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Research Article

H_∞ Filtering for Networked Systems with Bounded Measurement Missing

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This paper is concerned with the H_∞ filtering problem for networked systems with bounded measurement missing. A switched linear system model is proposed to describe the considered filtering error system. A sufficient condition is derived for the filtering error system to be exponentially stable and achieve a prescribed H_∞ filtering performance level. The obtained condition establishes quantitative relations among the H_∞ performance level and two parameters characterizing the measurement missing, namely, the measurement missing rate bound and the maximal number of consecutive measurement missing. A convex optimization problem is presented to design the linear H_∞ filters. Finally, an illustrative example is given to show the effectiveness of the proposed results.

1. Introduction

A sensor network consists of spatially distributed autonomous sensors to cooperatively monitor physical or environmental conditions. The purpose of a sensor network is to provide users with the information of interest from data gathered by spatially distributed sensors, and it has found applications in a variety of areas, such as moving target localization and tracking, environment monitoring, intelligent transportation systems, and sensor-actuator network-based control (see [1, 2] and the literature therein). Thus, it is not surprising that signal estimation has been one of the most fundamental collaborative information processing problems in sensor networks [3–10]. In the environment of sensor networks, the measurements may be unavailable to the estimators intermittently. The corresponding estimation problem is usually termed estimation with missing, incomplete, or intermittent measurements and has received increasing research attention; see, for example, [11–17] and the references therein.

Measurement missing degrades the performance of the filtering system, and one may be concerned with the problem about how much the filtering performance declines for a certain amount of measurement missing, what is the maximal number of consecutive measurement missing that the filtering system can tolerant to guarantee the desired filtering performance, and so forth. This gives rise to the idea of revealing the relations, especially the quantitative relations, among the filtering performance level and some parameters characterizing the measurement missing, such as the missing rate/probability bound and the maximal number of consecutive measurement missing. Such relations may provide useful guidelines for designing the filtering systems with measurement missing. For example, in the sensor-network-based filtering system, one may purposely suspend some sensor nodes intermittently to save node power without destroying the stability and desired performance level of the filtering systems; in some network-based fast sampling filtering systems, one may consider dropping certain amount of data packets to ensure that the set of the network-based filtering systems is schedulable and meanwhile to guarantee that the overall network-based filtering systems are stable and achieve prescribed filtering performance level and so forth. In these applications, establishing the relations among the filtering performance level and the parameters characterizing the measurement missing is of prior importance. Existing results on this topic can be generally classified into two frameworks, namely, the stochastic framework [18–22] and the deterministic framework [23].

A general structure of the filtering system with measurement missing is shown in Figure 1, where y is the measurement and $\hat{y}$ is the filter input. In the stochastic

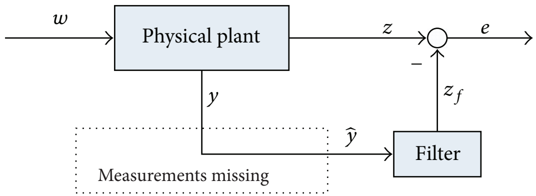

FIGURE 1: Diagram of the filtering problem with measurement missing.

framework, the measurement missing is usually described by a Bernoulli sequence or a Markov chain. For those results using the Bernoulli sequence, such as [14, 15], the filter input is usually described as $\hat{y}(k) = \gamma(k)y(k) + (1 - \gamma(k))\hat{y}(k-1)$ or as $\hat{y}(k) = \gamma(k)y(k)$ by introducing the binary random variable $\gamma(k)$, where $\text{Prob}\{\gamma(k) = 1\} = \gamma$, and γ is usually called the measurement arrival probability while $1 - \gamma$ is called the measurement missing probability. For those results using Markov chain technique, such as [12, 21], the measurement missing and the measurement arrival are defined as two modes of the Markov chain, and the mode transition probabilities are assumed to be known. In these ways, the stochastic parameters (either the Bernoulli sequence or the Markov chain) are incorporated into the filtering system models, and the relation between the filtering performance level and the measurement missing probability is implicitly established. Note that the maximal number of consecutive measurement missing is an important parameter that characterizes the measurement missing, and it also plays an important role in affecting the filtering performance. However, the relation between the filtering performance level and the maximal number of consecutive measurement missing is not established in all the aforementioned results using stochastic framework. Being different from the stochastic framework, the effects of the measurement missing on the filtering system are treated as time delays in the filter input in the deterministic framework, and the filtering error system with bounded measurement missing is usually described as a deterministic system with bounded time-varying delays. In this way, the relation between the filtering performance level and the maximal number of consecutive measurement missing is established. Some results on this topic can be found, for example, in [7, 23], where the H_∞ filtering was investigated for linear continuous-time systems with bounded measurement missing and delays. However, the relation between the filtering performance level and the measurement missing rate/probability is not established in the existing results using deterministic framework.

By a closer inspection, it is found that the relations among the filtering performance level and the parameters characterizing the measurement missing revealed in the existing results using either the stochastic framework or the deterministic framework are quite implicit. Actually, besides of determining the maximal allowable measurement missing rate/probability bound, one may be interested more in the quantitative results about how much the filtering performance declines for a certain amount of measurement missing and for a certain extent of increase on the maximal number of consecutive measurement missing, or in other words, the quantitative relations among the performance level and the parameters characterizing the measurement missing. Specifically, this gives rise to the problem of how to express the filtering performance level as a function of the parameters characterizing the measurement missing. To the best of the authors' knowledge, such quantitative relations for the filtering systems with bounded measurement missing have not yet been established in the existing results, which motivates the presented research.

In this paper, the H_∞ filtering problem is investigated for networked systems with bounded measurement missing. The filtering error system is described as a switched linear system by using the augmentation technique and by including the numbers of consecutive measurement missing as switching parameters. Based on the obtained switched system model, a sufficient condition is derived for the filtering system to be exponentially stable and achieve a prescribed H_∞ performance level. A convex optimization problem is also presented to design the linear H_∞ filters which guarantee that the considered filtering system achieves a suboptimal H_∞ performance level. The main contributions of the paper are summarized as follows. (1) A switched system model with finite number of subsystems is established to describe the networked filtering system with bounded missing measurements. (2) A quantitative relation is established between the filtering performance and parameters characterizing the measurement missing. Specifically, the exponential decay rate of the filtering error system is given as a monotonic increasing function of the measurement missing rate bound, while the H_∞ performance level is given as a monotonic increasing function of both the measurement missing rate bound and the maximal number of consecutive measurement missing. (3) The established relation gives quantitative results about how much the H_∞ filtering performance level declines for a certain amount of measurement missing and for a certain extent of increase on the maximal number of consecutive measurement missing. They theoretically reveal that measurement missing degrades the H_∞ filtering performance.

2. Modelling of the Filtering System

The considered filtering problem is shown in Figure 1, where the physical plant is described by

$$\begin{aligned} x(k+1) &= A_p x(k) + B_p w(k), \\ y(k) &= C_p x(k) + D_p w(k), \\ z(k) &= L_p x(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathfrak{R}^n$ is the system state, $y(k) \in \mathfrak{R}^p$ is the measured output, $z(k) \in \mathfrak{R}^q$ is the signal to be estimated, and $w(k) \in \mathfrak{R}^m$ is the noise signal which belongs to $l_2[0, \infty)$. A_p, B_p, C_p, D_p, and L_p are constant matrices with appropriate dimensions, and A_p is assumed to be stable. The full-order linear filter used to estimate the signal $z(k)$ is given by

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f \hat{y}(k), \\ z_f(k) &= C_f x_f(k), \end{aligned} \tag{2}$$

where $x_f(k) \in \mathfrak{R}^n$ is the filter state, $\hat{y}(k) \in \mathfrak{R}^p$ is the filter input, and $z_f(k) \in \mathfrak{R}^q$ is the estimated signal. A_f, B_f, and C_f are filter matrices to be determined.

Since the measurements may be missed during the transmission from the sensor to the filter, the filter input $\hat{y}$ may not be equal to the measured output y. Suppose that the number of consecutive measurement missing i is upper bounded by $i \le d$ and the filter input holds at its last available value if a measurement is missed, where d is a known constant. Then, we have

$$\hat{y}(k) = y(k-i) \tag{3}$$

if there are i consecutive measurement losses before the time step $k+1$, where $i \in Z_0 = \{0, 1, \ldots, d\}$. It can be seen from (3) that the system model of the filter varies over different sampling intervals since the number of consecutive measurement missing is time varying over different sampling periods. By defining the filtering error signal as $e(k) = z(k) - z_f(k)$ and denoting

$$\begin{aligned} \xi(k) &= \begin{bmatrix} x^T(k) & x^T(k-1) & \cdots & x^T(k-d) & x_f^T(k) \end{bmatrix}^T, \\ v(k) &= \begin{bmatrix} w^T(k) & w^T(k-1) & \cdots & w^T(k-d) \end{bmatrix}^T, \end{aligned} \tag{4}$$

we obtain the following filtering error system model:

$$S_{\sigma(k)} : \begin{cases} \xi(k+1) = A_{\sigma(k)} \xi(k) + B_{\sigma(k)} v(k) \\ e(k) = C \xi(k), \end{cases} \tag{5}$$

where $\sigma(k) \in Z_0$, and

$$A_{\sigma(k)} = \begin{bmatrix} \begin{bmatrix} A_p \Lambda_a \\ \Pi \end{bmatrix} & \begin{bmatrix} 0 \\ O_a \end{bmatrix} \\ B_f C_p I_{a\sigma(k)} & A_f \end{bmatrix},$$

$$B_{\sigma(k)} = \begin{bmatrix} \begin{bmatrix} B_p \Lambda_b \\ O_b \end{bmatrix} \\ B_f D_p I_{b\sigma(k)} \end{bmatrix},$$

$$C = \begin{bmatrix} \begin{bmatrix} L_p & O_c \end{bmatrix} & -C_f \end{bmatrix},$$

$$\Lambda_a = \begin{bmatrix} I_1 & O_a^T \end{bmatrix}, \qquad \Lambda_a = \begin{bmatrix} I_3 & O_d^T \end{bmatrix},$$

$$\Pi = \begin{bmatrix} I_2 & O_a \end{bmatrix}, \qquad O_a = 0 \in R^{dn \times n},$$

$$O_b = 0 \in R^{dn \times (d+1)n},$$

$$O_c = 0 \in R^{p \times dn}, \qquad O_d = 0 \in R^{dm \times m},$$

$$I_1 = I \in R^{n \times n}, \qquad I_2 = I \in R^{dn \times dn},$$

$$I_3 = I \in R^{m \times m},$$

$$I_{a\sigma(k)} = \begin{bmatrix} \Delta_{a0} & \Delta_{a1} & \cdots & \Delta_{ad} \end{bmatrix},$$

$$I_{b\sigma(k)} = \begin{bmatrix} \Delta_{b0} & \Delta_{b1} & \cdots & \Delta_{bd} \end{bmatrix},$$

$$\Delta_{ai} = \begin{cases} I_1, & \sigma(k) = i \\ 0, & \sigma(k) \ne i, \end{cases} \qquad \Delta_{bi} = \begin{cases} I_3, & \sigma(k) = i \\ 0, & \sigma(k) \ne i. \end{cases} \tag{6}$$

By applying the augmentation technique, the filtering error system is finally described as a nondelayed switched system by including the numbers of consecutive measurement missing as switching parameters. $\sigma(k)$ serves as the switching signal of system (5) which contains $d+1$ subsystems. The switching of the subsystems is determined by the measurement missing status. Specifically, for $i = 0, 1, \ldots, d$, we have $\sigma(k) = i$ and system (5) resides in the subsystem S_i during the interval $[k, k+1)$ if there is i consecutive measurement missing before time step $k+1$. This is illustrated in Figure 2, where $\bullet$ represents the measurement that is successfully received by the filter, while $\circ$ stands for the one that is missed. At time step $k-1$, the measurement $y(k-1)$ is successfully received by the filter. Then, $\hat{y}(k-1) = y(k-1)$, and $S_{\sigma(k)}$ resides in S_0. At time step k, the measurement $y(k)$ is missed and the filter input holds at its previous value; that is, $\hat{y}(k)$ takes $y(k-1)$, and $S_{\sigma(k)}$ resides in S_1. Then, at time step $k+1$, the measurement $y(k+1)$ is missed again, and the filter input remains at $y(k-1)$; that is, $\hat{y}(k+1) = y(k-1)$, and $S_{\sigma(k)}$ resides in S_2. The rest can be deduced by analogy. It is seen from the above analysis that the filtering error system $S_{\sigma(k)}$ takes on the following characteristics.

(a) For $i = 1, \ldots, d-1$, subsystem S_{i+1} always appears after the subsystem S_i, and subsystem S_0 always appears before S_1 or after S_d.

(b) Switching between the subsystems of $S_{\sigma(k)}$ occurs only when the subsystems S_j $(j = 1, \ldots, d)$ appear, and the number of switches of $S_{\sigma(k)}$ is determined by the total activation times of S_j. Moreover, at most two switches may be involved when S_i appears one time.

For $i = 0, 1, \ldots, d$ and $0 \le k_1 \le k \le k_2$, let $N_\sigma[k_1, k_2]$ denote the number of switches of $\sigma(k)$ and $n_i[k_1, k_2]$ the times that the subsystem S_i is activated on the interval $[k_1, k_2]$. Then, it follows from the characteristics of $S_{\sigma(k)}$ that the following relation holds:

$$N_\sigma[k_1, k_2] \le 2 \sum_{i=1}^{d} n_i[k_1, k_2]. \tag{7}$$

Definition 1. For any switching signal $\sigma(k)$ and $0 \le k_1 \le k \le k_2$, let $N_\sigma[k_1, k_2]$ denote the number of switches of $\sigma(k)$ on

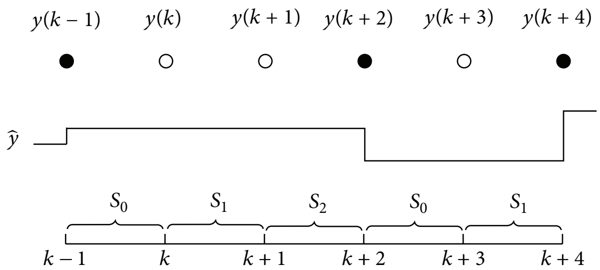

Figure 2: An example of measurement missing.

the interval $[k_1, k_2]$. Then, $\theta[k_1, k_2] = N_\sigma[k_1, k_2]/(k_2 - k_1)$ is called the switching frequency of $\sigma(k)$ on the interval $[k_1, k_2]$.

Definition 2. For $i \in Z_0$ and $0 \le k_1 < k_2$, let $\alpha_i[k_1, k_2] = n_i[k_1, k_2]/(k_2 - k_1)$. Then $\alpha_i[k_1, k_2]$ is called the occurrence rate of the subsystem S_i on the interval $[k_1, k_2]$.

The subsystem S_j, $j \in \{1, \ldots, d\}$ is activated when a measurement missing occurs, so the measurement missing rate on any interval $[k_1, k_2]$ can be defined as $\alpha[k_1, k_2] = \sum_{i=1}^{d} \alpha_i[k_1, k_2]$. Then, we have by relation (7) and Definitions 1 and 2 that

$$\theta\left[k_1, k_2\right] \le 2\alpha\left[k_1, k_2\right]. \tag{8}$$

Moreover, we have by the characteristics of $S_{\sigma(k)}$ that there exists a constant $\alpha < 1$ such that the following relations are true for any $t \ge 0$ and $l \ge 0$:

$$\begin{aligned} \alpha\left[t, t+l\right] &\le 1, \quad l \le d \\ \alpha\left[t, t+l\right] &\le \alpha < 1, \quad l > d. \end{aligned} \tag{9}$$

Remark 3. The considered filtering system with bounded measurement missing is finally described as a switched linear system $S_{\sigma(k)}$ with $d + 1$ subsystems, where S_0 describes the filtering system without measurement missing, while S_j ($j = 1, \ldots, d$) describes those with measurement missing. It is reasonable to see that the filtering system will achieve a better H_∞ performance level if S_0 appears with higher frequency (or, in other words, the measurement missing rate is lower). This is the motivation of the proposed switched system model of the filtering error system, which may enable us to establish the quantitative relation between the H_∞ filtering performance and the measurement missing rate bound. Note that the augmentation technique is used to obtain the switched system model, which increases computational cost. From the energy-efficient perspective, the proposed method is applicable to the case where the number of consecutive packet losses is bounded and not too large.

The following definition and assumptions are needed in the derivation of the main results.

Definition 4. System (5) is said to be exponentially stable with decay rate $\lambda < 1$ if, for any finite initial state $\xi(0) \in \mathfrak{R}^{(d+2)n}$, there exist a constant $c > 0$ such that $\|\xi(k)\| \le c\lambda^k \|\xi(0)\|$ holds.

Assumption 5. For any $t \ge 0$ and $l > d$, $\alpha[t, t + l]$ is upper bounded by a constant measurement missing rate bound α; that is, $\alpha[t, t + l] \le \alpha$, where $0 \le \alpha < 1$.

Assumption 6. $\xi(k) = 0$ and $w(k) = 0$, for all $k \le 0$.

The H_∞ filtering problem to be addressed in this paper is expressed as follows.

Problem HFBMM (H_∞ Filtering with Bounded Measurement Missing). For the filtering problem in Figure 1 and a given system (1), determine the matrices A_f, B_f, and C_f in filter (2), such that the system (5) with $v(k) = 0$ is exponentially stable, and the estimation error e satisfies a prescribed H_∞ performance level γ (i.e., $\|e\|_2 \le \gamma\|v\|_2$) under zero initial conditions (i.e., $\xi(k) = 0$, for all $k \le 0$) for all admissible bounded measurement missing.

3. H_∞ Filtering Analysis

An H_∞ performance condition for the filtering error system (5) is presented in the following theorem.

Theorem 7. *For given scalars $\mu > 1$ and $0 < \lambda < 1$ satisfying $\mu\lambda < 1$, if there exist matrices $P_i > 0$, $i = 0, 1, \ldots, d$, and a scalar $\gamma_0 > 0$, such that the following inequalities hold:*

$$\Omega_i = \begin{bmatrix} -\lambda^2 P_i + C^T C & 0 \\ 0 & -\gamma_0^2 I \end{bmatrix} + \begin{bmatrix} A_i^T \\ B_i^T \end{bmatrix} P_i \left[A_i \ \ B_i\right] < 0, \tag{10}$$

$$P_\alpha \le \mu P_\beta, \quad \alpha, \beta \in Z_0, \tag{11}$$

then system (5) is exponentially stable with decay rate $\delta(\alpha) = \lambda\mu^\alpha$ and achieves a prescribed H_∞ performance level $\gamma(\alpha, d) = \gamma_0((1-\lambda^2)(1-(\mu\lambda)^{2(d+1)})/(1-(\mu\lambda)^2)+(1-\lambda^2)(\mu^\alpha\lambda)^{2(d+1)}/(1-(\mu^\alpha\lambda)^2))^{1/2}$, where α is the measurement missing rate bound.

Proof. Choose the Lyapunov function $V_{\sigma(k)}(k) = \xi^T(k) P_{\sigma(k)} \xi(k)$ and define $J(k) = \|e(k)\|^2 - \gamma_0^2 \|v(k)\|^2$. For $i \in Z_0$, it follows from (5) and (10) that

$$V_i\left(k+1\right) - \lambda V_i\left(k\right) + J\left(k\right) = \eta^T\left(k\right)\Omega_i \eta\left(k\right) < 0, \tag{12}$$

where $\eta(k) = \left[\xi^T(k) \ \ v^T(k)\right]^T$. For any given integer $k \ge 1$, let $k_1 < \cdots < k_i$, $i \ge 1$ denotes the switching instants of the switching signal $\sigma(k)$ on the interval $[0, k]$. Then, we have by (11) that

$$\begin{aligned} V_{\sigma(k_i)}\left(k_i\right) &= \xi^T\left(k_i\right) P_{\sigma(k_i)} \xi\left(k_i\right) \le \mu \xi^T\left(k_i\right) P_{\sigma(k_i - 1)} \xi\left(k_i\right) \\ &= \mu V_{\sigma(k_i - 1)}\left(k_i\right), \quad \forall i \ge 1. \end{aligned} \tag{13}$$

It follows from (12) and (13) that

$$\begin{aligned} V_{\sigma(k)}(k) &\le \lambda^{2(k-k_i)} V_{\sigma(k_i)}(k_i) - \sum_{t=k_i}^{k-1} \lambda^{2(k-1-t)} J(t) \\ &\le \lambda^{2(k-k_i)} \cdot \mu \cdot V_{\sigma(k_i-1)}(k_i) - \sum_{t=k_i}^{k-1} \lambda^{2(k-1-t)} J(t) \\ &\le \lambda^{2(k-k_i)} \cdot \mu \cdot \left[\lambda^{2(k_i-k_{i-1})} V_{\sigma(k_{i-1})}(k_{i-1}) - \sum_{t=k_{i-1}}^{k_i-1} \lambda^{2(k_i-1-t)} J(t) \right] \\ &\quad - \sum_{t=k_i}^{k-1} \lambda^{2(k-1-t)} J(t) \\ &\ \ \vdots \\ &\le \mu^{N_\sigma[0,k-1]} \lambda^{2(k-k_i)} \lambda^{2(k_i-k_{i-1})} \cdots \lambda^{2k_1} V_{\sigma(0)}(0) \\ &\quad - \Gamma(J(t)), \end{aligned} \tag{14}$$

where

$$\begin{aligned} \Gamma(J(t)) = {} & \mu^{N_\sigma[0,k-1]} \lambda^{2(k-k_i)} \lambda^{2(k_i-k_{i-1})} \cdots \lambda^{2(k_2-k_1)} \\ & \times \sum_{t=0}^{k_1-1} \lambda^{2(k_1-1-t)} J(t) \\ & + \mu^{N_\sigma[0,k-1]-1} \lambda^{2(k-k_i)} \lambda^{2(k_i-k_{i-1})} \cdots \lambda^{2(k_3-k_2)} \\ & \times \sum_{t=k_1}^{k_2-1} \lambda^{2(k_2-1-t)} J(t) \\ & \ \vdots \\ & + \mu^0 \sum_{t=k_i}^{k-1} \lambda^{2(k-1-t)} J(t) \\ = {} & \Gamma\left(\|e(t)\|^2\right) - \gamma_0^2 \Gamma\left(\|v(t)\|^2\right). \end{aligned} \tag{15}$$

First, we consider the exponential stability of system (5) for $v(k) = 0$. It follows from (8), (14), $v(k) = 0$, and $N_\sigma[0, k-1] = (k-1) \cdot \theta[0, k-1] \le k \cdot \theta[0, k-1]$ that

$$\begin{aligned} V_{\sigma(k)}(k) &\le \mu^{N_\sigma[0,k-1]} \lambda^{2(k-k_i)} \lambda^{2(k_i-k_{i-1})} \cdots \lambda^{2k_1} V_{\sigma(0)}(0) \\ &\le \mu^{2k\cdot(\theta[0,k-1]/2)} \lambda^{2k} V_{\sigma(0)}(0) \\ &\le \left(\mu^{\alpha[0,k-1]} \lambda\right)^{2k} V_{\sigma(0)}(0). \end{aligned} \tag{16}$$

It is reasonable in practice that the time horizon k is larger than d. So, it can be further obtained from (16) by assumption 1 that

$$V_{\sigma(k)}(k) \le \left(\mu^{\alpha} \lambda\right)^{2k} V_{\sigma(0)}(0) = \delta^{2k} V_{\sigma(0)}(0) \tag{17}$$

which yields $\|\xi(k)\| \le \sqrt{\pi_M/\pi_m}\,\delta^k \|\xi(0)\|$, where $\pi_M = \max_{i\in Z_0}\{\lambda_{\max}(P_i)\}$ and $\pi_m = \min_{i\in Z_0}\{\lambda_{\min}(P_i)\}$. $\lambda_{\max}(\star)$ and $\lambda_{\min}(\star)$ represent the maximum and the minimum eigenvalues of the matrix $\star$, respectively. On the other hand, $\alpha < 1$ and $\mu\lambda < 1$ guarantee that $\delta < 1$. Thus, it is concluded by Definition 4 that system (5) is exponentially stable with decay rate δ.

Next, to prove the H_∞ performance, we consider $v(k) \ne 0$. Then, under zero initial condition, we have, by (14) and the fact that $V_{\sigma(k)}(k) \ge 0$, that $\Gamma(J(t)) = \Gamma(\|e(t)\|^2) - \gamma_0^2 \Gamma(\|v(t)\|^2) \le 0$, and thus

$$\begin{aligned} \sum_{t=0}^{k-1} \mu^{N_\sigma[t,k-1]} \lambda^{2(k-1-t)} \|e(t)\|^2 &= \Gamma\left(\|e(t)\|^2\right) \\ &\le \gamma_0^2 \Gamma\left(\|v(t)\|^2\right) \\ &= \gamma_0^2 \sum_{t=0}^{k-1} \mu^{N_\sigma[t,k-1]} \lambda^{2(k-1-t)} \|v(t)\|^2. \end{aligned} \tag{18}$$

It follows from Definition 1 that $N_\sigma[t, k-1] = (k-1-t) \cdot \theta[t, k-1] \le 2(k-1-t)\alpha[t, k-1]$, which leads to

$$1 \le \mu^{N_\sigma[t,k-1]} \le \mu^{2(k-1-t)\alpha[t,k-1]}. \tag{19}$$

We have by (18) and (19) that

$$\sum_{t=0}^{k-1} \lambda^{2(k-1-t)} \|e(t)\|^2 \le \gamma_0^2 \sum_{t=0}^{k-1} \left(\mu^{\alpha[t,k-1]} \lambda\right)^{2(k-1-t)} \|v(t)\|^2. \tag{20}$$

Summing both sides of (20) from $k = 1$ to $k = +\infty$ and changing the order of the summation, taking (9) into account, we obtain

$$\begin{aligned} \left(1-\lambda^2\right)^{-1} \sum_{t=0}^{+\infty} \|e(t)\|^2 &= \sum_{t=0}^{+\infty} \|e(t)\|^2 \cdot \left(\sum_{k=t+1}^{+\infty} \lambda^{2(k-1-t)} \right) \\ &= \sum_{k=1}^{+\infty} \sum_{t=0}^{k-1} \lambda^{2(k-1-t)} \|e(t)\|^2 \\ &\le \gamma_0^2 \sum_{k=1}^{+\infty} \sum_{t=0}^{k-1} \left(\mu^{\alpha[t,k-1]} \lambda\right)^{2(k-1-t)} \|v(t)\|^2 \\ &= \gamma_0^2 \sum_{t=0}^{+\infty} \|v(t)\|^2 \\ &\quad \cdot \left(\sum_{k=t+1}^{+\infty} \left(\mu^{\alpha[t,k-1]} \lambda\right)^{2(k-1-t)} \right) \\ &= \gamma_0^2 \sum_{t=0}^{+\infty} \|v(t)\|^2 \cdot \left(\sum_{i=0}^{d} \left(\mu^{\alpha[t,t+i]} \lambda\right)^{2i} \right. \\ &\quad \left. + \sum_{i=d+1}^{+\infty} \left(\mu^{\alpha[t,t+i]} \lambda\right)^{2i} \right) \end{aligned}$$

$$\le \gamma_0^2 \sum_{t=0}^{+\infty} \|v(t)\|^2 \cdot \left(\sum_{i=0}^{d} (\mu\lambda)^{2i} + \sum_{i=d+1}^{+\infty} (\mu^\alpha \lambda)^{2i} \right) = \gamma_0^2 \left(\frac{1-(\mu\lambda)^{2(d+1)}}{1-(\mu\lambda)^2} + \frac{(\mu^\alpha\lambda)^{2(d+1)}}{1-(\mu^\alpha\lambda)^2} \right) \times \sum_{t=0}^{+\infty} \|v(t)\|^2. \tag{21}$$

It follows from (21) that $\|e\|_2 \le \gamma(\alpha, d)\|v\|_2$, which implies that the filtering error system (5) achieves the H_∞ performance level $\gamma(\alpha, d)$. The proof is completed. □

Remark 8. Theorem 7 establishes the quantitative relations among the H_∞ filtering performance level, the measurement missing rate bound, and the maximal number of consecutive measurement missing. Specifically, the H_∞ filtering performance level $\gamma(\alpha, d)$ is given as a monotonic increasing function of both the measurement missing rate bound α and the maximal number of consecutive measurement missing d, which implies that some smaller measurement missing rate and maximal number of consecutive measurement missing lead to a better H_∞ filtering performance.

Remark 9. It is obtained from (12) that

$$V_i(k) < \lambda^k V(0) - \sum_{t=0}^{k-1} \lambda^{(k-1-t)} J(t). \tag{22}$$

It follows from (22) that $V_i(k) < \lambda^k V(0)$ for $v(k) = 0$, which implies that the subsystem S_i ($i \in Z_0$) is exponentially stable with decay rate λ. On the other hand, we have by (22) that

$$\sum_{t=0}^{k-1} \lambda^{(k-1-t)} J(t) < 0 \tag{23}$$

under zero initial condition. Summing both sides of (23) from $k = 1$ to $k = +\infty$, we obtain $(1-\lambda^2)^{-1} \sum_{t=0}^{+\infty} J(t) < 0$, which yields $\|e\|_2 < \gamma_0 \|v\|_2$. It implies that the subsystem S_i achieves the H_∞ performance level γ_0. In summary, if (10) is true for $0 < \lambda < 1$, then the subsystem S_0 is exponentially stable with decay rate λ and achieves the H_∞ performance level γ_0. Note that if there is no measurement missing, then $S_{\sigma(k)}$ resides in S_0, and we have $\alpha = 0$, $d = 0$ and $\mu = 1$. In this case, $\delta(\alpha)$ and $\gamma(\alpha, d)$ in Theorem 7 are, respectively, reduced to λ and γ_0, which are just the decay rate and the H_∞ performance level of S_0, respectively. So, Theorem 7 theoretically reveals that measurement missing degrades the H_∞ filtering performance and presents the quantitative result about the effect of the measurement missing on the H_∞ filtering performance.

4. H_∞ Filter Design

Theorem 10. *Consider the filtering problem in Figure 1. For given scalars $\mu > 1$ and $0 < \lambda < 1$ satisfying $\mu\lambda < 1$, if there exist matrices $Q_{i11} > 0$, $Q_{i22} > 0$, Q_{i12}, $i = 0, 1, \ldots, d$, U_1, U_2, U_3, $\widehat{A}_f$, $\widehat{B}_f$, and $\widehat{C}_f$ of appropriate dimensions and a scalar $\gamma_0 > 0$, such that the following linear matrix inequalities hold:*

$$\begin{bmatrix} \lambda^2 Q_{i11} & \lambda^2 Q_{i12} & 0 & \Delta_a^T U_1 + I_{ai}^T C_p^T \widehat{B}_f^T E^T & \Delta_a^T U_3 + I_{ai}^T C_p^T \widehat{B}_f^T & \left[L_p O_c\right]^T \\ * & \lambda^2 Q_{i22} & 0 & \widehat{A}_f^T E^T & \widehat{A}_f^T & -\widehat{C}_f^T \\ * & * & \gamma_0^2 I & \Delta_b^T U_1 + I_{bi}^T D_p^T \widehat{B}_f^T E^T & \Delta_b^T U_3 + I_{bi}^T D_p^T \widehat{B}_f^T & 0 \\ * & * & * & U_1 + U_1^T - Q_{i11} & U_3 + E U_2 - Q_{i12} & 0 \\ * & * & * & * & U_2 + U_2^T - Q_{i22} & 0 \\ * & * & * & * & * & I \end{bmatrix} > 0, \tag{24}$$

$$Q_\alpha \le \mu Q_\beta, \quad \alpha, \beta \in Z_0, \tag{25}$$

then the filters of form (2) *guarantee that the filtering error system* (5) *is exponentially stable with decay rate $\delta(\alpha)$ and achieves a prescribed H_∞ performance level $\gamma(\alpha, d)$, and the filter matrices are given by*

$$A_f = U_2^{-1} \widehat{A}_f, \qquad B_f = U_2^{-1} \widehat{B}_f, \qquad C_f = \widehat{C}_f, \tag{26}$$

where $\delta(\alpha)$ and $\gamma(\alpha, d)$ are given in Theorem 7, and

$$\Delta_a = \begin{bmatrix} A_p \Lambda_a \\ \Pi \end{bmatrix}, \qquad \Delta_b = \begin{bmatrix} B_p \Lambda_b \\ O_b \end{bmatrix}, \qquad Q_i = \begin{bmatrix} Q_{i11} & Q_{i12} \\ Q_{i12}^T & Q_{i22} \end{bmatrix}, \qquad E = \begin{bmatrix} I_{n\times n} \\ 0_{dn\times dn} \end{bmatrix}. \tag{27}$$

Proof. By Lemma 1 in [24], (10) is true if and only if there exists a matrix G such that the following inequality holds:

$$\begin{bmatrix} \begin{bmatrix} \lambda_i^2 P_i - C^T C & 0 \\ 0 & \gamma^2 I \end{bmatrix} & \begin{bmatrix} A_i^T \\ B_i^T \end{bmatrix} G \\ * & G + G^T - P_i \end{bmatrix} > 0, \tag{28}$$

On the other hand, (24) indicates that U_2 is nonsingular, and thus we can always find square and nonsingular matrices $M \in R^{n\times n}$ and $G_{22} \in R^{n\times n}$ satisfying $U_2 = M^T G_{22}^{-1} M$. Now, introduce the variables $U_1 \in R^{(d+1)n\times(d+1)n}$ and $G_{12} \in R^{(d+1)n\times n}$, and let

$$G = \begin{bmatrix} U_1 & G_{12} \\ ME^T & G_{22} \end{bmatrix}, \qquad U_3 = G_{12} G_{22}^{-1} M,$$
$$J_1 = \begin{bmatrix} I & 0 \\ 0 & G_{22}^{-1} M \end{bmatrix}, \tag{29}$$
$$\begin{bmatrix} A_f & B_f \\ C_f & 0 \end{bmatrix} = \begin{bmatrix} M^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \widehat{A}_f & \widehat{B}_f \\ \widehat{C}_f & 0 \end{bmatrix} \begin{bmatrix} M^{-1} G_{22} & 0 \\ 0 & I \end{bmatrix}.$$

Then, pre- and postmultiplying (28) and (11) by $\mathrm{diag}\{J_1, I, J_1\}$ and J_1, respectively, following some routine matrix manipulations, yield (24) and (25). Furthermore, denote the filter transfer function from $\widehat{y}(k)$ to $z_f(k)$ by $T_f(z) = C_f(zI - A_f)^{-1} B_f$. Substituting the filter matrices with (29) and taking $U_2 = M^T G_{22}^{-1} M$ into account, we obtain $T_f(z) = \widehat{C}_f(zI - U_2^{-1}\widehat{A}_f)^{-1} U_2^{-1} \widehat{B}_f$. So, the filter matrices are given by (26). Then, it follows from Theorem 7 that Theorem 10 is true. The proof is completed. □

The condition in Theorem 10 is convex in γ_0^2. So, the convex optimization problem

$$\begin{aligned} \min \quad & \rho \\ \text{s.t.} \quad & (24) \text{ and } (25) \text{ with } \rho = \gamma_0^2 \end{aligned} \tag{30}$$

can be formulated to design the H_∞ filters. If ρ^* is the solution of the optimization problem (30), then the designed filters guarantee that system (5) achieves the following H_∞ performance level:

$$\gamma^*(\alpha, d) = \left(\frac{\rho^* \left(1 - \lambda^2\right) \left(1 - (\mu\lambda)^{2(d+1)}\right)}{1 - (\mu\lambda)^2} + \frac{\rho^* \left(1 - \lambda^2\right) (\mu^\alpha \lambda)^{2(d+1)}}{1 - (\mu^\alpha \lambda)^2} \right)^{1/2}. \tag{31}$$

Remark 11. Note that $\gamma^*(\alpha, d)$ is monotonic increasing on μ and λ. Therefore, μ and λ should be chosen as small as possible to obtain a better H_∞ filtering performance and meanwhile to make the condition $\mu\lambda < 1$ easier to be satisfied. Nevertheless, it can be seen from (24) and (25) that some smaller μ and λ will result in a larger value of γ_0 and ultimately yield a larger value of $\gamma^*(\alpha, d)$. The following algorithm provides a method on determining the parameters μ and λ that will result in a suboptimal H_∞ performance level $\gamma^*(\alpha, d)$.

Algorithm 12.

Step 1. Choose some sufficiently large initial μ and λ such that $\mu\lambda < 1$ holds. Set $\mu^0 = \mu$ and $\lambda^0 = \lambda$. Solve the optimization problem (30) and calculate $\gamma^*(\alpha, d, \mu^0, \lambda^0)$ for given α and d.

Step 2. Decrease λ^0 by a certain step length; say, $\Delta\lambda$; that is, set $\lambda^0 = \lambda^0 - \Delta\lambda$. Then, solve the optimization problem (30) and calculate $\gamma^*(\alpha, d, \mu^0, \lambda^0)$ for given α and d.

Step 3. If $\gamma^*(\alpha, d, \mu^0, \lambda^0) < \gamma^*(\alpha, d, \mu^0, \lambda^0 + \Delta\lambda)$, then return to Step 2. Otherwise, exit and set $\lambda^* = \lambda^0 + \Delta\lambda$.

Step 4. Decrease μ^0 by a certain step length; say, $\Delta\mu$; that is, set $\mu^0 = \mu^0 - \Delta\mu$. Then, solve the optimization problem (30) and calculate $\gamma^*(\alpha, d, \mu^0, \lambda^*)$ for the obtained λ^* and given α and d.

Step 5. If $\gamma^*(\alpha, d, \mu^0, \lambda^*) < \gamma^*(\alpha, d, \mu^0 + \Delta\mu, \lambda^*)$, then return to Step 4. Otherwise, exit and set $\mu^* = \mu^0 + \Delta\mu$. Then, μ^* and λ^* are the desired values of the parameters μ and λ, respectively, and $\gamma^*(\alpha, d, \mu^*, \lambda^*)$ is the desired suboptimal H_∞ performance level for given α and d.

5. Illustrative Examples

Example 1. Consider a mechanical system with two masses and two springs as that studied in [7]. Its state-space model is given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_1 + k_2}{m_1} & \dfrac{k_2}{m_1} & -\dfrac{c}{m_1} & 0 \\ \dfrac{k_2}{m_2} & -\dfrac{k_2}{m_2} & 0 & -\dfrac{c}{m_2} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{m_1} \\ 0 \end{bmatrix} w(t), \tag{32}$$

where x_1 and x_2 are the positions of masses m_1 and m_2, respectively. Choose $m_1 = 1$, $m_2 = 0.5$, $k_1 = k_2 = 1$, and $c = 0.5$, and set $T = 200$ ms; then we obtain the discrete-time system model (1) with

$$A_p = \begin{bmatrix} 0.9617 & 0.0191 & 0.1878 & 0.001228 \\ 0.03697 & 0.9629 & 0.002456 & 0.1789 \\ -0.3732 & 0.1853 & 0.8678 & 0.01787 \\ 0.3528 & -0.3553 & 0.03574 & 0.784 \end{bmatrix},$$
$$B_p = \begin{bmatrix} 0.01922 \\ 0.000125 \\ 0.1878 \\ 0.002456 \end{bmatrix}. \tag{33}$$

Suppose that x_1 is measured by a device with noise $w(k)$ and x_2 is to be estimated by using the H_∞ filters; then we have $C_p = [1\ 0\ 0\ 0]$ and $L_p = [0\ 1\ 0\ 0]$. Furthermore, we choose $D_p = 0.1$.

Choose $\mu = 1.01$ and $\lambda = 0.96$. Suppose that $d = 2$ and $\alpha = 50\%$. Then, by solving the optimization problem (23),

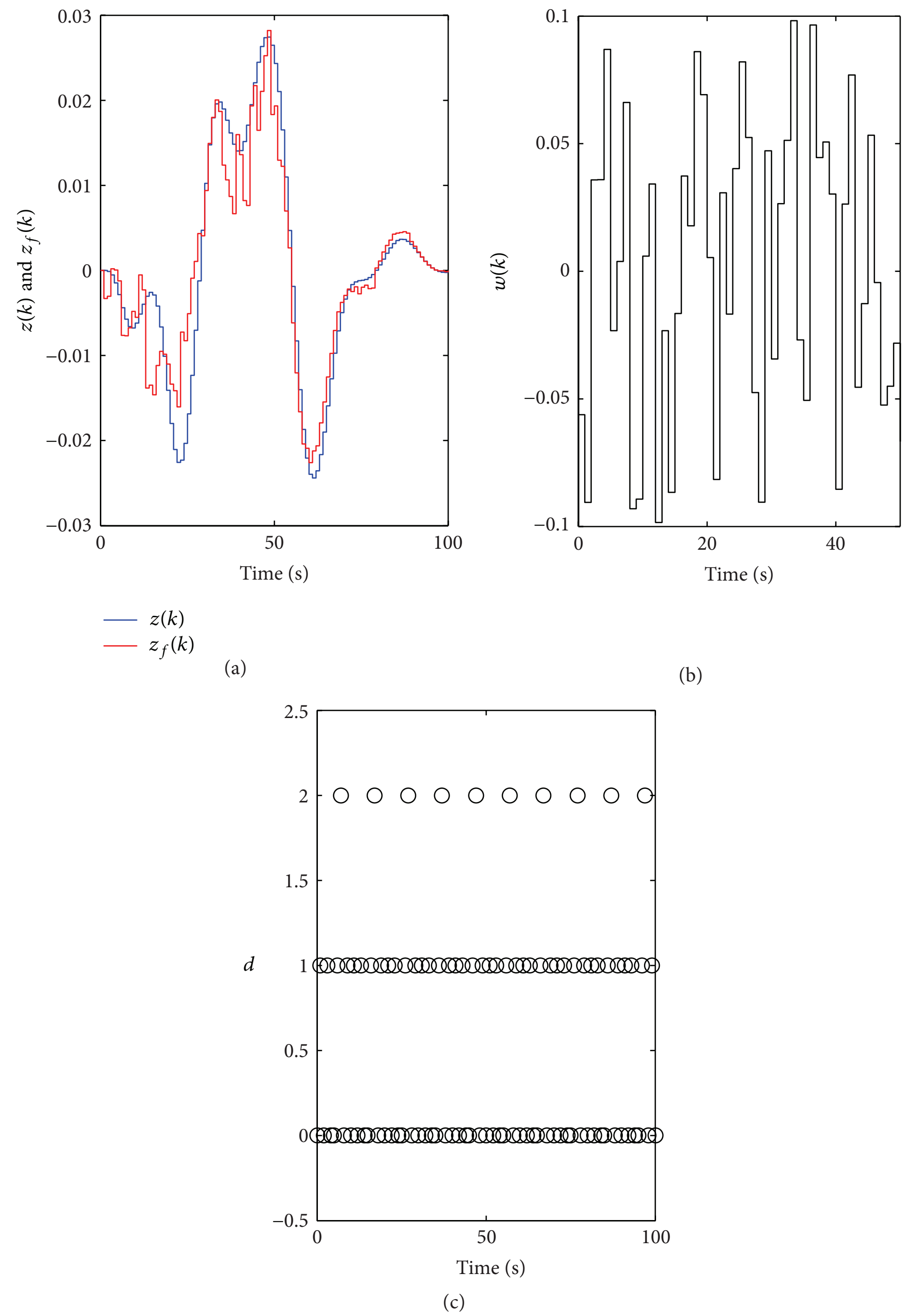

FIGURE 3: Trajectories of $z(k)$ and $z_f(k)$, the noise signal $w(k)$, and the measurement missing status.

we obtain the H_∞ performance level $\gamma^* = 0.8001$ with the following filter gain matrices:

$$A_f = \begin{bmatrix} -0.0691 & -0.0666 & 0.3304 & -0.0409 \\ -0.5288 & 0.9180 & 0.0793 & 0.1596 \\ -1.1896 & 0.1687 & 1.0783 & -0.0035 \\ 0.8709 & -0.3034 & -0.0219 & 0.8090 \end{bmatrix},$$
$$B_f = \begin{bmatrix} -1.0655 \\ -0.5858 \\ -0.7988 \\ 0.5298 \end{bmatrix}, \qquad C_f = \begin{bmatrix} 0.0003 \\ -1.0002 \\ -0.0002 \\ -0.0001 \end{bmatrix}. \tag{34}$$

In the simulation, the noise signal $w(k)$ is assumed to be uniformly distributed with $[-0.1, 0.1]$ for the interval $[0, 50]$. The measurement missing is generated randomly and is shown in Figure 3(b), where the measurement missing rate is 50%. The trajectories of $z(k)$ and $z_f(k)$ are depicted in Figure 3(a). By calculation, we obtain from the results in Figure 3(a) that $\gamma_a = \|e\|_2/\|v\|_2 = 0.0460 < \gamma^* = 0.8001$, showing the effectiveness of the H_∞ filter design. In what follows, we will show the relations among the H_∞ performance level, the measurement missing rate bound, and the maximal number of consecutive measurement missing.

Table 1: The relation between the H_∞ performance level and the measurement missing rate bound for $d = 2$.

α	0%	20%	50%	80%
γ^*	0.7525	0.7704	0.8001	0.8339
γ_a	0.0470	0.0475	0.0565	0.0926

Table 2: The relation between the H_∞ performance level and the maximal number of consecutive measurement missing for $\alpha = 50\%$.

d	1	2	3
γ^*	0.5529	0.8001	1.0027
γ_a	0.0218	0.0424	0.0506

For $\mu = 1.01$, $\lambda = 0.96$, and $d = 2$, the relation between the H_∞ performance level and the measurement missing rate bound is given in Table 1 by solving the optimization problem (30). For $\mu = 1.01$, $\lambda = 0.96$, and $\alpha = 50\%$, the relation between the H_∞ performance level and the maximal number of consecutive measurement missing is presented in Table 2 by solving the optimization problem (30). In calculating the actual noise attenuation level γ_a, the noise signal $w(k)$ in Figure 3 is adopted, and the measurement missing is generated randomly under the constraint of the corresponding missing rate bound. It can be seen from Table 1 that the larger the missing rate bound, the worse the H_∞ filtering performance. Similarly, Table 2 shows that the larger the maximal number of consecutive measurement missing, the worse the H_∞ filtering performance. These verify the statements in Theorem 7. Moreover, γ_a is always smaller than γ^* for different values of α in Table 1, and it is always smaller than γ^* for different values of d in Table 2, showing the effectiveness of the H_∞ filter design. On the other hand, it can also be seen from Tables 1 and 2 that γ^* is much larger than γ_a for different values of α and d, which indicates that there exists certain extent of conservatism in the H_∞ filter design and the estimation of the H_∞ performance level γ^*.

6. Conclusions

In this paper, the H_∞ filtering problem was investigated for networked systems with bounded measurement missing. A switched system model was proposed to describe the considered system, which helped establish the quantitative relation between the H_∞ performance level and two parameters, namely, the measurement missing rate bound and the maximal number of consecutive measurement missing. It has been shown by the example that there exists certain extent of conservatism in the proposed H_∞ filter design, and the conservatism is partly introduced by the bounding on the switching frequency of the switched filtering error system given in (8). A tighter bounding on the switching frequency may help reduce the conservatism, which requires more detailed online information about the measurement missing status.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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