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Research Article

Wirtinger-Type Inequality and the Stability Analysis of Delayed Lur'e System

Zixin Liu,[1,2] Jian Yu,[3] Daoyun Xu,[1] and Dingtao Peng[3]

[1] *College of Computer Science and Information, GuiZhou University, Guiyang, Guizhou 550025, China*
[2] *School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang, Guizhou 550004, China*
[3] *College of Science, Guizhou University, Guiyang, Guizhou 550025, China*

Correspondence should be addressed to Zixin Liu; xinxin905@163.com

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This paper proposes a new delay-depended stability criterion for a class of delayed Lur'e systems with sector and slope restricted nonlinear perturbation. The proposed method employs an improved Wirtinger-type inequality for constructing a new Lyapunov functional with triple integral items. By using the convex expression of the nonlinear perturbation function, the original nonlinear Lur'e system is transformed into a linear uncertain system. Based on the Lyapunov stable theory, some novel delay-depended stability criteria for the researched system are established in terms of linear matrix inequality technique. Three numerical examples are presented to illustrate the validity of the main results.

1. Introduction

Lur'e control system is an important nonlinear control system. Since the notion of absolute stability was first time introduced by Lur'e in [1], the problem of the absolute stability of Lur'e control system has been widely studied for several decades (see [2–6]). However, because of the existence of time delays, stochastic disturbances, parameter uncertainties, and so on, the convergence of Lur'e system may often be destroyed. This makes the design or performance for the corresponding closed-loop systems become difficult. Therefore, the stability analysis of delayed Lur'e system becomes very important. Up to now, various stability conditions have been obtained, and many excellent papers and monographs have been available (see [7–12]).

Recently, a great deal of effort has been done to the stability analysis of delayed Lur'e system with sector and slope restricted nonlinearities. To enlarge the feasibility region of the stability criteria, by introducing variables in cross-term, Park researched a new bounding technique in [13]. Concerning the descriptor method for delayed system, an extensive work was developed by Fridman and Shaked in [14]. By employing linear matrix inequality and matrix decomposing technique, Cao and Zhong [6] researched the absolute stability problem of Lur'e control systems with multiple time delays and nonlinearities and established some improved delay-dependent criteria. In order to further reduce stability criterion's conservatism, sector bounds and slope bounds are employed to a Lyapunov-Krasovskii functional through convex representation of the nonlinearities so that some new improved criteria were established by Lee and Park in [12] and Yin et al. in [15], respectively.

On the other hand, these previous works only focused on the relationship between $\int_{t-\tau}^{t} x^T(s)Qx(s)ds$ and $(\int_{t-\tau}^{t} x(s)ds)^T Q(\int_{t-\tau}^{t} x(s)ds)$ or between $\int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)Qx(s)ds\,d\theta$ and $(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)ds\,d\theta)^T Q(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)ds\,d\theta)$. One natural question is whether there exists a relationship among $\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{z}(s)^T R\dot{z}(s)ds\,d\theta$, $\int_{-\tau}^{0}\int_{t+\theta}^{t} z(s)ds\,d\theta$, $\int_{t-\tau}^{t} \dot{z}^T(s)R\dot{z}(s)ds$, $z(t-\tau)$, and $\int_{t-\tau}^{t} z(s)ds$. This idea motivates this study. By using the Jensen integral inequality, we first establish some improved vector Wirtinger-type inequalities. On the basis of these new established inequalities, a new Lyapunov functional including triple integral items is proposed, and some less conservative delay-dependent stability criteria are derived. Finally, three numerical examples are presented to illustrate the validity of the main results.

Notation. The notations are used in our paper except where otherwise specified. R, R^n are real and n-dimension real number sets, respectively; $\mathrm{diag}(\cdots)$ denotes the block diagonal matrix. I is identity matrix; $*$ represents the elements below the main diagonal of a symmetric block matrix; real matrix $P > 0(<0)$ denotes P is a positive-definite (negative-definite) matrix.

2. Preliminaries

Consider the following delayed Lur'e system:

$$\begin{aligned} \dot{y}(t) &= Ay(t) + By(t-\tau(t)) + Cf(\sigma(t)), \\ \sigma(t) &= H^T y(t), \quad \forall t \geq 0, \\ y(s) &= \varphi(s), \quad s \in [-\tau_u, 0], \end{aligned} \tag{1}$$

where $y(t) \in R^n$ denotes the state vector; $\sigma(t) \in R^m$ is the output vector; $H = (h_1, h_2, \ldots, h_m)_{n\times m} \in R^{n\times m}$; $A, B,$ and C are constant known matrices of appropriate dimensions. The delay $\tau(t)$ is assumed to satisfy

$$0 < \tau_l \leq \tau(t) \leq \tau_u, \quad \dot{\tau}(t) \leq \tau < 1. \tag{2}$$

$f(\sigma(t)) \in R^m$ denotes the nonlinear function in feedback path, which has the following form:

$$\begin{aligned} f(\sigma(t)) &= [f_1(\sigma_1(t)), f_2(\sigma_2(t)), \ldots, f_m(\sigma_m(t))]^T \\ \sigma(t) &= [\sigma_1(t), \sigma_2(t), \ldots, \sigma_m(t)]^T \\ &\triangleq [h_1^T y(t), h_2^T y(t), \ldots, h_m^T y(t)]^T, \end{aligned} \tag{3}$$

which satisfies a sector condition with $f_i(\cdot), (i = 1, 2, \ldots, m)$ belonging to sector $[l_i^-, l_i^+]$, where l_i^-, l_i^+ are known constant scalars; that is,

$$l_i^- \leq \frac{f_i(\sigma_i(t))}{\sigma_i(t)} \leq l_i^+, \quad i = 1, 2, \ldots, m. \tag{4}$$

Notice that the nonlinear function $f_i(\cdot)$ can be written as a convex combination of the sector bounds as follows:

$$f_i(\sigma_i(t)) = (\lambda_i(\sigma_i(t)) l_i^- + (1 - \lambda_i(\sigma_i(t))) l_i^+) \sigma_i(t), \quad i = 1, 2, \ldots, m, \tag{5}$$

where $\lambda_i(\sigma_i) = (f_i(\sigma_i(t)) - l_i^-\sigma_i(t))/(l_i^+ - l_i^-)\sigma_i(t)$ satisfying $0 \leq \lambda_i(\sigma_i) \leq 1$. Namely, $f_i(\sigma(t)) = \Lambda_i(\sigma_i(t))\sigma_i(t)$, where $\Lambda_i(\sigma_i(t))$ is an element of a convex hull $\mathrm{Co}\{l_i^-, l_i^+\}$. Set $L = \mathrm{diag}\{l_1, l_2, \ldots, l_m\}$, where $l_i = \max\{|l_i^-|, |l_i^+|\}$. Obviously, $-1 \leq (\lambda_i(\sigma_i(t))l_i^- + (1 - \lambda_i(\sigma_i(t)))l_i^+)/l_i \leq 1$. Define $\Delta_i = (\lambda_i(\sigma_i(t))l_i^- + (1 - \lambda_i(\sigma_i(t)))l_i^+)/l_i$, $i = 1, 2, \ldots, m$, $\Delta = \mathrm{diag}(\Delta_1, \Delta_2, \ldots, \Delta_m)$ then nonlinear function $f(\sigma(t))$ can be expressed as $f(\sigma(t)) = L\Delta H^T y(t)$, where Δ satisfies $\Delta^T\Delta \leq I$. And the system (1) can be rewritten as the following delayed uncertain system:

$$\begin{aligned} \dot{y}(t) &= \left(A + CL\Delta H^T\right) y(t) + By(t - \tau(t)), \\ y(s) &= \varphi(s), \quad s \in [-\tau_u, 0]. \end{aligned} \tag{6}$$

Remark 1. Different from previous work [6, 9, 10, 12, 15], in this paper, by using the convex expression $f_i(\sigma(t)) = \Lambda_i(\sigma_i(t))\sigma_i(t)$, we transform the original nonlinear system (1) into a linear uncertain system (6). As a result, the stability problem of nonlinear Lur'e system (1) can be transformed into the robust stability problem of linear uncertain system (6).

Let $L^- = \mathrm{diag}(l_1^-, l_2^-, \ldots, l_m^-)$ and $L^+ = \mathrm{diag}(l_1^+, l_2^+, \ldots, l_m^+)$. For further discussion, the following lemmas are needed.

Lemma 2 (see [16]). *For any positive definite symmetric constant matrix Q and scalar $\tau > 0$, such that the following integrations are well defined, then*

$$\begin{aligned} &-\int_{-\tau}^{0}\int_{t+\theta}^{t} y^T(s) Qy(s)\, ds\, d\theta \\ &\leq -\frac{1}{\tau^2}\left(\int_{-\tau}^{0}\int_{t+\theta}^{t} y(s)\, ds\, d\theta\right)^T Q \left(\int_{-\tau}^{0}\int_{t+\theta}^{t} y(s)\, ds\, d\theta\right). \end{aligned} \tag{7}$$

Lemma 3 (see [17]). *Given symmetric matrix P_1 and any real matrices P_2, P_3 of appropriate dimensions,*

$$P_1 + P_2\Delta P_3 + P_3^T\Delta^T P_2^T < 0, \tag{8}$$

for all $\Delta \in \Theta$ satisfying $\Delta^T\Delta \leq I$ if and only if there exists $S \in S_\Delta$ such that

$$\begin{bmatrix} P_1 + P_3^T S P_3 & P_2 \\ P_2^T & -S \end{bmatrix} < 0, \tag{9}$$

where $S_\Delta =: \{\mathrm{diag}(s_1 I, \ldots, s_k I, S_1, \ldots, S_l) : S_i > 0, k, l \in N\}$.

On the basis of Jensen integral inequality, we first give out an improved Wirtinger-type vector inequality as follows.

Lemma 4. *Let $z(t) \in R^n$ have continuous derived function $\dot{z}(t)$ on interval $[a, b]$. Assume that $z(a) = 0$; then for any $n\times n$-matrix $R > 0$, the following inequality holds:*

$$\int_a^b z^T(s) Rz(s)\, ds \leq \frac{(b-a)^2}{2}\int_a^b \dot{z}^T(s) R\dot{z}(s)\, ds. \tag{10}$$

Proof. Since $z(a) = 0$, one can get that $z(s) = \int_a^s \dot{z}(t)dt$,

$$\begin{aligned}
&\int_a^b z^T(s) Rz(s)\, ds \\
&= \int_a^b \left(\int_a^s \dot{z}(t)\, dt\right)^T R\left(\int_a^s \dot{z}(t)\, dt\right) ds \\
&\le \int_a^b (s-a) \int_a^s \dot{z}^T(t) R\dot{z}(t)\, dt\, ds \\
&= \int_a^b \int_t^b (s-a)\, \dot{z}^T(t) R\dot{z}(t)\, ds\, dt \\
&= \int_a^b \dot{z}^T(t) R\dot{z}(t) \left(\frac{(b-a)^2}{2} - \frac{(t-a)^2}{2}\right) dt \\
&\le \frac{(b-a)^2}{2} \int_a^b \dot{z}^T(t) R\dot{z}(t)\, dt.
\end{aligned} \tag{11}$$

This completes the proof. □

On the basis of Lemma 4, we further give out some improved Wirtinger-type inequalities as follows.

Lemma 5. *Let $z(t) \in R^n$ have continuous derived function $\dot{z}(t)$ on interval $[a, b]$. Then for any $n\times n$-matrix $R > 0$, scalar $\tau > 0$, the following inequality holds:*

(1)

$$\begin{aligned}
&-\int_{t-\tau}^t \dot{z}^T(s) R\dot{z}(s)\, ds \\
&\le -\frac{2}{\tau^3}\left(\int_{t-\tau}^t z(s)\, ds\right)^T R\left(\int_{t-\tau}^t z(s)\, ds\right) \\
&\quad - \frac{2}{\tau} z^T(t-\tau) Rz(t-\tau) + \frac{4}{\tau^2}\left(\int_{t-\tau}^t z(s)\, ds\right)^T Rz(t-\tau);
\end{aligned} \tag{12}$$

(2)

$$\begin{aligned}
&-\int_{-\tau}^0 \int_{t+\theta}^t \dot{z}(s)^T R\dot{z}(s)\, ds\, d\theta \\
&\le -\frac{2}{\tau^4}\left(\int_{-\tau}^0 \int_{t+\theta}^t z(s)\, ds\, d\theta\right)^T R\left(\int_{-\tau}^0 \int_{t+\theta}^t z(s)\, ds\, d\theta\right) \\
&\quad - \frac{2}{\tau^2}\left(\int_{t-\tau}^t z(s)\, ds\right)^T R\left(\int_{t-\tau}^t z(s)\, ds\right) \\
&\quad + \frac{4}{\tau^3}\left(\int_{-\tau}^0 \int_{t+\theta}^t z(s)\, ds\, d\theta\right)^T R\left(\int_{t-\tau}^t z(s)\, ds\right).
\end{aligned} \tag{13}$$

Proof. Set $x(s) = z(s) - z(t-\tau)$, $s \in [t-\tau, t]$. Notice that $x(t-\tau) = z(t-\tau) - z(t-\tau) = 0$; from Lemma 4, we have

$$\begin{aligned}
&\int_{t-\tau}^t x^T(s) Rx(s)\, ds \\
&= \int_{t-\tau}^t (z(s) - z(t-\tau))^T R(z(s) - z(t-\tau))\, ds \\
&\le \frac{\tau^2}{2} \int_{t-\tau}^t \left(\frac{d(z(s) - z(t-\tau))}{ds}\right)^T \\
&\qquad \times R\left(\frac{d(z(s) - z(t-\tau))}{ds}\right) ds \\
&= \frac{\tau^2}{2} \int_{t-\tau}^t \dot{z}(s)^T R\dot{z}(s)\, ds.
\end{aligned} \tag{14}$$

Additionally, from Jensen inequality, one can get

$$\begin{aligned}
&\int_{t-\tau}^t (z(s) - z(t-\tau))^T R(z(s) - z(t-\tau))\, ds \\
&\ge \frac{1}{\tau}\left(\int_{t-\tau}^t (z(s) - z(t-\tau))\, ds\right)^T \\
&\quad \times R\left(\int_{t-\tau}^t (z(s) - z(t-\tau))\, ds\right).
\end{aligned} \tag{15}$$

Thus, we have

$$\begin{aligned}
&-\int_{t-\tau}^t \dot{z}(s)^T R\dot{z}(s)\, ds \\
&\le -\frac{2}{\tau^3}\left(\int_{t-\tau}^t (z(s) - z(t-\tau))\, ds\right)^T \\
&\quad \times R\left(\int_{t-\tau}^t (z(s) - z(t-\tau))\, ds\right) \\
&= -\frac{2}{\tau^3}\left(\int_{t-\tau}^t z(s)\, ds\right)^T R\left(\int_{t-\tau}^t z(s)\, ds\right) \\
&\quad - \frac{2}{\tau} z^T(t-\tau) Rz(t-\tau) + \frac{4}{\tau^2}\left(\int_{t-\tau}^t z(s)\, ds\right)^T Rz(t-\tau).
\end{aligned} \tag{16}$$

Similarly, let $x(s) = z(s) - z(t+\theta)$; from inequality (16), Lemmas 2, and 4, one can obtain

$$\begin{aligned}
&-\int_{-\tau}^0 \int_{t+\theta}^t \dot{z}(s)^T R\dot{z}(s)\, ds\, d\theta \\
&\le -\int_{-\tau}^0 \int_{t+\theta}^t \frac{2}{(-\theta)^2} x(s)^T Rx(s)\, ds\, d\theta \\
&\le -\frac{1}{\tau^2}\left(\int_{-\tau}^0 \int_{t+\theta}^t \frac{\sqrt{2}\,(z(s) - z(t+\theta))}{-\theta} ds\, d\theta\right)^T \\
&\quad \times R\left(\int_{-\tau}^0 \int_{t+\theta}^t \frac{\sqrt{2}\,(z(s) - z(t+\theta))}{-\theta} ds\, d\theta\right) \\
&\le -\frac{1}{\tau^2}\left(\int_{-\tau}^0 \int_{t+\theta}^t \frac{\sqrt{2} z(s)}{\tau} ds\, d\theta - \sqrt{2}\int_{t-\tau}^t z(s)\, ds\right)^T \\
&\quad \times R\left(\int_{-\tau}^0 \int_{t+\theta}^t \frac{\sqrt{2} z(s)}{\tau} ds\, d\theta - \sqrt{2}\int_{t-\tau}^t z(s)\, ds\right)
\end{aligned}$$

$$= -\frac{2}{\tau^4}\left(\int_{-\tau}^{0}\int_{t+\theta}^{t} z(s)\,ds\,d\theta\right)^T R\left(\int_{-\tau}^{0}\int_{t+\theta}^{t} z(s)\,ds\,d\theta\right) - \frac{2}{\tau^2}\left(\int_{t-\tau}^{t} z(s)\,ds\right)^T R\left(\int_{t-\tau}^{t} z(s)\,ds\right) + \frac{4}{\tau^3}\left(\int_{-\tau}^{0}\int_{t+\theta}^{t} z(s)\,ds\,d\theta\right)^T R\left(\int_{t-\tau}^{t} z(s)\,ds\right). \quad (17)$$

This completes the proof. □

Remark 6. Compared with traditional Wirtinger-type integral inequality, Jensen integral inequality [18], and double integral Jensen inequality [16], Lemma 5 gives out a transitional form among $\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{z}(s)^T R\dot{z}(s)ds\,d\theta$, $\int_{-\tau}^{0}\int_{t+\theta}^{t} z(s)ds\,d\theta$, $\int_{t-\tau}^{t}\dot{z}^T(s)R\dot{z}(s)ds$, $z(t-\tau)$, and $\int_{t-\tau}^{t} z(s)ds$. These inequality relationships can be used as a handy tool to deal with the stability problem.

3. Main Results

In this section, we attempt to establish some new practically computable stability criteria for system (1). By constructing a new Lyapunov functional including triple integral items, we obtain the following stability result.

Theorem 7. *For given scalars $\tau_l > 0$, $\tau_u > 0$, $\tau < 1$, $L^- = \mathrm{diag}(l_1^-, l_2^-, \ldots, l_m^-)$, $L^+ = \mathrm{diag}(l_1^+, l_2^+, \ldots, l_m^+)$, system (1) is globally asymptotically stable if there exist positive definite diagonal matrices $D_1 = \mathrm{diag}\{d_1, d_2, \ldots, d_m\}$, $D_2 = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, $D_3 = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, symmetric positive definite matrices Q_1, Q_2, Q_3, Q_4, and arbitrary matrices M_1, M_2 of appropriate dimensions such that the following condition holds:*

$$\begin{bmatrix} \Xi + \Psi^T S \Psi & \Phi \\ * & -S \end{bmatrix} < 0, \quad (18)$$

where $S \in S_\Delta$, $\Xi = (\Xi_{ij})$, $\Phi, \Psi \in R^{7m\times 7m}$, $i, j = 1, 2, \ldots, 7$,

$$\Xi_{11} = \left(\tau_l^2 + \tau_u^2\right) HD_3\left(L^+ - L^-\right)H^T + M_1 A + A^T M_1^T + Q_4, \qquad \Xi_{12} = M_1 B,$$

$$\Xi_{13} = -M_1 + A^T M_2^T + Q_1 - L^- HH^T D_1 + L^+ HH^T D_2,$$

$$\Xi_{22} = -(1-\tau)Q_4, \qquad \Xi_{23} = B^T M_2^T,$$

$$\Xi_{33} = -M_2 - M_2^T + \frac{\tau_l^2}{2}Q_2 + \frac{\tau_u^2}{2}Q_3,$$

$$\Xi_{44} = -\frac{2}{\tau_l^2}Q_2, \qquad \Xi_{46} = \frac{4}{\tau_l^3}Q_2,$$

$$\Xi_{55} = -\frac{2}{\tau_l^2}Q_3, \qquad \Xi_{57} = \frac{4}{\tau_u^3}Q_3,$$

$$\Xi_{66} = -\frac{2}{\tau_l^4}Q_2 - \frac{2}{\tau_l^2}\left(HD_3\left(L^+ - L^-\right)H^T\right),$$

$$\Xi_{77} = -\frac{2}{\tau_u^4}Q_3 - \frac{2}{\tau_u^2}\left(HD_3\left(L^+ - L^-\right)H^T\right),$$

$$\Phi = \mathrm{diag}\left\{M_1 CL, 0, H, 0, 0, 0, 0\right\},$$

$$\Psi = \left(H, 0, M_2 CL + \left(D_1 - D_2\right)HL, 0, 0, 0, 0 \atop O\right). \quad (19)$$

Proof. Choose a new class of Lyapunov functional candidate as follows:

$$V(y(t)) = V_1(y(t)) + V_2(y(t)) + V_3(y(t)) + V_4(y(t)), \quad (20)$$

where

$$V_1(y(t)) = y^T(t) Q_1 y(t) + 2\sum_{i=1}^{m}\left\{\int_0^{h_i^T y(t)} \lambda_i\left(f_i(s) - l_i^- s\right)ds + \int_0^{h_i^T y(t)} d_i\left(l_i^+ s - f_i(s)\right)ds\right\};$$

$$V_2(y(t)) = 2\sum_{i=1}^{m}\left\{\int_{-\tau_l}^{0}\int_{\theta}^{0}\int_{t+\mu}^{t}\alpha_i\sigma_i(s)\left[f_i(\sigma_i(s)) - l_i^-\sigma_i(s)\right] \times ds\,d\mu\,d\theta\right\} + 2\sum_{i=1}^{m}\left\{\int_{-\tau_l}^{0}\int_{\theta}^{0}\int_{t+\mu}^{t}\alpha_i\sigma_i(s)\left[l_i^+\sigma_i(s) - f_i(\sigma_i(s))\right] \times ds\,d\mu\,d\theta\right\} + 2\sum_{i=1}^{m}\left\{\int_{-\tau_u}^{0}\int_{\theta}^{0}\int_{t+\mu}^{t}\alpha_i\sigma_i(s)\left[f_i(\sigma_i(s)) - l_i^-\sigma_i(s)\right] \times ds\,d\mu\,d\theta\right\} + 2\sum_{i=1}^{m}\left\{\int_{-\tau_u}^{0}\int_{\theta}^{0}\int_{t+\mu}^{t}\alpha_i\sigma_i(s)\left[l_i^+\sigma_i(s) - f_i(\sigma_i(s))\right] \times ds\,d\mu\,d\theta\right\};$$

$$V_3(y(t)) = \int_{-\tau_l}^{0}\int_{\theta}^{0}\int_{t+\mu}^{t}\dot{y}^T(s) Q_2 \dot{y}(s)\,ds\,d\mu\,d\theta + \int_{-\tau_u}^{0}\int_{\theta}^{0}\int_{t+\mu}^{t}\dot{y}^T(s) Q_3 \dot{y}(s)\,ds\,d\mu\,d\theta;$$

$$V_4(y(t)) = \int_{t-\tau(t)}^{t} y^T(s) Q_4 y(s)\,ds. \quad (21)$$

The time derivative of $V(y(t))$ along the trajectory of system (1) is given as

$$\dot{V}(y(t)) = \dot{V}_1(y(t)) + \dot{V}_2(y(t)) + \dot{V}_3(y(t)) + \dot{V}_4(y(t)), \tag{22}$$

where

$$\begin{aligned}
&\dot{V}_1(y(t)) \\
&\quad = 2y^T(t) Q_1 \dot{y}(t) + 2\left[f^T(\sigma(t)) H^T \right.\\
&\qquad \left. - y^T(t) L^- HH^T\right] D_1 \dot{y}(t) \\
&\qquad + 2\left[y^T(t) L^+ HH^T - f^T(\sigma(t)) H^T\right] D_2 \dot{y}(t), \\
&\quad = 2y^T(t)\left(Q_1 - L^- HH^T D_1 + L^+ HH^T D_2\right)\dot{y}(t) \\
&\qquad + 2y^T(t) H\Delta L H^T (D_1 - D_2)\dot{y}(t).
\end{aligned} \tag{23}$$

Consider

$$\begin{aligned}
&\dot{V}_2(y(t)) \\
&\quad = 2\sum_{i=1}^{m}\left\{\int_{-\tau_l}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t)\left[f_i(\sigma_i(t)) - l_i^-\sigma_i(t)\right]d\mu\, d\theta\right. \\
&\qquad - \int_{-\tau_l}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t+\mu)\left[f_i(\sigma_i(t+\mu))\right. \\
&\qquad \left.\left. - l_i^-\sigma_i(t+\mu)\right]d\mu\, d\theta\right\} \\
&\quad + 2\sum_{i=1}^{m}\left\{\int_{-\tau_l}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t)\left[l_i^+\sigma_i(t) - f_i(\sigma_i(t))\right]d\mu\, d\theta\right. \\
&\qquad - \int_{-\tau_l}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t+\mu)\left[l_i^+\sigma_i(t+\mu)\right. \\
&\qquad \left.\left. - f_i(\sigma_i(t+\mu))\right]d\mu\, d\theta\right\} \\
&\quad + 2\sum_{i=1}^{m}\left\{\int_{-\tau_u}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t)\left[f_i(\sigma_i(t)) - l_i^-\sigma_i(t)\right]d\mu\, d\theta\right. \\
&\qquad - \int_{-\tau_u}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t+\mu)\left[f_i(\sigma_i(t+\mu))\right. \\
&\qquad \left.\left. - l_i^-\sigma_i(t+\mu)\right]d\mu\, d\theta\right\} \\
&\quad + 2\sum_{i=1}^{m}\left\{\int_{-\tau_u}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t)\left[l_i^+\sigma_i(t) - f_i(\sigma_i(t))\right]d\mu\, d\theta\right. \\
&\qquad - \int_{-\tau_u}^{0}\int_{\theta}^{0}\alpha_i\sigma_i(t+\mu)\left[l_i^+\sigma_i(t+\mu)\right. \\
&\qquad \left.\left. - f_i(\sigma_i(t+\mu))\right]d\mu\, d\theta\right\} \\
&\quad = 2\sum_{i=1}^{m}\left\{\alpha_i\sigma_i(t)\left[f_i(\sigma_i(t)) - l_i^-\sigma_i(t)\right]\frac{\tau_l^2}{2}\right. \\
&\qquad \left. - \int_{-\tau_l}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[f_i(\sigma_i(s)) - l_i^-\sigma_i(s)\right]ds\, d\theta\right\} \\
&\quad + 2\sum_{i=1}^{m}\left\{\alpha_i\sigma_i(t)\left[l_i^+\sigma_i(t) - f_i(\sigma_i(t))\right]\frac{\tau_l^2}{2}\right. \\
&\qquad \left. - \int_{-\tau_l}^{0}\int_{t+\theta}^{t}\alpha_i y_i(s)\left[l_i^+ y_i(s) - f_i(\sigma_i(s))\right]ds\, d\theta\right\} \\
&\quad + 2\sum_{i=1}^{m}\left\{\alpha_i\sigma_i(t)\left[f_i(\sigma_i(t)) - l_i^-\sigma_i(t)\right]\frac{\tau_u^2}{2}\right. \\
&\qquad \left. - \int_{-\tau_u}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[f_i(\sigma_i(s)) - l_i^-\sigma_i(s)\right]ds\, d\theta\right\} \\
&\quad + 2\sum_{i=1}^{m}\left\{\alpha_i\sigma_i(t)\left[l_i^+\sigma_i(t) - f_i(\sigma_i(t))\right]\frac{\tau_u^2}{2}\right. \\
&\qquad \left. - \int_{-\tau_u}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[l_i^+\sigma_i(s) - f_i(\sigma_i(s))\right]ds\, d\theta\right\} \\
&\quad = 2\sum_{i=1}^{m}\left\{\frac{\alpha_i\tau_l^2}{2}\sigma_i(t)\left[l_i^+ - l_i^-\right]\sigma_i(t)\right. \\
&\qquad + \sum_{i=1}^{n}\frac{\alpha_i\tau_u^2}{2}\sigma_i(t)\left[l_i^+ - l_i^-\right]\sigma_i(t) \\
&\qquad - \int_{-\tau_l}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[l_i^+ - l_i^-\right]\sigma_i(s)\, ds\, d\theta \\
&\qquad \left. - \int_{-\tau_u}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[l_i^+ - l_i^-\right]\sigma_i(s)\, ds\, d\theta\right\}.
\end{aligned} \tag{24}$$

Notice that

$$\begin{aligned}
&\sum_{i=1}^{m}\alpha_i\tau_l^2\sigma_i(t)\left[l_i^+ - l_i^-\right]\sigma_i(t) \\
&\qquad = \tau_l^2 y^T(t) HD_3\left[L^+ - L^-\right]H^T y(t), \\
&\sum_{i=1}^{m}\alpha_i\tau_u^2\sigma_i(t)\left[l_i^+ - l_i^-\right]\sigma_i(t) \\
&\qquad = \tau_u^2 y^T(t) HD_3\left[L^+ - L^-\right]H^T y(t).
\end{aligned} \tag{25}$$

From Lemma 2, we have

$$\begin{aligned}
&2\sum_{i=1}^{m}\left\{-\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[l_i^+ - l_i^-\right]\sigma_i(s)\, ds\, d\theta\right\} \\
&\quad = -2\int_{-\tau_l}^{0}\int_{t+\theta}^{t} y^T(s) HD_3\left[L^+ - L^-\right]H^T y(s)\, ds\, d\theta \\
&\quad \le -\frac{2}{\tau_l^2}\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t} y(s)\, ds\, d\theta\right)^T HD_3 \\
&\qquad \times\left[L^+ - L^-\right]H^T\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t} y(s)\, ds\, d\theta\right).
\end{aligned} \tag{26}$$

Similarly,

$$2\sum_{i=1}^{m}\left\{-\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\alpha_i\sigma_i(s)\left[l_i^+-l_i^-\right]\sigma_i(s)\,ds\,d\theta\right\}$$
$$\le -\frac{2}{\tau_u^2}\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T HD_3 \qquad (27)$$
$$\times\left[L^+-L^-\right]H^T\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right).$$

Namely,

$$\dot V_2(y(t))$$
$$\le y^T(t)\left[\left(\tau_l^2+\tau_u^2\right)HD_3\left(L^+-L^-\right)H^T\right]y(t)$$
$$-\frac{2}{\tau_l^2}\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T\left[HD_3\left(L^+-L^-\right)H^T\right]$$
$$\times\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)$$
$$-\frac{2}{\tau_u^2}\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T\left[HD_3\left(L^+-L^-\right)H^T\right]$$
$$\times\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right);$$

$$\dot V_3(y(t))$$
$$=\int_{-\tau_l}^{0}\int_{\theta}^{0}\left[\dot y^T(t)Q_2\dot y(t)-\dot y^T(t+\mu)Q_2\dot y(t+\mu)\right]d\mu\,d\theta$$
$$+\int_{-\tau_u}^{0}\int_{\theta}^{0}\left[\dot y^T(t)Q_3\dot y(t)-\dot y^T(t+\mu)Q_3\dot y(t+\mu)\right]d\mu\,d\theta$$
$$=\frac{\tau_l^2}{2}\dot y^T(t)Q_2\dot y(t)+\frac{\tau_u^2}{2}\dot y^T(t)Q_3\dot y(t)$$
$$-\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot y^T(s)Q_2\dot y(s)\,ds\,d\theta$$
$$-\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot y^T(s)Q_3\dot y(s)\,ds\,d\theta$$
$$\le\frac{\tau_l^2}{2}\dot y^T(t)Q_2\dot y(t)+\frac{\tau_u^2}{2}\dot y^T(t)Q_3\dot y(t)$$
$$-\frac{2}{\tau_l^4}\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T Q_2\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)$$
$$-\frac{2}{\tau_l^2}\left(\int_{t-\tau_l}^{t}y(s)\,ds\right)^T Q_2\left(\int_{t-\tau_l}^{t}y(s)\,ds\right)$$
$$+\frac{4}{\tau_l^3}\left(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T Q_2\left(\int_{t-\tau_l}^{t}y(s)\,ds\right)$$
$$-\frac{2}{\tau_u^4}\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T Q_3\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)$$
$$-\frac{2}{\tau_u^2}\left(\int_{t-\tau_u}^{t}y(s)\,ds\right)^T Q_3\left(\int_{t-\tau_u}^{t}y(s)\,ds\right)$$
$$+\frac{4}{\tau_u^3}\left(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)\,ds\,d\theta\right)^T Q_3\left(\int_{t-\tau_u}^{t}y(s)\,ds\right),$$

$$\dot V_4(y(t))$$
$$=y^T(t)Q_4y(t)-(1-\dot\tau(t))y^T(t-\tau(t))Q_4y(t-\tau(t))$$
$$\le y^T(t)Q_4y(t)-(1-\tau)y^T(t-\tau(t))Q_4y(t-\tau(t)). \qquad (28)$$

Additionally, for arbitrary matrices M_1, M_2 of appropriate dimensions, we have

$$2y(t)^TM_1\left[-\dot y(t)+\left(A+CL\Delta H^T\right)y(t)+By(t-\tau(t))\right]=0,$$
$$2\dot y^T(t)M_2\left[-\dot y(t)+\left(A+CL\Delta H^T\right)y(t)+By(t-\tau(t))\right]=0. \qquad (29)$$

Hence, $\dot V(y(t))\le\xi^T(t)(\Xi+\Phi\widetilde\Delta\Psi+\Psi^T\widetilde\Delta^T\Phi^T)\xi(t)$, where $\xi(t)=[y^T(t),y^T(t-\tau(t)),\dot y^T(t),\int_{t-\tau_l}^{t}y^T(s)ds,\int_{t-\tau_u}^{t}y^T(s)ds,\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y^T(s)ds\,d\theta,\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y^T(s)ds\,d\theta]^T$, and $\widetilde\Delta=\widetilde\Delta^T=\mathrm{diag}(\Delta,0,\Delta,0,0,0,0)$. Since $\Delta^T\Delta\le I$, thus $\widetilde\Delta^T\widetilde\Delta\le I$. From Lemma 3, if the conditions in Theorem 7 are satisfied, then, $\Xi+\Phi\widetilde\Delta\Psi+\Psi^T\widetilde\Delta^T\Phi^T<0$; by Lyapunov stable theory, the delayed Lur'e system (1) is asymptotically stable, which completes the proof. □

Remark 8. Different from previous work, in the proof of Lemma 5, we establish the relationship among $\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot z(s)^TR\dot z(s)ds\,d\theta$, $\int_{-\tau}^{0}\int_{t+\theta}^{t}z(s)ds\,d\theta$, and $\int_{t-\tau}^{t}z(s)ds$. On the basis of this new relationship, in Theorem 7, items $\int_{t-\tau_l}^{t}y^T(s)ds$, $\int_{t-\tau_u}^{t}y^T(s)ds$, $\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y^T(s)ds\,d\theta$, and $\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y^T(s)ds\,d\theta$ are introduced as the state of $\xi(t)$; this may reduce criterion's conservatism.

For further use of the information of nonlinear function $f(\sigma(t))$, let us define $W(t)=(w_{ij}(t))_{m\times n}=\Lambda(t)H^T$, where $\Lambda(t)=\mathrm{diag}(\Lambda_1(\sigma_1(t)),\Lambda_2(\sigma_2(t)),\ldots,\Lambda_m(\sigma_m(t)))$. Since $\Lambda_i(\sigma_i(t))=\lambda_i(\sigma_i(t))l_i^-+(1-\lambda_i(\sigma_i(t)))l_i^+\in[l_i^-,l_i^+]$, $\forall i\in N$, there must exist $\underline{W}=(\underline{w}_{ij})_{m\times n}$, $\overline{W}=(\overline{w}_{ij})_{m\times n}$ such that $\underline{w}_{ij}\le w_{ij}(t)\le\overline{w}_{ij}$, $\forall t\in[t_0,+\infty),i,j\in N$. Set $\widetilde H=(\underline{W}+\overline{W})/2=(\widetilde h_{ij})_{m\times n}$, $\widehat H=(\overline{W}-\underline{W})/2=(\widehat h_{ij})_{m\times n}$. From the result obtained in [19], $W(t) = (w_{ij}(t))_{m\times n} = \Lambda(t)H^T$ can be rewritten as $\Lambda(t)H^T = \widetilde{H} + E\Sigma F$, where

$$E = \left[\sqrt{\widehat{h}_{11}}\, e_1, \ldots, \sqrt{\widehat{h}_{1m}}\, e_1, \sqrt{\widehat{h}_{m1}}\, e_m, \ldots, \sqrt{\widehat{h}_{mm}}\, e_m\right] \in R^{m\times m^2},$$
$$F = \left[\sqrt{\widehat{h}_{11}}\, e_1, \ldots, \sqrt{\widehat{h}_{1m}}\, e_m, \sqrt{\widehat{h}_{m1}}\, e_1, \ldots, \sqrt{\widehat{h}_{mm}}\, e_m\right] \in R^{m\times m^2}, \tag{30}$$

e_i $(i = 1, 2, \ldots, m)$ denotes the ith column vectors of the $m\times m$ identity matrix, and $\Sigma = \mathrm{diag}(\varepsilon_{11}, \ldots, \varepsilon_{1m}, \ldots, \varepsilon_{m1}, \ldots, \varepsilon_{mm},)$, where ε_{ij} $(i, j = 1, 2, \ldots, m)$ satisfies $|\varepsilon_{ij}| \le 1$. This means that system (1) can be rewritten as

$$\dot{y}(t) = \left[\left(A + C\widetilde{H}\right) + CE\Sigma F\right] y(t) + By(t - \tau(t)),$$
$$y(s) = \varphi(s), \quad s \in [-\tau_u, 0], \tag{31}$$

where Σ satisfies $\Sigma^T\Sigma = \Sigma\Sigma^T \le I$. From Theorem 7, we can get the following result.

Theorem 9. *For given scalars $\tau_l > 0, \tau_u > 0, \tau < 1$, $L^- = \mathrm{diag}(l_1^-, l_2^-, \ldots, l_m^-)$, $L^+ = \mathrm{diag}(l_1^+, l_2^+, \ldots, l_m^+)$, system (1) is globally asymptotically stable if there exist positive definite diagonal matrices $D_1 = \mathrm{diag}\{d_1, d_2, \ldots, d_m\}$, $D_2 = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, $D_3 = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, symmetric positive definite matrices Q_1, Q_2, Q_3, Q_4, and arbitrary matrices M_1, M_2 of appropriate dimensions such that the following condition holds:*

$$\begin{bmatrix} \Xi' + \Psi'^T S \Psi' & \Phi' \\ * & -S \end{bmatrix} < 0, \tag{32}$$

where $S \in S_\Delta, \Xi' = (\Xi'_{ij}), \Phi', \Psi' \in R^{7m\times 7m}$, $i, j = 1, 2, \ldots, 7$,

$$\Xi'_{11} = \left(\tau_l^2 + \tau_u^2\right) HD_3\left(L^+ - L^-\right) H^T + M_1\left(A + C\widetilde{H}\right) + \left(A + C\widetilde{H}\right)^T M_1^T + Q_4, \quad \Xi'_{12} = M_1 B,$$
$$\Xi'_{13} = -M_1 + \left(A + C\widetilde{H}\right)^T M_2^T + Q_1 + \left(\widetilde{H}^T H^T - L^- HH^T\right) D_1 + \left(L^+ HH^T - \widetilde{H}^T H^T\right) D_2,$$
$$\Xi'_{22} = -(1 - \tau) Q_4, \quad \Xi'_{23} = B^T M_2^T,$$
$$\Xi'_{33} = -M_2 - M_2^T + \frac{\tau_l^2}{2} Q_2 + \frac{\tau_u^2}{2} Q_3,$$
$$\Xi'_{44} = -\frac{2}{\tau_l^2} Q_2, \quad \Xi'_{46} = \frac{4}{\tau_l^3} Q_2,$$
$$\Xi'_{55} = -\frac{2}{\tau_l^2} Q_3, \quad \Xi'_{57} = \frac{4}{\tau_u^3} Q_3,$$
$$\Xi'_{66} = -\frac{2}{\tau_l^4} Q_2 - \frac{2}{\tau_l^2}\left(HD_3\left(L^+ - L^-\right) H^T\right),$$
$$\Xi'_{77} = -\frac{2}{\tau_u^4} Q_3 - \frac{2}{\tau_u^2}\left(HD_3\left(L^+ - L^-\right) H^T\right),$$
$$\Phi' = \mathrm{diag}\left\{M_1 CE, 0, F^T, 0, 0, 0, 0\right\},$$
$$\Psi' = \begin{pmatrix} F^T, 0, M_2 CE + \left(D_1 - D_2\right) HE, 0, 0, 0, 0 \\ O \end{pmatrix}. \tag{33}$$

Remark 10. Due to the existence of items $\Psi^T S\Psi$ and $\Psi'^T S\Psi'$, the results established in Theorems 7 and 9 are not LMI criteria. In order to overcome this flaw, by using the lemma derived in [20], we further establish the following more practicable stable rules.

Corollary 11. *For given scalars $\tau_l > 0, \tau_u > 0, \tau < 1$, $L^- = \mathrm{diag}(l_1^-, l_2^-, \ldots, l_m^-)$, $L^+ = \mathrm{diag}(l_1^+, l_2^+, \ldots, l_m^+)$, system (1) is globally asymptotically stable if there exist positive definite diagonal matrices $D_1 = \mathrm{diag}\{d_1, d_2, \ldots, d_m\}$, $D_2 = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, $D_3 = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, symmetric positive definite matrices Q_1, Q_2, Q_3, Q_4, positive scalar $\delta > 0$, and arbitrary matrices M_1, M_2 of appropriate dimensions such that the following condition holds:*

$$\begin{bmatrix} \delta\Xi & \Phi & \delta\Psi^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0. \tag{34}$$

Corollary 12. *For given scalars $\tau_l > 0, \tau_u > 0, \tau < 1$, $L^- = \mathrm{diag}(l_1^-, l_2^-, \ldots, l_m^-)$, $L^+ = \mathrm{diag}(l_1^+, l_2^+, \ldots, l_m^+)$, system (1) is globally asymptotically stable if there exist positive definite diagonal matrices $D_1 = \mathrm{diag}\{d_1, d_2, \ldots, d_m\}$, $D_2 = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$, $D_3 = \mathrm{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, symmetric positive definite matrices Q_1, Q_2, Q_3, Q_4, positive scaler $\delta > 0$, and arbitrary matrices M_1, M_2 of appropriate dimensions such that the following condition holds:*

$$\begin{bmatrix} \delta\Xi' & \Phi' & \delta\Psi'^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0. \tag{35}$$

4. Numerical Example

In order to show the effectiveness of the technique proposed in this paper, we revisit the example in [21], and compare our criteria with existing delay-dependent criteria.

Example 1. In order to compare with preview results easily, consider the delayed system (1) with parameters given by

$$A = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.2 \end{bmatrix},$$
$$C = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, \qquad H = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}. \tag{36}$$

Time-varying delay $\tau(t) = \widetilde{\tau}$ is a constant.

This system has been investigated in [19, 22–24]. And the maximum values of $\widetilde{\tau}_{\max}$ for the stability of system (1) are

$\tilde{\tau}_{\max} = 0.3053$, $\tilde{\tau}_{\max} = 0.3230$, $\tilde{\tau}_{\max} = 0.9278$, and $\tilde{\tau}_{\max} = 2.055$, respectively. In order to deduce the conservatism of those criteria established in [19, 22–24], Tian et al. derived a new improved result by decomposing matrix B as $B = B_{11} + B_{12}$ in [21], where $B_{11} = \begin{bmatrix} -0.05 & -0.2 \\ 0.1 & -0.1 \end{bmatrix}$, $B_{12} = \begin{bmatrix} -0.15 & -0.3 \\ 0.4 & -0.1 \end{bmatrix}$ and gave out the maximum delay bound as $\tilde{\tau}_{\max} = 3.7272$. However, one can see that, when time-varying delay $\tau(t)$ is not a constant, then the results established in [19, 21–24] are invalid. If $C = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.3 \end{bmatrix}$, $H = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}$ $f(\sigma(t)) = [f_1(\sigma_1(t)), f_2(\sigma_2(t))]^T$, $f_i(\sigma_i(t)) = 0.5(|\sigma_i(t) + 1| - |\sigma_i(t) - 1|)$, $i = 1, 2$. Obviously, nonlinear function $f(\sigma(t))$ satisfies $\sigma(t)f(\sigma(t)) \geq 0$, $f(0) = 0$, and $0 \leq f_i(\sigma_i(t))/\sigma_i(t) \leq 0.5$. If $\tau_l = 4$, $\tau_u = 8$, $\dot{\tau}(t) = 0.9$. Let $\delta = 0.1$; one can get that the results obtained in Corollaries 11 and 12 are feasibility. This mean that the results obtained in this paper are more general and less conservative than those in [19, 21–24].

Example 2. Consider the system described in (1) with parameters given by

$$A = \begin{bmatrix} -1.2 & 0 \\ 0.8 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$C = \begin{bmatrix} -1 & 0.6 \\ -0.6 & -1 \end{bmatrix}, \qquad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{37}$$

$l_1^- = l_2^- = 0$, $l_1^+ = 1$, $l_2^+ = 3$.

This system has been investigated in [25, 26], and the maximum value of τ_u for the stability of system (1) are $\tau_u^{\max} = 0.5805$, and $\tau_u^{\max} = 0.6780$, respectively. In [27], by using the sector bounds and slope bounds to the Lyapunov-Krasovskii functional through convex representation of the nonlinearities, Choi et al. improved the upper bound of $\tau(t)$ to 1.1131. Let $\delta = 0.1$, $\tau_l = 0.01$; by using the results obtained in Corollaries 11 and 12, one can get the maximum values of $\tau_u^{\max}$ are 1.2103 and 1.2314, respectively. This means that the results obtained in this paper are less conservative than those in [25–27].

Example 3. Consider the system described in (1) with parameters given by

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \qquad H = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}; \tag{38}$$

$f(\sigma(t)) = (0.35+0.15\sin(t))\sigma(t)$. Time-varying delay $\tau(t) = \tilde{\tau}$ is a constant.

Obviously, the bounds of the sector nonlinearity are $l_1^- = 0.2$, $l_1^+ = 0.5$. For this system, the maximum values of $\tilde{\tau}_{\max}$ for the stability of system (1) are $\tilde{\tau}_{\max} = 2.4859$ and $\tilde{\tau}_{\max} = 2.5049$ in [5, 26], respectively. By using various convex optimization techniques, the results obtained in [5, 26] were further improved by Lee and Park in [12], and $\tilde{\tau}_{\max} = 2.5361$. From Corollary 11, we find the maximum allowable time delay bound can be 2.5812, which means that the result established in Corollary 11 is less conservative than the ones obtained in [5, 26].

5. Conclusions

Combined with Lyapunov stable theory and double integral inequality, this paper researches a class of delayed Lur'e systems with interval time-varying delays. Different from previous work on this topic, this paper first establishes some new vector Wirtinger-type inequalities. Then, by using the property of convex function, the original nonlinear Lur'e system is transformed into a linear uncertain system. At last, by constructing a new Lyapunov functional including triple integral items, some new less conservative delay-dependent stability criteria are established. Numerical examples show that the new criteria derived in this paper are less conservative than some previous results obtained in the references cited therein.

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