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Research Article

Robust H_∞ Filter Design for Itô Stochastic Pantograph Systems

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The problem of robust H_∞ filter design is investigated for stochastic pantograph systems governed by linear Itô differential equation. First, a sufficient condition for asymptotic mean-square stability of stochastic pantograph systems is presented by means of Lyapunov approach. Then, based on matrix inequalities, the H_∞ filtering problem for this kind of systems is studied and a sufficient condition for the existence of the H_∞ filter is derived. Furthermore, the explicit expression of the desired filter parameters is characterized. Finally, an example is given to illustrate the results.

1. Introduction

Stochastic pantograph system which is treated as a special class of time-delay systems has also attracted more and more researchers [1–5]. Reference [1] gave the necessary analytical theory for the existence and uniqueness of a strong solution of the linear stochastic pantograph differential equation and presented the strong approximations to the solution obtained by a continuous extension of θ-Euler scheme. Reference [2] investigated the asymptotic growth and delay properties of solution of linear stochastic pantograph equation and gave the sufficient conditions on parameters when the solution grows at a polynomial rate in pth mean and almost sure sense. Reference [3] studied the αth moment stability for stochastic pantograph equation by using Razumikhin technique. Reference [4] investigated the convergence of the Euler method of stochastic pantograph equations and proved that the Euler approximation solution converges to the analytic solution in probability under weaker conditions. Reference [5] studied the almost surely asymptotic stability of the nonlinear stochastic pantograph differential equations with Markovian switching under the weakened linear growth condition. At present, most literatures on stochastic pantograph equation focus on the existence, uniqueness, and convergence of the numerical solution produced by kinds of approximate methods.

On the other hand, due to great many applications of robust H_∞ control and filtering in real world, the problems on these two have been studied extensively [6–14]. Compared with classical Kalman filter, one does not need to know the exact statistic information about the external disturbance in the H_∞ filtering design. H_∞ filtering requires one to design a filter such that the L_2 gain from the external disturbance to the estimation error is below a prescribed level $\gamma > 0$. Reference [10] studied the problem of H_∞ filtering for general continuous-time linear stochastic systems and gave a necessary and sufficient condition for the existence of H_∞ filter and furthermore designed H_2/H_∞ filter. Reference [11] gave a necessary and sufficient condition for reduced-order H_∞ filter of linear continuous and discrete-time stochastic systems. Reference [12] investigated the robust H_∞ filtering problem for nonlinear stochastic systems and gave a sufficient condition for the existence of H_∞ filter. Reference [13] studied the mixed H_2/H_∞ filtering for a class of nonlinear stochastic systems. Reference [14] considered the finite-time H_∞ filter design for a class of nonlinear stochastic systems. Nevertheless, to the best of our knowledge, the issue on the H_∞ filtering for stochastic linear pantograph systems with state-dependent noise has not been investigated in previous literatures.

In this paper, we first consider the problem on the asymptotic mean-square stability and give a test criterion for stochastic pantograph systems by the Lyapunov approach. On this basis, a sufficient condition of the asymptotic mean square stability is obtained, which can be available for studying the H_∞ filtering of stochastic pantograph systems. Moreover, the H_∞ filter design is investigated and a sufficient condition for the existence of H_∞ filter is obtained in the form of linear matrix inequality. Finally, an example is given to illustrate our proposed methods.

This paper is organized as follows. Section 2 discusses the asymptotic mean-square stability of stochastic pantograph systems and presents a sufficient condition of stability by means of the Lyapunov approach. The H_∞ filtering problem of stochastic pantograph systems is investigated in Section 3. Section 4 provides a numerical example to demonstrate the effectiveness and applicability of the proposed methods. Section 5 concludes this paper.

2. Asymptotic Mean-Square Stability

Consider the following linear stochastic pantograph system:

$$dx(t) = (Ax(t) + A_1 x(qt))\,dt + (Cx(t) + C_1 x(qt))\,dw(t)$$
$$x(0) = x_0, \tag{1}$$

where $x(t) \in R^n$ is system state; $0 < q < 1$; $w(t)$ is a one-dimension standard Wiener process defined on a complete probability space (Ω, F, F_t, P) with $F_t = \sigma(w(s) : 0 \le s \le t)$; A, A_1, C, C_1 are all constant matrices of $R^{n\times n}$. For initial value $x_0 \in R^n$ and $T > 0$, there exists a unique solution $x(t) \in L_F^2(R_T, R^n)$ [1].

Definition 1. The stochastic pantograph system (1) is said to be asymptotically mean square stable if for any initial value x_0, the corresponding state satisfies

$$\lim_{t\to\infty} \mathbb{E}\|x(t)\|^2 = 0. \tag{2}$$

Next, a test criterion for asymptotically mean-square stable of stochastic pantograph systems is given.

Lemma 2. *Stochastic pantograph system* (1) *is asymptotically mean-square stable if there exist some positive constant scalars $k_1 > 0$, $k_2 > 0$, and $k_3 > 0$ and a Lyapunov function $V(t,x)$ satisfying*

$$k_1\|x(t)\|^2 \le V(t,x) \le k_2\|x\|^2, \tag{3}$$

$$LV(t,x) \le -k_3\|x\|^2, \tag{4}$$

where

$$LV(t,x) = \frac{\partial V(t,x)}{\partial t} + \frac{\partial V'(t,x)}{\partial x}(Ax(t) + A_1x(qt)) + \frac{1}{2}(Cx(t) + C_1x(qt))'\frac{\partial^2 V(t,x)}{\partial x^2}(Cx(t) + C_1x(qt)). \tag{5}$$

Proof. Expressing the difference $V(t,x(t)) - V(0,x_0)$ by means of Itô formula [15], calculating expectations, we get

$$\mathbb{E}V(t,x(t)) - V(0,x_0) = \int_0^t \mathbb{E}LV(s,x(s))\,ds. \tag{6}$$

Differentiating this equality with respect to t and using (3), (4), we see that

$$\frac{d}{dt}\mathbb{E}V(t,x(t)) \le -\frac{k_3}{k_2}\mathbb{E}V(t,x(t)). \tag{7}$$

This implies the estimate

$$\mathbb{E}V(t,x(t)) \le V(0,x_0)\exp\left\{-\frac{k_3}{k_2}t\right\}. \tag{8}$$

Together with (3), this estimate yields

$$\mathbb{E}\|x(t)\|^2 \le \frac{1}{k_1}V(0,x_0)\exp\left\{-\frac{k_3}{k_2}t\right\}. \tag{9}$$

Let $t \to \infty$; then (2) is obtained. This proof is complete. □

On the basis of Lemma 2, the following theorem gives a sufficient condition of the asymptotic mean-square stability is obtained, which can be available for studying the H_∞ filtering of stochastic pantograph systems.

Theorem 3. *If the following linear matrix inequality*

$$PA + A'P + P + 2C'PC + \frac{1}{q}\left(A_1'PA_1 + 2C_1'PC_1\right) < 0 \tag{10}$$

has a solution $P > 0$, then stochastic pantograph system (1) *is asymptotically mean-square stable.*

Proof. Take a Lyapunov function $V(t,x) = x'Px$, where $P > 0$ is the solution of (10). Applying Itô formula and by Cauchy inequality $X'PY + Y'PX \le X'PX + Y'PY$, we obtain

$$\begin{aligned} dV(x) &= \Big[x'\left(PA + A'P + C'PC\right)x \\ &\quad + x'(qt)A_1'Px + x'PA_1x(qt) + x'(qt)C_1'PCx \\ &\quad + x'C'PC_1x(qt) + x'(qt)C_1'PC_1x(qt)\Big]dt \end{aligned}$$

$$\begin{aligned}
&+ \left[(Cx + C_1 x(qt))' Px + x' P (Cx + C_1 x(qt)) \right] dw(t) \\
&\leq \left[x' \left(PA + A'P + C'PC \right) x \right. \\
&\quad + x'(qt) A_1' P A_1 x(qt) + x' P x + x'(qt) C_1' P C_1 x(qt) \\
&\quad \left. + x' C' P C x + x'(qt) C_1' P C_1 x(qt) \right] dt \\
&\quad + \left[(Cx + C_1 x(qt))' Px + x' P (Cx + C_1 x(qt)) \right] dw(t) \\
&\leq \left[x' \left(PA + A'P + P + 2C'PC \right) x + x'(qt) Q x(qt) \right] dt \\
&\quad + \left[(Cx + C_1 x(qt))' Px + x' P (Cx + C_1 x(qt)) \right] dw(t),
\end{aligned} \tag{11}$$

where $Q = (A_1' P A_1 + 2C_1' P C_1) \geq 0$ due to $P > 0$. So for any $0 \leq s \leq t$, taking integral from s to t, we have

$$\begin{aligned}
&\mathbb{E}V(x(t)) - \mathbb{E}V(x(s)) \\
&\quad \leq \mathbb{E} \int_s^t x'(u) \left(PA + A'P + P + 2C'PC \right) x(u) \, du \\
&\quad\quad + \mathbb{E} \int_s^t x'(qu) Q x(qu) \, du,
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&\mathbb{E} \int_s^t x'(qu) Q x(qu) \, du \\
&\quad = \frac{1}{q} \mathbb{E} \int_s^{qt} x'(u) Q x(u) \, du \leq \frac{1}{q} \mathbb{E} \int_s^t x'(u) Q x(u) \, du.
\end{aligned} \tag{13}$$

The above last inequality is valid because of $Q \geq 0$ and $0 < q < 1$, so

$$\begin{aligned}
&\mathbb{E}V(x(t)) - \mathbb{E}V(x(s)) \\
&\quad \leq \mathbb{E} \int_s^t x'(u) \left(PA + A'P + P + 2C'PC + \frac{1}{q} Q \right) x(u) \, du.
\end{aligned} \tag{14}$$

Multiplying $1/(t-s)$ by both sides simultaneously and letting $t \to s$, we obtain

$$\mathbb{E}dV(x(t)) \leq x'(t) \left(PA + A'P + P + 2C'PC + \frac{1}{q} Q \right) x(t) \, dt. \tag{15}$$

Therefore, the infinitesimal generator of stochastic pantograph system (1) satisfies

$$\begin{aligned}
LV(x(t)) &\leq x'(t) \left(PA + A'P + P + 2C'PC + \frac{1}{q} Q \right) x(t) \\
&\leq -k_3 \|x\|^2,
\end{aligned} \tag{16}$$

where $PA + A'P + P + 2C'PC + (1/q)Q \leq -k_3 < 0$ for some $k_3 > 0$. By Lemma 2, the asymptotic mean-square stability of (1) is derived, which completes the proof. □

Remark 4. Inequality (10) is a linear matrix inequality, which provides more convenience to test the asymptotic mean-square stability of stochastic pantograph system (1).

Remark 5. When $A_1 = C_1 = 0$, the pantograph system (1) becomes normal stochastic linear system

$$dx(t) = Ax(t) \, dt + Cx(t) \, dw(t), \tag{17}$$

and (10) is simplified by

$$PA + A'P + P + 2C'PC < 0, \tag{18}$$

which implies $PA + A'P + P + C'PC < 0$, so (18) can also guarantee the asymptotic mean-square stability of (17) [15].

Remark 6. Let $\tau(t) = t - qt$; system (1) becomes

$$\begin{aligned}
dx(t) &= \left[Ax(t) + A_1 x(t - \tau(t)) \right] dt \\
&\quad + \left[Cx(t) + C_1 x(t - \tau(t)) \right] dw(t).
\end{aligned} \tag{19}$$

System (19) is a time-vary delay system. Condition (10) guarantees that system (19) is asymptotically mean-square stable.

3. Robust H_∞ Filter Design

Based on the asymptotic mean-square stability of pantograph system discussed in the above section, we are in a position to deal with the H_∞ filtering problem for stochastic pantograph system.

Consider the following stochastic linear perturbed pantograph system with measurement output:

$$\begin{aligned}
dx(t) &= \left(Ax(t) + A_1 x(qt) + Bv(t) \right) dt \\
&\quad + \left(Cx(t) + C_1 x(qt) \right) dw(t) \\
dy(t) &= \left(A_2 x(t) + B_1 v(t) \right) dt + C_2 x(t) \, dw(t) \\
z(t) &= Mx(t),
\end{aligned} \tag{20}$$

where $x(t) \in R^n$, $y(t) \in R^{n_y}$, $v(t) \in R^{n_v}$, and $z(t) \in R^{n_z}$ are the system state, the exogenous disturbance signal, the measurement output, and the state combination to be estimated, respectively. A, A_1, A_2, B, B_1, C, C_1, C_2, and M are constant matrices of suitable dimension. Here we suppose $v(t) \in L_F^2(R_+, R^{n_v})$, which guarantees that the system (20) has a unique solution $x(t) \in L_F^2(R_T, R^n)$ for any $T > 0$.

The so-called H_∞ filtering problem is to design an estimator to estimate the unknown state combination $z(t)$ via output measurement $y(t)$, which guarantees the L_2 gain (from the external disturbance to estimation error) to be less than a prescribe level $\gamma > 0$, and the extended system is internally stable. Here we construct the following linear pantograph filter via output measurement for the estimation of $z(t)$:

$$\begin{aligned}
d\hat{x}(t) &= \left(A_f \hat{x}(t) + B_f \hat{x}(qt) \right) dt + C_f dy(t) \\
\hat{x}(0) &= \hat{x}_0 \\
\hat{z}(t) &= M_f \hat{x}(t),
\end{aligned} \tag{21}$$

where $\widehat{x}(t) \in R^n$, $A_f \in R^{n\times n}$, $B_f \in R^{n\times n}$, $C_f \in R^{n\times n_y}$, and $M_f \in R^{n_z\times n}$ are constant matrices to be determined subsequently. Let $\eta(t) = (x'(t)\ \ \widehat{x}'(t))'$, $\widetilde{z}(t) = z(t) - \widehat{z}(t)$; then the extended system is

$$d\eta(t) = \left(\widetilde{A}\eta(t) + \widetilde{A}_1\eta(qt) + \widetilde{B}v(t)\right)dt + \left(\widetilde{C}\eta(t) + \widetilde{C}_1\eta(qt)\right)dw(t), \quad (22)$$
$$\widetilde{z}(t) = \widetilde{M}\eta(t),$$

where

$$\widetilde{A} = \begin{bmatrix} A & 0 \\ C_fA_2 & A_f \end{bmatrix}, \qquad \widetilde{A}_1 = \begin{bmatrix} A_1 & 0 \\ 0 & B_f \end{bmatrix},$$
$$\widetilde{B} = \begin{bmatrix} B \\ C_fB_1 \end{bmatrix}, \qquad \widetilde{C} = \begin{bmatrix} C & 0 \\ C_fC_2 & 0 \end{bmatrix}, \quad (23)$$
$$\widetilde{C}_1 = \begin{bmatrix} C_1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \widetilde{M} = \begin{bmatrix} 0 & -M_f \end{bmatrix}.$$

For a given disturbance attenuation level $\gamma > 0$ and $v(t) \in L_F^2(R_+, R^{n_v})$, define the associated H_∞ filtering performance of (22) as

$$J_\infty = \mathbb{E}\int_0^\infty \|\widetilde{z}(t)\|^2 dt - \gamma^2 \mathbb{E}\int_0^\infty \|v(t)\|^2 dt. \quad (24)$$

As in [10], the H_∞ filtering problem is formulated as follows.

Stochastic H_∞ Filtering Problem. Given $\gamma > 0$, find an estimator $\widehat{x}$ of the form (21) leading (22) to being internally stable. Moreover, for any $v(t) \neq 0$, $v(t) \in L_F^2(R_+, R^{n_v})$ with $\eta(0) = 0$, there always is $J_\infty < 0$.

In what follows, we will give the main result of H_∞ filtering problem and provide a technique to determine matrices A_f, B_f, C_f, M_f of filter (22).

Theorem 7. *If the following matrix inequality*

$$P\widetilde{A} + \widetilde{A}'P + P + 2\widetilde{C}'P\widetilde{C} + \frac{1}{q}\left(\widetilde{A}_1'P\widetilde{A}_1 + 2\widetilde{C}'P\widetilde{C}\right) + \widetilde{M}'\widetilde{M} + \gamma^{-2}P\widetilde{B}\widetilde{B}'P < 0 \quad (25)$$

has a solution $P > 0$, then system (22) with $v(t) = 0$ is asymptotically mean-square stable, and $J_\infty < 0$ holds for any $v(t) \neq 0$, $v(t) \in L_F^2(R_+, R^{n_v})$ when $\eta(0) = 0$.

Proof. When $v(t) = 0$, from (25) we obtain

$$P\widetilde{A} + \widetilde{A}'P + P + 2\widetilde{C}'P\widetilde{C} + \frac{1}{q}\left(\widetilde{A}_1'P\widetilde{A}_1 + 2\widetilde{C}'P\widetilde{C}\right) < -\widetilde{M}'\widetilde{M} - \gamma^{-2}P\widetilde{B}\widetilde{B}'P < 0, \quad (26)$$

so system (22) is asymptotical mean-square stable according to Theorem 3.

Next, we prove $J_\infty < 0$ for any nonzero $v(t) \in L_F^2(R_+, R^{n_v})$ with $\eta(0) = 0$, taking the Lyapunov function $V(\eta) = \eta'P\eta$, where $P > 0$ is a solution of (25), and following the outline of the proof in Theorem 3, we obtain that the infinitesimal generator of (22) satisfies

$$LV(\eta) \le \eta'\left(P\widetilde{A} + \widetilde{A}'P + P + 2\widetilde{C}'P\widetilde{C}\right. \left. + \frac{1}{q}\left(\widetilde{A}_1'P\widetilde{A}_1 + 2\widetilde{C}'P\widetilde{C}\right)\right)\eta + v'\widetilde{B}'P\eta + \eta'P\widetilde{B}v. \quad (27)$$

Note that for $T > 0$,

$$\begin{aligned} J_T(\eta, v) &= \mathbb{E}\int_0^T \|\widetilde{z}(t)\|^2 dt - \gamma^2\mathbb{E}\int_0^T \|v(t)\|^2 dt \\ &= \mathbb{E}\int_0^T \|\widetilde{z}(t)\|^2 dt - \gamma^2\mathbb{E}\int_0^T \|v(t)\|^2 dt + d\left(\eta'P\eta\right) - d\left(\eta'P\eta\right) \\ &= -\mathbb{E}\eta'(T)P\eta(T) + \mathbb{E}\int_0^T \left(\eta'\widetilde{M}'\widetilde{M} - \gamma^2v'v + LV(\eta)\right)dt \\ &\le \mathbb{E}\int_0^T \left(\eta\widetilde{M}'\widetilde{M} - \gamma^2v'v + LV(\eta)\right)dt \\ &\le \mathbb{E}\int_0^T \begin{bmatrix} \eta \\ v \end{bmatrix}' N \begin{bmatrix} \eta \\ v \end{bmatrix} dt, \end{aligned} \quad (28)$$

where

$$N = \begin{bmatrix} P\widetilde{A} + \widetilde{A}'P + P + 2\widetilde{C}'P\widetilde{C} + \frac{1}{q}\left(\widetilde{A}_1'P\widetilde{A}_1 + 2\widetilde{C}'P\widetilde{C}\right) + \widetilde{M}'\widetilde{M} & P\widetilde{B} \\ \widetilde{B}'P & -\gamma^{-2}I \end{bmatrix}. \quad (29)$$

If $N < 0$, then there exists $\epsilon > 0$, such that

$$J_T(\eta, v) \le -\epsilon\mathbb{E}\int_0^T \left(\|\eta\|^2 + \|v\|^2\right)dt \le -\epsilon\mathbb{E}\int_0^T \|v\|^2 dt. \quad (30)$$

Let $T \to \infty$; then $J_\infty(\eta, v) \le -\epsilon\mathbb{E}\int_0^\infty \|v\|^2 dt < 0$. By Schur Complement, $N < 0$ is equivalent to (25), which ends the proof. □

It is difficult to solve the inequality (25) because of its nonlinearity, so Theorem 7 cannot be directly available for designing the filter. Next we will give a sufficient condition easy to be solved.

Theorem 8. *If the following LMI*

$$\begin{bmatrix} P_{11}A + A'P_{11} + P_{11} + \frac{2}{q}C_1'P_{11}C_1 & A_2'Z_1' & 2C'P_{11} & 2C_1'Z_1' & M' & P_{11}B & A_1'P_{11} & 0 \\ * & Z_3 + Z_3' + P_{22} & 0 & 0 & -M_f' & Z_1B_1 & 0 & Z_2' \\ * & * & -2P_{11} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -2P_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & -qP_{11} & 0 \\ * & * & * & * & * & * & * & -qP_{22} \end{bmatrix} < 0 \quad (31)$$

has solutions $P_{11} > 0$, $P_{22} > 0$, $Z_1 \in R^{n\times n_y}$, $Z_2 \in R^{n\times n}$, $Z_3 \in R^{n\times n}$, M_f, then system (22) is internally asymptotically mean-square stable, and filtering performance $J_\infty < 0$ holds for any $v(t) \neq 0$, $v(t) \in L_F^2(R_+, R^{n_v})$ with $\eta(0) = 0$. The corresponding H_∞ filter (21) can be formulated by

$$\begin{aligned} d\hat{x}(t) &= \left(P_{22}^{-1}Z_3\hat{x}(t) + P_{22}^{-1}Z_2\hat{x}(qt)\right)dt + P_{22}^{-1}Z_1 dy(t), \\ \hat{x}(0) &= \hat{x}_0, \\ \hat{z}(t) &= M_f\hat{x}(t). \end{aligned} \quad (32)$$

Proof. By Schur Complement, (25) is equivalent to

$$\begin{bmatrix} P\widetilde{A} + \widetilde{A}'P + P + \frac{2}{q}\widetilde{C}_1'P\widetilde{C}_1 & 2\widetilde{C}'P & \widetilde{M}' & P\widetilde{B} & \widetilde{A}_1'P \\ * & -2P & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -qP \end{bmatrix} < 0. \quad (33)$$

Taking $P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix}$ and substituting (23) into (31), after a series computations, we have

$$\begin{bmatrix} P_{11}A + A'P_{11} + P_{11} + \frac{2}{q}C_1'P_{11}C_1 & A_2'C_f'P_{22} & 2C'P_{11} & 2C_1'C_f'P_{22} & M' & P_{11}B & A_1'P_{11} & 0 \\ * & P_{22}A_f + A_f'P_{22} + P_{22} & 0 & 0 & -M_f' & P_{22}C_fB_1 & 0 & B_f'P_{22} \\ * & * & -2P_{11} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -2P_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & -qP_{11} & 0 \\ * & * & * & * & * & * & * & -qP_{22} \end{bmatrix} < 0. \quad (34)$$

Setting $P_{22}C_f = Z_1$, $P_{22}B_f = Z_2$, $P_{22}A_f = Z_3$, then (34) turns out to be (31). Therefore, $A_f = P_{22}^{-1}Z_3$, $B_f = P_{22}^{-1}Z_2$, $C_f = P_{22}^{-1}Z_1$; then the proof is complete. □

Remark 9. In the proof of Theorem 8, the matrix P is chosen as diag$\{P_{11}, P_{22}\}$ for simplicity. In order to reduce the conservatism of the conditions, the matrix P can also be chosen as $\begin{bmatrix} P_{11} & P_{12} \\ P_{12}' & P_{22} \end{bmatrix}$. However, this case will increase the complexity of computation.

Remark 10. In many engineering applications, the performance constraint is often specified a priori. In Theorem 8, the filter is designed after H_∞ performance is prescribed. In fact, we can obtain an improved performance by optimization method. In addition, inequality (31) may be no feasible solution for very small q, that is, very large time delay. However, the smallest q can be found by numerical algorithm. The results in Theorem 8 suggest the following optimization problems.

(OP1): The optimal H_∞ filtering problem for stochastic pantograph systems is defined by

$$\min_{P_{11}>0, P_{22}>0, Z_1, Z_2, Z_3, M_f} \chi \quad \text{subject to (31) with } \chi = \gamma^2. \quad (35)$$

Then the minimum value of optimal H_∞ performance γ^* is given by $\gamma^* = (\min \chi)^{1/2}$.

(OP2): The minimum value of γ corresponding to the different values of q in the interval $(0, 1)$ can be found.

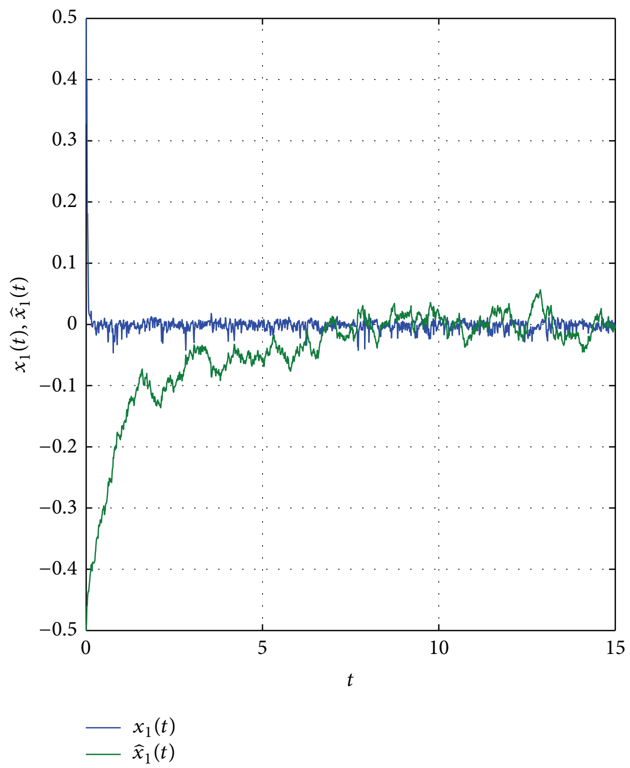

FIGURE 1: The trajectories of $x_1(t)$ and $\hat{x}_1(t)$.

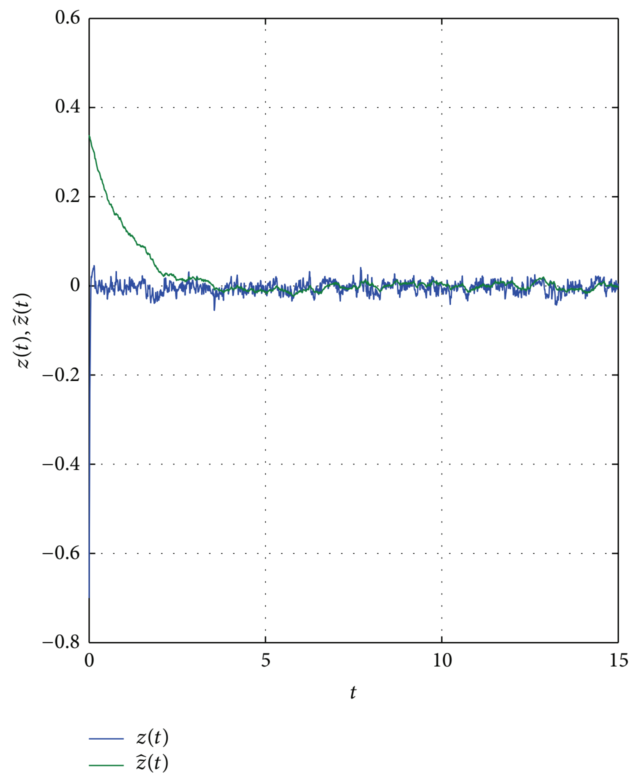

FIGURE 3: The trajectories of $z(t)$ and $\hat{z}(t)$.

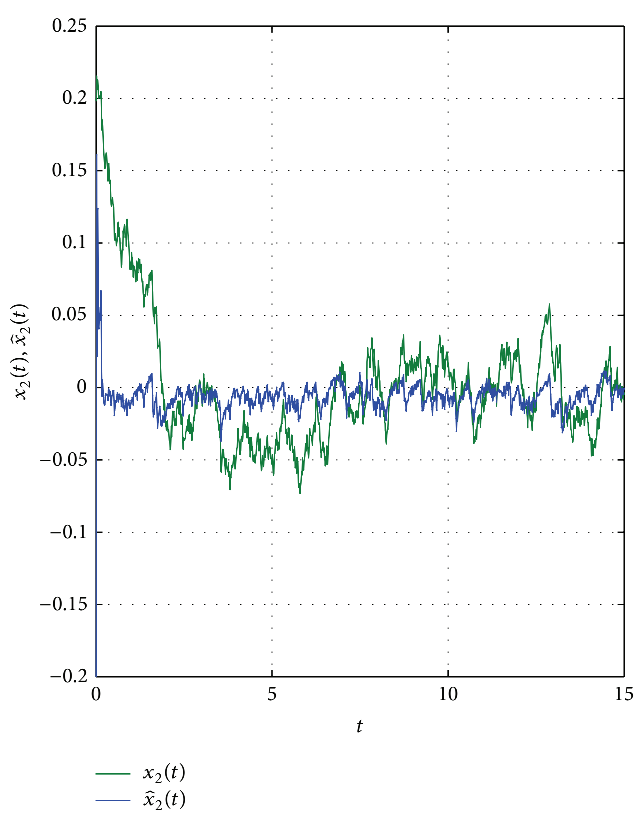

FIGURE 2: The trajectories of $x_2(t)$ and $\hat{x}_2(t)$.

Algorithm I. Consider the following steps.

Step 1. By simple search algorithms, if we find a series of q_i $(i = 1, \ldots, n)$ to make (31) have feasible solutions, then go to Step 2. Otherwise, go to Step 6.

Step 2. Set $i = 1$, take a q_i.

Step 3. Solving the following optimization problem OP1.

Step 4. Set $i = i + 1$, if $i + 1 > n$, then go to Step 5; otherwise $q_i = q_{i+1}$, go to Step 3.

Step 5. (31) has feasible solutions. Stop.

Step 6. (31) has no feasible solutions. Stop.

Remark 11. The smallest q may be obtained by Algorithm I.

4. Numerical Example

In this section, a numerical example is provided to demonstrate the effectiveness and applicability of the proposed methods. Consider the following Itô stochastic pantograph system:

$$dx(t) = (Ax(t) + A_1 x(qt) + Bv(t))\,dt + (Cx(t) + C_1 x(qt))\,dw(t)$$

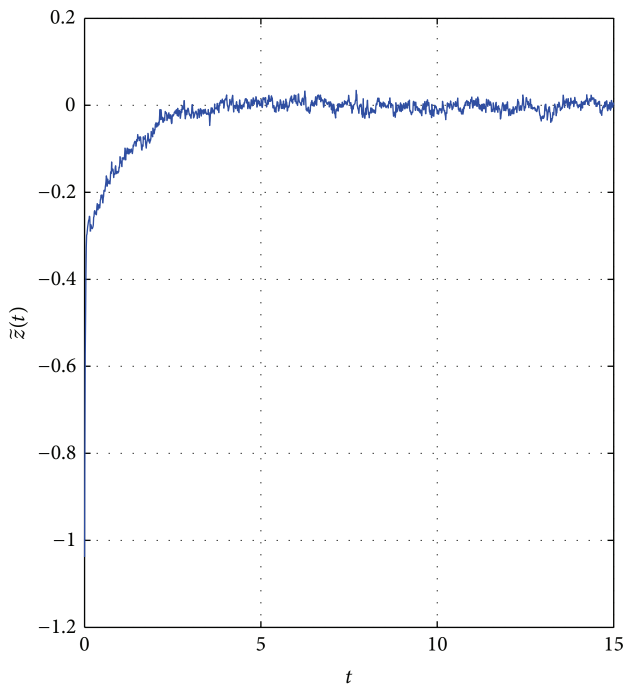

FIGURE 4: The trajectories of $\tilde{z}(t)$.

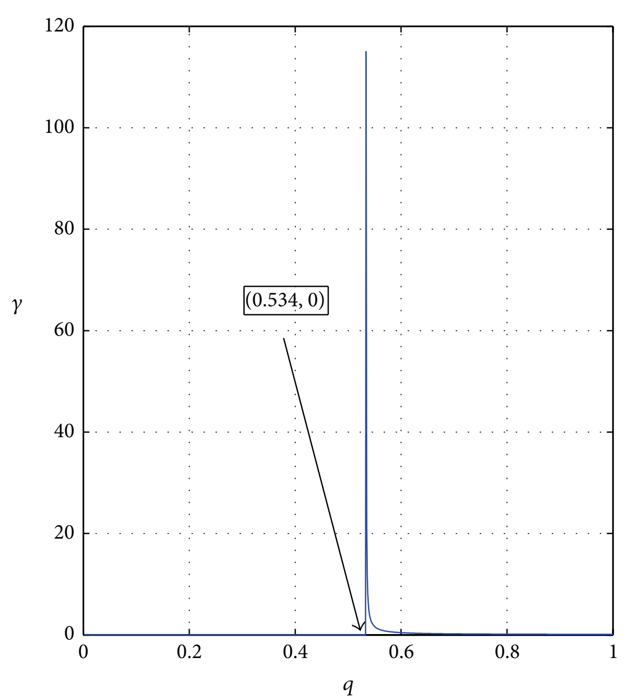

FIGURE 5: The minimum value of γ versus q in $(0, 1)$.

$$dy(t) = (A_2 x(t) + B_1 v(t))\, dt + C_2 x(t)\, dw(t)$$
$$z(t) = Mx(t), \tag{36}$$

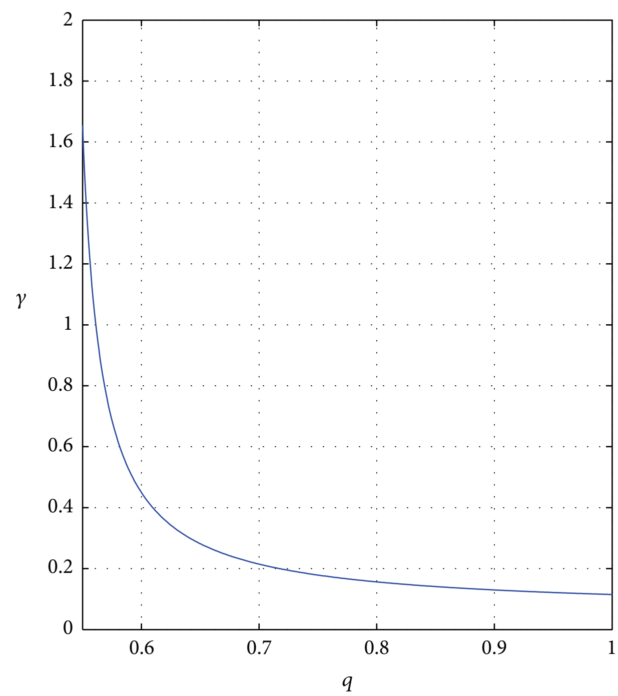

FIGURE 6: The minimum value of γ versus q in $(0.55, 1)$.

where

$$\begin{aligned}
A &= \begin{bmatrix} -50 & 0.8 \\ 0.4 & -10 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 2 & 1.5 \\ 0.2 & -2 \end{bmatrix}, \\
B &= \begin{bmatrix} 0.3 \\ -0.5 \end{bmatrix}, \qquad C = \begin{bmatrix} 4 & 0.5 \\ 3 & 1 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} -0.8 & 1.2 \\ 0.8 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 15.6 & 1.1 \\ 0.6 & 8 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1.6 & 2 \\ 0 & 1 \end{bmatrix}, \\
M &= \begin{bmatrix} -1 & 1 \end{bmatrix}, \qquad q = 0.9.
\end{aligned} \tag{37}$$

Consider the following filter for estimation of $z(t)$:

$$\begin{aligned}
d\hat{x}(t) &= (A_f \hat{x}(t) + B_f \hat{x}(qt))\, dt + C_f dy(t), \\
\hat{x}(0) &= \hat{x}_0, \\
\hat{z}(t) &= M_f \hat{x}(t).
\end{aligned} \tag{38}$$

According to OP1, the minimum value of γ is 0.13 and the corresponding estimation gains of H_∞ filter are derived from theorem

$$\begin{aligned}
A_f &= \begin{bmatrix} -1.4414 & 0.0518 \\ 0.0512 & -1.5298 \end{bmatrix}, \qquad B_f = \begin{bmatrix} 0.5184 & -0.0095 \\ -0.0093 & 0.5418 \end{bmatrix}, \\
C_f &= \begin{bmatrix} 0.0610 & -0.0221 \\ -0.0219 & -0.0915 \end{bmatrix}, \qquad M_f = \begin{bmatrix} -0.3870 & 0.7240 \end{bmatrix}.
\end{aligned} \tag{39}$$

The initial condition in the simulation is assumed to be $\eta(0) = [0.5 \quad -0.2 \quad -0.5 \quad 0.2]$. Figures 1 and 2 show the trajectories of $x_1(t)$, $\hat{x}_1(t)$, $x_2(t)$, and $\hat{x}_2(t)$ by using the proposed H_∞ filter. Figure 3 shows the response of real state $z(t)$ and its estimation $\hat{z}(t)$. Figure 4 is the simulation result of the estimation error response of $\tilde{z}(t) = z(t) - \hat{z}(t)$, which demonstrates that the estimation error is asymptotically mean-square stable.

By the OP2, the minimum value of q can be given by $q = 0.534$. Figure 5 shows the minimum value of γ corresponding to different q in the interval $(0, 1)$. From Figure 5, we see that (31) has no feasible solution when q is in $(0, 0.534)$. In order to see the relationship between γ and q more clearly, Figure 6 gives the minimum value of γ corresponding to different q in the interval $(0.55, 1)$.

5. Conclusion

This paper has discussed infinite horizon H_∞ filtering for stochastic linear pantograph systems with state-dependent noise, which has not been studied for pantograph system in the previous literatures. A sufficient condition for asymptotic mean-square stability of stochastic linear pantograph systems is presented and a sufficient condition for the existence of the H_∞ filter is given in the form of linear matrix inequality. The results obtained in this paper may be significant in studying the other control/filtering problem such as H_2, H_2/H_∞ control/filtering for linear/nonlinear stochastic pantograph systems.

Notations

A':	The transpose of A
$A > 0$ $(A \geq 0)$:	A is positive (nonnegative)
R^n:	The n-dimensional Euclidean space with $\|\cdot\|_2$
$R^{m\times n}$:	The set of all $m \times n$ matrices
R_+	$= [0, \infty)$
R_T	$= [0, T]$ for $T > 0$
I:	Identify matrix
$\mathbb{E}(\cdot)$:	The mathematical expectation operator
$L^2_F(R_r, R^k)$:	The space of nonanticipative square integrable stochastic processes $y(t) \in L^2(\Omega, R^k)$ with respect to an increasing σ-algebra satisfying F_t-measurable and $\mathbb{E}\int_0^T \|\cdot\|^2 dt < \infty$
$C^{1,2}(R_+ \times R^k; R_+)$:	The family of all nonnegative functions $V(t, x)$ on which are continuously once differentiable in t and twice differentiable in x.

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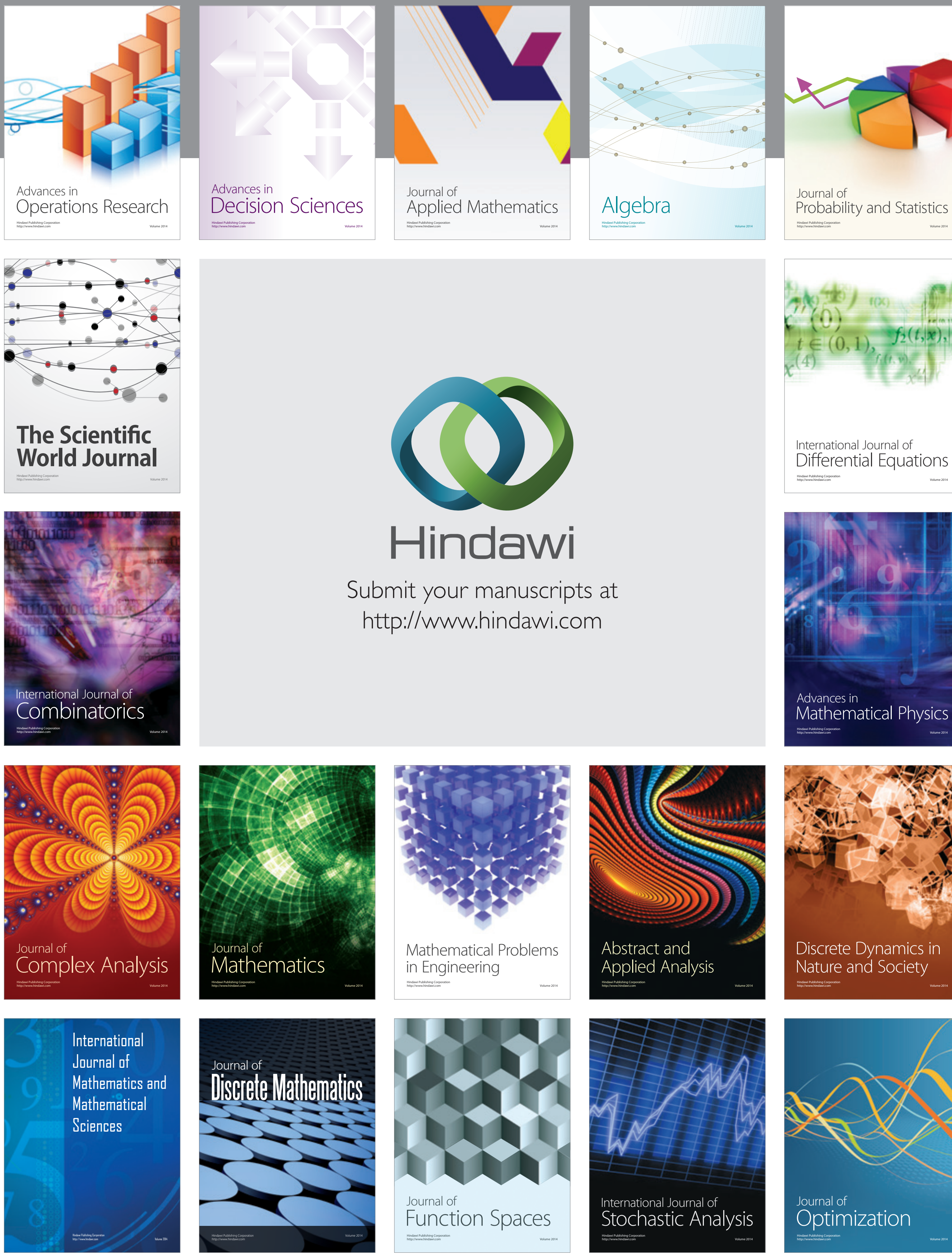
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