

Research Article

Cubic B-Spline Curves with Shape Parameter and Their Applications

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The present studies on the extension of B-spline mainly focus on Bezier methods and uniform B-spline and are confined to the adjustment role of shape parameters to curves. Researchers pay little attention to nonuniform B-spline. This paper discusses deeply the extension of the quasi-uniform B-spline curves. Firstly, by introducing shape parameters in the basis function, the spline curves are defined in matrix form. Secondly, the application of the shape parameter in shape design is analyzed deeply. With shape parameters, we get another means for adjusting the curves. Meanwhile, some examples are given. Thirdly, we discuss the smooth connection between adjacent B-spline segments in detail and present the adjusting methods. Without moving the control points position, through assigning appropriate value to the shape parameter, C^1 continuity of combined spline curves can be realized easily. Results show that the methods are simple and suitable for the engineering application.

1. Introduction

B-spline methods are very popular in computer-aided geometric design and associated fields because of their distinct advantages. In recent years, some other methods have been presented for representing curves and surfaces. Papers [1–7] presented successively C-curves, T-curves, TC-curves, and β -spline in trigonometric functions space. In order to improve the flexibility of product design, researchers give further consideration to introduce shape parameters. Through the parameters, designers can adjust flexibly the shape of curves and surfaces. Wang et al. [8, 9] introduced successively shape parameters to uniform quadratic TC-B-spline curves and quadratic TC-Bezier curves. Xiong et al. [10] discussed extension of uniform C-B-spline curves and surfaces. Bashir et al. [11] researched the G^2 and C^2 rational quadratic trigonometric Bezier curve with two shape parameters, and Liu et al. [12] discussed further hyperbolic polynomial uniform B-spline curves and surfaces with shape parameter. In recent years, researchers also paid attention to extension of traditional B-spline methods. But they mainly concentrated on Bezier curves [13], quadratic and cubic uniform B-spline curves [14–19]. Uniform B-splines can represent overall continuity closed curves and surfaces. But they use equally spaced knots;

the spline does not interpolate the first and last control points. Because a nonuniform B-spline uses repeated knots technology, the curves have clamped property. The designers can locate more easily the two end points of the curve and achieve smooth connection between adjacent B-spline segments. So it has more practical significance for us to study extension of nonuniform B-spline curves and surfaces.

This paper discusses mainly cubic B-spline curves with shape parameter and presents the matrix representation and analyzes the influence of shape parameter on the curve shape. The application of the shape parameter in shape design is discussed deeply. With shape parameters, we get another means for adjusting the curves. In the end, we focus on discussions about how to realize C^1 continuity between adjacent B-spline segments by only adjusting the value of the shape parameters without changing the position of the control points. Results show that the methods given by this paper are simple and suitable for the engineering application.

2. Definition of Basis Functions and Curves

Given control points d_0, d_1, \dots, d_n , let the knot vector be $U = [0, 0, 0, 1, 2, \dots, n-2, n-1, n-1, n-1]$. Then we have

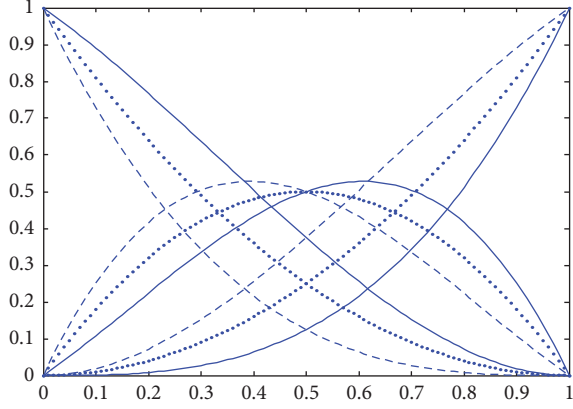


FIGURE 1: The influence of the parameter on the spline shape ($n = 2$).

a piecewise representation of the quadratic open uniform B-spline with parameter

$$\mathbf{p}(u) = \sum_{j=i-2}^i \mathbf{d}_j N_{j,2}(\lambda, u), \quad (1)$$

$$u \in [i, i+1] \subset [0, n-1], \quad i = 2, 3, \dots, n,$$

where $-1 \leq \lambda \leq 1$ is called shape parameter and $N_{j,2}(\lambda, u)$ ($j = 0, 1, \dots, n$) are basis functions.

(1) When $n = 2$, the curve only has one segment.

$$\mathbf{p}(t) = \sum_{j=0}^2 \mathbf{d}_j N_{j,2}^0(\lambda, t), \quad t \in [0, 1], \quad (2)$$

where $N_{0,2}^0(\lambda, t)$, $N_{1,2}^0(\lambda, t)$, and $N_{2,2}^0(\lambda, t)$ are

$$\begin{aligned} N_{0,2}^0(\lambda, t) &= (1 + \lambda t)(1 - t)^2, \\ N_{1,2}^0(\lambda, t) &= (2 - \lambda + 2\lambda t)(1 - t)t, \\ N_{2,2}^0(\lambda, t) &= (1 - \lambda + \lambda t)t^2. \end{aligned} \quad (3)$$

Also it can be expressed in matrix form:

$$\mathbf{p}(t) = [1 \ t \ t^2 \ t^3] M(\lambda) \begin{pmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \\ \mathbf{d}_2 \end{pmatrix}, \quad t \in [0, 1], \quad (4)$$

$$M = \begin{pmatrix} 1 + \lambda & 0 & 0 \\ -2(1 + \lambda) & 2 - \lambda & 0 \\ 1 + \lambda & 3\lambda - 2 & 1 - \lambda \\ 0 & -2\lambda & \lambda \end{pmatrix}.$$

Figure 1 shows the influence of the parameter λ on $N_{0,2}(\lambda, u)$, $N_{1,2}(\lambda, u)$, and $N_{2,2}(\lambda, u)$, where solid line, dash line, and dotted line correspond to $\lambda = 1$, $\lambda - 1$, and $\lambda = 0$.

(2) When $n = 3$, the spline curve is composed of two segments.

$$\begin{aligned} \mathbf{p}_1(t) &= \sum_{j=0}^2 \mathbf{d}_j N_{j,2}^1(\lambda, t), \\ \mathbf{p}_2(t) &= \sum_{j=1}^3 \mathbf{d}_j N_{j,2}^2(\lambda, t), \end{aligned} \quad (5)$$

$$t \in [0, 1]$$

$$\begin{aligned} N_{0,2}^1(\lambda, t) &= (1 + \lambda t)(1 - t)^2, \\ N_{1,2}^1(\lambda, t) &= \frac{1}{4}(2 + \lambda t)(2 - t)t \\ &\quad + (1 - \lambda(1 - t))t(1 - t), \\ N_{2,2}^1(\lambda, t) &= \frac{1}{4}(2(1 - \lambda) + \lambda t)t^2, \\ N_{1,2}^2(\lambda, t) &= \frac{1}{4}(2 + \lambda + \lambda t)(1 - t)^2, \\ N_{2,2}^2(\lambda, t) &= \frac{1}{4}(2 - \lambda(1 - t))(1 - t)(1 + t) \\ &\quad + (1 + \lambda t)t(1 - t), \end{aligned} \quad (6)$$

$$N_{3,2}^2(\lambda, t) = (\lambda t - \lambda + 1)t^2.$$

Also it can be expressed in the following matrix form:

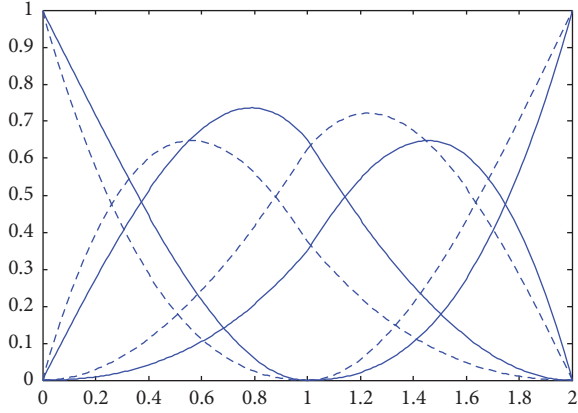
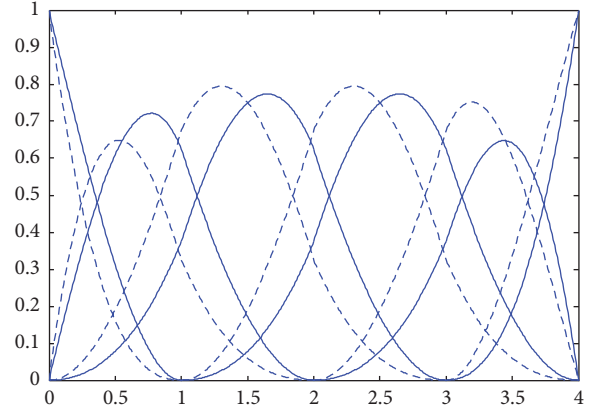
$$\mathbf{p}_1(t) = [1 \ t \ t^2 \ t^3] M_1(\lambda) \begin{pmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \\ \mathbf{d}_2 \end{pmatrix}, \quad t \in [0, 1],$$

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ \lambda - 2 & 2 - \lambda & 0 \\ 1 - 2\lambda & \frac{5}{2}\lambda - \frac{3}{2} & \frac{1}{2}(1 - \lambda) \\ \lambda & -\frac{5}{4}\lambda & \frac{1}{4}\lambda \end{pmatrix}, \quad (8)$$

$$\mathbf{p}_2(t) = [1 \ t \ t^2 \ t^3] M_2(\lambda) \begin{pmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{d}_3 \end{pmatrix}, \quad t \in [0, 1],$$

$$M_2 = \begin{pmatrix} \frac{1}{2} + \frac{1}{4}\lambda & \frac{1}{4}(2 - \lambda) & 0 \\ -1 - \frac{1}{4}\lambda & 1 + \frac{1}{4}\lambda & 0 \\ \frac{1}{2} - \frac{1}{4}\lambda & \frac{5}{4}\lambda - \frac{3}{2} & 1 - \lambda \\ \frac{1}{4}\lambda & -\frac{5}{4}\lambda & \lambda \end{pmatrix}.$$

Figure 2 shows the influence of λ on the shape of $N_{0,2}(\lambda, u)$, $N_{1,2}(\lambda, u)$, $N_{2,2}(\lambda, u)$, and $N_{3,2}(\lambda, u)$, where solid line and dash line correspond to $\lambda = 0.6$ and $\lambda = -0.5$.


 FIGURE 2: The influence of the parameter on the spline shape ($n = 3$).

 FIGURE 3: The influence of the parameter on the spline shape ($n = 5$).

(3) When $n > 3$, the spline curve is composed of $n - 1$ segments.

$$\begin{aligned} \mathbf{p}_1(t) &= \sum_{j=0}^2 d_j N_{j,2}^1(\lambda, t), \\ \mathbf{p}_{n-1}(t) &= \sum_{j=n-2}^n \mathbf{d}_j N_{j,2}^{n-1}(\lambda, t), \\ \mathbf{p}_{i-1}(t) &= \sum_{j=i-2}^i d_j N_{j,2}^3(\lambda, t), \quad i = 3, \dots, n-1, \\ & t \in [0, 1], \end{aligned} \quad (9)$$

where $N_{0,2}^1$, $N_{1,2}^1$, and $N_{2,2}^1$ are shown in (6).

$$\begin{aligned} N_{n-2,2}^{n-1}(\lambda, t) &= \frac{1}{4} (2 + \lambda + \lambda t) (1 - t)^2, \\ N_{n-1,2}^{n-1}(\lambda, t) &= \frac{1}{4} (2 - \lambda (1 - t)) (1 - t) (1 + t) \\ &\quad + (1 + \lambda t) t (1 - t), \\ N_{n,2}^{n-1}(\lambda, t) &= (\lambda t - \lambda + 1) t^2, \\ N_{i-2,2}^3(\lambda, t) &= \frac{1}{4} (2 + \lambda + \lambda t) (1 - t)^2, \\ N_{i-1,2}^3(\lambda, t) &= \frac{1}{4} (2 - \lambda (1 - t)) (1 - t) (1 + t) \\ &\quad + \frac{1}{4} (2 + \lambda t) t (2 - t), \\ N_{i,2}^3(\lambda, t) &= \frac{1}{4} (\lambda t - 2\lambda + 2) t^2, \end{aligned} \quad (10)$$

$$i = 3, 4, \dots, n-1.$$

Also it can be expressed in the following matrix form:

$$\begin{aligned} \mathbf{p}_{n-1}(t) &= [1 \ t \ t^2 \ t^3] M_{n-1}(\lambda) \begin{pmatrix} \mathbf{d}_{n-2} \\ \mathbf{d}_{n-1} \\ \mathbf{d}_n \end{pmatrix}, \\ M_{n-1} &= \begin{pmatrix} \frac{1}{2} + \frac{1}{4}\lambda & \frac{1}{4}(2 - \lambda) & 0 \\ -1 - \frac{1}{4}\lambda & 1 + \frac{1}{4}\lambda & 0 \\ \frac{1}{2} - \frac{1}{4}\lambda & \frac{5}{4}\lambda - \frac{3}{2} & 1 - \lambda \\ \frac{1}{4}\lambda & -\frac{5}{4}\lambda & \lambda \end{pmatrix}, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbf{p}_{i-1}(t) &= [1 \ t \ t^2 \ t^3] M_{i-1}(\lambda) \begin{pmatrix} \mathbf{d}_{i-2} \\ \mathbf{d}_{i-1} \\ \mathbf{d}_i \end{pmatrix}, \\ M_{i-1} &= \begin{pmatrix} \frac{1}{4}(2 + \lambda) & \frac{1}{4}(2 - \lambda) & 0 \\ -\frac{1}{4}(4 + \lambda) & \frac{1}{4}(4 + \lambda) & 0 \\ \frac{1}{4}(2 - \lambda) & \frac{1}{4}(-4 + 3\lambda) & \frac{1}{2}(1 - \lambda) \\ \frac{1}{4}\lambda & -\frac{1}{2}\lambda & \frac{1}{4}\lambda \end{pmatrix}. \end{aligned}$$

Figure 3 shows the influence of the parameter λ on the shape of basis functions $N_{0,2}(\lambda, u)$, $N_{1,2}(\lambda, u)$, $N_{2,2}(\lambda, u)$, $N_{3,2}(\lambda, u)$, $N_{4,2}(\lambda, u)$, and $N_{5,2}(\lambda, u)$, where solid line and dash line correspond to $\lambda = 0.5$ and $\lambda = -0.7$.

3. The Properties of Basis Functions and Spline Curves

(1)

$$N_i^j(\lambda, u) \geq 0, \quad \forall u \in [0, 1], \lambda \in [-1, 1]. \quad (12)$$

(2)

$$\begin{aligned}
\sum_{j=0}^2 N_{j,2}^0(\lambda, u) &= 1, \\
\sum_{j=0}^2 N_{j,2}^1(\lambda, u) &= 1, \\
\sum_{j=1}^3 N_{j,2}^2(\lambda, u) &= 1, \\
\sum_{j=n-2}^n N_{j,2}^{n-1}(\lambda, u) &= 1 \\
\sum_{j=i-2}^i N_{j,2}^3(\lambda, u) &= 1, \quad i = 3, 4, \dots, n-1.
\end{aligned} \tag{13}$$

(3)

$$\begin{aligned}
N_{0,2}^j(\lambda, t) &\begin{cases} 1 & t = 0 \\ > 0 & t \in (0, 1) \\ 0 & t = 1, \end{cases} \\
& \qquad \qquad \qquad j = 0, 1, \\
N_{2,2}^0(\lambda, t) &\begin{cases} 0 & t = 0 \\ > 0 & t \in (0, 1) \\ 1 & t = 1, \end{cases} \\
N_{3,2}^2(\lambda, t) &\begin{cases} 0 & t = 0 \\ > 0 & t \in (0, 1) \\ 1 & t = 1, \end{cases} \\
N_{n,2}^{n-1}(\lambda, t) &\begin{cases} 0 & t = 0 \\ > 0 & t \in (0, 1) \\ 1 & t = 1, \end{cases}
\end{aligned} \tag{14}$$

where $\lambda = 0$, that is, quadratic open B-spline.

(4)

$$\begin{aligned}
\mathbf{p}(0) &= \mathbf{d}_0, \\
\mathbf{p}(n-1) &= \mathbf{d}_n.
\end{aligned} \tag{15}$$

(5)

$$\begin{aligned}
\left. \frac{d\mathbf{p}}{du} \right|_{u=0} &= \left. \frac{d\mathbf{p}_1}{dt} \right|_{t=0} = (2 - \lambda)(\mathbf{d}_1 - \mathbf{d}_0), \\
\left. \frac{d\mathbf{p}}{du} \right|_{u=n-1} &= \left. \frac{d\mathbf{p}_{n-1}}{dt} \right|_{t=1} = (2 + \lambda)(\mathbf{d}_n - \mathbf{d}_{n-1}).
\end{aligned} \tag{16}$$

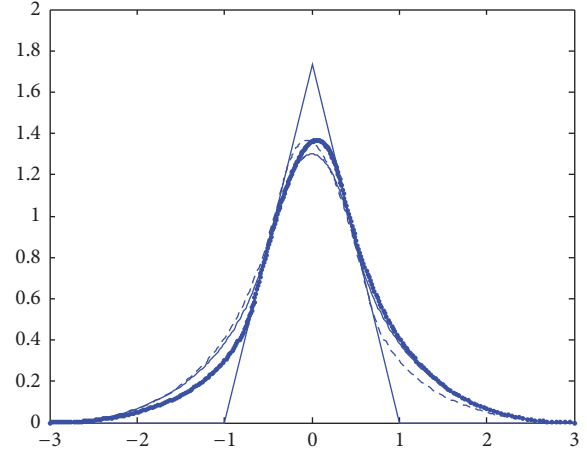


FIGURE 4: $\alpha = -0.65, 0$, and 0.65 corresponded, respectively, to the thick line, the thin line, and the dash line.

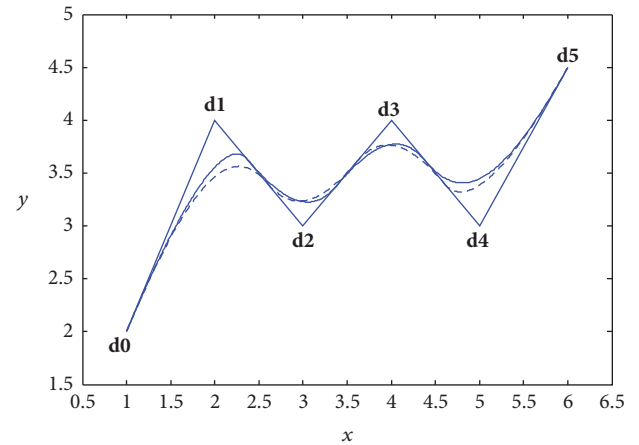


FIGURE 5: $\lambda = 0.5$ corresponds to the solid line and $\lambda = -0.4$ the dash line.

(6)

$$\begin{aligned}
\mathbf{p}'_{i-1}(\lambda, 1) &= \left(1 - \frac{1}{4}\lambda\right)(\mathbf{d}_i - \mathbf{d}_{i-1}), \\
\mathbf{p}'_i(\lambda, 0) &= \left(1 + \frac{1}{4}\lambda\right)(\mathbf{d}_i - \mathbf{d}_{i-1}), \\
& \qquad \qquad \qquad i = 2, 3, \dots, n-2.
\end{aligned} \tag{17}$$

That is to say, the curve is G1 at inner knots. Especially when $\lambda = 0$, the curve is C1.

4. Application in Shape Design

Nonuniform B-spline methods have important applications in shape design. By modifying the shape parameters, the designers get additional choice in two-dimensional design.

Figures 4–6 illustrate the influence of the parameter λ on the shape of curve, where Figure 4 shows $\alpha = -0.65, 0$, and 0.65 corresponding, respectively, to the thick line, the thin line, and the dash line. Figure 5 shows $\lambda = 0.5$ corresponding

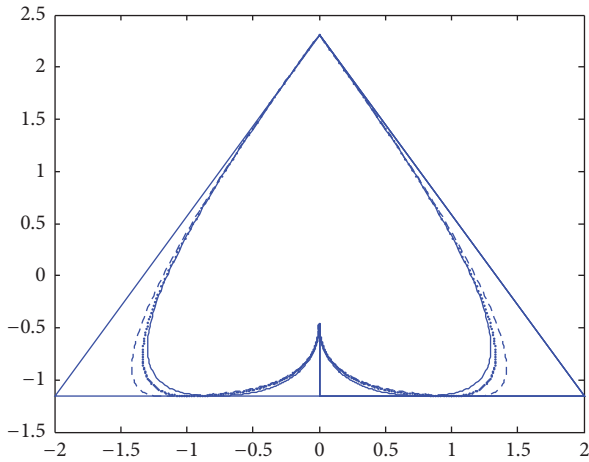


FIGURE 6: $\alpha = -0.4, 0,$ and 0.4 corresponded, respectively, to the solid line, the dotted line, and the dash line.

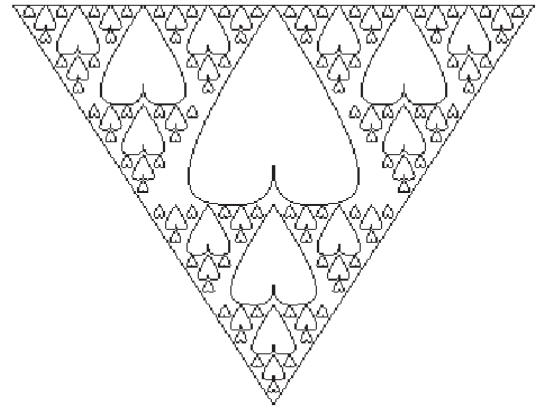


FIGURE 9: Fractal curve generated by the spline curve in this paper ($\alpha = -0.55$).

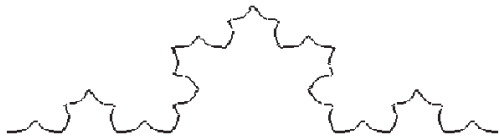


FIGURE 7: Fractal curve generated by the spline curve in this paper ($\alpha = -0.6, 4$ iterations).



FIGURE 8: Fractal curve generated by the spline curve in this paper ($\alpha = -0.6, 5$ iterations).

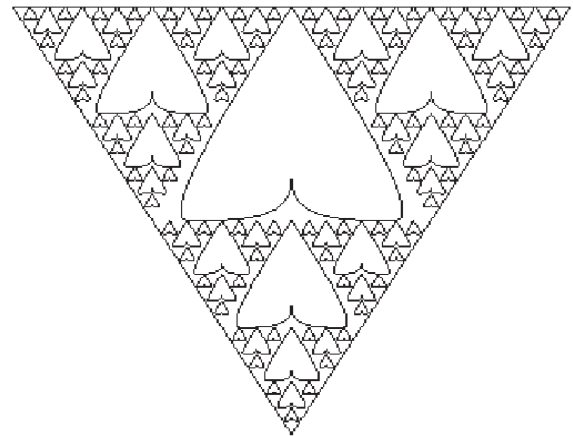


FIGURE 10: Fractal curve generated by the spline curve in this paper ($\alpha = -0.8$).

to the solid line and $\lambda = -0.4$ the dash line. Figure 6 shows $\alpha = -0.4, 0,$ and 0.4 corresponding, respectively, to the solid line, the dotted line, and the dash line. Figures 7–11 show the application of the spline curve in this paper in fractal modeling.

5. Composite Spline Curves

In the practical application, we usually construct composite spline curves that satisfy some smooth conditions. By adjusting shape parameters, designers can achieve the goal.

Two spline curves are given: The first one is

$$\mathbf{p}(u) = \sum_{j=i-2}^i \mathbf{p}_j N_{j,2}(\lambda_1, u), \tag{18}$$

$$u \in [i-2, i-1] \subset [0, n-1], \quad i = 2, 3, \dots, n, \quad \lambda_1 \in [-1, 1].$$

The knot vector $\mathbf{U} = [0, 0, 0, 1, 2, \dots, n-2, n-1, n-1, n-1]$.

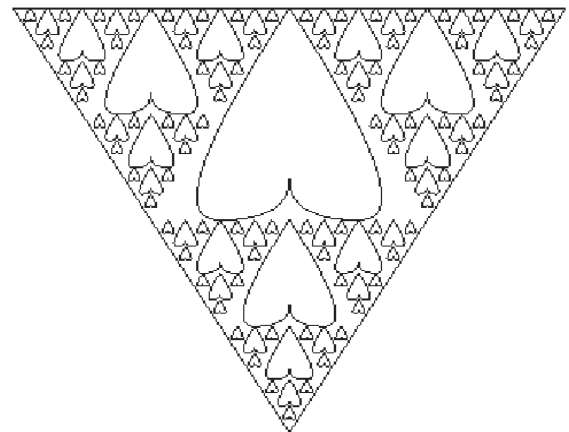


FIGURE 11: Fractal curve generated by the spline curve in this paper ($\alpha = 0$).

The second one is

$$\mathbf{q}(u) = \sum_{j=i-2}^i \mathbf{q}_j N_{j,2}(\lambda_2, v), \quad (19)$$

$$v \in [i-2, i-1] \subset [0, m-1], \quad i = 2, 3, \dots, m, \quad \lambda_2 \in [-1, 1].$$

The knot vector $\mathbf{V} = [0, 0, 0, 1, 2, \dots, m-2, m-1, m-1, m-1]$.

From the equations above, $\mathbf{p}_n \equiv \mathbf{q}_0$.

If curves $\mathbf{p}(t)$ and $\mathbf{q}(t)$ satisfy G^1 continuity that $\exists \delta > 0$, $\Delta \mathbf{q}_0 = \delta \mathbf{p}_{n-1}$, we can modify the parameter to enable the two curves' C^1 continuity without changing the position of the control points. The adjusting methods are shown below.

(1) If $[2 - 3/\delta, 2 - 1/\delta] \cap [-1, 1] \neq \Phi$, we choose $\lambda_2 \in [2 - 3/\delta, 2 - 1/\delta] \cap [-1, 1]$ and modify $\lambda_1 = (2 - \lambda_2)\delta - 2$.

(2) If $[\delta - 2, 3\delta - 2] \cap [-1, 1] \neq \Phi$, we choose $\lambda_1 \in [\delta - 2, 3\delta - 2] \cap [-1, 1]$ and adjust $\lambda_2 = 2 - (2 + \lambda_1)/\delta$.

We only prove that (1) and (2) can be gotten in the same way.

If $[2 - 3/\delta, 2 - 1/\delta] \cap [-1, 1] \neq \Phi$, we can choose freely the value of λ_2 as long as $\lambda_2 \in [2 - 3/\delta, 2 - 1/\delta] \cap [-1, 1]$. From $2 - 3/\delta \leq \lambda_2 \leq 2 - 1/\delta$, we find $-1 \leq (2 - \lambda_2)\delta - 2 \leq 1$ and adjust $\lambda_1 = (2 - \lambda_2)\delta - 2$.

$$\begin{aligned} \left. \frac{d\mathbf{q}}{dv} \right|_{v=0} &= (2 - \lambda_2) (\mathbf{Q}_1 - \mathbf{Q}_0) \\ &= (2 - \lambda_2) \delta (\mathbf{p}_n - \mathbf{p}_{n-1}), \end{aligned} \quad (20)$$

$$\left. \frac{d\mathbf{p}}{du} \right|_{u=n-1} = (2 + \lambda_1) (\mathbf{p}_n - \mathbf{p}_{n-1}).$$

From $\lambda_1 = (2 - \lambda_2)\delta - 2$, we find $\delta = (2 + \lambda_1)/(2 - \lambda_2)$. So $(d\mathbf{q}/dv)|_{v=0} = (d\mathbf{p}/du)|_{u=n-1}$.

Two spline curves are given.

The first one is $\mathbf{p}(u) = \sum_{j=0}^5 \mathbf{p}_j N_{j,2}(\lambda_1, u) = \sum_{j=i-2}^i \mathbf{p}_j N_{j,2}(\lambda_1, u)$, $u \in [i-2, i-1] \subset [0, 4]$, $i = 2, 3, 4, 5$, $\lambda_1 \in [-1, 1]$. The knot vector $\mathbf{U} = [0, 0, 0, 1, 2, 3, 4, 4, 4]$.

The second one is $\mathbf{q}(u) = \sum_{j=0}^4 \mathbf{q}_j N_{j,2}(\lambda_2, v) = \sum_{j=i-2}^i \mathbf{q}_j N_{j,2}(\lambda_2, v)$, $v \in [i-2, i-1] \subset [0, 3]$, $i = 2, 3, 4$, $\lambda_2 \in [-1, 1]$. The knot vector $\mathbf{V} = [0, 0, 0, 1, 2, 3, 3, 3]$, where $\lambda_1 = 0.6$ and $\lambda_2 = 0.4$ and the control points are

$$\begin{aligned} &\mathbf{p}_0 (0.5, 1), \\ &\mathbf{p}_1 (1, 2), \\ &\mathbf{p}_2 (2, 2.25), \\ &\mathbf{p}_3 (3, 1.5), \\ &\mathbf{p}_4 (4, 2.25), \\ &\mathbf{p}_5 (5, 1.25) \\ &\mathbf{q}_0 (5, 1.25), \\ &\mathbf{q}_1 (5.5, 0.75), \\ &\mathbf{q}_2 (6.5, 2), \\ &\mathbf{q}_3 (7, 1), \\ &\mathbf{q}_4 (8, 2.5). \end{aligned} \quad (21)$$

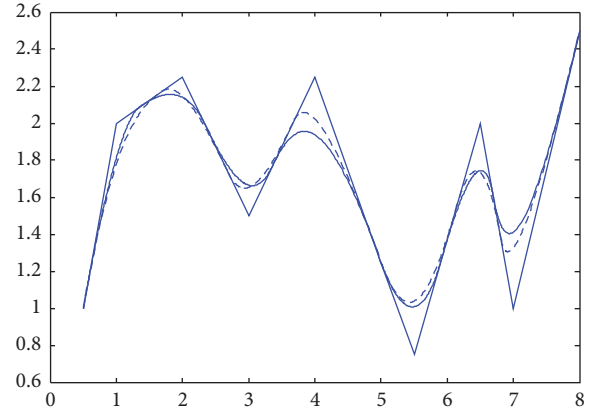


FIGURE 12: C^1 composite spline curve.

Looking at the control points, we see that $\mathbf{p}_5 \equiv \mathbf{q}_0$ and $\Delta \mathbf{q}_0 = \delta \mathbf{p}_4$, $\delta = 1/2$.

Because $[2 - 3/\delta, 2 - 1/\delta] \cap [-1, 1] = [-4, 0] \cap [-1, 1] \neq \Phi$, we can choose $\lambda_2 \in [-4, 0] \cap [-1, 1]$. Here we let $\lambda_2 = -0.5$. From the above method, we adjust $\lambda_1 = 0.6$ to $\lambda_1 = (2 - \lambda_2)\delta - 2 = -0.75$. Now, the curves $\mathbf{p}(t)$ and $\mathbf{q}(t)$ are of C^1 continuity, as shown in Figure 12, where the solid line is t , the original curve, and the dash line is the adjusted curve.

On one hand, the adjusting methods are very simple to do, as shown in (1) and (2) in Section 5. On the other hand, the adjusting methods let the designers achieve easily smooth connection between adjacent B-spline segments without moving the control points. So it is suitable for the engineering application.

6. Conclusion

This paper proposes a class of cubic spline curves with parameter. It is actually the extension of quadratic open B-spline. Through the parameter, we can adjust flexibly the shape of spline curves. With different parameter values, the curve is dragged near or pushed away from the curve. The spline curve is global G^1 and keeps clamped property. We can realize C^1 smooth connection between the spline curves by only modifying the shape parameter value without changing the control points. The adjusting methods proposed in this paper have very important application value.

Conflicts of Interest

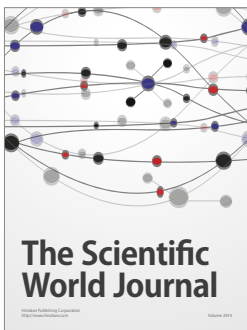
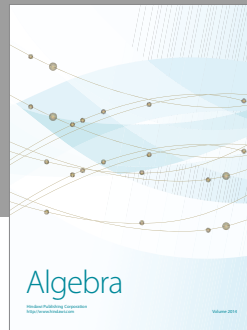
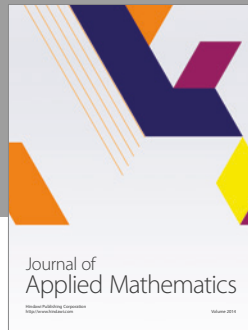
The authors declare that they have no conflicts of interest.

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