=\int_0^t H(\sigma(t)\,\mathrm{d}y;t,s)\,\lambda(s)\,\mathrm{d}s, \tag{32}$$

$$\begin{aligned}b'(t) &= \int_R \frac{y}{1+y^2}\nu'(t,\mathrm{d}y)-\frac{\mu(t)}{\sigma(t)}\\ &= \int_R \frac{y}{1+y^2}\nu(t,\sigma(t)\,\mathrm{d}y)-\frac{\mu(t)}{\sigma(t)},\end{aligned} \tag{33}$$

$$\delta'^2(t)=\lim_{\varepsilon\to 0}\int_{|y|<\varepsilon}y^2\nu(t,\sigma(t)\,\mathrm{d}y). \tag{34}$$

Thus, the Lévy-Khintchin representation of $Z_h(t)$ is given by

$$\begin{aligned}&\mathrm{i}\theta b'(t)-\frac{1}{2}\theta^2\delta'^2(t)\\ &\quad+\int_{R\setminus\{0\}}\left(\mathrm{e}^{\mathrm{i}\theta y}-1-\frac{\mathrm{i}\theta y}{1+y^2}\right)\nu(t,\sigma(t)\,\mathrm{d}y).\end{aligned} \tag{35}$$

Proof. The characteristic function of $Z_h(t)$, denoted by $C_Z(\theta)$, can be written as

$$\begin{aligned}C_Z(\theta) &= E\left[\mathrm{e}^{\mathrm{i}\theta Z_h(t)}\right]=E\left[\mathrm{e}^{\mathrm{i}\theta(\Delta X_h(t)-\mu(t))/\sigma(t)}\right]\\ &= \mathrm{e}^{-\mathrm{i}\theta\mu(t)/\sigma(t)}C_\Delta\left(\frac{\theta}{\sigma(t)}\right).\end{aligned} \tag{36}$$

Then,

$$\begin{aligned}\log C_Z(\theta) &= -\mathrm{i}\theta\frac{\mu(t)}{\sigma(t)}+\int_R\left(\mathrm{e}^{\mathrm{i}\theta y/\sigma(t)}-1\right)\nu(t,\mathrm{d}y)\\ &= -\mathrm{i}\theta\frac{\mu(t)}{\sigma(t)}+\int_R\left(\mathrm{e}^{\mathrm{i}\theta z}-1\right)\nu(t,\sigma(t)\,\mathrm{d}z),\end{aligned} \tag{37}$$

where $z=y/\sigma(t)$. The rest of proof is the same as the last part proof of Proposition 4. □

Because the weak limit of the sequence of infinitely divisible random variables is also infinitely divisible, we can obtain the following weak convergence results of the regularized process $\{Z_h(t)\}$ based on Theorem 5 and Proposition 4.

Corollary 6. *Under assumptions* (A_1)–(A_5)*, there exists an infinitely divisible random variable* $Z \approx (\delta_0^2, \nu_0(\cdot), b_0)$ *with* $\cdot \subset R$ *and constants* b_0 *and* $\delta_0^2 \geq 0$*, such that the sufficient and necessary conditions for* $Z_h(t) \xrightarrow{d} Z$ *as* $t \to \infty$ *are*

$$\nu'(t,\cdot) \longrightarrow \nu_0(\cdot), \qquad b'(t) \longrightarrow b_0, \qquad \delta'^2(t) \longrightarrow \delta_0^2, \tag{38}$$

where $\nu'(t,\cdot) \to \nu_0(\cdot)$ *means* $\int_R f(x)\nu'(t,\mathrm{d}x) \to \int_R f(x)\nu_0(\mathrm{d}x)$ *for all bounded and continuous functions* $f : R \to R \setminus \{0\}$.

4.2. Convergence to α-Stable Distribution. Now, we present the conditions for $\{Z_h(t)\}$ to converge weakly to an α-stable distribution. Let F denote the common distribution function of secondary shocks Y_{ij}. We consider the case that F has a regular-tailed distribution with index $\alpha \in (0,2)$ and then have the following theorem.

Theorem 7. *Under assumptions* (A_1)–(A_5)*, if* $F(y) = y^{-\alpha}L(y)$*, where* $\alpha \in (0,2)$ *and* L *is some slowly varying function, then* $\sigma(t)$ *and* $\mu(t)$ *can be chosen as*

$$\sigma(t) = F^{-1}\left(1 - \frac{1}{\beta(t)}\right), \tag{39}$$

$$\mu(t) = \begin{cases} 0, & \text{if } \alpha \in (0,1]; \\ \sigma(t)\displaystyle\int_{|y|<1} y\nu(t,\sigma(t)\,\mathrm{d}y), & \text{if } \alpha \in (1,2), \end{cases} \tag{40}$$

respectively, such that as $t \to \infty$,

$$Z_h(t) = \frac{\Delta X_h(t) - \mu(t)}{\sigma(t)} \xrightarrow{d} G_\alpha, \tag{41}$$

where $\beta(t) = \int_0^t E[M_1(t-s-h,t-s)]\lambda(s)\mathrm{d}s$ *and* G_α *is the* α*-stable distribution with index* $\alpha \in (0,2)$.

Proof. The selected $\sigma(t)$ implies that $\sigma(t) \to \infty$ as $t \to \infty$. From Theorem 5, we know that $Z_h(t) \approx (\delta'^2(t), \nu'(t,\cdot), b'(t))$; according to Corollary 6 and Subsection 2.4 of Klüppelberg et al. [25], to prove this theorem, we need to show, as $t \to \infty$,

(i) for each $A \subset R$, $\nu'(t,A) \to \nu_0(A)$, and for $y > 0$, $\nu_0(\cdot)$ satisfies the following homogeneous conditions:

$$\begin{aligned} \nu_0(y,\infty) &= y^{-\alpha}\nu_0(1,\infty), \\ \nu_0(-\infty,-y) &= y^{-\alpha}\nu_0(-\infty,-1); \end{aligned} \tag{42}$$

(ii) $\delta'^2(t) \to 0$;

(iii) $b'(t) \to b_0$;

note that (i) holds if we show, for $y > 0$,

$$\nu'(t,(y,\infty)) \longrightarrow \nu_0(y,\infty), \tag{43}$$

where the convergence is in the sense of Corollary 6. From (32) and (23), we have

$$\begin{aligned} \nu'(t,(y,\infty)) &= \nu(t,(\sigma(t)y,\infty)) \\ &= \int_0^t \overline{H}(\sigma(t)y;t,s)\lambda(s)\,\mathrm{d}s \\ &= \int_0^t P\{\psi((t-h,t),s,\gamma_1) > \sigma(t)y\}\lambda(s)\,\mathrm{d}s \\ &= \int_0^t P\left\{cX_1\mathbf{1}_{\{t-h<s\leq t\}} + \sum_{j=1}^{M_1} Y_{1j} > \sigma(t)y\right\}\lambda(s)\,\mathrm{d}s \\ &= \int_0^t \sum_{m=0}^{\infty} p_m P\left\{cX_1\mathbf{1}_{\{t-h<s\leq t\}} + \sum_{j=1}^{m} Y_{1j} > \sigma(t)y\right\}\lambda(s)\,\mathrm{d}s, \end{aligned} \tag{44}$$

where $M_1 = M_1(t-s-h,t-s)$, $p_m = P\{M_1 = m\}$. Due to the existence of the integral above, the finite variance of X_1, and the regular-tailed distribution property of $\sum_{j=1}^m Y_{1j}$, we have $P\{|cX_1| > \sigma(t)y\} = o(P\{\sum_{j=1}^m Y_{1j} > \sigma(t)y\})$. It follows from Lebesgue's dominated convergence theorem that

$$\begin{aligned} \nu'(t,(y,\infty)) &\sim \int_0^t \sum_{m=0}^{\infty} p_m P\left\{\sum_{j=1}^{m} Y_{1j} > \sigma(t)y\right\}\lambda(s)\,\mathrm{d}s \\ &\sim \int_0^t \sum_{m=0}^{\infty} m p_m P\{Y_{11} > \sigma(t)y\}\lambda(s)\,\mathrm{d}s \\ &= P\{Y_{11} > \sigma(t)y\}\int_0^t E[M_1(s)]\lambda(s)\,\mathrm{d}s \\ &= P\{Y_{11} > \sigma(t)y\}\beta(t). \end{aligned} \tag{45}$$

From (39), we have $\beta(t) = 1/\overline{F}(\sigma(t)) = 1/P\{Y_{11} > \sigma(t)\}$. Using the properties of regular-tailed distributions and slowly varying functions, we obtain

$$\begin{aligned} \nu'(t,(y,\infty)) &\sim \frac{P\{Y_{11} > \sigma(t)y\}}{P\{Y_{11} > \sigma(t)\}} \\ &= \frac{(\sigma(t)y)^{-\alpha}}{(\sigma(t))^{-\alpha}} \cdot \frac{L(\sigma(t)y)}{L(\sigma(t))} = y^{-\alpha}. \end{aligned} \tag{46}$$

Let $\nu_0(y,\infty) = y^{-\alpha}$; then, $\nu_0(\cdot)$ is a Lévy measure; it implies (43) and also satisfies the homogeneous conditions (42). Thus, (i) has been proved.

For showing (ii), we note from (46) that, as $t \to \infty$,

$$\begin{aligned}\nu(t,\sigma(t)\,\mathrm{d}y) &= \nu(t,(\sigma(t)\,y,\infty)) - \nu(t,(\sigma(t)(y+\mathrm{d}y),\infty)) \\ &\sim y^{-\alpha} - (y+\mathrm{d}y)^{-\alpha},\end{aligned} \tag{47}$$

and using (34),

$$\begin{aligned}\delta'^2(t) &= \lim_{\varepsilon\to 0}\int_{|y|<\varepsilon} y^2\nu(t,\sigma(t)\,\mathrm{d}y) \\ &\sim \lim_{\varepsilon\to 0}\int_{|y|<\varepsilon} y^2\left(y^{-\alpha} - (y+\mathrm{d}y)^{-\alpha}\right).\end{aligned} \tag{48}$$

Since $\alpha < 2$, the integral above converges to zero, thus $\delta'^2(t) \to 0$.

Finally, we show (iii). From (33) and (40), when $\alpha \in (1,2)$,

$$\begin{aligned}b'(t) &= \int_R \frac{y}{1+y^2}\nu(t,\sigma(t)\,\mathrm{d}y) - \frac{\mu(t)}{\sigma(t)} \\ &= \int_R \frac{y}{1+y^2}\nu(t,\sigma(t)\,\mathrm{d}y) - \int_{|y|<1} y\nu(t,\sigma(t)\,\mathrm{d}y) \\ &= \int_{|y|\geq 1} \frac{y}{1+y^2}\nu(t,\sigma(t)\,\mathrm{d}y) \\ &\quad - \int_{|y|<1} \frac{y^3}{1+y^2}\nu(t,\sigma(t)\,\mathrm{d}y) \\ &\sim \int_{|y|\geq 1} \frac{y}{1+y^2}\left(y^{-\alpha} - (y+\mathrm{d}y)^{-\alpha}\right) \\ &\quad - \int_{|y|<1} \frac{y^3}{1+y^2}\left(y^{-\alpha} - (y+\mathrm{d}y)^{-\alpha}\right) \\ &:= I(\alpha).\end{aligned} \tag{49}$$

Since all the integrands are bounded and continuous over their integration regions, the above integrals exist. Let $b_0 = I(\alpha)$, thus we have $b'(t) \to b_0$.

When $\alpha \in (0,1]$,

$$\begin{aligned}b'(t) &= \int_R \frac{y}{1+y^2}\nu(t,\sigma(t)\,\mathrm{d}y) \\ &\sim \int_R \frac{y}{1+y^2}\left(y^{-\alpha} - (y+\mathrm{d}y)^{-\alpha}\right) \\ &:= I'(\alpha).\end{aligned} \tag{50}$$

Similarly, the above integral exists, and we also have $b'(t) \to b_0$ by choosing $b_0 = I'(\alpha)$. □

We remark that the another representation of $\beta(t)$ in Theorem 7 is

$$\beta(t) = \Lambda(t)\,E[M_1] = E[N(t)]\,E[M_1], \tag{51}$$

where $M_1 = M_1(t-U_1-h, t-U_1)$. Note that

$$\beta(t) = \frac{1}{P\{Y_{11} > \sigma(t)\}} = \frac{\sigma^\alpha(t)}{L(\sigma(t))}, \tag{52}$$

which implies

$$\sigma(t) = \Lambda^{1/\alpha}(t)\,L_1(\Lambda(t)), \tag{53}$$

where L_1 is a slowly varying function.

The theorem below gives the conditions for the regularized process $\{Z_h(t)\}$ to converge to normal distribution. This result is natural.

Theorem 8. *Under assumptions (A_1)–(A_5), if* $\mathrm{Var}(Y_{11})$ *exists and is finite, we can choose*

$$\begin{aligned}\sigma^2(t) &= \int_0^t E\left[\psi^2((t-h,t),s,\gamma_1)\right]\lambda(s)\,\mathrm{d}s, \\ \mu(t) &= \int_0^t E\left[\psi((t-h,t),s,\gamma_1)\right]\lambda(s)\,\mathrm{d}s,\end{aligned} \tag{54}$$

such that, as $t \to \infty$,

$$Z_h(t) = \frac{\Delta X_h(t) - \mu(t)}{\sigma(t)} \xrightarrow{\mathrm{d}} \Phi, \tag{55}$$

where Φ represents standard normal distribution.

5. Summary and Discussion

The local cumulative shock model defined by (6), (7), and (8) is a generalization of the classical extreme shock models. There are two types of shocks in this model called primary shocks and secondary shocks, respectively. The primary shocks are generated by a nonhomogeneous Poisson process and each of them causes a series of secondary shocks. Therefore, the shock process has a cluster structure and can be considered as the superposition of two streams of shock processes. In this model, the local cumulative shock process $\{\Delta X_h(t);\ t \geq h\}$ plays the core role in determining the lifetime behavior of the system.

This paper discusses the limit properties of $\{\Delta X_h(t)\}$. Based on the theories of infinite divisibility and stable distributions, we establish the central limit theorems of $\{\Delta X_h(t)\}$, and find the conditions under which the regularized process of $\{\Delta X_h(t)\}$ converges to infinitely divisible, α-stable or normal distributions. Our work provides a foundation for the next work of studying lifetime property and failure behavior of the local cumulative shock model.

$\{\Delta X_h(t)\}$ is a very novel stochastic process and has not been found in the available literature. The new failure mechanism defined by $\{\Delta X_h(t)\}$ is more reasonable than the classical models to explain some reliability phenomenons such as the bankruptcy behavior in insurance and financial industry, where the fateful risk is not necessarily long-history dependent but more like short-term determined. For such a situation, the classical models may not provide realistic risk measures. There are two main factors that make our model more reasonable in describing the risk behavior for the operations of an insurance or a financial firm. The first one is that the cluster structure is based on the event sequence of purchase and claims of each insurance policy. The second is that the local cumulative shock is more realistic and flexible in modeling the bankrupt condition for modern insurance services.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

J. M. Bai's work is supported by the National Natural Science Foundation of China Grant nos. 71171103 and 71072070. C. Yuan's work is supported by Humanities & Social Sciences Project of the Ministry of Education of China Grant no. 13YJC630215 and partly by the National Natural Science Foundation of Guangdong Province Grant no. S2012040007369. The authors would like to thank the editor and the anonymous referees for their valuable comments and suggestions, and also they are grateful to Dr. Zhe George Zhang in the Department of Decision Sciences, Western Washington University, and Professor Zehui Li of the School of Mathematics and Statistics, Lanzhou University, for their useful suggestions and modifications.

References

[1] G. K. Agrafiotis and M. Z. Tsoukalas, "On excess-time correlated cumulative processes," *Journal of the Operational Research Society*, vol. 46, no. 10, pp. 1269–1280, 1995.

[2] J.-M. Bai, Z.-H. Li, and X.-B. Kong, "Generalized shock models based on a cluster point process," *IEEE Transactions on Reliability*, vol. 55, no. 3, pp. 542–550, 2006.

[3] A. Gut, "Cumulative shock models," *Advances in Applied Probability*, vol. 22, no. 2, pp. 504–507, 1990.

[4] A. Gut, "Mixed shock models," *Bernoulli*, vol. 7, no. 3, pp. 541–555, 2001.

[5] A. Gut and J. Hüsler, "Extreme shock models," *Extremes*, vol. 2, pp. 293–305, 1999.

[6] N. Igaki, U. Sumita, and M. Kowada, "Analysis of Markov renewal shock models," *Journal of Applied Probability*, vol. 32, no. 3, pp. 821–831, 1995.

[7] G. Skoulakis, "A general shock model for a reliability system," *Journal of Applied Probability*, vol. 37, no. 4, pp. 925–935, 2000.

[8] M. Finkelstein and F. Marais, "On terminating Poisson processes in some shock models," *Reliability Engineering and System Safety*, vol. 95, no. 8, pp. 874–879, 2010.

[9] S. Mercier and H. H. Pham, "A bivariate random shock model for a two-component system with dependence," in *Proceedings of the Workshop ANR AMMSI*, INSA, Toulouse, France, January 2014.

[10] E. Omey and R. Vesilo, "Local limit theorems for shock models," HUB Research Papers 2011/23, 2011.

[11] U. Sumita and J. S. Zuo, "Correlated multivariate shock models associated with a renewal sequence and its application to analysis of browsing behavior of Internet users," *Journal of the Operations Research Society of Japan*, vol. 53, no. 2, pp. 119–135, 2010.

[12] Z. L. Wang, H.-Z. Huang, Y. F. Li, and N.-C. Xiao, "An approach to reliability assessment under degradation and shock process," *IEEE Transactions on Reliability*, vol. 60, no. 4, pp. 852–863, 2011.

[13] J.-M. Bai, Z. G. Zhang, and Z.-H. Li, "Lifetime properties of a cumulative shock model with a cluster structure," *Annals of Operations Research*, vol. 212, pp. 21–41, 2014.

[14] Y. Ogata, "Exploratory analysis of earthquake clusters by likelihood-based trigger models," *Journal of Applied Probability*, vol. 38, pp. 202–212, 2001.

[15] D. J. Daley and D. Vere-Jones, *An Introduction to the Theory of Point Processes*, vol. 1, Springer, New York, NY, USA, 2nd edition, 2003.

[16] N. Hohn, D. Veitch, and P. Abry, "Cluster processes: a natural language for network traffic," *IEEE Transactions on Signal Processing*, vol. 51, no. 8, pp. 2229–2244, 2003.

[17] G. Faÿ, B. González-Arévalo, T. Mikosch, and G. Samorodnitsky, "Modeling teletraffic arrivals by a Poisson cluster process," *Queueing Systems. Theory and Applications*, vol. 54, no. 2, pp. 121–140, 2006.

[18] T. Rolski, H. Schmidli, V. Schmidt, and J. Teugels, *Stochastic Processes for Insurance and Finance*, John Wiley & Sons, Chichester, UK, 1999.

[19] M. Denuit, E. Frostig, and B. Levikson, "Shifts in interest rate and common shock model for coupled lives," *Belgian Actuarial Bulletin*, vol. 6, pp. 1–4, 2006.

[20] F. Lindskog and A. J. McNeil, "Common Poisson shock models: applications to insurance and credit risk modelling," *Astin Bulletin*, vol. 33, no. 2, pp. 209–238, 2003.

[21] R. Cont and P. Tankov, *Financial Modelling with Jump Processes*, Chapman & Hall/CRC Press, Boca Raton, Fla, USA, 2004.

[22] K.-I. Sato, *Levy Processes and Infinitely Divisible Distributions*, Cambridge University Press, Cambridge, UK, 1999.

[23] Z. Y. Lin, C. R. Lu, and Z. G. Su, *Foundations of Probability Limit Theory*, Higher Education Press, Beijing, China, 1999, (Chinese).

[24] P. Embrechts, C. Klüppelberg, and T. Mikosch, *Modelling Extremal Events (for Insurance and Finance)*, Springer, Berlin, Germany, 1997.

[25] C. Klüppelberg, T. Mikosch, and A. Schärf, "Regular variation in the mean and stable limits for Poisson shot noise," *Bernoulli*, vol. 9, no. 3, pp. 467–496, 2003.