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1974 CONJECTURE OF ANDREWS ON PARTITIONS

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The case $k = a$ of the 1974 conjecture of Andrews on two partition functions $A_{\lambda,k,a}(n)$ and $B_{\lambda,k,a}(n)$ was proved by the first author and Sudha (1993) and the case $k = a+1$ was established by the authors (2000). In this paper, we prove that the conjecture is false and give a revised conjecture for a particular case when λ is even.

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1. Introduction. Andrews [3] proved a general theorem from which the well-known Rogers-Ramanujan identities, Gordon's theorem [7], the Göllnitz-Gordon identities [6] and their generalization [1], Schur's theorem and its generalization [10] could be deduced. In 1969, Andrews [2] proved the following theorem.

Theorem 1.1 [2, Theorem 2]**.** *If λ, k, and a are positive integers with $\lambda/2 \le a \le k$, $k \ge 2\lambda - 1$, then for every positive integer,*

$$A_{\lambda,k,a}(n) = B_{\lambda,k,a}(n), \tag{1.1}$$

where $A_{\lambda,k,a}(n)$ and $B_{\lambda,k,a}(n)$ are defined as follows.

Definition 1.2. For an even integer λ, let $A_{\lambda,k,a}(n)$ denote the number of partitions of n into parts such that no part which is not equivalent to $0 (\bmod \lambda+1)$ may be repeated and no part is equivalent to $0, \pm(a-\lambda/2)(\lambda+1) \bmod[(2k-\lambda+1)(\lambda+1)]$. For an odd integer λ, let $A_{\lambda,k,a}(n)$ denote the number of partitions of n into parts such that no part which is not equivalent to $0(\bmod((\lambda+1)/2))$ may be repeated, no part is equivalent to $\lambda+1 (\bmod 2\lambda+2)$, and no part is equivalent to $0, \pm(2a-\lambda)((\lambda+1)/2) \bmod[(2k-\lambda+1)(\lambda+1)]$.

Definition 1.3. Let $B_{\lambda,k,a}(n)$ denote the number of partitions of n of the form $b_1 + \cdots + b_s$ with $b_i \ge b_{i+1}$, no part which is not equivalent to $0(\bmod \lambda+1)$ is repeated, $b_i - b_{i+k-1} \ge \lambda+1$ with strict inequality if $\lambda+1/b_i$, $\sum_{i=j}^{\lambda-j+1} f_i \le a-j$ for $1 \le j \le (\lambda+1)/2$, and $f_1 + \cdots + f_{\lambda+1} \le a-1$, where f_j is the number of appearances of j in the partition.

Since Schur's theorem [10] is the case $\lambda = k = a = 2$, it is not a particular case of Theorem 1.1 as $k \ge 2\lambda - 1$ is not satisfied. Hence Andrews [2] conjectured that Theorem 1.1 may be still true if $k \ge \lambda$. In fact, he gave a proof of this result [4].

In the conclusion of [4], Andrews stated the following two conjectures.

Conjecture 1.4. *For $\lambda/2 < a \le k < \lambda$, let $n^c = (k+\lambda-a+1)(k+\lambda-a)/2+(k-\lambda+1)(\lambda+1)$. Then*

$$B_{\lambda,k,a}(n) = A_{\lambda,k,a}(n) \quad \text{for } 0 \le n < n^c, \qquad B_{\lambda,k,a}(n) = A_{\lambda,k,a}(n)+1 \quad \text{for } n = n^c. \tag{1.2}$$

Conjecture 1.5. *For all positive integers n, $A_{4,3,3}(n) = B^0_{4,3,3}(n)$, where $B^0_{4,3,3}(n)$ denotes the number of partitions of n enumerated by $B_{4,3,3}(n)$ with the added restrictions:*

$$\begin{aligned} f_{5j+2}+f_{5j+3} &\le 1 \quad \text{for } j \ge 0,\\ f_{5j+4}+f_{5j+6} &\le 1 \quad \text{for } j \ge 0,\\ f_{5j-1}+f_{5j}+f_{5j+5}+f_{5j+6} &\le 3 \quad \text{for } j \ge 1. \end{aligned} \tag{1.3}$$

Conjecture 1.5 is designed to show that when the condition $k \ge \lambda$ is removed with some additional restrictions on the summands, some partition identities can be obtained in a few cases. In 1994, Andrews et al. [5] proved Conjecture 1.5.

The first author and Sudha [9] have proved the case $k = a$ of Conjecture 1.4 while the authors in [8] have established the case $k = a+1$ of Conjecture 1.4. The objective of the present paper is to prove that Conjecture 1.4 is false if n exceeds $(2k-a-\lambda/2+1)(\lambda+1)$ for even λ and $k \ge a+2$. For odd λ, we have verified and checked that Conjecture 1.4 is false when $\lambda = 11$, $k = 9$, and $a = 6$. We also give the following revised conjecture for a particular case when λ is even.

Revised Conjecture 1.6. *Let λ be even, $a-\lambda/2 = 1$, $\theta = k-a$, $\theta(\theta-1)/2 < [a-\lambda/2](\lambda+1)$, and $0 \le \theta \le \lambda/2-3$. Then*

$$\begin{aligned} B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) \quad \text{for } n < \left(2k-a-\frac{\lambda}{2}+1\right)(\lambda+1),\\ B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n)+B_{\lambda,k,a}(x),\\ \text{where } n &= \left(2k-a-\frac{\lambda}{2}+1\right)(\lambda+1)+x,\ 0 \le x \le \frac{\theta(\theta-1)}{2}. \end{aligned} \tag{1.4}$$

These results support (i) Andrews' contention that $k \ge \lambda$ is essential for the truth of Theorem 1.1 and (ii) his belief that Theorem 1.1 was the best possible one, but his conjecture about first counterexamples when $k \ge \lambda$ is false.

2. Preliminaries. Let $P_{B_{\lambda,k,a}}(n)$ and $P_{A_{\lambda,k,a}}(n)$ denote the sets of partitions enumerated by $B_{\lambda,k,a}(n)$ and $A_{\lambda,k,a}(n)$, respectively. Let $P'_A(n)$ (resp., $P'_B(n)$) denote the set of partitions enumerated by $A_{\lambda,k,a}(n)$ (resp., $B_{\lambda,k,a}(n)$) but not by $B_{\lambda,k,a}(n)$ (resp., $A_{\lambda,k,a}(n)$).

$\pi \in P'_A(n)$ implies that it violates one of the conditions on $f's$ or $b's$. Let S_j ($j = 1,2,\dots,\lambda/2$) denote the condition $\sum_{i=j}^{\lambda-j+1} f_i \le a-j$, let S denote the condition $\sum_{i=1}^{\lambda+1} f_i \le a-1$, and let S^* be the condition on $b's$.

Let $(2k-a-\lambda/2+1)(\lambda+1) \le n < (2k-a-\lambda/2+1)(\lambda+1)+\theta(\theta-1)/2$, where $\theta(\theta-1)/2 < (a-\lambda/2)(\lambda+1)$ and $\theta = k-a$. Then

$$P'_B(n) = Q^1 \cup \cdots \cup Q^{a-1} \cup R(n), \tag{2.1}$$

where for $1 \le i \le a-1$,

$$Q^i = \left\{\pi \in P_B'(n) : \left(a - \frac{\lambda}{2}\right)(\lambda+1) \text{ appears } i \text{ times}\right\},$$
$$R(n) = \left\{\left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) + \pi : \pi \text{ is a partition of } n - \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) \text{ into parts with } C\right\}. \tag{2.2}$$

Here C stands for "subjected to the conditions in the definition of B." Clearly, $\#R(n) = B_{\lambda,k,a}[n-(2k-a-\lambda/2+1)(\lambda+1)]$.

From the method explained in [8, 9], it follows that the partitions violating $S_1,\dots,S_{\lambda/2}$ will be mapped onto $Q^1 \cup \cdots \cup Q^{a-1}$. If $a-\lambda/2 = 1$, then S reduces to S_1. As such, any contribution to $R(n)$ can come only from those partitions of P_A' which violate S^* but do not violate any of $S_1,\dots,S_{\lambda/2}$. For the counterexample in Section 3, we enumerate separately the partitions counted by $R(n)$. If there are no partitions of n violating only S^*, then for such n, we have that $P_A'(n)$ is the union of the partitions violating $S_1,\dots,S_{\lambda/2}$ and Q^{a-1} is the set containing $a-1$ times $\lambda+1$. This set is identified with the first stage of S_1 where all the parts from $1,\dots,\lambda$ appear. Q^{a-2} will be the union of the two sets, one containing $a-2$ times $\lambda+1$ and the other containing $a-2$ times $\lambda+1$ plus a part between 1 and λ. These two sets are, respectively, identified with the first stage of S_2 where all the parts from $2,\dots,\lambda-1$ appear, and the second stage of S_1 in which all the parts except one part from $1,\dots,\lambda$ appear and so on.

3. Counterexample.

Let $\lambda = 12$, $k = 11$, $a = 7$, $\theta = 4$, $a-\lambda/2 = 1$, $\theta(\theta-1)/2 = 6 < (a-\lambda/2)(\lambda+1) = 13$, and $n^c = 136$. In this case,

$$\begin{gathered} S_{\lambda/2} = S_6 : f_7 + f_6 \le 1, \qquad S_5 : f_8 + f_7 + f_6 + f_5 \le 2, \qquad S_4 : f_9 + \cdots + f_4 \le 3, \\ S_3 : f_{10} + \cdots + f_3 \le 4, \qquad S_2 : f_{11} + \cdots + f_2 \le 5, \qquad S_1 : f_{12} + \cdots + f_1 \le 6, \\ S : f_{13} + \cdots + f_1 \le 6; \\ P_B'(n) = Q^1 \cup \cdots \cup Q^6 \cup R(n), \end{gathered} \tag{3.1}$$

where $Q^i = \{\pi \in P_B'(n) : 13 \text{ appears } i \text{ times}\}$, $1 \le i \le 6$, and $R(n) = \{130 + \pi : \pi \text{ is a partition of } n-130 \text{ into parts with } C\}$. Here $\#R(n) = B_{12,11,7}(x)$, where $x = n - 130$. We now prove

$$B_{12,11,7}(n) = A_{12,11,7}(n), \quad n < 130, \tag{3.2}$$

$$B_{12,11,7}(n) = A_{12,11,7}(n) + B_{12,11,7}(x), \quad n = 130 + x,\ 0 \le x < 6, \tag{3.3}$$

$$B_{12,11,7}(136) = A_{12,11,7}(136) + B_{12,11,7}(6) - 1 = A_{12,11,7}(136) + 3, \tag{3.4}$$

since $B(6) = 4$ as 6, $5+1$, $4+2$, and $3+2+1$ are the only relevant partitions of 6 enumerated by B.

Proof of (3.2), (3.3), and (3.4). Equation (3.2) follows from [8]. We now prove that for $1 \le n < 136$, there are no partitions of n violating only S^* and that

$$18+17+16+15+14+12+11+10+9+8+6 \tag{3.5}$$

is the only partition of 136 violating only S^*.

In [8, 9] we have shown that for $n < 130$, if a partition violates S^*, then it violates either S or S_1. However, for $130 \le n \le 136$, we now investigate such partitions.

If a partition violates S^*, then there exist a partition

$$n = b_1 + \cdots + b_i + \cdots + b_{i+10} + \cdots + b_s \tag{3.6}$$

and an integer i with $b_i - b_{i+10} < 13$. We get the following possibilities.

Case 1. If $b_{i+10} \ge 13$, then the number being partitioned is greater than or equal to

$$\begin{aligned} &(12+x_{11}) + \cdots + (12+x_1) + \cdots, \\ &11(12+1), \quad \text{where } x_{11} - x_1 < 13. \end{aligned} \tag{3.7}$$

If (3.7) contains the part 13 more than 6 times, then it violates S. Let $x \le 6$ be the number of $13's$ and let y denote the number of terms greater than 13 in (3.7) so that $x + y = 11$. Then (3.7) becomes

$$13x + (12+2) + \cdots + (12+11-x) = 11(13) + \frac{(11-x)(11-x-1)}{2}. \tag{3.8}$$

Let n^c denote the n in the conjecture. If $k = a + \theta$, then

$$\begin{aligned} n^c &= \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) + \frac{\theta(\theta-1)}{2} \\ &= k(\lambda+1) + \left(k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) + \frac{(k-a)(k-a-1)}{2} \\ &< k(\lambda+1) + \left(k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) + \frac{(k-x)(k-x-1)}{2} \\ &< k(\lambda+1) + \frac{(k-x)(k-x-1)}{2} \quad \text{since } k - a - \frac{\lambda}{2} + 1 < 0. \end{aligned} \tag{3.9}$$

In this case, we have that $n^c < 11(13) + (11-x)(11-x-1)/2$.

Case 2. Let $b_{i+10} < 13$ and $b_i < 13$. Then (3.6) violates S_1.

Case 3. Let $b_{i+10} < 13$ and $b_i \ge 13$. Let β denote the number of parts among $1, 2, \ldots, 13$. If $\beta \ge 7$, then (3.6) violates S or S_1. Hence, $1 \le \beta \le 6$. Let α denote the number of parts 13 so that $5 \le \alpha \le 10$ and $\alpha + \beta = 11$. Then the number being partitioned is

$$(12+x_\alpha) + \cdots + (12+x_1) + y_1 + \cdots + y_\beta. \tag{3.10}$$

Since $(12+x_\alpha) - y_\beta < 13$, we have $x_\alpha = y_\beta$. Now, $x_1 \ge 2$, $x_2 \ge 3, \ldots, x_\alpha \ge \alpha + 1$. Thus, $y_\beta \ge \alpha + 1, \ldots, y_1 \ge \alpha + \beta = 11$. Hence, (3.10) is greater than or equal to

$$(12+\alpha+1) + \cdots + (12+2) + (\alpha+\beta) + \cdots + (\alpha+1) \tag{3.11}$$

and equals

$$\frac{13\alpha+(\alpha+\beta)(\alpha+\beta+1)}{2}. \tag{3.12}$$

Let $\beta=1,2,3,4,5$. Then (3.10) is, respectively, 196, 183, 170, 157, and 144, all of which are greater than $136=n^c$.

Now let $\beta=6$. Since we have to choose 6 parts from $1,2,\dots,13$ and 5 parts greater than 13 for a partition violating S^* (and not violating any of $S,S_1,\dots,S_6$), it is clear that the minimum part should be 6. Let $S_1^*=\{6,7\}$ and $S_2^*=\{8,9,10,11,12,13\}$. Since $f_6+f_7\le 1$, we can choose either 6 or 7 from S_1^* and the other five must be from S_2^*. Also there are 5 parts greater than 13. In this case, the minimum value of n will be

$$6+8+9+10+11+12+14+15+16+17+18=136. \tag{3.13}$$

Thus for all $130\le n<136$, there are no partitions of n violating only S^*. It is easy to see that when $n=136$,

$$18+\cdots+14+12+\cdots+8+6 \tag{3.14}$$

is the only partition of 136 violating only S^*. Thus we find

$$P_A'(n)=\{\text{union of the partitions violating } S_1,\dots,S_6\}\quad \text{for } 1\le n<136 \tag{3.15}$$

while

$$P_A'(136)=\{\text{union of the partitions violating } S_1,\dots,S_6\}+1. \tag{3.16}$$

We now establish a bijection of $Q^1\cup\cdots\cup Q^6$ onto $P_A'(n)$ which is explained in Table 3.1. This also proves (3.3) and (3.4). Before writing the table, we observe that for a partition

$$\pi+13\times i+\alpha_1+\cdots+\alpha_j,\quad 1\le i\le 6, \tag{3.17}$$

belonging to P_B', π is a partition of $(n-13\times i-\alpha_1-\cdots-\alpha_j)$ into parts greater than 13 with C, where $1\le\alpha_j<\cdots<\alpha_1\le 12$, and for a partition

$$\pi+\beta_1+\cdots+\beta_j \tag{3.18}$$

belonging to P_A', π is a partition of $(n-\beta_1-\cdots-\beta_j)$ into parts greater than β_1 such that 13 is not a part, where $1\le\beta_j<\cdots<\beta_1\le 12$.

Remark 3.1. In Table 3.1, some partitions in Q^2 are not covered. They are

$$\begin{aligned}&\{\pi+13\times 2+x_1+x_2+1: 2\le x_2\le 11,\ 3\le x_1\le 12,\ (x_1,x_2)\ne(7,6)\}\\ &\quad\cup\{\pi+13\times 2+12+x_1+x_2: 3\le x_1\le 11,\ 2\le x_2\le 10,\ (x_1,x_2)\ne(7,6)\}.\end{aligned} \tag{3.19}$$

Here we split 13×2 into pairs (α,β) and (γ,δ) in the following order:

$$(7,6)\ (8,5)\ (9,4)\ (10,3)\ (11,2)\ (12,1). \tag{3.20}$$

TABLE 3.1

$P'_{B_{12,11,7}}(n)$	$P'_{A_{12,11,7}}(n)$
$Q^6=\{\pi+13\times 6\}$	1st stage of $S_1=\{\pi+12+\cdots+1\}$
$Q^5=\{\pi+13\times 5\}$	1st stage of $S_2=\{\pi+11+\cdots+2\}$
$\cup\{\pi+13\times 5+(13-x_1):1\le(13-x_1)\le 12\}$	2nd stage of $S_1=\{\pi+12+\cdots+(x_1+1)+(x_1-1)+\cdots+2+1:1\le x_1\le 12\}$
$Q^4=\{\pi+13\times 4+x:x=0,1,2,12\}$	1st stage of $S_3=\{\pi+10+\cdots+3+x:x=0,1,2,12\}$
$\cup\{\pi+13\times 4+(13-x_1):2<(13-x_1)\le 11\}$	2nd stage of $S_2=\{\pi+11+\cdots+(x_1+1)+(x_1-1)+\cdots+2:2\le x_1<11\}$
$\cup\{\pi+13\times 4+(13-x_1)+(13-x_2):1\le(13-x_2)<(13-x_1)\le 12\}$	3rd stage of $S_1=\{\pi+12+\cdots+(x_1+1)+(x_1-1)+\cdots+(x_2+1)+(x_2-1)+\cdots+1:1\le x_2<x_1\le 12,(x_i,x_j)\ne(7,6)\}$ **Note 1.** If $(x_i,x_j)=(7,6)$, then it will be covered in the 3rd stage of S_2.
$Q^3=\{\pi+13\times 3+x:x=0,1,2,11,12\}$	1st stage of $S_4=\{\pi+9+\cdots+4+x:x=0,1,2,11,12\}$
$\cup\{\pi+13\times 3+(13-x_1):3\le(13-x_1)\le 10\}$	2nd stage of $S_3=\{\pi+10+\cdots+(x_1+1)+(x_1-1)+\cdots+3:3\le x_1\le 10\}$
$\cup\{\pi+13\times 3+(13-x_1)+(13-x_2):2\le(13-x_2)<(13-x_1)\le 11\}$	3rd stage of $S_2=\{\pi+11+\cdots+(x_1+1)+(x_1-1)+\cdots+(x_2+1)+(x_2-1)+\cdots+2:2\le x_2<x_1\le 11,(x_i,x_j)\ne(7,6)\}$ **Note 2.** If $(x_i,x_j)=(7,6)$, then it will be covered in the 3rd stage of S_3.
$\cup\{\pi+13\times 3+x+y:(x,y)$ = all possible pairs of $1,2,11,12$ except $(11,2)\}$	4th stage of $S_4=\{\pi+9+\cdots+4+x+y:(x,y)$ = all possible pairs of $1,2,11,12$ except $(11,2)\}$
$\cup\{\pi+13\times 3+(13-x_1)+\cdots+(13-x_3):1\le(13-x_3)<(13-x_2)<(13-x_1)\le 12\}$	4th stage of $S_1=\{\pi+12+\cdots+(x_1+1)+(x_1-1)+\cdots+(x_3+1)+(x_3-1)+\cdots+1:1\le x_3<x_2<x_1\le 12,(x_i,x_j)\ne(7,6)\}$ **Note 3.** If $(x_i,x_j)=(7,6)$, then it will be covered in the 4th stage of S_2.
$Q^2=\{\pi+13\times 2+x:x=0,1,2,3,10,11,12\}$	1st stage of $S_5=\{\pi+8+7+6+5+x:x=0,1,2,3,10,11,12\}$
$\cup\{\pi+13\times 2+(13-x_1):4\le(13-x_1)\le 9\}$	2nd stage of $S_4=\{\pi+9+\cdots+(x_1+1)+(x_1-1)+\cdots+4:4\le x_1\le 9\}$
$\cup\{\pi+13\times 2+(13-x_1)+(13-x_2):3\le(13-x_2)<(13-x_1)\le 10\}$	3rd stage of $S_3=\{\pi+10+\cdots+(x_1+1)+(x_1-1)+\cdots+(x_2+1)+(x_2-1)+\cdots+3:3\le x_2<x_1\le 10,(x_i,x_j)\ne(7,6)\}$

TABLE 3.1. Continued.

$P'_{B_{12,11,7}}(n)$	$P'_{A_{12,11,7}}(n)$
	Note 4. If $(x_i, x_j) = (7,6)$, then it will be covered in the 3rd stage of S_4.
$\cup\{\pi + 13 \times 2 + x + y : (x,y)$ = all possible pairs of $1,2,3,10$ $11,12$ except $(10,3)\}$	4th stage of $S_5 = \{\pi + 8 + 7 + 6 + 5 + x + y : (x,y)$ = all possible pairs of $1,2,3,10,11,12$ except $(10,3)\}$
$\cup\{\pi + 13 \times 2 + (13 - x_1) + \cdots + (13 - x_3) : 2 \le (13 - x_3) < (13 - x_2) < (13 - x_1) \le 11\}$	4th stage of $S_2 = \{\pi + 11 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + (x_3 + 1) + (x_3 - 1) + \cdots + 2 : 2 \le x_3 < x_2 < x_1 \le 11\}$ $(x_i, x_j) \ne (7,6)\}$
	Note 5. If $(x_i, x_j) = (7,6)$, then it will be covered in the 4th stage of S_3.
$\cup\{\pi + 13 \times 2 + (13 - x_1) + \cdots + (13 - x_4) : 1 \le (13 - x_4) < \cdots < (13 - x_1) \le 12\}$	5th stage of $S_1 = \{\pi + 12 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + (x_4 + 1) + (x_4 - 1) + \cdots + 1 : 1 \le x_4 < \cdots < x_1 \le 12\}$ $(x_i, x_j) \ne (7,6)\}$
	Note 6. If $(x_i, x_j) = (7,6)$, then it will be covered in the 5th stage of S_2.
$Q^1 = \{\pi + 13 + x : x = 0,1,2,3,4,9,10,11,12\}$	1st stage of $S_6 = \{\pi + 7 + 6 + x : x = 0,1,2,3,4,9,10,11,12\}$
$\cup\{\pi + 13 + (13 - x_1) : 5 \le (13 - x_1) \le 8\}$	2nd stage of $S_5 = \{\pi + 8 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + 5 : 5 \le x_1 \le 8\}$
$\cup\{\pi + 13 + (13 - x_1) + (13 - x_2) : 4 \le (13 - x_2) < (13 - x_1) \le 9\}$	3rd stage of $S_4 = \{\pi + 9 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + (x_2 + 1) + (x_2 - 1) + \cdots + 4 : 4 \le x_2 < x_1 \le 9\}$
$\cup\{\pi + 13 + x + y : (x,y)$ = all possible pairs of $1,2,3,4,9,$ $10,11,12$ except $(9,4)\}$	4th stage of $S_5 = \{\pi + 7 + 6 + x + y : (x,y)$ = all possible pairs of $1,2,$ $3,4,9,10,11,12$ except $(9,4)\}$
$\cup\{\pi + 13 + (13 - x_1) + \cdots + (13 - x_3) : 3 \le (13 - x_3) < (13 - x_2) < (13 - x_1) \le 10\}$	4th stage of $S_3 = \{\pi + 10 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + (x_3 + 1) + (x_3 - 1) + \cdots + 3 : 3 \le x_3 < x_2 < x_1 \le 10\}$
$\cup\{\pi + 13 + (13 - x_1) + \cdots + (13 - x_4) : 2 \le (13 - x_4) < \cdots < (13 - x_1) \le 11\}$	5th stage of $S_2 = \{\pi + 11 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + (x_4 + 1) + (x_4 - 1) + \cdots + 2 : 2 \le x_4 < \cdots < x_1 \le 11\}$
$\cup\{\pi + 13 + (13 - x_1) + \cdots + (13 - x_5) : 1 \le (13 - x_5) < \cdots < (13 - x_1) \le 12\}$	6th stage of $S_1 = \{\pi + 12 + \cdots + (x_1 + 1) + (x_1 - 1) + \cdots + (x_5 + 1) + (x_5 - 1) + \cdots + 1 : 1 \le x_5 < \cdots < x_1 \le 12\}$

We arrange $\pi+\alpha+\beta+\gamma+\delta+x_1+x_2+y$ ($y = 12$ or 1) in the decreasing order and associate it to the rearranged partition π^* which belongs to P'_A.

A similar procedure is adopted for some partitions in Q^1 which are also not covered in Table 3.1. This completes the proof of (3.3) and (3.4). □

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