

Research Article

Chaos Synchronization of Financial Chaotic System with External Perturbation

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This paper investigates the problem of two kinds of function projective synchronization of financial chaotic system with definite integration scaling function, which are not fully considered in the existing research. Different from the previous methods, in this paper, the following two questions are investigated: (1) two kinds of the definite integration scaling function projective synchronization are given; (2) the upper and lower limit of the definite integral scaling function are the bound dynamical systems. Finally, illustrative example is provided to show the effectiveness of this method.

1. Introduction

In nonlinear areas, researchers are striving to utilize the theory of nonlinear dynamics, especially the chaos theory, to study the complexity of economic and financial systems in recent years [1–6]. Since Strotz et al. have done the pioneering work in this area [7], various economics chaotic models have been proposed, such as the Kaldorian model [8], the IS-LM model [9–11], the hyperchaotic finance system [12], and other nonlinear dynamical models [13–16].

It is well known that economic chaotic systems are inevitably influenced by external disturbances stemmed from environmental interference [17–22], and external disturbances may lead to the destabilization of economic and financial chaotic systems and cause undesirable results. It is necessary to study the global stabilization of economic and financial chaotic systems under the presence of external disturbance. Some results have been reported about stabilization of complex systems with external disturbance [23, 24].

In the past few decades, projective synchronization of chaotic systems has attracted a great deal of attention, because projective synchronization can obtain the effective results faster. Later, with the deepening of the research, various projective synchronization methods were discussed.

For example, in [25–29], the authors investigated function projective synchronization of chaotic systems, and the scaling function is selected to be constant or unity. In [30–33], the authors discussed function projective synchronization of chaotic systems with the unary scaling function. Up to now, most of research efforts mentioned above have concentrated on studying the presetting scaling function in numerical examples yet. About the problem of the definite integral scaling function, to the best of the authors' knowledge, it has rarely been investigated, which still remains open. Motivated by the existing works, in the present paper, we intend to derive some scaling function projective synchronization criteria for the chaotic financial system. Different from the previous methods, our main contributions are as follows: (1) we give two kinds of the new scaling function projective synchronization, that is, the definite integration scaling function projective synchronization and adaptive definite integration scaling function projective synchronization; (2) we consider that the upper and lower limit of the definite integral scaling function are the bound dynamical systems, which can be the stable equilibrium point, the stable periodic orbit, or the chaotic attractor.

The rest of this paper is organized as follows: Model and preliminaries are presented in Section 2. The sufficient

conditions of synchronization are given in Section 3. Section 4 presents an example and relates simulation results. The conclusions are given in Section 5.

2. Model and Preliminaries

In 2001, a new dynamical model of the financial system was proposed. The model is described by the following three-dimensional system:

$$\begin{aligned}\dot{x}_1 &= x_3 + (x_2 - a)x_1, \\ \dot{x}_2 &= 1 - bx_2 - x_1^2, \\ \dot{x}_3 &= -x_1 - cx_3,\end{aligned}\quad (1)$$

where x_1 is the interest rate, x_2 is the investment demand, and x_3 is the price index. $a > 0$ represents the saving amount, $b > 0$ represents the cost per investment, and $c > 0$ represents the elasticity of demand of commercial markets. When parameters $a = 0.9$, $b = 0.2$, and $c = 1.5$, system (1) exhibits chaotic behavior [14].

Now, we consider the following system with external perturbation and system (1) changes into the following system:

$$\begin{aligned}\dot{y}_1 &= y_3 + (y_2 - a_2)y_1, \\ \dot{y}_2 &= 1 - b_2y_2 - y_1^2, \\ \dot{y}_3 &= -y_1 - c_2y_3 + hv,\end{aligned}\quad (2)$$

where $\dot{v} = -1.2y_1y_2 - 0.5v$.

In essence, three-dimensional system (2) with external perturbation can be seen as the following four-dimensional system without external perturbation:

$$\begin{aligned}\dot{y}_1 &= y_3 + (y_2 - a_2)y_1, \\ \dot{y}_2 &= 1 - b_2y_2 - y_1^2, \\ \dot{y}_3 &= -y_1 - c_2y_3 + hv, \\ \dot{v} &= -1.2y_1y_2 - 0.5v,\end{aligned}\quad (3)$$

and system (3) exhibits chaotic behavior for the parameters $a = 0.9$, $b = 0.2$, $c = 1.5$, and $h = -5$ (see Figures 1–6).

The bifurcation diagram would be far better to summarize all of the possible behaviors as the parameter varies on one diagram. For $-14 \leq h \leq 1.5$, the bifurcation diagram of four-dimensional system (3) shows the complicated bifurcation phenomena in Figure 7.

Remark 1. The above analysis shows that the chaotic financial system remained as chaotic characteristics in the appropriate external perturbation. More detailed analysis on chaotic system (2) will be reported in a forthcoming paper.

3. Main Results

In this section, we give two kinds of integral proportional function synchronization definition and discuss the synchronization of the chaotic finance system.

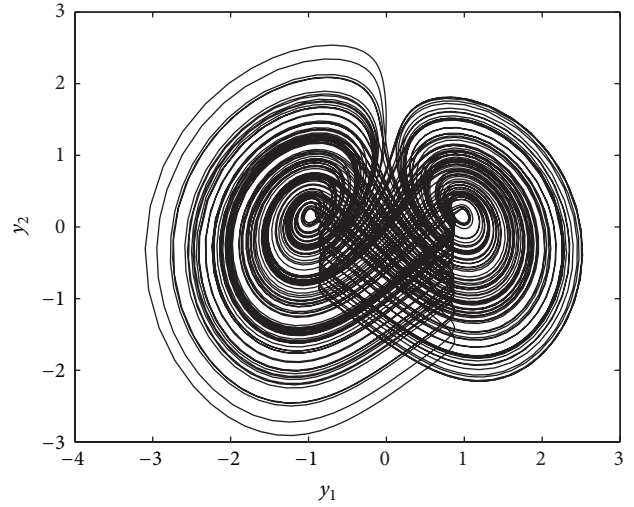


FIGURE 1: y_1 - y_2 phase plane, the chaotic attractor.

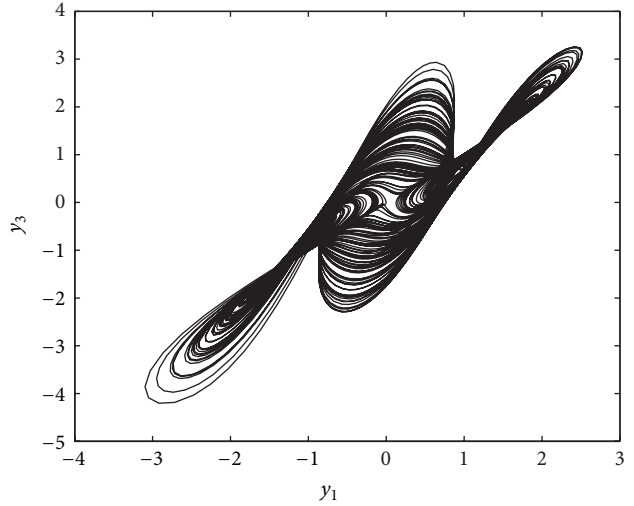


FIGURE 2: y_1 - y_3 phase plane, the chaotic attractor.

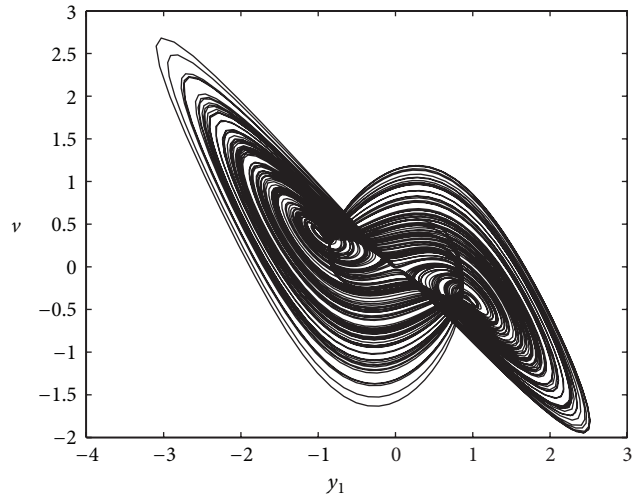


FIGURE 3: y_1 - v phase plane, the chaotic attractor.

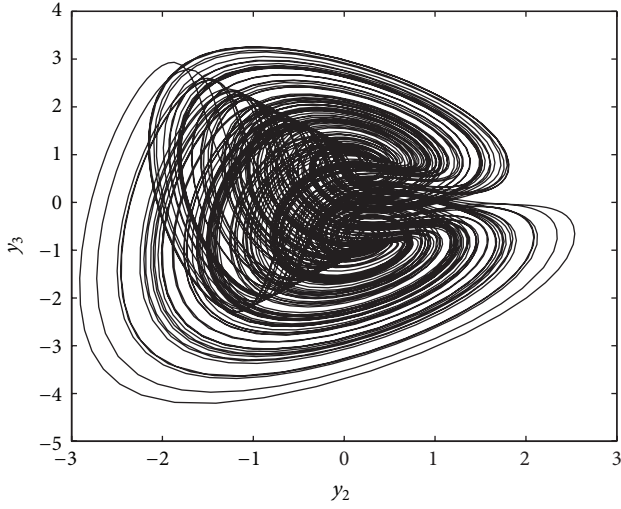


FIGURE 4: y_2 - y_3 phase plane, the chaotic attractor.

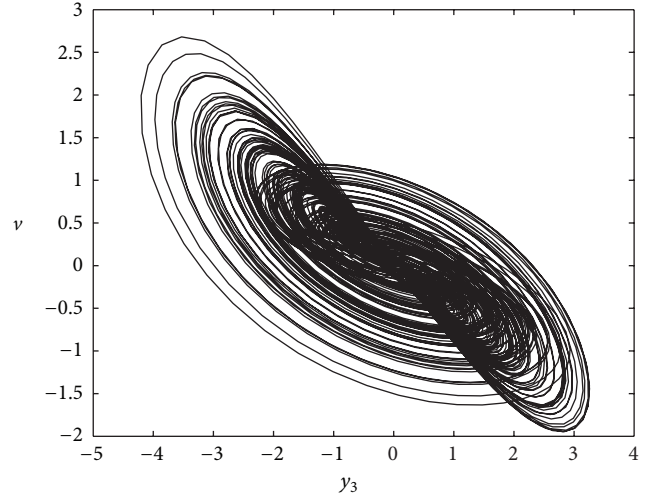


FIGURE 6: y_3 - v phase plane, the chaotic attractor.

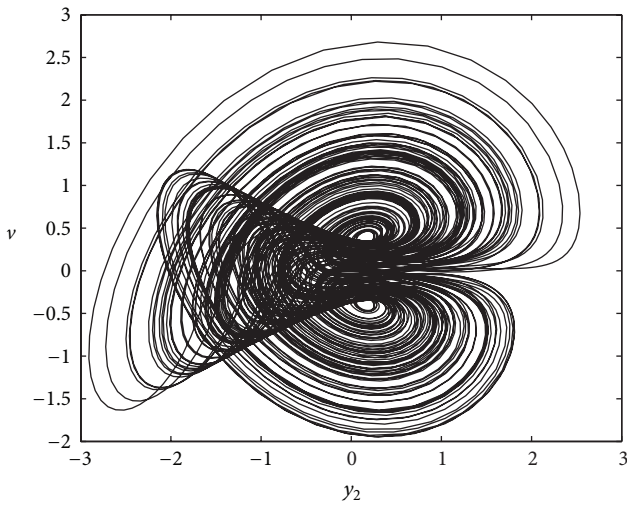


FIGURE 5: y_2 - v phase plane, the chaotic attractor.

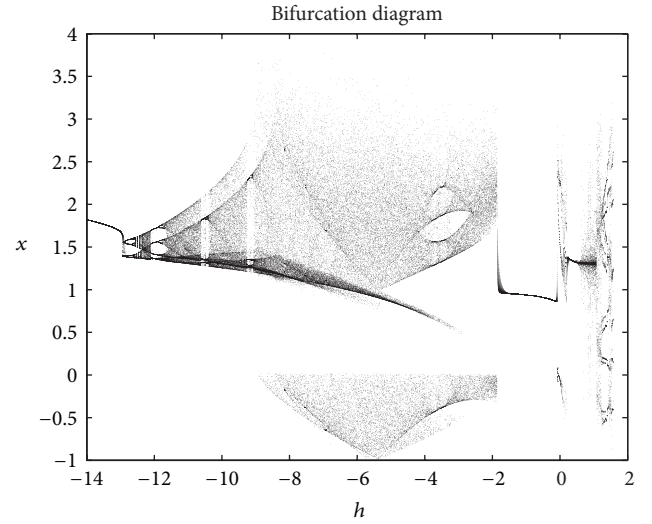


FIGURE 7: Bifurcation diagram of system (3).

The drive system is given by system (1), and the response system with external perturbation is the following controlled system:

$$\begin{aligned} \dot{y}_1 &= y_3 + (y_2 - a)y_1 + u_1 \\ \dot{y}_2 &= 1 - by_2 - y_1^2 + u_2 \\ \dot{y}_3 &= -y_1 - cy_3 + hv + u_3, \end{aligned} \quad (4)$$

where $\dot{v} = -1.2y_1y_2 - 0.5v$.

Definition 2. For systems (1) and (4), if

$$\lim_{t \rightarrow \infty} \|e_i\| = \lim_{t \rightarrow \infty} \left\| y_i - \left(\int_{\delta_i}^{w_i} s ds \right) x_i \right\| = 0, \quad i = 1, 2, \dots, n, \quad (5)$$

then the status of the response system and the drive system is called adaptive definite integration scaling function projective synchronization (ADISFPS), where $\dot{w}_i = f(w_i, t)$ and $\dot{\delta}_i = g(\delta_i, t)$ are bounded dynamical systems.

Remark 3. For Definition 2, when the upper and lower limit of integration of the scaling function are boundless, one problem should be taken into account if $e_i = y_i - \left(\int_{\delta_i}^{w_i} s ds \right) x_i$ is equal to infinite value. To achieve synchronization, that is, $e_i = y_i - \left(\int_{\delta_i}^{w_i} s ds \right) x_i$ is equal to 0 in the process of synchronization, we let the upper and lower limit of integration of the scaling function have the boundary; thus, we assume that for the dynamical systems $\dot{w}_i = f(w_i, t)$ and $\dot{\delta}_i = g(\delta_i, t)$ there exists the stable equilibrium point, the stable periodic orbit, or the chaotic attractor in the phase space, which satisfies the assumption.

Remark 4. Synchronization of the financial systems shows they can maintain consistency; that is, the economic systems in two different areas are to keep synchronized development by applying the appropriate control conditions. In economic activities, the driving system can be understood as a virtual economic goal, the response system is thought as a controlled object, and the manager's goal is to control a controlled object to a virtual economic goal. In practice, due to the complexity of the economic environment, the financial systems are not always completely synchronization; some complex synchronization methods should be considered. Therefore, the study of projective synchronization is meaningful in real economic activities.

Theorem 5. *System (1) and system (4) are asymptotically synchronized with the following adaptive control mechanism:*

$$\begin{aligned} u_1 &= (w_1 \dot{w}_1 - \delta_1 \dot{\delta}_1) x_1 - \gamma_1 y_2 + \left(\int_{\delta_1}^{w_1} s ds \right) x_1 x_2 + k_1 e_1 \\ u_2 &= (w_2 \dot{w}_2 - \delta_2 \dot{\delta}_2) x_2 \\ &\quad + \int_{\delta_2}^{w_2} s ds - 1 + \gamma_1^2 - \left(\int_{\delta_2}^{w_2} s ds \right) x_1^2 + k_2 e_2 \\ u_3 &= (w_3 \dot{w}_3 - \delta_3 \dot{\delta}_3) x_3 - hv + k_3 e_3, \end{aligned} \quad (6)$$

$$\dot{v} = -1.2\gamma_1 y_2 - 0.5v, \quad (7)$$

$$\dot{w}_i = f(w_i, t), \quad i = 1, 2, 3, \quad (8)$$

$$\dot{\delta}_i = g(\delta_i, t), \quad i = 1, 2, 3, \quad (9)$$

$$\dot{k}_i = -\lambda_i e_i^2, \quad i = 1, 2, 3. \quad (10)$$

Proof. The error system between driving system (1) and response system (4) can be expressed by

$$\begin{aligned} \dot{e}_1 &= \gamma_3 + (\gamma_2 - a_2) \gamma_1 + u_1 - \left(\int_{\delta_1}^{w_1} s ds \right) (x_3 + (x_2 - a) x_1) \\ &\quad - (w_1 \dot{w}_1 - \delta_1 \dot{\delta}_1) x_1, \\ \dot{e}_2 &= 1 - b_2 \gamma_2 - \gamma_1^2 + u_2 - \left(\int_{\delta_2}^{w_2} s ds \right) (1 - bx_2 - x_1^2) \\ &\quad - (w_2 \dot{w}_2 - \delta_2 \dot{\delta}_2) x_2, \\ \dot{e}_3 &= -\gamma_1 - c_2 \gamma_3 + hv + u_3 - \left(\int_{\delta_3}^{w_3} s ds \right) (-x_1 - cx_3) \\ &\quad - (w_3 \dot{w}_3 - \delta_3 \dot{\delta}_3) x_3. \end{aligned} \quad (11)$$

Substituting (6) into (11) yields

$$\begin{aligned} \dot{e}_1 &= e_3 - ae_1 + k_1 e_1, \\ \dot{e}_2 &= -be_2 + k_2 e_2, \\ \dot{e}_3 &= -e_1 - ce_3 + k_3 e_3. \end{aligned} \quad (12)$$

Choose a candidate Lyapunov function as follows:

$$V = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2) + \frac{1}{2} \sum_{i=1}^3 \frac{1}{\lambda_i} (k_i + L)^2, \quad (13)$$

where $L > \max\{-a, -b, -c\}$.

Then the differentiation of V along the trajectories of (12) is

$$\begin{aligned} \dot{V} &= e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + \sum_{i=1}^3 \frac{1}{\lambda_i} (k_i + L) \dot{k}_i \\ &= e_1 (e_3 - ae_1 + ke_1) + e_2 (-be_2 + ke_2) \\ &\quad + e_3 (-e_1 - ce_3 + ke_3) - \sum_{i=1}^3 (k_i + L) e_i^2 \\ &= e_1 (e_3 - ae_1) - be_2^2 + e_3 (-e_1 - ce_3) - \sum_{i=1}^3 L e_i^2 \\ &= -(L + a) e_1^2 - (L + b) e_2^2 - (L + c) e_3^2 < 0. \end{aligned} \quad (14)$$

It is clear that V is positive definite and \dot{V} is negative definite. According to the Lyapunov stability theorem, the trivial solution of error system (11) is asymptotically stable, which implies that the synchronization of systems (1) and (4) is achieved. The proof of the theorem is completed. \square

Remark 6. Synchronization conditions of the chaotic financial system with external perturbation are derived from Theorem 5. When the chaotic financial system with external perturbation is thought as the four-dimensional system without external perturbation, similar to Theorem 5, we may get the following synchronization condition.

Remark 7. In [25–33], research efforts have concentrated on studying a presetting scaling function with numerical examples. In this paper, the scaling function can be the definite integration, and the upper and lower limit of the definite integral scaling function are the bound dynamical systems.

The drive system is given by system (1), and the response system without external perturbation is the following controlled system:

$$\begin{aligned} \dot{y}_1 &= \gamma_3 + (\gamma_2 - a_2) \gamma_1 + u_1, \\ \dot{y}_2 &= 1 - b_2 \gamma_2 - \gamma_1^2 + u_2, \\ \dot{y}_3 &= -\gamma_1 - c_2 \gamma_3 + hv + u_3, \\ \dot{v} &= -1.2\gamma_1 y_2 - 0.5v + u_4. \end{aligned} \quad (15)$$

Theorem 8. System (1) and system (15) are asymptotically synchronized with the following adaptive control mechanism:

$$\begin{aligned}
u_1 &= (w_1 \dot{w}_1 - \delta_1 \dot{\delta}_1) x_1 - y_1 y_2 + \left(\int_{\delta_1}^{w_1} s ds \right) x_1 x_2 + k_1 e_1, \\
u_2 &= (w_2 \dot{w}_2 - \delta_2 \dot{\delta}_2) x_2 + \int_{\delta_2}^{w_2} s ds \\
&\quad - 1 + y_1^2 - \left(\int_{\delta_2}^{w_2} s ds \right) x_1^2 + k_2 e_2, \\
u_3 &= (w_3 \dot{w}_3 - \delta_3 \dot{\delta}_3) x_3 - hv + k_3 e_3, \\
u_4 &= (w_4 \dot{w}_4 - \delta_4 \dot{\delta}_4) x_4 + 1.2 y_1 y_2 + 0.5 v + k_4 e_4, \\
\dot{w}_i &= f(w_i, t), \quad i = 1, 2, 3, 4, \\
\dot{\delta}_i &= g(\delta_i, t), \quad i = 1, 2, 3, 4, \\
\dot{k}_i &= -\lambda_i e_i^2, \quad i = 1, 2, 3, 4.
\end{aligned} \tag{16}$$

The proofs of Theorem 8 follow directly from Theorem 5. Thus we leave out their proofs here.

Definition 9. For systems (1) and (4), if

$$\lim_{t \rightarrow \infty} \|e_i\| = \lim_{t \rightarrow \infty} \left\| y_i - \left(\int_0^t \kappa_i(s) ds \right) x_i \right\| = 0, \tag{17}$$

then the status of the response system and the drive system is called the definite integration scaling function projective synchronization (DISFPS).

Remark 10. Similar to Remark 3, $\kappa_i(t)$ should be bound function; otherwise, one problem should be taken into account if e_i is equal to infinite value.

According to Definition 9, similar to Theorems 5 and 8, we may get Theorems 11 and 12.

Theorem 11. System (1) and system (4) are asymptotically synchronized with the following adaptive control mechanism:

$$\begin{aligned}
u_1 &= \kappa_1 x_1 - y_1 y_2 + \left(\int_0^t \kappa_1(s) ds \right) x_1 x_2 + k_1 e_1, \\
u_2 &= \kappa_2 x_2 + \int_0^t \kappa_2(s) ds - 1 + y_1^2 \\
&\quad - \left(\int_0^t \kappa_2(s) ds \right) x_1^2 + k_2 e_2, \\
u_3 &= \kappa_3 x_3 - hv + k_3 e_3, \\
\dot{v} &= -1.2 y_1 y_2 - 0.5 v, \\
\dot{k}_i &= -\lambda_i e_i^2, \quad i = 1, 2, 3.
\end{aligned} \tag{18}$$

Theorem 12. System (1) and system (15) are asymptotically synchronized with the following adaptive control mechanism:

$$\begin{aligned}
u_1 &= \kappa_1 x_1 - y_1 y_2 + \left(\int_0^t \kappa_1(s) ds \right) x_1 x_2 + k_1 e_1, \\
u_2 &= \kappa_2 x_2 + \int_0^t \kappa_2(s) ds - 1 \\
&\quad + y_1^2 - \left(\int_0^t \kappa_2(s) ds \right) x_1^2 + k_2 e_2, \\
u_3 &= \kappa_3 x_3 - hv + k_3 e_3, \\
u_4 &= \kappa_4 x_4 + 1.2 y_1 y_2 + 0.5 v + k_4 e_4, \\
\dot{k}_i &= -\lambda_i e_i^2, \quad i = 1, 2, 3, 4.
\end{aligned} \tag{19}$$

The proofs of Theorems 11 and 12 follow directly from Theorem 5. Thus we leave out their proofs here.

4. Simulation and Results

In this section, computer simulations are used to verify and demonstrate the effectiveness of Theorem 5.

The dynamical systems $\dot{w}_i = f(w_i, t)$ and $\dot{\delta}_i = g(\delta_i, t)$ are the following chaotic systems, and two chaotic systems are bounded:

$$\begin{aligned}
\dot{w}_1 &= w_2 w_3 - 2w_1, \\
\dot{w}_2 &= (w_3 - 5) w_1 - 2w_2, \\
\dot{w}_3 &= 1 - w_1 w_2, \\
\dot{\delta}_1 &= 36(\delta_2 - \delta_1), \\
\dot{\delta}_2 &= 20\delta_2 - \delta_1 \delta_3, \\
\dot{\delta}_3 &= -3\delta_3 + \delta_1 \delta_2.
\end{aligned} \tag{20}$$

In the simulations, the initial values of the drive and response systems are taken as (3, 1, 2) and (-2, 3, -3), respectively, the initial values of the dynamical systems $\dot{w}_i = f(w_i, t)$ and $\dot{\delta}_i = g(\delta_i, t)$ are taken as (6, 7, 1) and (5, 3, 7) and the initial values of external perturbation system $v = 2$ and $k_i = 1, i = 1, 2, 3$, respectively. The simulated results about the function project synchronization are shown in Figures 1 and 2. In Figure 8, three state errors versus time are shown and it is also shown that the state errors tend to zero asymptotically as time evolves. Figure 9 shows the feedback gain $k_i, i = 1, 2, 3$.

Numerical simulations of Theorems 8, 11, and 12 can be illustrated in a similar way as shown in Theorem 5; thus we leave out numerical simulations here.

5. Conclusion

First, we have discussed that the three-dimensional chaotic system with external perturbation can be seen as the four-dimensional chaotic system without external perturbation. Second, the definite integral scaling function synchronization

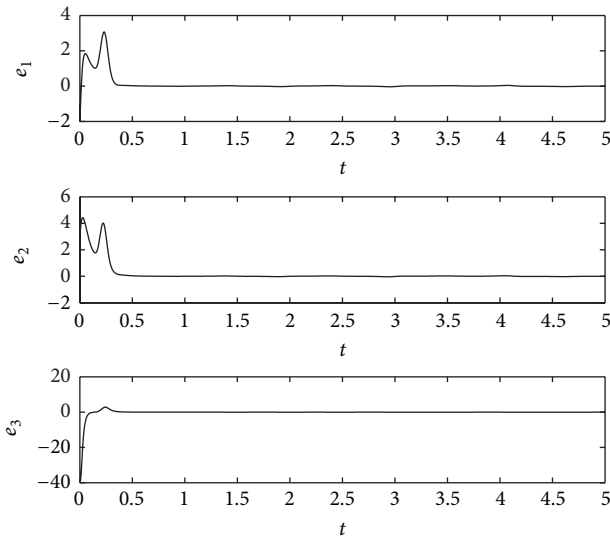


FIGURE 8: Synchronization errors with time t .

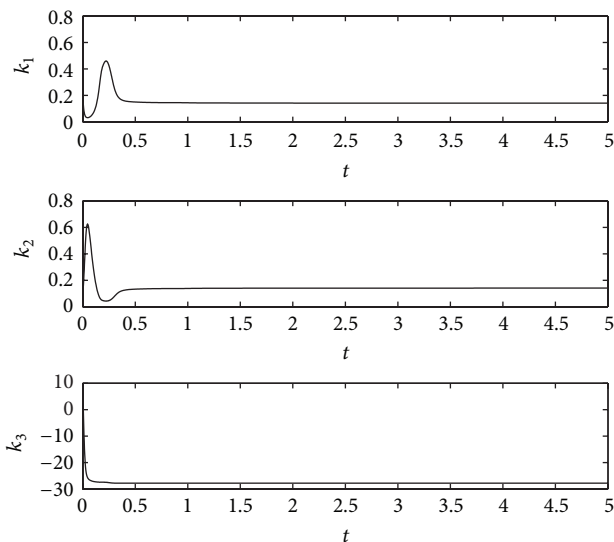


FIGURE 9: Dynamic curve of the feedback gain k_i , $i = 1, 2, 3$.

problem of financial chaotic systems is investigated. Finally, numerical simulations are then given to verify the effectiveness of the proposed adaptive schemes.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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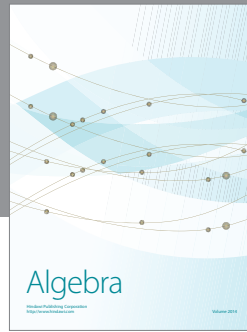
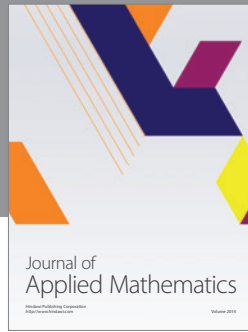
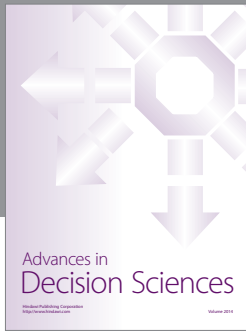
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